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# New Approaches for On-line Tuning of the Linear Sliding Surface Slope in Sliding Mode Controllers

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**Abstract**

*The main objective of this study is to propose new approaches for on-line tuning of the linear sliding surface slope in sliding mode controllers. The new approaches are developed for a class of second order systems on new coordinate axes, one of which is the classical sliding surface and the other one is naturally chosen to be orthogonal to it. The control input of the sliding mode control law is then modified accordingly by applying the Lyapunov stability condition. The adjustment of the linear sliding surface slope defined in the new coordinate axes is achieved by tuning a new parameter using different methods. First, an adaptive sliding surface with a rotation scheme is constructed by interpreting the classical delta neighbourhood approach. Next, the rotation process is achieved by using a fuzzy tuning mechanism that uses the new coordinates as its input variables and generates an incremental change in the new parameter value as an output. Numerical simulations are performed on a second order nonlinear system model with parameter uncertainties and bounded external disturbance. The two newly proposed on-line tuning approaches are then compared with the rotation mechanism that uses an empirical function defined on the same new coordinate axes. Moreover, the new approaches are compared with classical sliding mode controllers having a constant sliding surface and the delta neighbourhood approach developed in the classical coordinate axes. Results have shown improved performances of the proposed approaches in terms of a decrease in the reaching and settling times and robustness to disturbances as compared with the classical sliding mode controller.*

**Key Words:** *Sliding mode control, sliding surface design, time-varying sliding surface*

## 1. Introduction

Variable structure systems (VSS) theory was first proposed in the early 1950s and has been extensively developed since then with the invention of high speed control devices. It provides a systematic solution to the problem of maintaining stability and consistent performance in the face of bounded disturbances. The most popular operation regime associated with VSS is known as sliding mode control, which is a nonlinear deterministic control with a high speed, nonlinear feedback that switches discontinuously in time on a prescribed sliding surface [1]. Many practical applications of sliding mode control have been reported in the control literature such as flight control, robotic manipulators and servo systems. The reason for this popularity is the attractive properties of sliding mode control; it is robust to external disturbances and parameter variations [2] and also it provides a fast error convergence characteristic for nonlinear systems by emulating a prescribed reduced order system [3,4].

In general, the design of a sliding mode controller (SMC) involves the determination of a sliding surface that represents the desired stable dynamics, the description of a control law that guarantees the reaching condition and sliding condition. The phase trajectory of an SMC can be investigated in two parts representing the two modes of the system [5]. The trajectories starting from a given initial condition off the sliding surface tend towards the sliding surface. This is known as the reaching or hitting phase and the system is sensitive to parameter variations in this part of the phase trajectory. When the convergence to the sliding surface occurs, the sliding phase starts. The trajectories are insensitive to parameter variations and disturbances in this phase [6]. Therefore, various methods have been suggested to eliminate or lessen the system sensitivity by minimizing or even removing the reaching phase [7]. The design problem in systems with discontinuous control laws can usually be reduced to selection of the parameters of the sliding surfaces in the state space for the control function to have discontinuities [8]. Obviously the distance between the system error states and the sliding surface can be decreased by using a time-varying sliding surface. Decreasing this distance may cause the discontinuous control law to become active and initiate sliding mode with high frequency chattering. Thus, most of the recent adaptive strategies presented in the literature to improve SMC performance are concerned with sliding surface design.

A successful sliding surface design method for improving SMC performance is to use time-varying linear sliding surfaces instead of constant surfaces of a classical SMC. The linear sliding surface can be moved by rotating or shifting in the state space in such a direction that the tracking behaviour can be improved.

A time-varying linear sliding surface in the state space was first introduced in [8] for a multi-input case to maintain the sliding mode during a tracking control by deriving the control laws for a nested chain of linear sliding surfaces on the assumption that the trajectory actually lies in the intersection of all preceding sliding surfaces. An interesting idea is proposed in [9,10], where a translation and rotation scheme is defined for second order systems with a delta-neighbourhood algorithm and the existence of sliding mode with the related linear time-varying sliding surface approach is proved. The time-varying linear sliding surface idea is then generalized in [11,12] for high order systems.

Fuzzy logic control and sliding mode control have been combined in a variety of ways for sliding surface design [13]. These approaches can be classified into two categories. The first approach taken by many researchers is to use fuzzy logic control for the determination of the movement of the sliding surface of a classical SMC. Inspired by the work given in [9], a fuzzy tuning approach to SMC design with a rotating and shifting scheme is proposed in [14]. In [15], a Takagi-Sugeno type fuzzy tuning algorithm is used for the movement of the sliding surface. In [16], two fuzzy approximators are employed. The movement of the sliding surface is managed by the first fuzzy approximator and the second fuzzy approximator regulates the behaviour of the states in the reaching phase. The second kind of approach is to directly fuzzify a sliding surface and is called fuzzy sliding mode controller. An example is given in [17], where the fuzzy rules are defined based on the fuzzified value of the sliding surface using a one-dimensional rule-base.

The rotation scheme proposed in [9,10] is considered in [18] and the moving sliding surface is obtained by using a time dependent function for defining the coefficient that determines the position of the sliding surface of a classical SMC. In addition, in [19] this coefficient is increased continuously as a function of system output where the coefficient is initially kept small to minimize the reaching phase duration. An alternative design method is proposed in [20], where a new sliding surface equation with a time-varying property is obtained. The proposed approach in [20] is developed on a new coordinate axes, one of which is the classical sliding surface and the other axis is naturally chosen to be orthogonal to it.

