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The $\mathcal{H}_\infty$ Model Matching Problem with One Degree of Freedom Static State Feedback

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Abstract

The aim of this paper is to develop a new approach for a solution of the continuous-time model matching problem by a static state feedback in the sense of $\mathcal{H}_\infty$ optimality criterion by using Linear Matrix Inequalities (LMIs). The main contribution could briefly be described as to reformulate the model matching problem in LMI formulation, to present the solvability conditions and to give a design procedure for the one degree of freedom static state feedback control law. Finally, the results are applied to an example system.

Key Words: *Model Matching Problem, Linear Matrix Inequalities, $\mathcal{H}_\infty$ Optimal Control Problem, One Degree of Freedom Static State Feedback.*

Introduction

The model matching problem is one of the most familiar in control theory, [1]. Let $T_m(s)$ and $T(s)$ be stable and proper transfer matrices. The continuous-time $\mathcal{H}_\infty$ model matching problem is to find a controller transfer matrix $R(s)$ that is stable and causal, that is $R(s)\in\mathcal{RH}_\infty$, to minimize the $\mathcal{H}_\infty$ norm of $T_m(s) - T(s)R(s)$. The interpretation is this: $T_m(s)$ and $T(s)$ are given as the model and the system transfer matrices respectively. The closed-loop performance $T(s)R(s)$ approximates the desired performance $T_m(s)$ such that,

$$\gamma_{opt} = \inf_{R(s)\in\mathcal{RH}_\infty} \|T_m(s) - T(s)R(s)\|_\infty \tag{1}$$

In the literature, there are some results on the $\mathcal{H}_\infty$ model matching problem: Two of them are based on the Nevanlinna-Pick Problem (NPP) [2] and Nehari Problem (NP) [3, 4]. In these studies, the $\mathcal{H}_\infty$ model matching problem is reduced to the one of these problems, and then by using the results on the solution of NPP or NP, first the value of γ_{opt} defined in (1) is found and then the controller transfer matrix $R(s)$ is obtained. Another solution of the $\mathcal{H}_\infty$ model matching problem is based on canonical spectral factorizations and the solutions of algebraic Riccati equations (ARE) [5].

In all these studies, the controller transfer matrix $R(s)$ is considered in the form of a precompensator in (1) and $R(s)$ is found as a stable and proper rational matrix. Since a precompensator which is a proper and stable rational matrix can generally be established by a dynamic state feedback in the feedback configuration [6], none of these results can directly be used in a one degree of freedom static state feedback control structure. However, the solutions of the continuous- and discrete-time $\mathcal{H}_\infty$ model matching problem by dynamic and static state feedbacks, with a two degrees of freedom control structure, via LMI optimization are given in [7, 8], respectively.

In this study, a special formulation is developed to solve the continuous-time $\mathcal{H}_\infty$ model matching problem by a one degree of freedom static state feedback. This formulation enables us to use the methods and the results presented for the solution of the continuous-time $\mathcal{H}_\infty$ optimal control problem and so the problem can be completely solved by LMI-based parameterization.

The paper is organized in the following way: In Section 2, we present a special formulation for the continuous-time $\mathcal{H}_\infty$ model matching problem by a one degree of freedom static state feedback in LMIs. In Section 3, the main result is given in Theorem 1.3, including the existence conditions. In Section 4, we construct the one degree of freedom static state feedback controller by using Theorem 1.3. In Section 5, there is a numerical example and then some conclusions are given in Section 6.

The following notation will be used throughout the paper: $KerM$ and ImM are the null space and range of the linear operator associated with M, respectively and N^T is the transpose of the matrix N. In addition, $P > 0$ denotes that the matrix P is positive definite.

The Continuous-time $\mathcal{H}_\infty$ Model Matching Problem in LMIs

In order to solve the continuous-time $\mathcal{H}_\infty$ model matching problem via LMI approach, the problem should be reformulated as the standard continuous-time $\mathcal{H}_\infty$ optimal control problem. Therefore, we shall assume that any state-space equations of the given system $T(s)$ and the model system $T_m(s)$ can be given as follows:

