

An Algorithm for Image Clustering and Compression

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Abstract

This paper presents a new approach to image compression based on fuzzy clustering. This new approach includes pre-filtering, and fuzzy logic image enhancing to reduce undesirable noise effects on segmentation result; separation of image into 4x4 blocks and two dimensional discrete cosine transform; obtaining of peak values of cosine membership functions by combining of performing the zig-zag method with discrete cosine transform coefficients; obtaining of membership values and cluster centroids; and finally, creation of segmented image and compression. After applying the new method on sample images at different number of clusters, better compression ratio, performing time and good validity measure was observed. Possibility to reach incorrect results and local minima is also prevented for clustering by this new method.

Key Words: *Image segmentation, Image compression, Fuzzy clustering, Discrete cosine transform.*

1. Introduction

Importance of image compression increases with advancing communication technology. Limited hardware and budget is also important in sending of data fast. The amount of data associated with visual information is so large that its storage requires enormous storage capacity. The storage and transmission of such data require large capacity and bandwidth, which could be very expensive. Image data compression techniques are concerned with reduction of the number of bits required to store or transmit images without any appreciable loss of information. Image transmission applications are in broadcast television; remote sensing via satellite, aircraft, radar, or sonar; teleconferencing; computer communications; and facsimile transmission. Image storage is required most commonly for educational and business documents, medical images. Because of their wide range of applications, data compression is of great importance in digital image processing [1,2,3].

It is widely believed that a picture is worth more than a thousands words. However, dealing with digital images requires far more computer memory and transmission time than that needed for plain text. Image compression is a process intended to yield a compact representation of an image, hence reducing the image storage/transmission requirements. Generally, images carry three main type of information: redundant, irrelevant, and useful. Redundant information is the deterministic part of the information which can be reproduced, without loss, from other information contained in the image (i.e., interpixel redundancy) : for example, low-varying background information. Irrelevant information is the part of information that has enormous details which is beyond the limit of perceptual significance (i.e., psychovisual redundancy). Useful information is the part of information which is neither redundant nor irrelevant. Decompressed images are

usually observed by human beings. Therefore, their fidelities are subject to the capabilities and limitations of the human visual system (HSV). A significant property of the HVS is the fact that it recognizes images by their regions and not by intensity value of their pixels [4]. In addition to this property, when an observer looks to an image, trying to understand it, he or she searches for distinguishing features such as edges (not pixel values) and mentally combines them together into recognizable groupings. This HVS property (recognizing images by their regions) can be exploited by segmenting images into regions based on the amount of information each region conveys to the viewer. Then, regions of each category could be encoded using a distinct encoding procedure. This encoding procedure should preserve the main visual characteristics of this particular category (focusing on useful information) while reducing the existing correlation (reducing redundant information), and neglecting some of the irrelevant details (omitting irrelevant information) [5].

Clustering is useful in several exploratory pattern-analysis, grouping, decision-making, and machine-learning situations, including data mining, document retrieval, image segmentation, and pattern classification. In image segmentation coding techniques, image is segmented to different regions separated with contours, and coded with different coding techniques. Region growing, c-means, and split and merge methods are used generally for image segmentation. Beside of this crisp classical segmentation methods, the fuzzy logic segmentation methods were also seen very effective for coding [5,6,7,8,9].

In this study, a new image clustering and compression method based on fuzzy logic and discrete cosine transform (DBIC) was introduced for gray scale images together with pre-filter and image enhancing based on fuzzy logic. This method was applied to different sample images and high compression ratios and good validity measures were observed.

The rest of this paper is organized as follows. Section 2 reviews discrete cosine transform, Section 3 reviews the fuzzy clustering; Section 4 reviews validity measure; Section 5 proposes a new image clustering and compression algorithm; Section 6 presents experimental results; and finally, Section 7 gives the conclusions.

