

## Free vibration of both-ends clamped wooden beams: is it potentially applicable as an in situ assessment method?

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**Abstract:** Considerable errors caused by shear deflection and rotary inertia in both-ends clamped flexural vibration make the modulus of elasticity hardly obtainable in flexurally excited beams with similar ending conditions. As both-ends clamped beams and columns are necessarily quality controlled in situ within the building structures, this study has attempted to identify some initial requirements for the dynamic responses of a sound both-ends clamped beam under flexural vibration. Accordingly, the dynamic responses of the both-ends clamped wooden beams in radial and tangential flexural vibration were compared to beams in a free-free condition while stepwise increasing axial compressions were applied to the beams. Both-ends clamped beams had the potential to be subjected to in situ longitudinal Young's modulus evaluations. The strong correlations among both-ends clamped and free-free beams in terms of evaluated moduli verified the possibility.

**Key words:** Assessment, beam, clamped, structure, vibration, Young's modulus

### 1. Introduction

Most wooden structural components in forms of beams and columns are situated in both-ends clamped conditions. In timber constructions, the load-bearing components may lose their efficiencies due to gradual deterioration or natural, biological, and mechanical damages. Frequent monitoring of the quality changes in load-bearing members, therefore, is essential. When a defect is visually detected, weak members are easily recognized and replaced. If the defect is hidden, however, it would be difficult to locate the damaged structural members. Strength is accurately measured by removing the wooden members in a building structure. A time-consuming possible mission, this method would be difficult to adopt. Hence, a testing method to appropriately measure the strength of a clamped timber in its building structure seems necessary. Naturally, tests should be conducted without rupturing the specimen (Kubojima et al. 2006).

X-ray digital scanning is perhaps the most accurate technique to identify damaged members (Pietikäinen 1996; Grundberg and Grönlund 1997; Wang et al. 1997; Burian 2006; Skog and Oja 2009; Oja et al. 2010). Difficulties in transporting and maintaining such equipment make it a more expensive method of quality control. The presence of an inexpensive rival method hardly justifies some current applications of the X-ray method. In the last 2 decades,

developing new and inexpensive methodologies, therefore, has been pursued by several researchers (Roohnia et al. 2011b), including those in the current study.

Young's modulus is a strength property that can be obtained without damaging the specimen (Kubojima et al. 2006). Widely used for its simplicity, the flexural vibration test is a popular method of measuring Young's modulus (Kubojima et al. 2006; Roohnia et al. 2011b). Research shows that defects alter the strength of structural timber. For example, a knot induces a resonance frequency shift of vibration (Nakayama 1974; Sobue and Nakano 2001; Brancheriau et al. 2006). Similarly, several proposals state that a local defect might alter the flexural curve of a beam in a static bending test (Nagai et al. 2007) or the flexural curve associated with a transfer function in flexural vibration (Yang et al. 2002; Choi et al. 2007). Considerable errors in shear deflection and rotary inertia in both-ends clamped flexural vibration make the modulus of elasticity hard to obtain in flexurally excited beams with similar ending conditions. While the effects of shear deflection and rotary inertia on dynamic modulus evaluations have been eliminated for free-free and cantilever beams in flexural vibrations (Harris and Piersol 2002; Turk et al. 2008), investigations on the both-ends clamped condition are in progress by the present research team. It was also indicated that altering gripping forces would change

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the resonance frequency in both-ends clamped beams to a considerable extent (Kubojima et al. 2012). Before proposing any new evaluation procedures, it would be necessary to study some initial requirements in dynamic responses of a sound both-ends clamped beam in flexural vibration. In both-ends clamped conditions, are the radial and the tangential dynamic responses comparable? Is there any influence made by axial loads (in a reasonable range) on the dynamic flexural responses of the columns and both-ends clamped beams? If there are any influences, are the radial or the tangential vibrations more influenced?

To eliminate the effects of shear deflection and rotary inertia on the evaluated modulus of elasticity, this study is concerned with initial inquiries and aims to develop an appropriate methodology. Hence, the slenderness ratio might not be the sole independent affecting factor.

