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Plane Wave Diffraction by a Dielectric Loaded Open Parallel Thick Plate Waveguide

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Abstract

A uniform asymptotic high-frequency solution for a two-dimensional diffraction problem of plane electromagnetic waves by a dielectric loaded open parallel thick plate waveguide is investigated rigorously using the Fourier transform technique in conjunction with the mode matching method. This mixed method of formulation gives rise to scalar modified Wiener-Hopf equations of the second kind for which the solution contains a set of infinitely many constants satisfying an infinite system of linear algebraic equations. A numerical solution of this system is obtained for various values of plate thickness, incidence angle and permittivity, and the effect of these parameters on the diffraction phenomenon is studied.

Key Words: *Diffraction, dielectric, waveguide, Wiener-Hopf equation, mode matching method.*

1. Introduction

The scattering of plane waves by a series of parallel plates constitutes an important class of a canonical problem in diffraction theory and has been studied for many years. Scattering by parallel plate waveguide [1,2] and infinite grating [3,4] was resolved earlier, but other configurations such as three parallel half-planes have long defied analysis. The reason is that the solution rests on the Wiener-Hopf technique. While the Wiener-Hopf method is well established for the scalar (single equation) case, a general method for tackling the matrix case is not yet available.

The diffraction of plane waves by three parallel infinitely thin soft half-planes was first considered by Jones, who formulated the problem as a three-dimensional matrix Wiener-Hopf equation [5]. These equations are converted into a pair involving a two-dimensional matrix and a scalar Wiener-Hopf equation. The three parallel half-planes problem was also considered by Abrahams [6], who presented a simpler approach to achieve the Wiener-Hopf factorization of the kernel matrix. In all of the studies cited above the system which guides the electromagnetic waves was considered to have an infinitely thin plate thickness. However, considering plates with a finite thickness will give much more realistic results about the diffraction of electromagnetic waves by these kind of geometries.

The aim of the present work is to observe the diffracted field contribution which comes from the results obtained in [7] when the lower plate of the waveguide mentioned in [7] has a thickness. To this end, we consider the uniform asymptotic high-frequency diffraction of E_z-polarized plane waves by a two-dimensional geometry which consists of two parallel thick conducting half-planes located over an electric wall. The region between the thick half-planes is filled by a simple dielectric material.

The traditional formulation of this problem leads to a kind of modified Wiener-Hopf equation which cannot be solved by considering the known techniques. In this work an alternative method of formulation, which is introduced in [8], will be used. The main idea of this method is first to expand the scattered field related to the waveguide region into a series of normal modes and then use the Fourier integral representation. This yields a scalar modified Wiener-Hopf equation of the second kind which can be solved by using the standard techniques. The solution of this Wiener-Hopf equation contains a set of infinitely many unknown constants satisfying an infinite system of linear algebraic equations. A numerical solution for these systems is obtained for various values of plate thickness, incident wave angle and permittivity ε_1 corresponding to simple material lying in the waveguide, through which the effect of these parameters on the diffraction phenomenon is observed.

A time factor e^{-iwt} with w being the angular frequency is assumed and suppressed throughout the paper.

2. Analysis

We consider the diffraction of an E_z-polarized plane wave by two parallel thick and infinitely thin half-planes defined by $\mathcal{S}_1 = \{(x,y,z);\ x \in (-\infty,0),\ y \in (a,b),\ z \in (-\infty,\infty)\}$, $\mathcal{S}_2 = \{(x,y,z);\ x \in (-\infty,0),\ y \in (0,c),\ z \in (-\infty,\infty)\}$ and $\mathcal{S}_3 = \{(x,y,z); x \in (0,\infty),\ y = 0,\ z \in (-\infty,\infty)\}$ respectively as depicted in Figure 1. The region between the half-planes $\mathcal{S}_1$ and $\mathcal{S}_2$ consists of a non-magnetic and non-conducting simple material having the permittivity ε_1. The region which exists outside the waveguide is considered as a free space.