$$T(s): \qquad \dot{x}(t) = Ax(t) + Bv(t) \tag{2}$$

$$y_s(t) = Cx(t) + Dv(t) \tag{3}$$

$$T_m(s): \qquad \dot{q}(t) = Fq(t) + Gw(t) \tag{4}$$

$$y_m(t) = Hq(t) + Jw(t) \tag{5}$$

where $x(t)\in\mathcal{R}^{n_s}$, $q(t)\in\mathcal{R}^{n_m}$, $v(t)\in\mathcal{R}^{m}$, $w(t)\in\mathcal{R}^{m}$, $y_s(t)\in\mathcal{R}^{p}$ and $y_m(t)\in\mathcal{R}^{p}$. The control input $u(t)$ can be generated by a static state feedback controller:

$$u(t) = Kx(t) \tag{6}$$

In Figure 1, the continuous-time $\mathcal{H}_\infty$ model matching problem with a static state feedback is given. We propose the one degree of freedom control structure in the control theory [9].

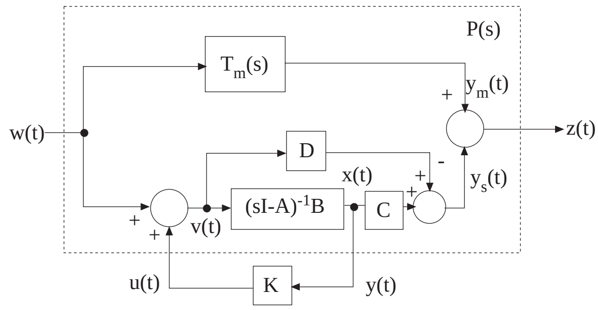

Figure 1

The plant $P(s)$ shown in Figure 1 can be given as

$$\begin{bmatrix} \dot{x}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} B \\ G \end{bmatrix} w(t) + \begin{bmatrix} B \\ 0 \end{bmatrix} u(t) \tag{7}$$

$$z(t) = \begin{bmatrix} -C & H \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} + (J-D)w(t) - Du(t) \tag{8}$$

$$y(t) = \begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ q(t) \end{bmatrix} \tag{9}$$

Let us define some matrices as follows:

$$\underline{A} = \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} \qquad B_1 = \begin{bmatrix} B \\ G \end{bmatrix} \qquad B_2 = \begin{bmatrix} B \\ 0_{n_m \times m} \end{bmatrix} \tag{10}$$

$$C_1 = \begin{bmatrix} -C & H \end{bmatrix} \qquad C_2 = \begin{bmatrix} I_{n_s} & 0_{n_s \times n_m} \end{bmatrix} \qquad D_1 = J - D \qquad D_2 = -D \tag{11}$$

As a result of the above formulation, the continuous-time $\mathcal{H}_\infty$ model matching problem with the one degree of freedom static state feedback is equivalent to the continuous-time $\mathcal{H}_\infty$ optimal control problem. Figure 2 shows this idea.

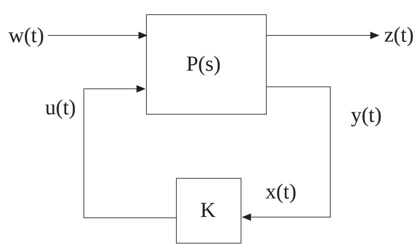

Figure 2

The closed-loop transfer matrix from $w(t)$ to $z(t)$ is

$$T_{zw}(s) = D_{cl} + C_{cl}(sI - A_{cl})^{-1}B_{cl} \tag{12}$$

where

$$A_{cl} = \underline{A} + B_2 K C_2 \tag{13}$$

$$B_{cl} = B_1 \tag{14}$$

$$C_{cl} = C_1 + D_2 K C_2 \tag{15}$$

$$D_{cl} = D_1 \tag{16}$$

The following lemma can be given on the internal stability of the closed-loop system (for the existence of a γ):

Lemma 0.1 *For the system described in* (7)*,* (8) *and* (9)*, there exists a matrix K such that the matrix $A_{cl} = \underline{A} + B_2KC_2$ is Hurwitz if and only if (A, B) is stabilizable and the matrix F is Hurwitz.*

Proof: When $\underline{A}$, B_2, C_2 and K are used in A_{cl}, the following relation is obtained:

$$A_{cl} = \begin{bmatrix} A & 0 \\ 0 & F \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} K \begin{bmatrix} I & 0 \end{bmatrix} = \begin{bmatrix} A + BK & 0 \\ 0 & F \end{bmatrix} \tag{17}$$

Therefore, the matrix A_{cl} is Hurwitz if and only if the matrix F is Hurwitz and there exists a matrix K such that the matrix $A + BK$ is Hurwitz, i.e. (A, B) is stabilizable. □

In order to guarantee the existence of a one degree of freedom static state feedback control law, i.e. the closed-loop system is internally stable, throughout the paper we assume that (A, B) is stabilizable and the matrix F is Hurwitz.