**2. Materials and methods**

Specimens were obtained from Silvestre pine green lumber with the nominal dimensions of 20 (radial) × 20 (tangential) × 360 (longitudinal) mm. Among the obtained bars, in accordance with ISO international standard No. 3129, only the visually clear ones were selected and stored in a climatic chamber at 65% relative humidity and 22 °C until the shrinkage dimension changes were stabilized. After applying free vibration in a free-free bar test and observing the correlation coefficients of Timoshenko’s flexural vibration equations (Timoshenko’s bending theory [1921] has been fitted initially to isotropic materials next to the clearest specimens. It benefits from a linear trend fitted to 3 or more points calculated from 3 or more consecutive modal frequencies. If a sample loses its homogeneity, the correlation coefficient of the fitted trend would decrease to values lower than 0.99.) with the relative differences in the radial and tangential flexural responses in terms of the evaluated longitudinal modulus of elasticity, the researchers selected 19 specimens as a set of suitable samples for further examination in this study. Roohnia et al. (2011a) certified that chances would exist only for small differences between the 2 evaluation series of longitudinal modulus of elasticity through tangential transverse (LT) and radial transverse (LR) vibrations in clear and sound beams<sup>1</sup>. Therefore, the introduced differences might be the defect indicators. The greater the observed difference in longitudinal modulus of elasticity evaluations of a proper bar, the larger the defect. The dimensions and mass of stabilized specimens were measured with a digital caliper with the accuracy of 0.01 mm and a digital balance with the accuracy of 0.01 g, respectively, while the frequencies

of the first mode of free flexural vibrations were evaluated in both-ends free versus both-ends clamped conditions using the fast Fourier transform spectrum obtained by MATLAB® 7.1 software.

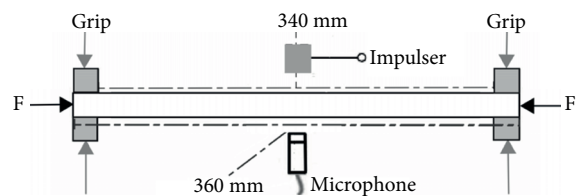
In the literature, the free flexural vibration of free-free beams was illustrated as a method for damping and evaluating modulus of elasticity measurements (Roohnia et al. 2010). The method was extended to include the both-ends clamped specimens in Figure 1. Sound recording and percussion were both done at the middle of the clamped beams. The capability of axial compress application was provided, too (one of the gripping mechanisms being free to displace parallel to the beam axis allowed the axial compress to be performed). A proper gripping force for both clamped ends was kept soft and constant using 2 similar jaws covered by hard leather, controlled by a proper torque wrench (Figure 2). However, the leather might reduce the full clamping of the ends. The gripping width was 1 cm as the translating beam span (free length) remained at 34 cm.

After evaluating the modulus of elasticity and estimating the shear moduli in Timoshenko bending equations for the both-ends free condition (Roohnia et al. 2010), the rest of the required mathematical calculations, to be used later for Eqs. (5) through (8), were followed in the Euler–Bernoulli elementary theory for both the free and clamped end vibrations, as below:

$$\left(\frac{E_d}{\rho}\right)_n = \left[\frac{4\pi^2 l^2 f_1^2}{a \cdot m_1^4}\right] \tag{1}$$

$$\alpha = \frac{I}{Al^2} \tag{2}$$

where  $E_d$  is the longitudinal dynamic modulus of elasticity (Pa);  $\rho$  is the stabilized density ( $\text{kg m}^{-3}$ );  $l$  is the vibrating free span (m);  $f_1$  is the fundamental frequency (Hz);  $a$  is a scalar constant related to the radius of gyration and the free length of the beam, calculated in Eq. (2); and  $m_1$  is a scalar constant related to the end-support condition of the beam, which is equal to 4.73 for the free-free beam