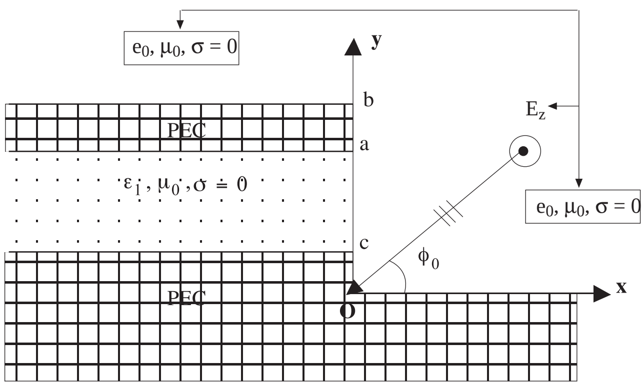

Figure 1. Geometry of the diffraction problem.

For analysis purposes, it is convenient to express the total field as follows:

$$u_T(x,y) = \begin{cases} u^i(x,y) + u^r(x,y) + u_1(x,y) \quad , \quad y > b \\ u_2(x,y) \quad , \quad c < y < a \ , \ x < 0 \\ u_3(x,y) \quad , \quad 0 < y < b \ , \ x > 0. \end{cases} \tag{1a}$$

Obviously, the total field in $y < 0$ and $y \in \{(0,c) \cup (a,b)\}$, $x < 0$ is identically zero. Here, $u^i(x,y)$ is the incident field given by

$$E_z^i = u^i(x,y) = \exp\{-ik[x\cos\phi_0 + y\sin\phi_0]\} \tag{1b}$$

while u^r denotes the field that would be reflected if the whole plane $y = b$ was perfectly conducting, namely

$$u^r(x,y) = -\exp\{-ik[x\cos\phi_0 - (y-2b)\sin\phi_0]\}. \tag{1c}$$

In $(1b,c)$, k is the free space wave number which is assumed to have a small positive imaginary part. $u_j(x,y)$, $j = 1,2,3$, which satisfy the Helmholtz equation in their corresponding regions, are to be determined with the aid of the following boundary and continuity conditions:

$$u_1(x,b) = 0 \quad , \quad x < 0 \tag{2a}$$

$$u_2(x,a) = 0 \quad , \quad x < 0 \tag{2b}$$

$$u_2(x,c) = 0 \quad , \quad x < 0 \tag{2c}$$

$$u_3(x,0) = 0 \quad , \quad x > 0 \tag{2d}$$

$$u_1(x,b) - u_3(x,b) = 0 \quad , \quad x > 0 \tag{2e}$$

$$\frac{\partial}{\partial y}u_1(x,b) - \frac{\partial}{\partial y}u_3(x,b) = 2ik\sin\phi_0 e^{-ikb\sin\phi_0}\, e^{-ikx\cos\phi_0} \quad , \quad x > 0 \tag{2f}$$

$$u_2(0,y) = u_3(0,y) \quad , \quad c < y < a \tag{2g}$$

$$\frac{\partial}{\partial x}u_2(0,y) = \frac{\partial}{\partial x}u_3(0,y) \quad , \quad c < y < a \tag{2h}$$

$$u_3(0,y) = 0 \quad , \quad y \in \{(0,c) \cup (a,b)\} \tag{2i}$$

Since $u_1(x,y)$ satisfies the Helmholtz equation in the range $x \in (-\infty,\infty)$, its Fourier transform with respect to x gives

$$\left[\frac{d^2}{dy^2} + (k^2 - \alpha^2)\right]F(\alpha,y) = 0 \tag{3a}$$

with

$$F(\alpha,y) = F_+(\alpha,y) + F_-(\alpha,y) \tag{3b}$$

where

$$F_\pm(\alpha,y) = \pm\int_0^{\pm\infty} u_1(x,y)e^{i\alpha x}dx. \tag{3c}$$

By taking into account the following asymptotic behaviours of u_1 for $x \to \pm\infty$

$$u_1(x,y) = \begin{cases} O(e^{-ikx}) \quad , \quad x \to -\infty \\ O(e^{-ikx\cos\phi_0}) \quad , \quad x \to +\infty \end{cases} \tag{4}$$

one can show that $F_+(\alpha, y)$ and $F_-(\alpha, y)$ are regular functions of α in the half-planes $\Im m(\alpha) > \Im m(k\cos\phi_0)$ and $\Im m(\alpha) < \Im m(k)$, respectively. The general solution of $(3a)$ satisfying the radiation condition for $y \to \infty$ reads

$$F_+(\alpha, y) + F_-(\alpha, y) = A(\alpha)\, e^{iK(\alpha)(y-b)} \tag{5a}$$

with

$$K(\alpha) = \sqrt{k^2 - \alpha^2}. \tag{5b}$$

The square-root function is defined in the complex α-plane cut along $\alpha = k$ to $\alpha = k + i\infty$ and $\alpha = -k$ to $\alpha = -k - i\infty$, such that $K(0) = k$.