1. Main Result

For a synthesis theorem on the LMI-based solution of the continuous-time $\mathcal{H}_\infty$ model matching problem, let us give the following lemmas. They will be used to prove the theorem that will be presented later. The first lemma is well known as **The Bounded Real Lemma** and can be used to turn the continuous-time $\mathcal{H}_\infty$ optimal control problem into an LMI:

Lemma 1.1 *Consider a continuous-time transfer matrix $T(s)$ of (not necessarily minimal) realization $T(s) = D + C(sI - A)^{-1}B$. The following statements are equivalent:*

i) $\|D + C(sI - A)^{-1}B\|_\infty < \gamma$ *and the matrix A is Hurwitz,*

ii) *there exists a solution $X > 0$ to the LMI:*

$$\begin{bmatrix} A^TX + XA & XB & C^T \\ B^TX & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0 \tag{18}$$

Proof: [10]. □

Lemma 1.2 *Suppose P, Q and H are matrices and that H is symmetric. The matrices N_P and N_Q are full rank matrices satisfying $ImN_P = KerP$ and $ImN_Q = KerQ$. Then there exists a matrix J such that,*

$$H + P^TJ^TQ + Q^TJP < 0 \tag{19}$$

if and only if the inequalities

$$N_P^T H N_P < 0 \qquad \textit{and} \qquad N_Q^T H N_Q < 0 \tag{20}$$

are both satisfied.

Proof: [11]. □

We can now present a synthesis theorem on the LMI-based solution of the problem.

Theorem 1.3 *A one degree of freedom static state feedback controller $K \in \mathcal{R}^{m \times n_s}$ exists for the continuous-time $\mathcal{H}_\infty$ model matching problem if and only if there exists a matrix*

$X_{cl} = \begin{bmatrix} X_1 & X_2 \\ X_2^T & X_3 \end{bmatrix} > 0$ *such that*

$$\begin{bmatrix} F^T X_3 + X_3 F & X_2^T B + X_3 G & H^T \\ B^T X_2 + G^T X_3 & -\gamma I_m & (J - D)^T \\ H & J - D & -\gamma I_p \end{bmatrix} < 0 \tag{21}$$

$$\begin{bmatrix} N_c & 0 \\ 0 & I_m \end{bmatrix}^T \begin{bmatrix} \begin{pmatrix} A & 0 \\ 0 & F \end{pmatrix} X_{cl}^{-1} + X_{cl}^{-1} \begin{pmatrix} A & 0 \\ 0 & F \end{pmatrix}^T & X_{cl}^{-1} \begin{pmatrix} -C^T \\ H^T \end{pmatrix} & \begin{pmatrix} B \\ G \end{pmatrix} \\ \begin{pmatrix} -C & H \end{pmatrix} X_{cl}^{-1} & -\gamma I_p & J - D \\ \begin{pmatrix} B^T & G^T \end{pmatrix} & (J - D)^T & -\gamma I_m \end{bmatrix} \begin{bmatrix} N_c & 0 \\ 0 & I_m \end{bmatrix} < 0 \tag{22}$$

where N_c is a full rank matrix with

$$Im N_c = Ker \begin{bmatrix} B^T & 0_{m \times n_m} & -D^T \end{bmatrix} \tag{23}$$

Proof: From The Bounded Real Lemma, $K \in \mathcal{R}^{m \times n_s}$ is the one degree of freedom static state feedback controller in Figure 2 if and only if the LMI

$$\begin{bmatrix} A_{cl}^T X_{cl} + X_{cl} A_{cl} & X_{cl} B_{cl} & C_{cl}^T \\ B_{cl}^T X_{cl} & -\gamma I & D_{cl}^T \\ C_{cl} & D_{cl} & -\gamma I \end{bmatrix} < 0 \tag{24}$$

holds for some $X_{cl} > 0$ in $\mathcal{R}^{(n_s+n_m)\times(n_s+n_m)}$. Using the expressions A_{cl}, B_{cl}, C_{cl} and D_{cl} in (13), (14), (15) and (16), this LMI can also be written follows:

$$H_{X_{cl}} + P_{X_{cl}}^T K Q + Q^T K^T P_{X_{cl}} < 0 \tag{25}$$

where

$$H_{X_{cl}} = \begin{bmatrix} \underline{A}^T X_{cl} + X_{cl} \underline{A} & X_{cl} B_1 & C_1^T \\ B_1^T X_{cl} & -\gamma I_m & D_1^T \\ C_1 & D_1 & -\gamma I_p \end{bmatrix} \tag{26}$$

$$Q = \begin{bmatrix} C_2 & 0_{n_s \times m} & 0_{n_s \times p} \end{bmatrix} \tag{27}$$

$$P_{X_{cl}} = \begin{bmatrix} B_2^T X_{cl} & 0_m & D_2^T \end{bmatrix} \tag{28}$$

We can use Lemma 3.2 to eliminate the matrix K in (25). Therefore, (25) holds for some K if and only if

$$N_{P_{X_{cl}}}^T H_{X_{cl}} N_{P_{X_{cl}}} < 0 \quad and \quad N_Q^T H_{X_{cl}} N_Q < 0 \tag{29}$$

where

$$ImN_{P_{X_{cl}}} = KerP_{X_{cl}} \tag{30}$$

$$ImN_Q = KerQ \tag{31}$$

$$X_{cl} > 0 \tag{32}$$

Then, the first inequality in (29) can be rewritten as $N_P^T T_{X_{cl}} N_P$ where the matrix N_P denotes any basis of $KerP$ and

$$P = \begin{bmatrix} B_2^T & 0_m & D_2^T \end{bmatrix} \tag{33}$$

We can take as

$$P_{X_{cl}} = P \begin{bmatrix} X_{cl} & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_p \end{bmatrix} \tag{34}$$

hence

$$N_{P_{X_{cl}}} = \begin{bmatrix} X_{cl}^{-1} & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_p \end{bmatrix} N_P \tag{35}$$

Consequently $N_{P_{X_{cl}}}^T H_{X_{cl}} N_{P_{X_{cl}}} < 0$ is equivalent to

$$N_P^T \left\{ \begin{bmatrix} X_{cl}^{-1} & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_p \end{bmatrix} H_{X_{cl}} \begin{bmatrix} X_{cl}^{-1} & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_p \end{bmatrix} \right\} N_P = N_P^T T_{X_{cl}} N_P < 0 \tag{36}$$

where

$$T_{X_{cl}} = \begin{bmatrix} \underline{A} X_{cl}^{-1} + X_{cl}^{-1} \underline{A}^T & B_1 & X_{cl}^{-1} C_1^T \\ B_1^T & -\gamma I_m & D_1^T \\ C_1 X_{cl}^{-1} & D_1 & -\gamma I_p \end{bmatrix} \tag{37}$$

Meanwhile, from (33) follows that bases of $KerP$ are

$$N_P = \begin{bmatrix} V_1 & 0 \\ 0 & I_m \\ V_2 & 0 \end{bmatrix} \tag{38}$$

where

$$N_c = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{39}$$

is any basis of the null space of $\begin{bmatrix} B_2^T & D_2^T \end{bmatrix}$. So $N_P^T T_{X_{cl}} N_P < 0$ can be reduced to

$$\begin{bmatrix} V_1 & 0 \\ 0 & I_m \\ V_2 & 0 \end{bmatrix}^T \begin{bmatrix} \underline{A}X_{cl}^{-1} + X_{cl}^{-1}\underline{A}^T & B_1 & X_{cl}^{-1}C_1^T \\ B_1^T & -\gamma I_m & D_1^T \\ C_1 X_{cl}^{-1} & D_1 & -\gamma I_p \end{bmatrix} \begin{bmatrix} V_1 & 0 \\ 0 & I_m \\ V_2 & 0 \end{bmatrix} < 0 \tag{40}$$

or equivalently

$$\begin{bmatrix} N_c & 0 \\ 0 & I_m \end{bmatrix}^T \begin{bmatrix} \underline{A}X_{cl}^{-1} + X_{cl}^{-1}\underline{A}^T & X_{cl}^{-1}C_1^T & B_1 \\ C_1 X_{cl}^{-1} & -\gamma I_p & D_1 \\ B_1^T & D_1^T & -\gamma I_m \end{bmatrix} \begin{bmatrix} N_c & 0 \\ 0 & I_m \end{bmatrix} < 0 \tag{41}$$