In the present study, the time-varying sliding surface design for a class of second order systems is undertaken by using the new coordinate axes proposed in [20]. First of all, the delta-neighbourhood algorithm in [9] is adapted to the sliding surface proposed in [20]. Then two-dimensional fuzzy rule tables are generated for the two stable regions of the state space to rotate the sliding surface in the new coordinate axes. Numerical simulations are performed on a second order nonlinear system model with parameter uncertainties and bounded external disturbance. The proposed rotation approaches are then compared with the mechanism using an empirical function based on the same new coordinate axes given in [20]. Moreover, the approaches based on the new coordinate axes are compared with sliding mode controllers having a constant sliding surface and a rotating sliding surface which is initially designed to pass arbitrary initial conditions and subsequently moves towards a predetermined sliding surface by rotating in discrete time using the delta neighbourhood approach [9].

## 2. Sliding Mode Controller

The SMC that has different structures on both sides of the sliding surface is a nonlinear controller for achieving the robust control characteristics [5]. The general state space expression of a typical second order non-autonomous nonlinear uncertain dynamic open-loop system with a single input can be described as [4]

$$\begin{aligned} &\dot{x}_1(t) = x_2(t) \\ &\dot{x}_2(t) = \sum_{i=1}^{q} (a_i + \Delta_i(t))\, f_i(\mathbf{x}, t) + b(\mathbf{x}, t)u(t) + d(t) \\ &x_1(t_0) = x_{10}, x_2(t_0) = x_{20} \end{aligned} \tag{1}$$

where $\mathbf{x}(t) = (x_1, x_2) \in \mathrm{R}^{2x1}$ is the state vector, $a_i$ $(i = 1...q)$ represent the constant parameters of the system whereas $\Delta_i(t)$ are the perturbations with bounded uncertainties, $u(t)$ is the input signal, $d(t)$ is a time-dependent disturbance with known upper bound and $f_i(\mathbf{x}, t)$ $(i = 1...q)$ and $b(\mathbf{x}, t)$ are functions determining the system characteristics. The control problem is to get $\mathbf{x}(t)$ to track a desired trajectory $\mathbf{x}_d(t) = (x_{d1}(t), x_{d2}(t))$. Since single input systems are considered, there is only one sliding surface $s(\mathbf{x}, t) = 0$ and for second order systems it can be defined as

$$s(\mathbf{x}, t) = e_2(t) + ce_1(t) \tag{2}$$

where $c$ is a strictly positive real number and the tracking error is defined as

$$\mathbf{e}(t) \hat{=} (e_1(t), e_2(t)) = (x_1(t) - x_{d1}(t), x_2(t) - x_{d2}(t)) \tag{3}$$

where $x_{di}$ is the desired trajectory of the i$^{th}$ state. Equation (2) gives a linear function in terms of error and the value of $c$ determines the slope of the sliding surface. Assuming that $\dot{e}_1 = e_2$, which can be obtained by taking $\dot{x}_{d1} = x_{d2}$, an homogeneous differential equation that has a unique solution $\mathbf{e} = 0$ could be obtained by setting $s = 0$. Thus, the error will asymptotically reach zero with an appropriate control law that could keep the trajectory on the sliding surface. Lyapunov's direct method could be used to obtain the control law that would maintain this goal and a candidate function is defined as [18]

$$V(s) = \frac{1}{2}s^2 \tag{4}$$

with $V(0) = 0$ and $V(s) > 0$ for $\forall s \neq 0$. It is aimed that the derivative of the Lyapunov function is negative definite. An efficient condition for the stability of the system in (1) can be satisfied if one can assure that [5]

$$\dot{V} = \frac{1}{2}\frac{d}{dt}s^2 \leq -\eta|s| \tag{5}$$

where $\eta$ is a strictly positive design scalar. Obtaining the inequality in (5) means that the system is stable and controlled in such a way that the system states always move towards the sliding surface and hits it. Therefore, (5) is called the reaching condition for the sliding surface. By substituting (2) into the reaching condition given in (5)

$$s.\dot{s} = s.\left(\sum_{i=1}^{q}(a_i + \Delta_i)\,f_i + bu + d - \dot{x}_{d2} + c\,(x_2 - \dot{x}_{d1})\right) \leq -\eta|s| \tag{6}$$

is obtained where the sign(.) denotes the signum function defined as

$$sign(x) = \begin{cases} -1 & if x < 0 \\ 0 & if x = 0 \\ 1 & if x > 0 \end{cases} \tag{7}$$

To formulate a SMC law, the uncertainties are assumed to be bounded such as

$$\Delta^- \leq \Delta_i(t) \leq \Delta^+ \tag{8a}$$

$$\gamma^- \leq d(t) \leq \gamma^+ \tag{8b}$$

Therefore, by inserting the perturbations in (8a) affecting the system parameters into the discontinuous control gain using the scaled relay structure [4], the control input satisfying the reaching condition can be chosen as

$$u = \underbrace{\left(-\sum_{i=1}^{q}a_i f_i + \dot{x}_{d2} - c\,(x_2 - \dot{x}_{d1})\right)/b}_{u_{eq}} - \underbrace{\left(k + \sum_{i=1}^{q}\left|\bar{\Delta}_i f_i\right|\right) sign\,(s)\,/b}_{u_{dis}} \tag{9}$$

where a conservative choice for $\bar{\Delta}$ is taken as [4]

$$\bar{\Delta} = \max\{\left|\Delta^-\right|, \left|\Delta^+\right|\} \tag{10}$$

Considering the external disturbances, the lower bound of $k$ can be written as

$$k > \eta + \max\left(|\gamma -|, \left|\gamma^{+}\right|\right). \tag{11}$$

and $k + \sum_{i=1}^{q} \left|\bar{\Delta}_i f_i\right|$ is the gain of the discontinuous control law, which is a strictly positive real function with a lower bound depending on the estimated system parameters and external disturbances. The control input in (10) consists of two parts. The first part, $u_{eq}$, is the continuous term that is known as equivalent control based on estimated system parameters and it compensates the estimated undesirable dynamics of the system. The second term with the signum function is the discontinuous control law, $u_{dis}$, which requires infinite switching on the part of the control signal and actuator at the intersection of error state trajectory and sliding surface. In this way, the trajectory is forced to always move towards the sliding surface [4].