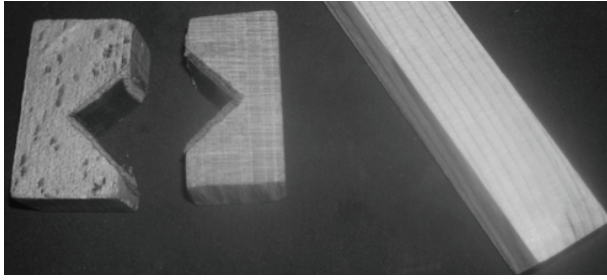


**Figure 1.** Free flexural vibration in a both-ends clamped beam bearing the axial compression, F.

<sup>1</sup> Assuming the equality of the longitudinal modulus obtained from LR and LT vibrations for an absolutely clear and sound beam, the observed differences in percentages due to defects was defined as ΔE%:

$$\Delta E_L = \left| \frac{E_{L,LT} - E_{L,LR}}{E_{L,LT}} \right| \times 100$$

where  $E_{L,LT}$  and  $E_{L,LR}$  represent the longitudinal modulus of elasticity obtained in LT and LR vibration tests, respectively.



**Figure 2.** Gripping jaws covered by hard leather (thickness = 3 mm).

(Bodig and Jayne 1989). Following the calculations for the described both-ends softly clamped beams, the value of  $m_1$  must coincide with a proper value between 3.142 and 4.730, from the simply supported (without gripping forces) to the actual both-ends fully clamped gripping condition, respectively (Bodig and Jayne 1989; Harris and Piersol 2002; Kubojima et al. 2006).

Theoretically, in an absolutely homogenized, isotropic, both-ends clamped beam, the fundamental modal frequency in terms of modulus of elasticity is calculated as below:

$$f_1 = \frac{m_1}{2\pi l^2} \sqrt{\frac{EI}{\rho A}} \tag{3}$$

Since flexural vibration is usually influenced by shear and rotary inertia, Timoshenko added these terms to the Euler–Bernoulli elementary theory of bending to develop the following differential equation of bending (Timoshenko 1921):

$$EI \frac{\partial^4 y}{\partial x^4} + \rho A \frac{\partial^2 y}{\partial t^2} - PI \left( 1 + \frac{sE}{G} \right) \frac{\partial^4 y}{\partial x^2 \partial t^2} + \frac{s\rho^2 I}{G} \frac{\partial^4 y}{\partial t^4} = 0 \tag{4}$$

where  $G$  is shear modulus and  $s = 1.18$  is the shear deflection coefficient (Nakao et al. 1984). When Eq. (4) is solved under the both-ends clamped condition, the resonance frequency  $f_1$  can be rewritten as follows:

$$f_1 = \frac{P_1}{2\pi^2} \sqrt{\frac{EI}{\rho A}} \tag{5}$$

The value of  $P_1$  is obtained by the following transcendental equations (Kubojima et al. 2006).

$$\frac{\tanh \frac{p_1}{2} \sqrt{\sqrt{B_i^2 p_i^4 + 1} - A_i p_i^2}}{\tan \frac{p_1}{2} \sqrt{\sqrt{B_i^2 p_i^4 + 1} + A_i p_i^2}} + \frac{\sqrt{\sqrt{B_i^2 p_i^4 + 1} + A_i p_i^2}}{\sqrt{\sqrt{B_i^2 p_i^4 + 1} - A_i p_i^2}} = 0 \tag{6}$$

$$A_i = \frac{I}{2Al^2} \left( 1 + \frac{sE}{G} \right) \tag{7}$$

$$B_i = \frac{I}{2Al^2} \left( -1 + \frac{sE}{G} \right) \tag{8}$$

In the present study, under the above-mentioned gripping mechanisms and the elastic moduli in the Timoshenko theory, the rebuilt value of  $p_1$  instead of  $m_1$  in Eq. (1) was introduced for a temporary account of the independent factors influencing the decrease in the both-ends clamped dynamic modulus of elasticity values, compared to the free-free methods. Eliminating the decreasing effects of the slenderness coefficient on the dynamic responses of the beams was not the researchers' concern here.