In the Fourier transform domain $(2a)$ takes the form

$$F_-(\alpha, b) = 0. \tag{6}$$

By considering $(5a)$ at $y = b$ with (6), one gets

$$F_+(\alpha, b) = A(\alpha). \tag{7}$$

In the region $0 < y < b$ and $x > 0$, $u_3(x, y)$ satisfies the Helmholtz equation

$$\Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\Big) u_3(x, y) = 0. \tag{8}$$

The half-range Fourier transform of (8) yields

$$\Big[\frac{d^2}{dy^2} + K^2(\alpha)\Big] G_+(\alpha, y) = f(y) + \alpha g(y) \tag{9a}$$

with

$$f(y) = \frac{\partial}{\partial x} u_3(0, y) \qquad , \qquad g(y) = -i u_3(0, y). \tag{9b, c}$$

$G_+(\alpha, y)$, which is defined by

$$G_+(\alpha, y) = \int_0^\infty u_3(x, y) e^{i\alpha x} dx, \tag{10}$$

is a function regular in the half-plane $\Im m(\alpha) > \Im m(-k)$. The general solution of $(9a)$ satisfying the Dirichlet boundary condition at $y = 0$ reads

$$G_+(\alpha, y) = B(\alpha) \sin\left[K(\alpha) y\right] + \frac{1}{K(\alpha)} \int_0^y \left[f(t) + \alpha g(t)\right] \sin\left[K(\alpha)(y - t)\right] dt. \tag{11}$$

By considering $(2e)$, one gets

$$F_+(\alpha, b) = G_+(\alpha, b) \tag{12}$$

and $B(\alpha)$ can be solved uniquely to give

$$\sin[K(\alpha) b] B(\alpha) = F_+(\alpha, b) - \frac{1}{K(\alpha)} \int_0^b [f(t) + \alpha g(t)] \sin[K(\alpha)(b - t)] dt. \tag{13}$$

Replacing (13) into (11) one obtains

$$\begin{aligned} G_+(\alpha,y) = \ & \frac{\sin[K(\alpha)y]}{\sin[K(\alpha)b]}\left\{F_+(\alpha,b) - \frac{1}{K(\alpha)}\int_0^b [f(t)+\alpha g(t)]\sin[K(\alpha)(b-t)]dt\right\} \\ & + \frac{1}{K(\alpha)}\int_0^y [f(t)+\alpha g(t)]\sin[K(\alpha)(y-t)]\,dt. \end{aligned} \tag{14}$$

Although the left-hand side of (14) is regular in the upper half-plane $\Im m(\alpha) > \Im m(-k)$, the regularity of the right-hand side is violated by the presence of simple poles occurring at the zeros of $\sin[K(\alpha)b]$, namely at $\alpha = \alpha_m$ given by

$$\alpha_m = \sqrt{k^2 - (m\pi/b)^2} \quad , \quad \Im m(\alpha_m) > \Im m(k) \quad , \quad m = 1, 2, ... \tag{15}$$

These poles can be eliminated by imposing that their residues are zero. This gives

$$F_+(\alpha_m, b) = \frac{(-1)^{m+1} b}{2K_m}[f_m + \alpha_m g_m] \tag{16a}$$

where K_m, f_m and g_m specify

$$K_m = K(\alpha_m) = \frac{m\pi}{b} \tag{16b}$$

$$\begin{bmatrix} f_m \\ g_m \end{bmatrix} = \frac{2}{b}\int_0^b \begin{bmatrix} f(t) \\ g(t) \end{bmatrix} \sin[K_m t]dt. \tag{16c}$$