# 3. The Proposed on-line Tuning Approaches

## 3.1. Illustration of the new coordinate axes

The original idea given in [20] is to transfer the classical sliding surface to a new plane within the classical $(e_1 - e_2)$ phase plane. This new plane is generated as follows:

i) one of the coordinates is taken as the original sliding surface $s$.

ii) the other coordinate is normally a perpendicular line to $s$ and it is given as

$$p = -\left(\frac{1}{c}\right) e_1(t) + e_2(t) \tag{12}$$

The new linear sliding surface can then be defined using (12) in the new $(s - p)$ plane as

$$\hat{s} = s - k_s.p \tag{13}$$

The proposed coordinate system provides valuable information about the distance to the conventional sliding surface. This information can be used in designing algorithms for on-line tuning of the major rotation parameter $k_{s.}$ It can also be used in designing non-linear sliding surfaces as it done in [21]. Moreover, it provides a compact and simple representation and formulation of the sliding mode equations

Assuming that $\dot{e}_1 = e_2$ by taking $\dot{x}_{d1} = x_{d2}$, it is straightforward to see solutions that make $e(t) = 0$ from the solution set of the differential equation given in (13) for $\hat{s} = 0$; that is,

$$\frac{c + {k_s}/{c}}{1 - k_s} > 0 \tag{14}$$

Therefore, (13) is a stable sliding surface if the inequality given in (14) is valid.

The controller is sensitive to uncertainties and disturbances acting on the system during the reaching phase. In addition, the trajectory of the system along the sliding surface may be inadequate for the desired performance requirements. Therefore, the sliding surface given in (13) should be adjusted to improve the system performance. The rotation of the sliding surface by adjusting the new parameter $k_s$ can be defined by different methods.

## 3.2. Adjustment of the new sliding surface

The adjustment of the linear sliding surface defined in (13) is achieved by basically adjusting the value of $k_s$ which gives the position of the new sliding surface. For negative values of $k_s$, the surface is over the classical sliding line given in (2) and for positive values it rotates to the opposite side. This property can be used to change the control law and to affect the system states. In order to obtain the control law given in (9), the value of the sliding surface is calculated using (13) instead of the classical linear constant surface function.

Mathematically, the allowable region of $k_s$ to obtain the inequality in (13), in terms of $c$ is

$$-c^2 < k_s < 1 \tag{15}$$

Obviously this includes the stable region of the error phase plane as shown in Fig.1. However, the upper limit in (15) cannot be reached in physical terms. Because, in mechanical systems the upper bound is generally determined by the frequency of the lowest unmodeled structural mode, the largest unmodeled time delay and the sampling rate [18]. Small values of $c$ will cause longer tracking times, and therefore the lower bound is related to the allowable tracking time. Therefore, the upper limit of $k_s$ is bounded by the mechanical properties of the related system. The rotation of the sliding surface with the change in the $k_s$ value is given in Figure 1. To rotate the sliding surface of a classical SMC in the same range obtained by (15), the sliding surface slope value changes between $[0;+\infty]$ in the same stable region. Therefore, the maximum allowable region of parameter $c$ may become large as it rotates clockwise.

The control law for the new sliding surface can be obtained by substituting (13) into the Lyapunov equation given in (4) and this can be rewritten as

$$V(\hat{s}) = \frac{1}{2}\hat{s}^2 > 0 \tag{16}$$

with $V(0) = 0$ and $V(\hat{s}) > 0$ for $\forall \hat{s} \neq 0$. By taking the derivative of (16) the reaching condition (5) with the new sliding surface can be obtained as

$$\dot{V}(\hat{s}) = \frac{1}{2}\frac{d}{dt}\hat{s}^2 \leq -\mu|\hat{s}| \tag{17}$$

From (17), a new control law providing the stability conditions can be chosen as

$$\begin{aligned} u(t) = & \left(-\sum_{i=1}^{q} a_i f_i + \dot{x}_{d2} - \frac{\left(c - \dot{k}_s + {}^{k_s}\!/_{c}\right)}{(1-k_s)} e_2 - \frac{\dot{k}_s}{c.(1-k_s)} e_1\right) /b \\ & - \left(\hat{k} + \sum_{i=1}^{q} \left|\bar{\Delta}_i f_i\right| + \right) sign(\hat{s})/b \hat{=} u_{eq} + u_{dis} \end{aligned} \tag{18}$$

The gain of the new discontinuous control law in (18) is

$$\hat{k} > \frac{\eta}{1-k_s} + \max\left(|\gamma-|, \left|\gamma^+\right|\right). \tag{19}$$

The parameter $k_s$ is a time-varying differentiable function. When $k_s$ is close to one, the discontinuous control gain will have a large amplitude. In order to hold this gain value similar to the gain in (9) and not to affect the system performance with the change in the discontinuous control gain, the strictly positive parameter $k$ in (18) is chosen as a time-varying function by taking

$$\eta = \hat{\eta}(1 - k_s), \forall t \tag{20}$$

where $\hat{\eta}$ is chosen as a strictly positive constant and $k_s < 1$. Therefore, $\eta$ is also strictly positive and by choosing a suitable $\hat{\eta}$ value, the stability condition in (17) is preserved. Thus, a linear rotating sliding surface could be obtained by adjusting the new parameter value $k_s$ using different methods. In [20], a shifted sigmoid function is chosen to adjust the value of $k_s$ which can be written as

$$k_s = \frac{k_s^+ - k_s^-}{1 + \exp(-bt + a)} + k_s^- \tag{21}$$

where $b$ and $a$ are parameters determining the shape of the function.This is an empirical relation. The maximum and minimum allowable values of $k_s$ that determine the region in which the sliding surface is adjusted are defined as $k_s^+$ and $k_s^-$, respectively. For different rotation directions, the parameters of the sigmoid function are chosen as