2. Discrete Cosine Transform

Discrete cosine transform (DCT) can be used at feature extraction, filtering, image compression and signal processing. Discrete cosine transform has been adopted by most emerging image and video compression standards including JPEG, H.261, MPEG, and the recent HDTV broadcasting. The various orthogonal transform, such as Karhunen-Loeve transform (KLT), discrete fourier transform (DFT), the Walsh-Hadamard transform (WHT), the Haar transform (HT), the Slant transform (ST) and discrete cosine transform (DCT), can provide a feature space. Discrete cosine transform includes the real arithmetic (versus complex arithmetic for DFT), has lower computational complexity (versus KLT), has good energy compact capability, and the existence of fast transform algorithms. Energy compaction performance of the DCT compares more closely to that of the KLT relative to the performances of the DFT, WHT, HT, and ST [9,10,11,12,13,14]. Two dimensional DCT transform of $f(x,y)$ is given as

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & \text{for } u = 0 \\ \sqrt{\frac{2}{N}} & \text{for } u = 1, 2, \dots, N - 1 \end{cases} \quad (1)$$

$$C(u, v) = \alpha(u) \alpha(v) \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x, y) \cos\left(\frac{(2x+1)u\pi}{2N}\right) \cos\left(\frac{(2y+1)v\pi}{2N}\right) \quad (2)$$

$x, y = 0, 1, 2, \dots, N-1$

$u, v = 0, 1, 2, \dots, N-1$

where $f(x,y)$ denotes a two dimensional sequence of $N \times N$ points and $C(u,v)$ denotes $N \times N$ points DCT of $f(x,y)$.

3. Fuzzy Clustering

Clustering analysis is based on partitioning a collection of data points into a number of subgroups, where the objects inside a cluster (a subgroup) show a certain degree of closeness or similarity. It has been playing an important role in solving many problems in pattern recognition and image processing. Clustering methods can be considered as either hard (crisp) or fuzzy depending on whether a pattern data belongs exclusively to a single cluster or to several clusters with different degrees. In hard clustering, a membership value of zero or one is assigned to each pattern data (feature vector), whereas in fuzzy clustering, a value between zero and one is assigned to each pattern by a membership function. In general, fuzzy clustering methods can be considered to be superior to that of its hard counterparts since they can represent the relationship between the input pattern data and clusters more naturally. Clustering algorithms such as hard c-means (HCM) and fuzzy c-means (FCM) are based on the “sum of intracluster distances” criterion. The fuzzy c-means clustering algorithm was first introduced by Dunn, and the related formulation and algorithm was extended by Bezdek [15,16,17,18,19,20,21].

The fuzzy c-means algorithm is based on minimization of the following objective function, with respect to μ , a fuzzy c-partition of the data set, and to v , a set of c prototypes :

$$J_{FCM} = \frac{1}{2} \sum_{x=1}^N \sum_{i=1}^c \mu_{x,i}^m d^2(z_x, v_i) \quad (3)$$

where $\mu_{x,i}$ ($x=1,2,\dots,N, i=1,2, \dots,c$) is membership value, it denotes fuzzy membership of data point x belonging to class i , v_i ($i=1, 2, \dots, c$) is centroid of each cluster and z_x ($x=1,2,\dots, N$) is data set (pixel values in image), m is fuzzification parameter, $d^2(z_x, v_i)$ is Euclidean distance between z_x and v_i , N is the number of data points, c is number of clusters.

Fuzzy partition is carried out through an iterative optimization of equation (3) according to [7] :

- 1) Choose primary centroids v_i (prototypes).
- 2) Compute the degree of membership of all data set in all the clusters:

$$\mu_{x,i} = \frac{\left(\frac{1}{d^2(z_x, v_i)}\right)^{1/(m-1)}}{\sum_{i=1}^c \left(\frac{1}{d^2(z_x, v_i)}\right)^{1/(m-1)}} \quad (4)$$

3) Compute new centroids v_i^1 :

$$v_i^1 = \frac{\sum_{x=1}^N \mu_{x,i}^m z_x}{\sum_{x=1}^N \mu_{x,i}^m} \quad (5)$$

and update the degree of memberships, $\mu_{x,i}$ to $\mu_{x,i}^1$ according to equation (4).

4)

$$\text{if } \max_{x,i} [|\mu_{x,i} - \mu_{x,i}^1|] < \varepsilon \text{ stop, otherwise go to step 3} \quad (6)$$

Where ε is a termination criterion between 0 and 1, and $d^2(z_x, v_i)$ is given by

$$d^2(z_x, v_i) = \|z_x - v_i\|^2 \quad (7)$$

4. Validity Measure

Quality of clustering is important together with increasing of importance of clustering. So validity criterion was created, and based on a validity function which identifies overall compact and separated clustering. Several validity functions such as partition coefficient (PC), classification entropy (CE), partition exponent (PE), csc (compact and separate clustering) index (S) and so on, have been used for measuring validity mathematically [22,23,24,25,26]. PC and CE have slightly larger domains than PE, and in this sense are more general. But PC, CE and PE validity measures are the lack of direct connection to geometrical property. S validity function also includes geometrical properties [18,19] and it is proportion of compactness to separation. A smaller S indicates a partition in which all the clusters are overall compact and separate to each other. S is given as

$$S = \frac{\pi}{s} \quad (8)$$

The compactness of fuzzy cluster c_i is computed as

$$\pi = \frac{\sigma}{N}, \quad \sigma = \sum \sigma_i, \quad i = 1, \dots, c \quad (9)$$