Consider now the region $c < y < a$, $x < 0$. c_n being a constant coefficient which will be determined by using the boundary conditions at $x = 0$, the total field in this region can be expressed in terms of Fourier series as

$$u_2(x,y) = \sum_{n=1}^{\infty} c_n \sin[\xi_n(y-c)]\ e^{-i\beta_n x} \tag{17a}$$

with

$$\xi_n = \frac{n\pi}{(a-c)} \quad , \quad n = 1, 2, ... \tag{17b}$$

and

$$\beta_n = k_1\sqrt{1 - \left[\frac{n\pi}{k_1(a-c)}\right]^2} \tag{17c}$$

where k_1 is the wave number of the simple dielectric material located in the waveguide. From the boundary conditions $(2g, h, i)$ and the relations $(9b, c)$ we get

$$f(y) = \frac{\partial}{\partial x}u_2(0,y) \quad , \quad c < y < a. \tag{18a}$$

and

$$g(y) = \begin{cases} 0 & , \quad a < y < b \\ -iu_2(0,y) & , \quad c < y < a \\ 0 & , \quad 0 < y < c. \end{cases} \tag{18b}$$

Owing to $(16c)$, $f(y)$ and $g(y)$ can be expanded into Fourier Sine series as follows:

$$\begin{bmatrix} f(y) \\ g(y) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} f_m \\ g_m \end{bmatrix} \sin(K_m y). \tag{19}$$

Substituting $(17a)$ and (19) into $(18a)$ and $(18b)$ we obtain respectively,

$$\sum_{m=1}^{\infty} f_m \sin(K_m y) = -i \sum_{n=1}^{\infty} c_n \beta_n \sin\left[\xi_n (y-c)\right] \quad , \quad c < y < a \tag{20}$$

and

$$\sum_{m=1}^{\infty} g_m \sin(K_m y) = \begin{cases} 0 & , \quad a < y < b \\ -i \sum\limits_{n=1}^{\infty} c_n \sin\left[\xi_n (y-c)\right] & , \quad c < y < a \\ 0 & , \quad 0 < y < c. \end{cases} \tag{21}$$

Let us multiply both sides of (20) by $\sin\left[\xi_\ell (y-c)\right]$ and integrate from $y=c$ to $y=a$ to get

$$c_\ell = (-1)^\ell \frac{2i}{(a-c)} \frac{\xi_\ell}{\beta_\ell} \sum_{m=1}^{\infty} \frac{f_m}{K_m^2 - \xi_\ell^2} \left[\sin(K_m a) + (-1)^{\ell+1} \sin(K_m c)\right] \quad , \quad \ell = 1, 2, ... \tag{22}$$

Similarly, the multiplication of both sides of (21) by $\sin(K_\ell y)$ and its integration from $y=0$ to $y=b$ yields

$$g_\ell = -i \frac{2}{b} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{c_n \xi_n}{(\xi_n^2 - K_\ell^2)} \left[\sin(K_\ell a) + (-1)^{n+1} \sin(K_\ell c)\right] \quad , \quad \ell = 1, 2, ... \tag{23}$$

Consider the continuity relation $(2f)$ which reads, in the Fourier transform domain,

$$\dot{F}_+(\alpha, b) - \dot{G}_+(\alpha, b) = -2k \sin\phi_0 \frac{e^{-ikb\sin\phi_0}}{(\alpha - k\cos\phi_0)} \quad . \tag{24}$$

where the dot (.) specifies the derivative with respect to y. Taking into account $(5a)$, (7) and (14) with (24) one obtains

$$\begin{aligned} \frac{e^{-iK(\alpha)b} K(\alpha)}{\sin[K(\alpha)b]} F_+(\alpha, b) + \dot{F}_-(\alpha, b) \quad &= 2k \sin\phi_0 \frac{e^{-ikb\sin\phi_0}}{(\alpha - k\cos\phi_0)} \\ &- \frac{1}{\sin[K(\alpha)b]} \int_0^b \left[f(t) + \alpha g(t)\right] \sin\left[K(\alpha)t\right] dt. \end{aligned} \tag{25}$$