$0 > k_s^- \geq \frac{s(0)}{p(0)}, \quad k_s^+ = 0$ and $b < 0$, if the rotation is clockwise,

$k_s^- = 0, \quad 0 < k_s^+ \leq \frac{s(0)}{p(0)}$ and $b > 0$, if the rotation is counter-clockwise.

Desired rotation of the sliding surface is obtained by choosing suitable values for the parameters $k_s^-$, $k_s^+$, $a$ and $b$. With the help of $k_s$, the rotation is carried out from the surface defined by the initial conditions to a predetermined slope value $c$.

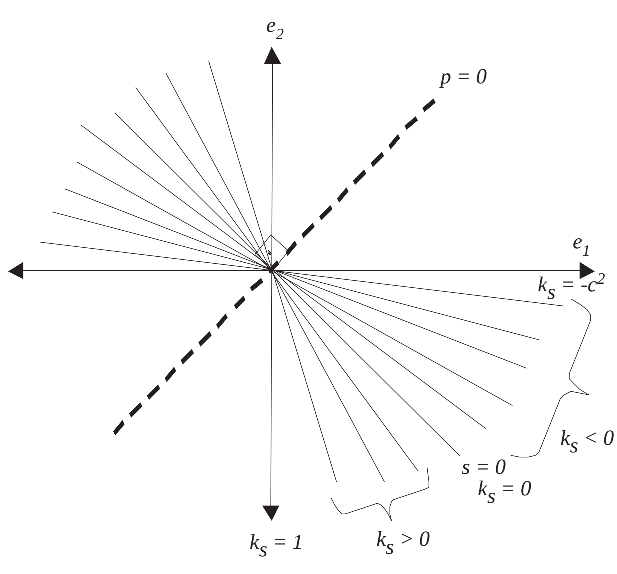



**Figure 1.** Time-varying sliding surfaces obtained with different values of $k_s$.

### 3.2.1. Delta neighbourhood approach

In the present study, two newly proposed approaches are given for adjusting the $k_s$ parameter. One alternative is to use the delta neighbourhood approach proposed in [9]. The basic philosophy of the rotation mechanism using this approach is that the sliding surface is initially chosen to pass arbitrary initial conditions and then is subsequently moved towards the predetermined sliding surface.

The rotation in [9] can be obtained by applying the delta neighbourhood approach to adjust the $k_s$ parameter, which, in return, changes the value of the sliding surface slope. This rotation algorithm using the delta neighbourhood approach for the proposed sliding surface may be adapted as follows:

**Step 1.** Determine an appropriate constant $d_r$ to adjust $k_s$ and define $d_{fr} = d_f + d_r$ where $d_f$ denotes the vicinity magnitude of the $k_s$ parameter.

**Step 2.** Calculate the initial value of $k_s(t_0) = s(t_0)/p(t_0)$.

**Step 3.** Determine the rotating direction from the sign of $k_s(t_0)$. As shown in Figure 1, if $k_s(t_0) < 0$, the initial conditions are over the sliding surface, therefore rotate clockwise (CW), if $k_s(t_0) > 0$ rotate counter-clockwise (CCW).

**Step 4.** Determine the new value of $k_s$ by solving the equation $|s(t_0)\text{-}k_s(t_1)p(t_0)| = d_{fr}$. The larger of the two solutions of $k_s(t_1)$ is chosen as the new $k_s$ value for CW rotation and the other for CCW rotation.

**Step 5.** If $k_s \geq 0$ for CW rotation or $k_s \leq 0$ for CCW rotation then fix $k_s = 0$.

The rotation is meaningful in the stable regions of the error phase plane. A shifting procedure is proposed in [9] for unstable regions. In this study, no adjustment is made in unstable regions by taking the $k_s$ parameter as zero. The algorithm given in this study is a new interpretation of the delta neighbourhood algorithm to the new sliding surface equation.

### 3.2.2. Fuzzy logic tuning approach

As mentioned in the introduction, a challenging approach in sliding surface design is to use fuzzy logic tuning. For this purpose, a fuzzy logic controller that uses $s, \hat{s}$ as input variables and generates $\Delta k_s$ as an output is proposed. Examining Figure 2, it can be seen that the signs of $s$, $\hat{s}$ and $p$ are sufficient to determine the region of the represantative point and information about the relative position of the representative point with respect to the classical and the proposed sliding surfaces. Thus, it is possible to designate linguistic rules to generate the variation of $k_s$ with this information. Two rule tables are generated as given in Table 1 for different values of $p$ that define in which region the representative point is. Since only the sign information of $p$ is evaluated, instead of using a rule table with three inputs, the rule table is separated into two regions for different signs of $p$. The input-output relationship of the rule table given in Table 1 is visualized in Figure 3. In the unstable regions, $k_s$ is taken as zero and no adjustment is made as in the previous approach.