The variation of fuzzy cluster i is defined as

$$\sigma_i = \sum (d_{x,i})^2, \quad x = 1, \dots, N \quad (10)$$

$d_{x,i}$ is called the fuzzy deviation of z_x from class i .

$$d_{x,i} = \mu_{x,i} \|v_i - z_x\| \quad (11)$$

s is separation of the fuzzy c-partition, where d_{min} is minimum distance between cluster centroids.

$$s = (d_{min})^2 \quad (12)$$

$$d_{min} = \min_{i,t} \|v_i - v_t\| \quad (13)$$

The compactness and separation validity function S is defined as the ratio of compactness to separation, and partition index is obtained by summing up this ratio over all clusters.

$$S = \frac{\pi}{s} = \frac{\left(\frac{\sigma}{N}\right)}{(d_{min})^2} \quad (14)$$

$$S = \frac{\sum_{i=1}^c \sum_{x=1}^N \mu_{x,i}^2 \|v_i - z_x\|^2}{N \min_{i,t} \|v_i - v_t\|^2} \quad (15)$$

Also other important validity functions are classification entropy (CE), and partition coefficient (PC). Minimum value of classification entropy and closing value of PC to one is shows better data classified. Classification entropy (CE) function is given by

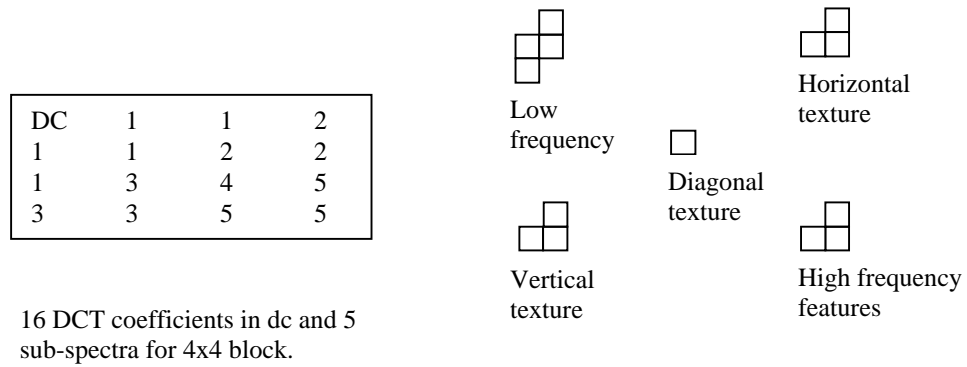
$$CE(\mu, c) = - \frac{\sum_{x=1}^N \sum_{i=1}^c \mu_{x,i} \ln \mu_{x,i}}{N} \quad (16)$$

Partition coefficient (PC) is given by

$$PC(\mu, c) = \frac{\sum_{x=1}^N \sum_{i=1}^c \mu_{x,i}^2}{N} \quad (17)$$

5. A New Image Clustering and Compression Algorithm

In this section, the new image clustering and compression method are introduced. New method includes a few stages: pre-filtering, fuzzification and enhancement of image, two-dimensional DCT, cosine membership function, and one dimensional run-length coding. In DCT domain, say, for a block of 4x4 coefficients with one dc component and fifteen ac components, the coefficients are divided into six categories: the dc level, the low frequency, horizontal, vertical, diagonal, and high frequency texture sub-spectra, as shown in Figure 1. Each of sub-spectral has different energy values. Human eyes are more sensitive to low frequency sub-spectral values than the other sub-spectral values. This sensitivity is reduced from horizontal sub-spectral values to high frequency sub-spectra values respectively. So, selection of cosine membership function peak values begins from dc and low frequency values according to number of cluster for reducing redundancy pixels in image.



1. Low frequency sub-spectra
2. Horizontal sub-spectra
3. Vertical sub-spectra
4. Diagonal sub-spectra
5. High frequency sub-spectra

Figure 1. Coarse spectral interpretation and features of 4 x 4 coefficients in DCT domain.