Substituting (19) in (25) and evaluating the resultant integral, one obtains the following modified Wiener-Hopf equation of the second kind valid in the strip $\Im m(k\cos\phi_0) < \Im m(\alpha) < \Im m(k)$

$$\begin{aligned} \frac{K(\alpha)}{N(\alpha)} F_+(\alpha, b) + \dot{F}_-(\alpha, b) = \quad & 2k \sin\phi_0 \frac{e^{-ikb\sin\phi_0}}{(\alpha - k\cos\phi_0)} \\ & + \sum_{m=1}^{\infty} \frac{(-1)^m K_m}{[\alpha^2 - \alpha_m^2]} (f_m + \alpha g_m) \end{aligned} \tag{26a}$$

with

$$N(\alpha) = e^{iK(\alpha)b} \sin\left[K(\alpha)b\right]. \tag{26b}$$

The formal solution of $(26a)$ can easily be obtained through the classical Wiener-Hopf procedure. The result is

$$\begin{aligned} \frac{\sqrt{\alpha+k}}{N_+(\alpha)} F_+(\alpha,b) = \quad & 2k\sin\phi_0 \frac{N_-(k\cos\phi_0)}{\sqrt{k-k\cos\phi_0}} \frac{e^{-ikb\sin\phi_0}}{(\alpha-k\cos\phi_0)} \\ & - \sum_{m=1}^{\infty} \frac{(-1)^m K_m}{2\alpha_m} \frac{N_+(\alpha_m)}{\sqrt{\alpha_m+k}} \frac{(f_m-\alpha_m g_m)}{(\alpha+\alpha_m)}. \end{aligned} \tag{27a}$$

Here $N_+(\alpha)$ and $N_-(\alpha)$ are the split functions, regular and free of zeros in the half-planes $\Im m(\alpha) > \Im m(-k)$ and $\Im m(\alpha) < \Im m(k)$ respectively, resulting from the Wiener-Hopf factorization of $(26b)$ as

$$N(\alpha) = N_+(\alpha)N_-(\alpha). \tag{27b}$$

The explicit expression of $N_\pm(\alpha)$ can be obtained by following the procedure outlined in [2]:

$$\begin{aligned} N_+(\alpha) = \quad & \sqrt{(1+\alpha/k)\sin(kb)} \exp\left\{\frac{bK(\alpha)}{\pi} \ln\left(\frac{\alpha+iK(\alpha)}{k}\right)\right\} \\ & \times \exp\left\{\frac{i\alpha b}{\pi}\left[1-\mathcal{C}+\ln\left(\frac{2\pi}{kb}\right)+i\frac{\pi}{2}\right]\right\} \prod_{m=1}^{\infty}\left(1+\frac{\alpha}{\alpha_m}\right)\exp\left(\frac{i\alpha b}{m\pi}\right) \end{aligned} \tag{28a}$$

$$N_-(\alpha) = N_+(-\alpha). \tag{28b}$$

In $(28a)$ $\mathcal{C}$ is the Euler's constant given by $\mathcal{C} = 0.57721...$ By considering $(27a)$ for $\alpha = \alpha_r$ with $(16a)$, (22) and (23) one gets infinitely many equations in an infinite number of unknowns which yield the constants f_r, $r = 1, 2, ..$ as follows:

$$(-1)^{r+1}\frac{b}{2}\frac{\sqrt{k+\alpha_r}}{K_r N_+(\alpha_r)} f_r + \sum_{m=1}^{\infty} Q_m(\alpha_r) f_m = 2k\sin\phi_0 \frac{N_-(k\cos\phi_0)}{\sqrt{k-k\cos\phi_0}} \frac{e^{-ikb\sin\phi_0}}{(\alpha_r-k\cos\phi_0)} \tag{29a}$$

with

$$\begin{aligned} Q_m(\alpha_r) = \quad & \frac{(-1)^m K_m}{2\alpha_m} \frac{N_+(\alpha_m)}{\sqrt{k+\alpha_m}(\alpha_r+\alpha_m)} \\ & -\frac{2}{b(a-c)} \sum_{n=1}^{\infty} \frac{\xi_n^2 \Omega_{mn}}{\beta_n \Psi_{mn}} \left\{ \frac{(-1)^r b\alpha_r \sqrt{k+\alpha_r}\,\Omega_{rn}}{K_r N_+(\alpha_r)\Psi_{rn}} + \sum_{s=1}^{\infty} \frac{(-1)^s K_s N_+(\alpha_s)\Omega_{sn}}{\sqrt{k+\alpha_s}\Psi_{sn}(\alpha_r+\alpha_s)} \right\} \end{aligned} \tag{29b}$$