Similarly, the condition $N_Q^T H_{X_{cl}} N_Q < 0$ is equivalent to

$$\begin{bmatrix} N_o & 0 \\ 0 & I_p \end{bmatrix}^T \begin{bmatrix} \underline{A}^T X_{cl} + X_{cl}\underline{A} & X_{cl}B_1 & C_1^T \\ B_1^T X_{cl} & -\gamma I_m & D_1^T \\ C_1 & D_1 & -\gamma I_p \end{bmatrix} \begin{bmatrix} N_o & 0 \\ 0 & I_p \end{bmatrix} < 0 \tag{42}$$

where

$$Im N_o = Ker \begin{bmatrix} C_2 & 0_{n_s \times m} \end{bmatrix} \tag{43}$$

Hence X_{cl} satisfies (25) if and only if X_{cl} satisfies (41) and (42). To complete the proof, it is sufficient to use (7), (8) and (9) into (42)

$$Im N_o = Ker \begin{bmatrix} C_2 & 0_{n_s \times m} \end{bmatrix} = Ker \begin{bmatrix} I_{n_s} & 0_{n_s \times n_m} & 0_{n_s \times m} \end{bmatrix} \tag{44}$$

and

$$N_o = \begin{bmatrix} 0_{n_s \times n_m} & 0 \\ I_{n_m} & 0 \\ 0 & I_m \end{bmatrix} \tag{45}$$

Therefore, the following inequalities can be derived

$$\begin{bmatrix} 0_{n_s \times n_m} & 0 & 0 \\ I_{n_m} & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_p \end{bmatrix}^T \begin{bmatrix} \begin{pmatrix} A & 0 \\ 0 & F \end{pmatrix}^T X_{cl} + X_{cl}\begin{pmatrix} A & 0 \\ 0 & F \end{pmatrix} & X_{cl}\begin{pmatrix} B \\ G \end{pmatrix} & \begin{pmatrix} -C^T \\ H^T \end{pmatrix} \\ \begin{pmatrix} B^T & G^T \end{pmatrix} X_{cl} & -\gamma I_m & (J-D)^T \\ \begin{pmatrix} -C & H \end{pmatrix} & J-D & -\gamma I_p \end{bmatrix} \begin{bmatrix} 0_{n_s \times n_m} & 0 & 0 \\ I_{n_m} & 0 & 0 \\ 0 & I_m & 0 \\ 0 & 0 & I_p \end{bmatrix} < 0 \tag{46}$$

and then

$$\begin{bmatrix} F^T X_3 + X_3 F & X_2^T B + X_3 G & H^T \\ B^T X_2 + G^T X_3 & -\gamma I_m & (J-D)^T \\ H & J-D & -\gamma I_p \end{bmatrix} < 0 \tag{47}$$

Finally, the condition (22) can easily be derived when (7), (8) and (9) are used in (41). □

2. Controller Construction

Although Theorem 1.3 is about the solvability conditions of the continuous-time $\mathcal{H}_\infty$ model matching problem with a one degree of freedom static state feedback, it also provides a construction procedure. Moreover, The MATLAB LMI Control Toolbox [12] should be used to solve LMIs:

Step 1: Find a solution $X_{cl} > 0$ to the LMIs (21) and (22) for γ_{opt} by decreasing γ.

Step 2: Find the one degree of freedom static state feedback control law $K \in \mathcal{R}^{m \times n_s}$ in (25).

3. Numerical Example

Consider the second-order unstable system

$$T(s) = \frac{s+1}{(s-1)(s+0.5)} \tag{48}$$

The model system is taken as

$$T_m(s) = \frac{1}{s+1} \tag{49}$$

The state-space equations of $T(s)$ are obtained as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v(t) \tag{50}$$

$$y_s(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{51}$$

The state-space equations of $T_m(s)$ are obtained as

$$\dot{q}(t) = -q(t) + w(t) \tag{52}$$

$$y_m(t) = q(t) \tag{53}$$

The state-space equations of $P(s)$ in Figure 2 can be given as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0.5 & 0.5 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u(t) \tag{54}$$

$$z(t) = \begin{bmatrix} -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} \tag{55}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ q(t) \end{bmatrix} \tag{56}$$