New algorithm is realized with six steps as shown extended block diagram at Figure 2. In the subsequent subsections, new algorithm is described in detail.

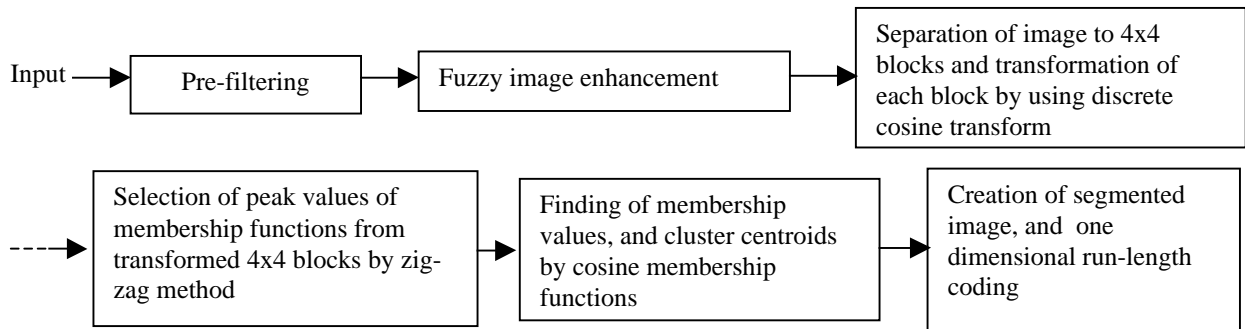


Figure 2. Block diagram of new image clustering and compression algorithm.

5.1. Pre-filtering

As a result of imperfect sampling processes, real images usually contain high frequency and low amplitude noise that is nearly invisible to humans. Human visual system removes this noise by nonlinear smoothing characteristics of the lens and retina. Pre-filter was applied to reduce undesirable noise effects on our segmentation results [27]. Characteristics of pre-filter is similar to lens and retina. Smoothing pre-filter is given as

$$z_x^2 = z_x^1 + \frac{d_{-2}(i) + 2d_{-1}(i) + 2d_{+1}(i) + d_{+2}(i)}{8} \tag{18}$$

$$z_x^1 = z_x + \frac{d_{-1}(i) + d_{+1}(i)}{4} \tag{19}$$

$$d_m(i) = \begin{cases} z_{(x+m)} - z_x & |z_{(x+m)} - z_x| < L \\ L & \text{otherwise} \end{cases} \quad (20)$$

where z_x is x th. pixel value in image. L is the filter constant and chosen to be 15.

5.2. Fuzzy image enhancement

Image can be considered as an array of fuzzy singletons, after pre-filtering, each with a value of membership denoting the degree of brightness level according to membership function in Figure 3 [28, 29]. Using notation of fuzzy sets, we can write image array as

$$Z_x \approx \begin{bmatrix} \mu_{11}/x_{11} & \mu_{12}/x_{12} & \cdots & \mu_{1n}/x_{1n} \\ \mu_{21}/x_{21} & \mu_{22}/x_{22} & \cdots & \mu_{2n}/x_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{m1}/x_{m1} & \mu_{m2}/x_{m2} & \cdots & \mu_{mm}/x_{mm} \end{bmatrix} \quad (21)$$

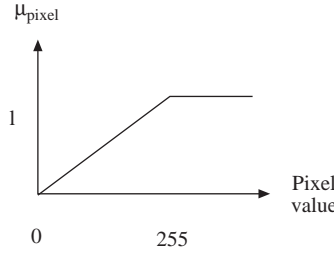


Figure 3. Membership function of pixels in image.

The obtaining new membership values μ_{mn}^1 of pixels for enhancing is shown as

$$\mu_{mn}^1 = \begin{cases} 2(\mu_{mn})^2 & 0 \leq \mu_{mn} \leq 0.5 \\ 1 - 2(1 - \mu_{mn})^2 & 0.5 < \mu_{mn} \leq 1 \end{cases} \quad (22)$$

Smoothing algorithm is used for reducing of noise by equation (23), and Figure 4.

$$\mu_{00} = \frac{\mu_{-10} + \mu_{10} + \mu_{01} + \mu_{0-1}}{4} \quad (23)$$

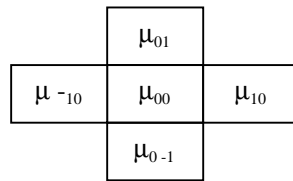


Figure 4. Pixels required around center pixel to use smoothing algorithm.