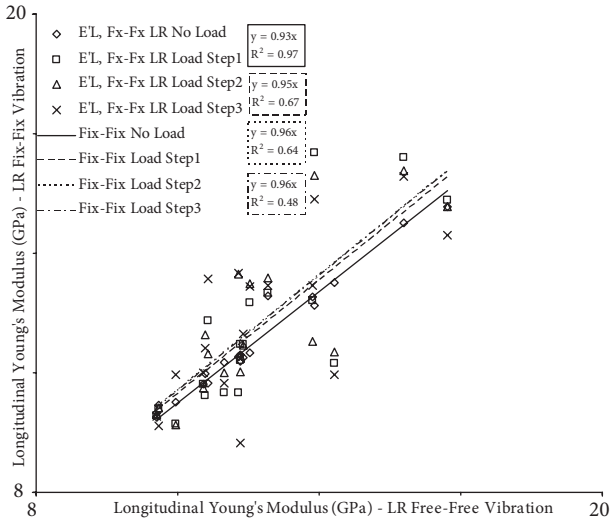
The axial compress force ( $F$ ) was performed by a manual wrench clamp to be scaled using an analog power meter in 3 steps of 50, 100 and 150<sup>N</sup>. Every single step was compared to the free-free and both-ends clamped without any axial forces conditions.

### 3. Results

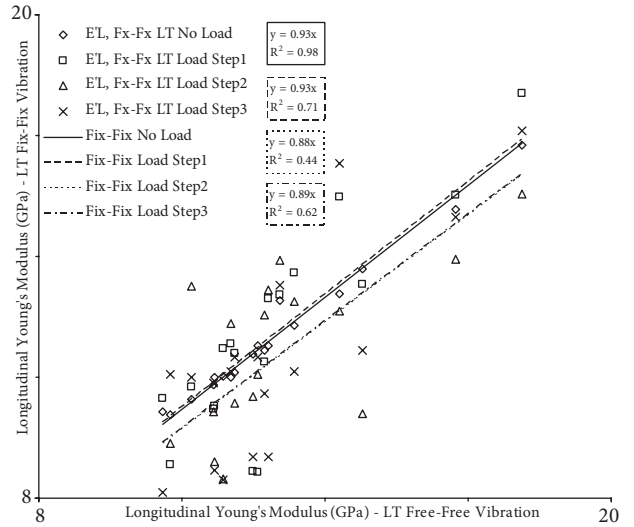
As described earlier, an estimation of the  $P_1$  value in gripping of both-ends clamped beams was separately recalculated for each individual beam (approximately 3.83) in Eq. (6). Hence, at least for the slenderness of these particular specimens, the effects of shear deflection and rotary motion were virtually compensated. Considering comparable results to those of Timoshenko's bending equations in a free-free flexural vibration test, an evaluation approach to longitudinal modulus of elasticity in both-ends clamped beams was initialized here.

Figures 3 and 4 show the correlation coefficients of evaluated longitudinal modulus of elasticity from both-ends clamped beams (shown here as  $F_x$ - $F_x$ ) in comparison with the both-ends free condition. The significance of the fitted correlation coefficients was statistically verified. Evaluated values in both-ends clamped beams in LT vibration decreased with the axial compression, whereas those of LR vibration remained constant without any noticeable shift. To assure the probable longitudinal Young's moduli shifts, the both-ends clamped beams put through the steps of axial compressions were compared to the preliminary unloaded conditions in terms of evaluated longitudinal moduli of elasticity in LR and LT flexural vibrations, as indicated in Figures 5 and 6, respectively. These correlation coefficients were also statistically verified.