When we search for a controller, γ_{opt}, the matrix X_{cl} and the one degree of freedom static state feedback controller are obtained as follows:

$$\gamma_{opt} = 0.67 \tag{57}$$

$$X_{cl} = \begin{bmatrix} 1.6087 & 0.7752 & -0.9248 \\ 0.7752 & 1.0993 & -0.6875 \\ -0.9248 & -0.6875 & 1.8633 \end{bmatrix} \tag{58}$$

$$K = \begin{bmatrix} -1.9875 & -2.3619 \end{bmatrix} \tag{59}$$

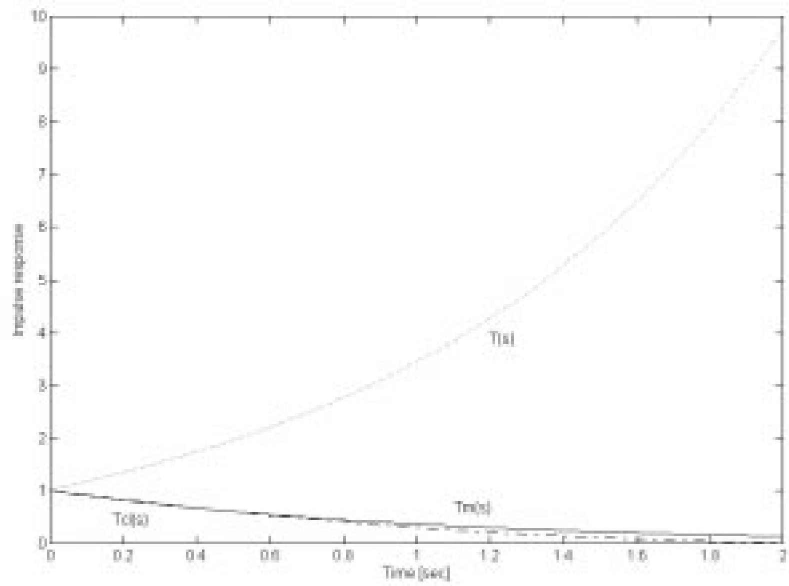

Figure 3

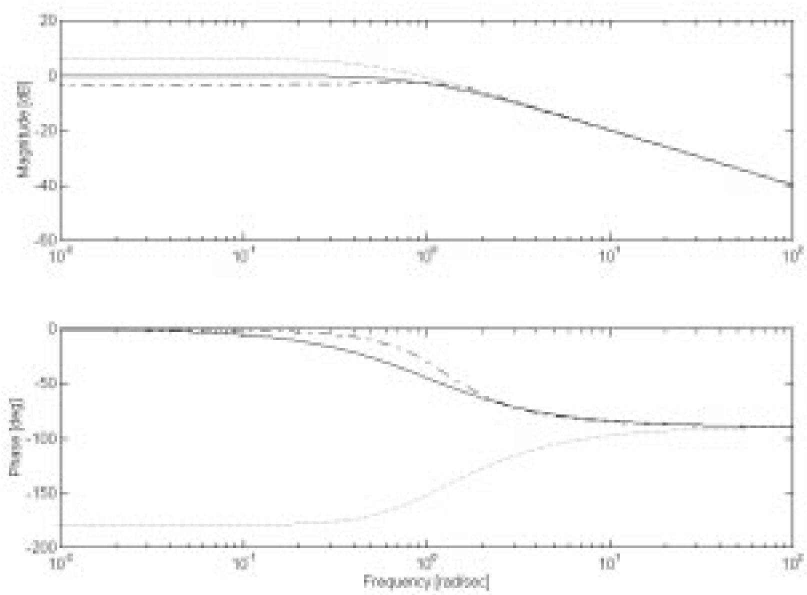

Figure 4 $T(s)$: ..., $T_m(s) : - - -$ and $T_{cl}(s) : -.-$

$T_{cl}(s)$ is the transfer matrix of the controlled system, i.e. T(s) with the one degree of freedom static state feedback controller. Figure 3 illustrates the impulse responses of $T(s)$, $T_m(s)$ and $T_{cl}(s)$. In Figure 4, the Bode diagrams of $T(s)$, $T_m(s)$ and $T_{cl}(s)$ are shown. They are well matched over γ_{opt}. As the figures indicate, the controlled system follows the dynamics of the target system: The magnitude and phase diagrams are matched to the stable frequence characteristics of the model system $T_m(s)$.

4. Conclusions

In this study, the continuous-time $\mathcal{H}_\infty$ model matching problem with the one degree of freedom static state feedback is investigated and an LMI-based solution of the problem is given. The existence conditions are formulated in Theorem 1.3 and a construction procedure is proposed in Section 4. In Section 5, there is a numerical example.

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