The amount of rotation is also related to the magnitude of $k_s$, because the same $\Delta k_s$ will result different amount of rotation for different $k_s$ values. To overcome this problem, instead of directly summing $\Delta k_s$ with the previous value of $k_s$, the new value is obtained by using the $\Delta k_s$ output of the fuzzy tuning approach according to the following recursive equation:

$$k_s(t) = k_s(t-1).\,(1 + sign(k_s(t-1).\Delta k_s) \tag{22}$$

In (22), as $\Delta k_s$ is multiplied by the magnitude of the previous value of $k_s$, the change in $k_s$ will become relatively small as $k_s$ approaches zero and this will improve the performance of the fuzzy tuning approach.

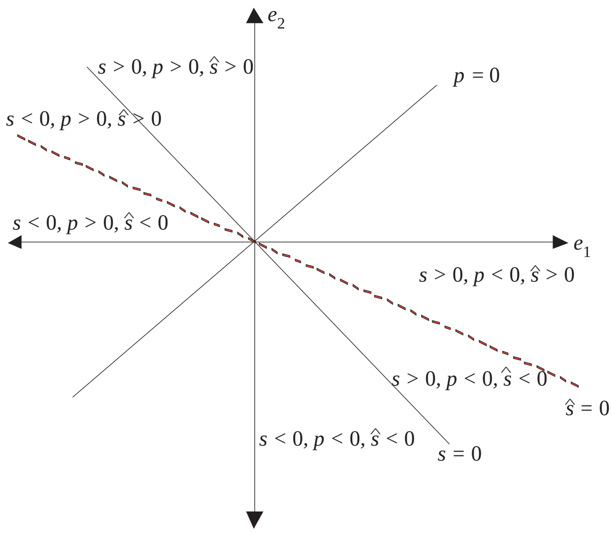



**Figure 2.** Control regions obtained by different signs of $s$, $\hat{s}$ and $p$.

**Table 1.** The rule tables used for the calculation of $\Delta k_s$ in the regions (a) $p < 0$, (b) $p > 0$.

$\hat{s}$

| | | **NB** | **NM** | **NS** | **ZE** | **PS** | **PM** | **PB** |
|---|---|---|---|---|---|---|---|---|
| | **NB** | ZE | ZE | ZE | NS | NS | NM | NB |
| | **NM** | PS | ZE | ZE | NS | NM | NM | NB |
| | **NS** | PM | PS | ZE | NS | NS | NM | NM |
| s | **ZE** | PM | PM | PS | ZE | ZE | NS | NM |
| | **PS** | PB | PM | PS | PS | ZE | NS | NM |
| | **PM** | PB | PM | PM | PS | ZE | ZE | NS |
| | **PB** | PB | PM | PM | PS | ZE | ZE | ZE |

(a)

$\hat{s}$

| | | **NB** | **NM** | **NS** | **ZE** | **PS** | **PM** | **PB** |
|---|---|---|---|---|---|---|---|---|
| | **NB** | ZE | ZE | ZE | PS | PM | PM | PB |
| | **NM** | NS | ZE | ZE | PS | PM | PM | PB |
| | **NS** | NM | NS | ZE | ZE | PS | PM | PB |
| s | **ZE** | NM | NS | ZE | ZE | PS | PM | PM |
| | **PS** | NM | NM | NS | NS | ZE | PS | PM |
| | **PM** | NB | NM | NM | NS | ZE | ZE | PS |
| | **PB** | NB | NM | NS | NS | ZE | ZE | ZE |

(b)

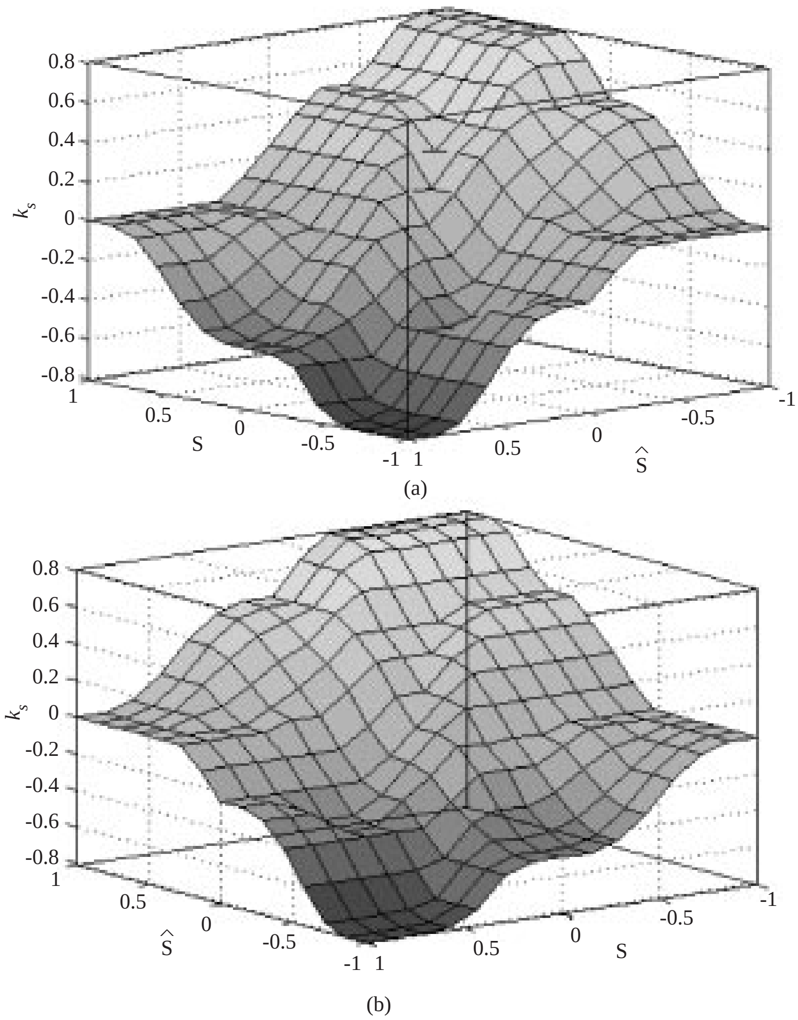



**Figure 3.** The input output relation of the fuzzy tuning approach in the regions with $a)$ $p < 0$ and $b)$ $p > 0$.

# 4. Simulation Results

The numerical simulations are performed on a second order model of a nonlinear spring damper system that is illustrated in Figure 4. The same model is also used in [9]. The state space representation of the spring damper system can be formulated as in the form of (1) with $q = 4$ by taking the system parameters as