After smoothing operation, image enhancement is applied again by using equation (22) and membership values are normalized between 0 and 255 to obtain crisp image.

5.3. Discrete cosine transform and creation segmented image

Image is separated to 4x4 blocks and two dimensional discrete cosine transform is applied to each block. After obtaining DCT coefficients (D_i , $i=1,2,\dots,16$), Coefficients are normalized by equation (24) for assigning new values between 0 and 255.

$$D_i^1(i) = \left(\frac{|\min_i(D_i)| + D_i}{\max_i(|\min_i(D_i)| + D_i)} \right) \times 255 \quad (24)$$

DCT coefficients are arranged in a row from low frequency to high frequency by zig-zag method (Figure 5) [30, 31, 32] as shown sub-spectral distribution of coefficients in Figure 1. Then these coefficients are compared with the coefficients in other blocks considering that they have same place in a block and the higher coefficients are chosen. Then they are put their own place in 4x4 blocks. Selection of peak values of membership functions is made by ordering of $D^1(1)$, $D^1(2)$, $D^1(3)$, ... , $D^1(16)$ coefficients according to number of cluster.

D(1)	D(2)	D(6)	D(7)
D(3)	D(5)	D(8)	D(13)
D(4)	D(9)	D(12)	D(14)
D(10)	D(11)	D(15)	D(16)

Figure 5. Zig-zag scan order.

Membership values of original image pixels are found by cosine membership function. Cosine membership function is given as

$$S(d, z_x, y, p) = \begin{cases} 1/2 + 1/2 \cos((z_x - d)\pi/(d - y)) & y \leq z_x \leq d \\ 1/2 + 1/2 \cos((z_x - d)\pi/(p - d)) & d \leq z_x \leq p \end{cases} \quad (25)$$

where d is the coordinate of the peak, z_x is the independent variable (image amplitudes) and y, p is the width of the band.

Cluster centroids v_i are calculated by following formula presented in [7].

$$v_i = \frac{\sum_{x=1}^N \mu_{x,i}^m z_x}{\sum_{x=1}^N \mu_{x,i}^m}; \quad 2 \leq i \leq c \quad (26)$$

After obtaining the membership values and the cluster centroids, image is created and it is coded by run-length coding.

6. Experimental Results

This new method (DBIC) was applied to 128x128 dimensional five sample gray scale images and compared with results of fuzzy c-means (FCM) and hard c-means (HCM) algorithms. Comparing parameters are compression ratio, csc index (S) validity measure and number of iterations. Comparison results are given at Table according to different number of clusters (c). Some of original images and segmented images by DBIC, FCM and HCM methods were given in Figures 6 – 11 respectively according to different number of clusters.

Table. Experimental results.

	Cameraman			Lena			Pepper			Brain Tomog.			Test Image		
	DBIC	FCM	HCM	DBIC	FCM	HCM	DBIC	FCM	HCM	DBIC	FCM	HCM	DBIC	FCM	HCM
c = 4															
Compression ratio	27.027	15.384	16.129	23.809	17.857	17.857	23.809	18.181	17.543	19.607	16.949	17.241	9.708	7.352	7.194
Validity measure	0.182	0.171	0.106	0.017	0.506	0.063	0.062	0.023	0.025	0.5092	0.014	0.014	0.099	0.032	0.037
Number of iteration	1	12	8	1	10	8	1	11	11	1	10	10	1	16	20
c = 5															
Compression ratio	25.641	13.157	15.873	19.120	14.925	14.705	21.052	14.492	13.888	18.215	14.084	14.492	8.865	6.134	6.172
Validity measure	0.183	0.158	0.059	0.069	0.058	0.085	0.041	0.052	0.063	0.0085	0.042	0.066	0.075	0.049	0.066
Number of iteration	1	26	28	1	15	8	1	5	5	1	21	10	1	9	14
c = 6															
Compression ratio	25.575	14.705	14.492	19.083	13.333	13.513	21.008	9.523	11.764	18.083	12.345	12.658	8.857	5.813	5.813
Validity measure	0.183	0.184	0.0894	0.069	0.087	0.074	0.042	0.145	0.074	0.5094	0.073	0.056	0.075	0.735	0.108
Number of iteration	1	26	14	1	28	7	1	16	8	1	20	9	1	23	17

Our method provides better image compression than other methods according to experimental results. It preserves intelligibility of images together with this high compression ratio. The images obtained by DBIC method has less small clusters, that generates noise effect, than images obtained by other methods. Also there is not any block effects in segmented images.