Here Ω_{pn} and Ψ_{pn} with $p = m, r, s$ stand for respectively,

$$\Omega_{pn} = \sin(K_p a) + (-1)^{n+1}\sin(K_p c) \tag{29c}$$

and

$$\Psi_{pn} = K_p^2 - \xi_n^2 \quad . \tag{29d}$$

By substituting (22) in (23) the constants g_r can be expressed in terms of f_r as

$$g_r = \frac{4}{b(a-c)} \sum_{m=1}^{\infty} f_m \left(\sum_{n=1}^{\infty} \frac{\xi_n^2 \Omega_{rn}\Omega_{mn}}{\beta_n \Psi_{rn}\Psi_{mn}} \right) \tag{30}$$

3. Analysis of the Diffracted Field

The scattered field u_1 in the region $y > b$ can be obtained by taking the inverse Fourier transform of $F(\alpha, y)$. By considering $(3b)$ and $(5a)$ one can write

$$u_1(x,y) = \frac{1}{2\pi}\int_{\mathcal{L}} A(\alpha)\, e^{iK(\alpha)(y-b)}\, e^{-i\alpha x} d\alpha \tag{31}$$

Here $\mathcal{L}$ is a straight line parallel to the real α-axis lying in the strip $\Im m(k\cos\phi_0) < \Im m(\alpha) < \Im m(k)$. The asymptotic evaluation of the integral in (31) through the saddle point technique enables us to write the uniform expression of the diffracted field in terms of the modified Fresnel integral

$$\mathcal{F}(z) = -2i\sqrt{z}e^{-iz}\int_{\sqrt{z}}^{\infty} e^{it^2} dt \tag{32}$$

as follows:

$$\begin{aligned} u_1^d(\rho,\phi) \quad \sim \frac{e^{ik\rho}}{\sqrt{k\rho}}\Bigg\{ & W(\phi,\phi_0)\left[\frac{\mathcal{F}\big(2k\rho\cos^2\frac{1}{2}(\phi+\phi_0)\big)}{\cos\frac{1}{2}(\phi+\phi_0)} + \frac{\mathcal{F}\big(2k\rho\cos^2\frac{1}{2}(\phi-\phi_0)\big)}{\cos\frac{1}{2}(\phi-\phi_0)}\right] \\ & + \frac{e^{i3\pi/4}}{2\sqrt{\pi}} N_-(k\cos\phi)\cos(\phi/2)\sum_{m=1}^{\infty}\frac{(-1)^m K_m}{\alpha_m}\frac{N_+(\alpha_m)}{\sqrt{1+\alpha_m/k}}\frac{(f_m-\alpha_m g_m)}{(\alpha_m - k\cos\phi)}\Bigg\} \end{aligned} \tag{33a}$$

with

$$W(\phi,\phi_0) = u_0\frac{e^{i3\pi/4}}{\sqrt{2\pi}} N_-(k\cos\phi) N_-(k\cos\phi_0) \tag{33b}$$

where

$$u_0 = e^{-ikb\sin\phi_0}. \tag{33c}$$

In $(33a)$ (ρ,ϕ) are the cylindrical polar coordinates defined by

$$x = \rho\cos\phi \quad , \quad y - b = \rho\sin\phi \ .$$

In the present work when we let $c = 0$ and $\varepsilon_1 = \varepsilon_0$ the geometry reduces exactly to that of the odd excitation case considered in [8] for conducting surfaces. It can be checked easily that the result obtained by setting $c = 0$, $\varepsilon_1 = \varepsilon_0$ and $\mathcal{F}(z) = 1$ $(z >> 1)$ in $(33a)$ is identical to that previously given in [8, formula (42c)] for $\eta_1 = \eta_2 = \eta_3 = 0$. The method presented in this study can also be extended to the H_z polarization case by considering that the total field u_T satisfies the homogeneous Neumann boundary condition on the conducting surfaces.

In order to see the accuracy and the effectiveness of different values of the parameters related to the geometry of the problem on the diffraction phenomenon, some numerical results concerning the variation of the diffracted field $\big(20\log|u_1^d.\sqrt{k\rho}|\big)$ versus the observation angle ϕ are presented. Figure 2 depicts the variation of the diffracted field with respect to the observation angle for different values of wall thickness $(b-a)$. As expected, the diffracted field increases with the increasing values of the wall thickness. Figure 3 gives the variation of the diffracted field versus the observation angle ϕ for different values of the incidence angle ϕ_0. As expected, the diffracted field decreases when the values of ϕ_0 increase. And finally Figure 4 shows the variation of the diffracted field versus the observation angle ϕ for different values of the permittivity

ε_1. It is observed that the diffracted field is not greatly affected by the permittivity. It increases slightly when the values of ε_1 increase.