$$\begin{aligned} & b = 1/m \\ & f_1 = x_1/m, \quad f_2 = x_1^3/m, \quad f_3 = x_2/m, \quad f_4 = x_2|x_2|/m \\ & a_1 = a_2 = -0.45, a_3 = a_4 = -0.25 \end{aligned} \tag{23}$$

where $m = 1$ kg is the mass of the damper. The parameter uncertainties and external disturbances are modelled as

$$\begin{aligned} & \Delta_1(t) = \Delta_2(t) = -0.25\sin(5\pi t) \\ & \Delta_3(t) = \Delta_4(t) = -0.15\sin(7\pi t) \\ & d(t) = 0.05 + 0.25\cos(3\pi t) \end{aligned} \tag{24}$$

The performances of all the SMCs with different sliding surface adjustment mechanisms are compared simultaneously. These controllers are the SMC with a constant sliding surface (SMC-Classical), the SMC with a rotating sliding surface using the delta neighbourhood method of Choi *et al.* presented in [9] (SMC-Choi), the proposed SMC with a rotating sliding surface applying the delta neighbourhood of [9] to the proposed sliding surface in (13) (SMC-Delta), the SMC with the fuzzy tuning approach (SMC-Fuzzy) and the SMC with the rotating sliding surface using a sigmoid function (SMC-Empirical) that is proposed in [20].

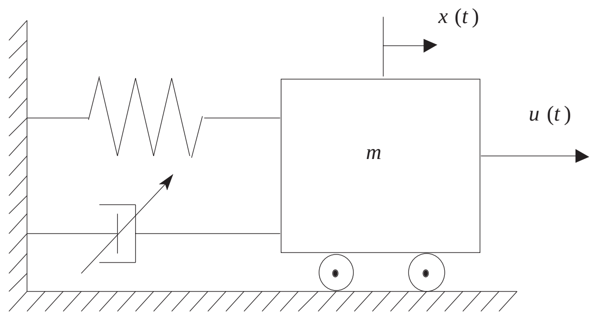



**Figure 4.** Spring-damper system.

The desired state trajectory is chosen as

$$\begin{aligned} x_{d1}(t) &= -0.5\cos(\pi t/5) \\ x_{d2}(t) &= \dot{x}_{d1}(t) = 0.1\pi\sin(\pi t/5) \end{aligned} \tag{25}$$

and all of the simulations are performed within the time interval of [0;10]. The control law given in (18) is used with (20) and the controller parameters for this case are taken as $\hat{k} = 0.5$ and $c = 7$. The input scaling factors for the $s$, $\hat{s}$ inputs and output scaling factor for $\Delta k_s$ output of the fuzzy logic controller in SMC-Fuzzy are chosen as 0.25, 4.7 and 0.125, respectively. The parameters for SMC-Choi defined in [8] are $d_f = 0.002$ and $d_r = 0.003$. and the parameters for SMC-Delta are chosen as $d_f = 0.002$ and $d_r = 0.03$. The sampling time is chosen to be T = 0.001 s for all the discretely rotating approaches, namely, SMC-Choi, SMC-Fuzzy and SMC-Delta. The sigmoid parameters for SMC-Empirical are taken as $k_s^- =$ s(0)/d(0) = -49, $k_s^+ = 0$, $a = 4$ and $b = 0$. The initial conditions for the states are $(x_1(0), x_2(0)) =$ (0,1) and the final conditions for the states are taken to be the origin. The control law for the SMC-Classical and SMC-Choi can be directly calculated from (18) by taking $k_s = \dot{k}_s = 0$.

The transient response of the system state $x_1(t)$ for the given initial conditions is shown in Figure 5; it can be seen that the settling times for the proposed approaches are all better than those for SMC-Classical. SMC-Delta has a response similar to that of SMC-Choi, which is an expected result as they both use the same method. The $d_r$ parameter of SMC-Delta is greater than that of SMC-Choi since the amount of change required in $k_s$ for SMC-Delta is larger than the change in $c$ of SMC-Choi. Figure 5 shows that SMC-Fuzzy has the minimum settling time and SMC-Empirical smoothly enters the desired trajectory as a result of the sigmoid structure used for adjusting $k_s$.

The control inputs are given in Figure 6. It is seen that the proposed approaches decrease the effect of disturbances by decreasing the reaching time. SMC-Fuzzy and SMC-Empirical have better reaching times than the other methods. The drastic chattering in the control inputs of all controllers during the sliding phase observed in Figure 6 may lead to high stress for the plant to be controlled. A well known method to handle this chattering problem is called the continuation method [22], in which the signum functions in the control inputs are replaced by the saturation functions.

The $(e_1\text{-}e_2)$ error phase plane trajectories of the related controllers are given in Figure 7. SMC-Fuzzy and SMC-Empirical are less effected by the external disturbances $d(t)$ since the discontinuous control laws of SMC-Fuzzy and SMC-Empirical become active much earlier than those of the other approaches. This can be observed in Figure 7 as the phase plane portraits of SMC-Classical, SMC-Choi and SMC-Delta have sinusoidal behaviour which directly reflects the effect of the sinusoidal disturbance $d(t)$ modelled as in (23). The phase plane trajectory is nonlinear and the states smoothly enter the sliding regime in all of the adaptive methods in reaching the sliding mode phase; whereas the sliding regime of SMC-Classical is linear.