DBIC method doesn't include mathematically iteration and it has less complex calculation than another methods. So it takes a little time to reach to the result. When examining clustering quality, Validity measures (S) of DBIC are very similar to validity measures of other methods at the most of point and also it is better at some of points.

Segmented image samples “cameraman” and “lena” by DBIC are shown at Figure 6 and 7 according to different number of clusters (c).

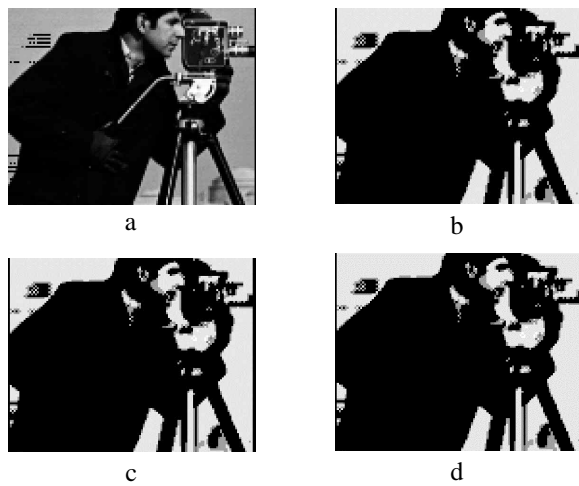


Figure 6. Cameraman image: a) original image; b) clustered image according to $c=4$; c) clustered image according to $c=5$; d) clustered image according to $c=6$.

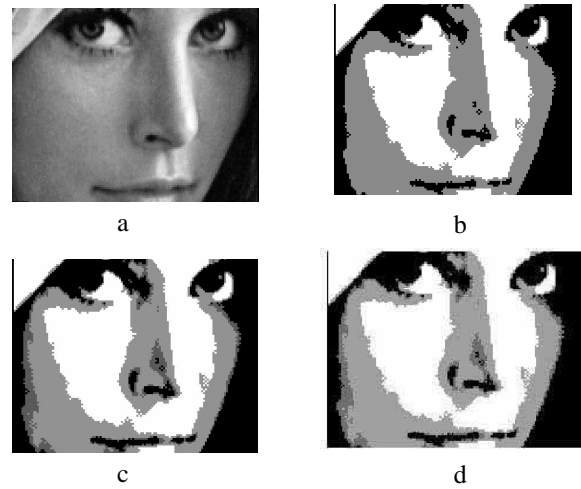


Figure 7. Lena image: a) original image; b) clustered image according to $c=4$; c) clustered image according to $c=5$; d) clustered image according to $c=6$.

Segmented image samples “cameraman” and “lena” by FCM are shown at Figure 8 and 9 according to different number of clusters (c), and fixed termination criterion ε ($\varepsilon = 0.1$) and fuzzification parameter m ($m = 2$).

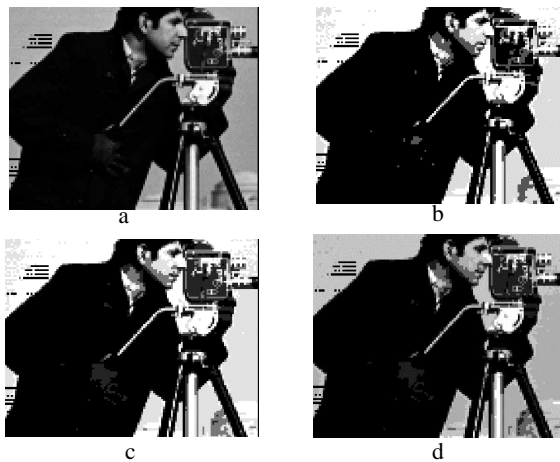


Figure 8. Cameraman image: a) original image; b) segmented image according to $c=4$; c) segmented image according to $c=5$; d) segmented image according to $c=6$.