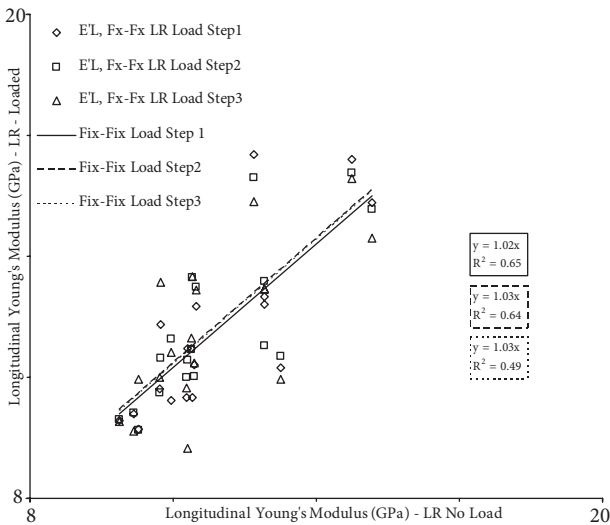
Similarly, evaluated values in both-ends clamped beams in LT vibration showed an obvious decrease in line with the axial compression, whereas those in LR vibration remained constant again without any significant shift. As the evaluations were done using similar calculations, the comparison among the different steps of loading compression and relative shifts of longitudinal Young's moduli were predicted to be valid.



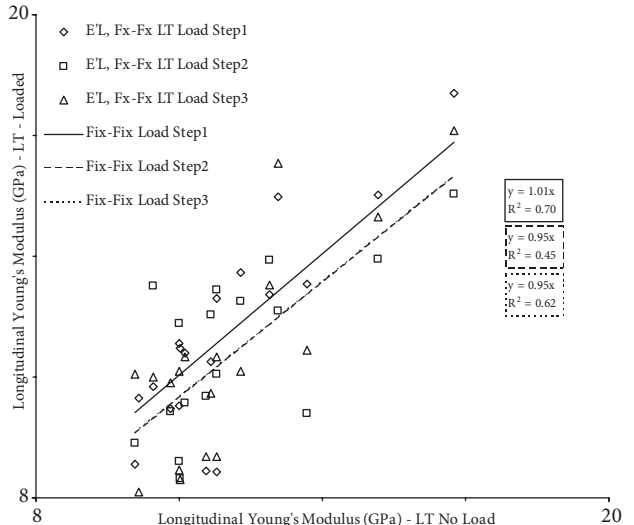
**Figure 3.** Longitudinal Young's modulus evaluated in both-ends clamped beams put through axial compression vs. the free-free condition (LR flexural vibration).



**Figure 4.** Longitudinal Young's modulus evaluated in both-ends clamped beams put through axial compression vs. the free-free condition (LT flexural vibration).



**Figure 5.** Longitudinal Young's modulus evaluated in both-ends clamped beams put through axial compression vs. no-loaded condition (LR flexural vibration).



**Figure 6.** Longitudinal Young's modulus evaluated in both-ends clamped beams put through axial compression vs. no-loaded condition (LT flexural vibration).

**4. Discussion**

The present study investigated the possibility of evaluating the longitudinal Young's modulus through free flexural vibrations in both-ends clamped beams and concluded that both-ends clamped beams had the potential for in situ longitudinal Young's modulus evaluation. The strong correlations between both-ends clamped and free-free beams in terms of evaluated moduli promised this possibility.

The effects of shear deflection and rotary inertia, initially excluded in the suggested procedure, need to be

developed in future studies. Meanwhile, the clamping forces and conditions that might reduce the scalar constant related to the end conditions of the beam have also remained as a concern for further studies.

Regarding Figure 3 (LR flexural vibration), there was no significant change in the longitudinal modulus of elasticity, but as the LT flexural vibration was taken into account in Figure 4, after axial loads greater than 100<sup>N</sup>, the modulus decreased by up to 10% when compared to the referenced both-ends free values. It was also concluded that the LT flexural vibration was more sensitive to axial

compression. Hence, the LR flexural vibration seemed promising for the beam and column evaluations under axial loads. If an entire LR plane would be out of reach, the modulus evaluations were predicted to be smaller in scope. If the elastic modulus is evaluated in slightly smaller values, it might not be a hazardous caution.

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