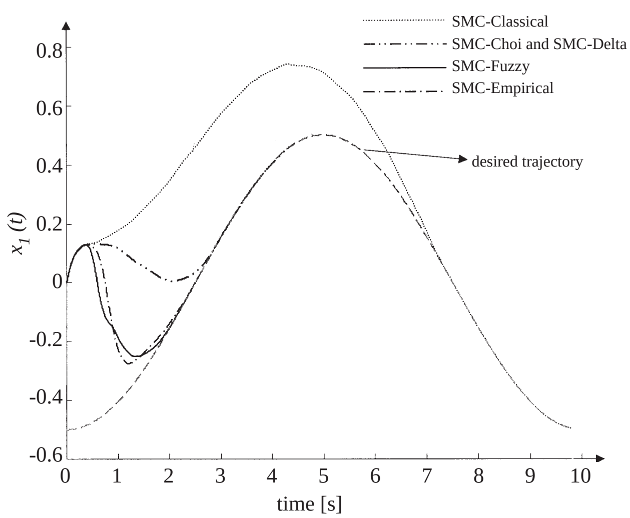
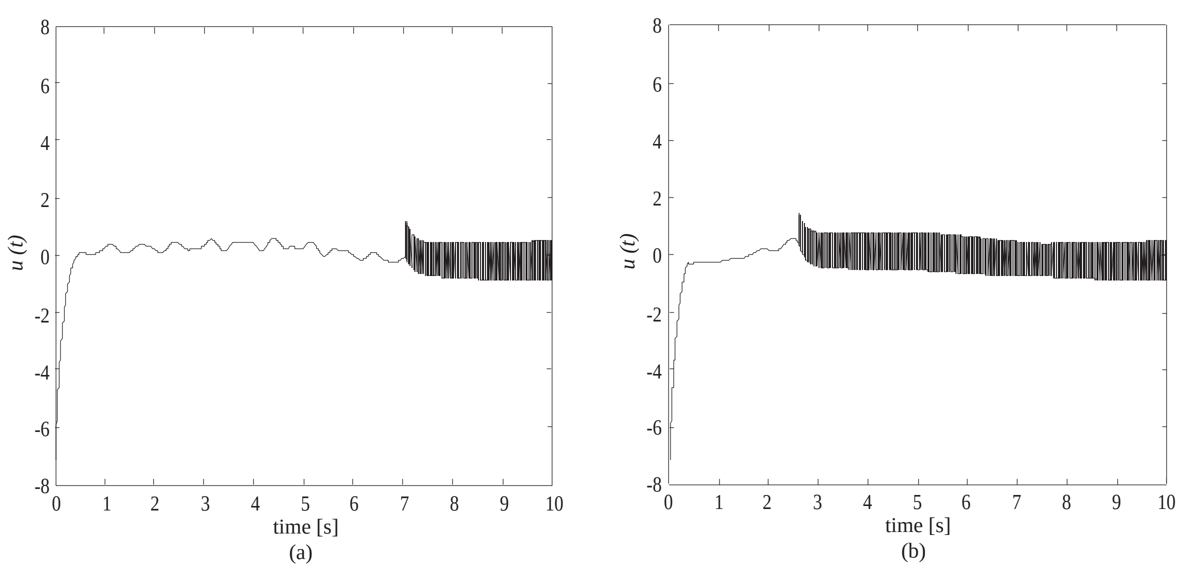