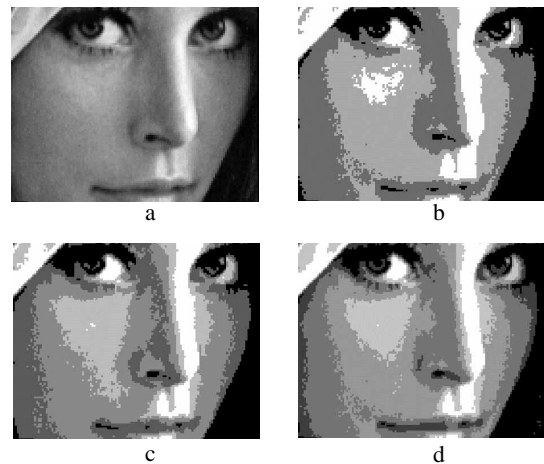


Figure 9. Lena image: a) original image; b) segmented image according to $c=4$; c) segmented image according to $c=5$; d) segmented image according to $c=6$.

Segmented image samples “cameraman” and “lena” by HCM are shown at Figure 10 and 11 according to different number of clusters (c), and fixed termination criterion ε ($\varepsilon = 0.1$).

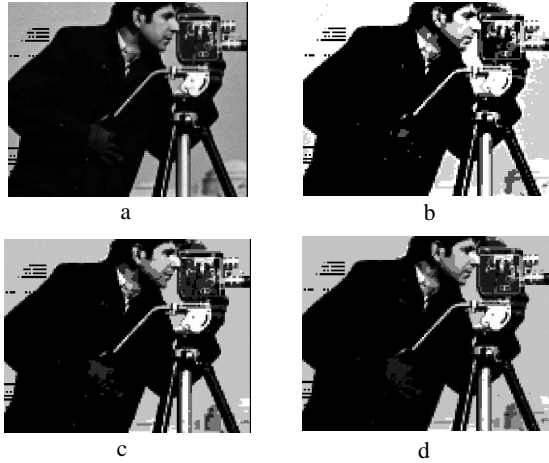


Figure 10. Cameraman image: a) original image; b) segmented image according to $c = 4$; c) segmented image according to $c = 5$; d) segmented image according to $c = 6$.

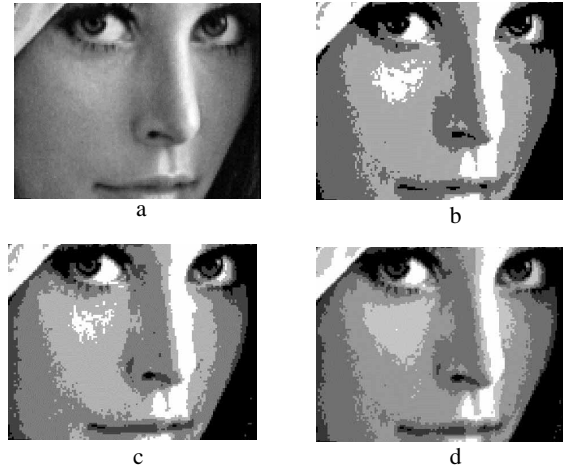


Figure 11. Lena image: a) original image; b) segmented image according to $c = 4$; c) segmented image according to $c = 5$; d) segmented image according to $c = 6$.

7. Conclusions

A new image clustering and compression method (DBIC) based on fuzzy logic and discrete cosine transform provides better compression ratio and performing time. DBIC method can reach solution with only one iteration, and it preserves intelligibility of images together with this high compression ratio. The DCT methodology in DBIC method can be equally applied to the Fast Fourier Transform (FFT) and other domains. So, according to different size of images, new method can be applied fast. This new method can be used for pattern recognition additionally, because it provides good validity measure. FCM is usually better at avoiding local minima than HCM, but FCM can still converge to local minima of the squared error criterion. So, selection initial values and the design of the membership functions are an important problem in fuzzy clustering. DBIC method does not need to selection of initial values and have complex membership functions, so there is not possibility to reach incorrect results and local minima. Because of these advantages, this new method is a good alternative method for image clustering, compression, and using at pattern recognition.

References

- [1] Rafael C. Gonzalez and Richard E. Woods, Digital Image Processing, Addison-Wesley Publishing, 1993.
- [2] M. Kunt, A. Ikonopoulou and M. Kocher, “Second Generation Image Coding Techniques”, Proc. IEEE 73 (4), pp. 549-575, 1985.
- [3] M. Kaya, The Development of Image Compression Techniques Using Fuzzy Image Compression, Ph.D. Thesis, Osmangazi University Electrical and Electronics Engineering Department, 2001.