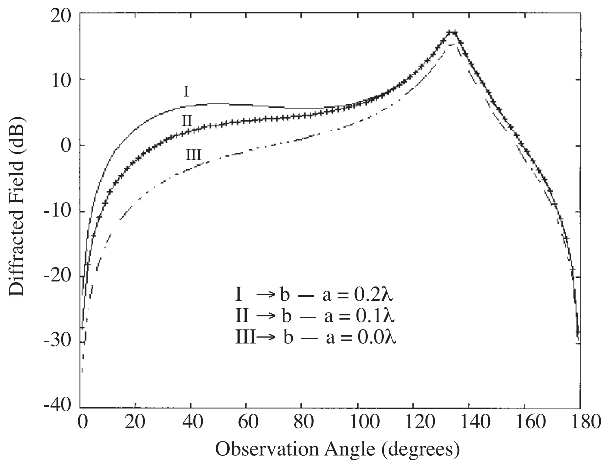

Figure 2. $20\log|u_1^d.\sqrt{k\rho}|$ versus the observation angle ϕ, for different values of $(b-a)$

$$\left(a = 2c = 0.2\lambda \quad , \quad \phi_0 = 45^o \quad , \quad \varepsilon_1 = \varepsilon_0 = \frac{1}{36\pi}10^{-9}\ F/m\right)$$

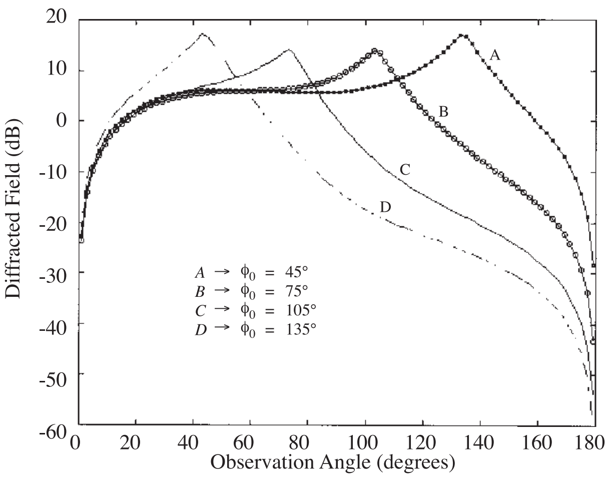

Figure 3. $20\log|u_1^d.\sqrt{k\rho}|$ versus the observation angle ϕ, for different values of ϕ_0

$$\left(b = 2a = 4c = 0.4\lambda \quad , \quad \varepsilon_1 = \varepsilon_0 = \frac{1}{36\pi}10^{-9}\ F/m\right)$$

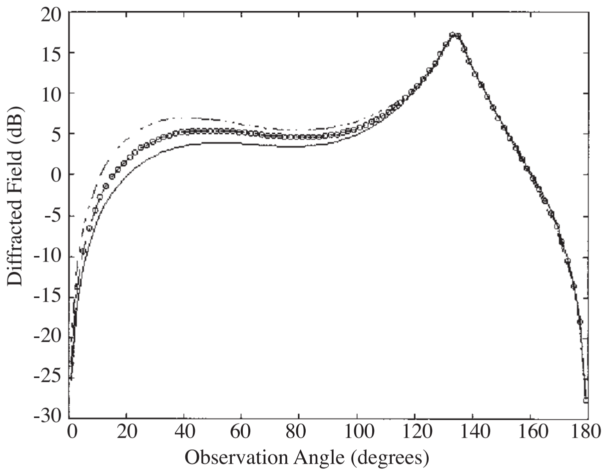

Figure 4. $20 \log |u_1^d.\sqrt{k\rho}|$ versus the observation angle ϕ, for different values of ε_1

$$\left(b = 2a = 4c = 0.4\lambda \quad , \quad \phi_0 = 45^o \quad , \quad \varepsilon_0 = \frac{1}{36\pi}10^{-9}\ F/m\right)$$

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