**Figure 5.** $x_1(t)$ state trajectories.



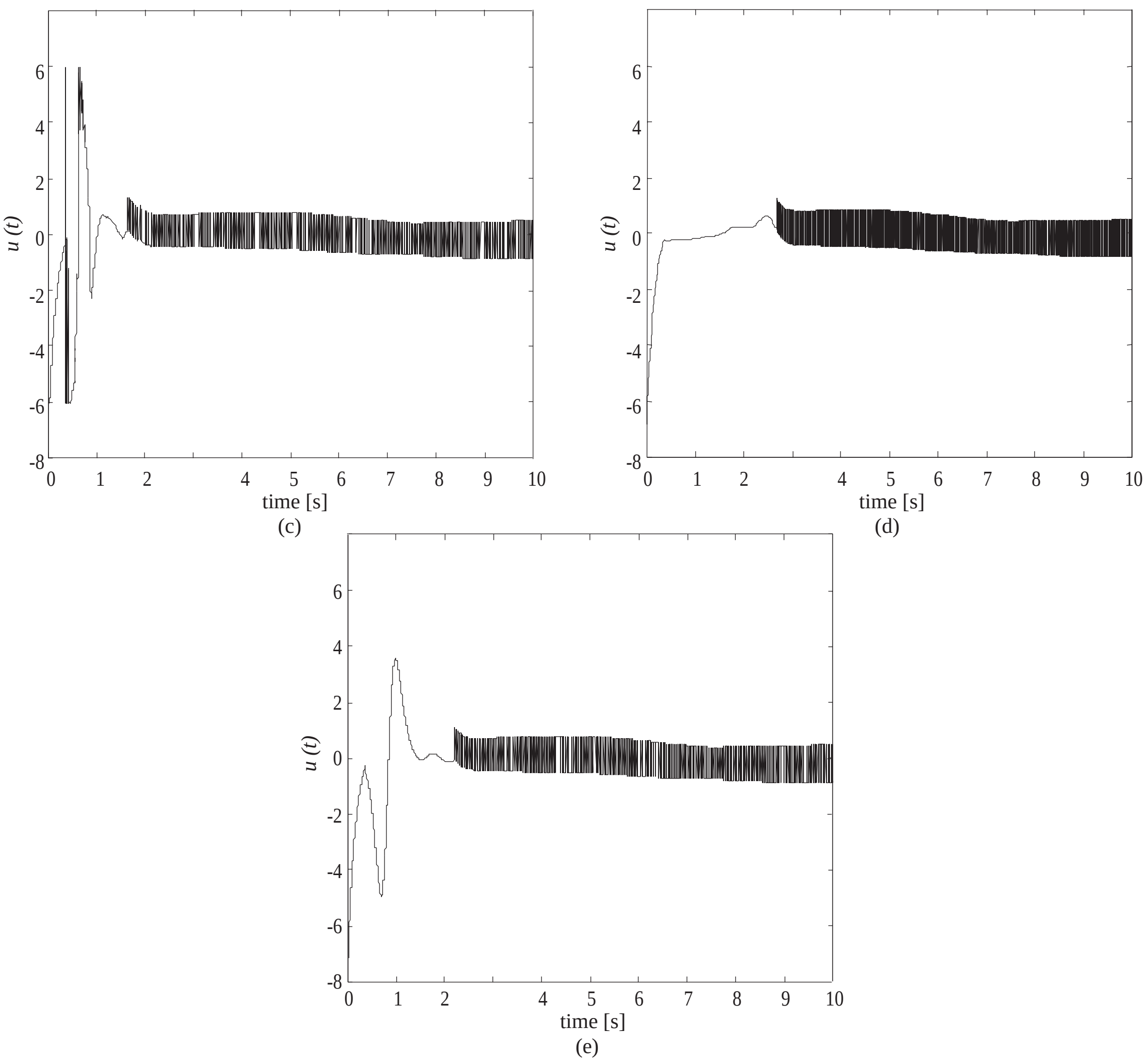



**Figure 6.** Control inputs for (a) SMC-Classical, (b) SMC-Choi, (c) SMC-Fuzzy, (d) SMC-Delta, and (e) SMC-Empirical.

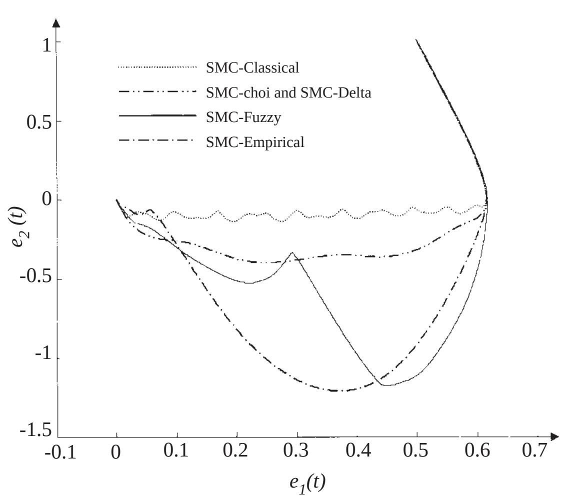



**Figure 7.** $(e_1$-$e_2)$ phase portraits of the control responses.

For a quantitative comparison of the controller performances, some performance measures such as integral absolute error (IAE), integral of time multiplied absolute error (ITAE) and reaching time ($t_{reach}$) are used. The two integral criteria IAE and ITAE are considered because using only visual observations of system response curves is not always sufficient for making a good comparison between different types of controllers [23]. IAE is a criterion which includes damping. If the IAE measure while following a disturbance is minimized, then it is known that offset is eliminated and damping is assured, otherwise IAE would continue to increase indefinitely with time. In ITAE the error term is multiplied by time and this penalizes long term errors [23]. Large errors contribute heavily to IAE; on the other hand, ITAE heavily penalizes errors that occur late in time. Thus, IAE and ITAE are viable measures that reflect the transient and steady-state characteristics of a control system, respectively [23]. The performance measures are given in Table 2. It is seen from the table that SMC-Fuzzy has the best performance and SMC-Empirical has values very close to those of SMC-Fuzzy. The similarity between SMC-Choi and SMC-Delta is very clear from the performance measures. The performance of all SMCs with adaptive sliding surfaces are improved with respect to SMC-Classical.

**Table 2.** Performance measures for the simulated results.

| | SMC-Classical | SMC-Choi | SMC-Delta | SMC-Fuzzy | SMC-Empirical |
|---|---|---|---|---|---|
| IAE($x_1$) | 2.450 | 0.980 | 0.981 | 0.543 | 0.576 |
| ITAE($x_1$) | 5.973 | 0.891 | 0.895 | 0.289 | 0.320 |
| $\boldsymbol{t_{reach}}$ | 7.108 | 2.679 | 2.701 | 1.837 | 2.171 |

## 5. Conclusion

In the present study, sliding mode controller design for uncertain dynamical second order systems is considered and two new approaches for on-line tuning of the linear sliding surface slope are proposed. The results have shown that both of the proposed approaches have improved the system transient response as compared to its counterparts by minimizing the settling time and lessening the effect of disturbances since the approaches have achieved a reasonable decrease in the reaching time. It is known that there is a trade-off between reaching time and settling time for a classical SMC; that is, when one of them is improved the other becomes worse. When a quantative comparison of the controller performances is considered, it is also observed that new approaches yield better results and, in fact, SMC-Fuzzy outperforms all the other approaches.

In this study, the parameters of the sigmoid function are set by the designer in an empirical function approach. A better performance for the proposed approach could have been obtained if a global optimization method such as genetic algorithms had been used in setting these parameters.

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