- [4] T. Cornsweet, Visual Perception, Academic Press, New York, 1971.
- [5] Mahmoud R. El-Sakka and Mohamed S. Kamel, "Adaptive Image Compression Based on Segmentation and Block Classification", Int. Journal of Imaging Systems and Technology, 10(1), pp. 33-46, 1999.
- [6] L.A. Zadeh, Fuzzy Sets, Inform. Control 8, pp. 338-353, 1965.
- [7] James C. Bezdek, "A Convergence Theorem for The Fuzzy ISODATA Clustering Algorithms", IEEE Transaction On Pattern Analysis And Machine Intelligence, Vol.pami-2(1), pp.1-8, 1980.
- [8] Seong – Gon Kong and Bart Kosko, "Image Coding with Fuzzy Image Segmentation", IEEE International Conference on Fuzzy Systems, SanDiego-USA, pp. 213-220, 1992.
- [9] Anil K. Jain, Fundamentals of Digital Image Processing, Prentice Hall, 1989.
- [10] J.S. Lin, "Fuzzy Possibilistic Neural Network to Vector Quantizer in Frequency Domains", Optical Engineering, pp. 839-847, Apr. 2002.
- [11] X. Lee, Y. Zhang and A. Leon-Garcia, "Information Loss Recovery for Block-Based Image Coding Techniques – A Fuzzy Logic Approach", IEEE Transactions on Image Processing, Vol. 4, No. 3 March 1995.
- [12] C. Hsieh, P. Lu, J. Chang and K. Chuan, "A Codebook Design Algorithm for Vector Quantization of Images", IEEE Region 10 Conference on Computer and Communication Systems, Hong Kong, September 1990.
- [13] N. Ahmed, T. Natarajan and K. R. Rao, "Discrete Cosine Transform", IEEE Transactions on Computers, January 1974.
- [14] H.J. Grosse, M.R. Varley, T.J. Terrell and Y.K. Chan, "Improved Coding of Transform Coefficients in JPEG - like Image Compression Schemes", Pattern Recognition Letters, Vol. 21, No. 12, pp. 1061-1069, 2000.
- [15] Hyun-Sook Rhee and Kyung-Whan Oh, "A Validity Measure for Fuzzy Clustering and Its Use in Selecting Optimal Number of Cluster", IEEE International Conference on Fuzzy Systems, New Orleans-USA, Vol. 2, pp. 1020-1025, 1996.
- [16] J.C. Dunn, "Well Separated Clusters and Optimal Fuzzy Partitions", J. Cybern., Vol. 4, No. 3, pp. 95-104, 1974.
- [17] J.C. Bezdek, Pattern Recognition with Fuzzy Objective Function Algorithms, Plenum Press, New York, 1981.
- [18] I. Gath and A.B. Geva, "Unsupervised Optimal Fuzzy Clustering", IEEE Trans. Pattern Anal. Machine Intell., Vol. 11, No. 7, pp. 773-781, 1989.
- [19] II Hong Suh, Jae-Hyun Kim and Frank Chung-Hoon Rhee, "Convex-set-based fuzzy clustering", IEEE Transaction on Fuzzy Systems Vol. 7, No. 3, pp. 271-285, 1999.
- [20] R. Krishnapuram and J. Kim, "Clustering algorithm based on volume criteria", IEEE Transactions on Fuzzy Systems Vol. 8, No. 2, pp. 228-236, 2000.
- [21] M. Kaya, "A New Image Clustering And Compression Method Based on Fuzzy Hopfield Neural Network", International Conference on Signal Processing, Çanakkale-Turkiye, pp. 11-16, September 2003.
- [22] M. P. Windham, "Cluster Validity for Fuzzy Clustering Algorithms", J. Fuzzy Sets and Systems, Vol. 5, pp. 177-185, 1981.

- [23] Xuanli Lisa Xie and Gerardo Beni, "A Validity Measure for Fuzzy Clustering", IEEE Trans. on Pattern Anal. Machine Intell., Vol. 13, No.8, pp. 841-847, 1991.
- [24] M. Sugeno and T. Yasukawa, "A Fuzzy-Logic-Based Approach to Qualitative Modeling", IEEE Trans. Fuzzy Syst., Vol. 1, pp. 7-31, 1993.
- [25] Amine M. Bensaid, Lawrence O. Hall, James C. Bezdek, Laurence P. Clarke, Martin L. Silbiger, John A. Arrington and Reed F. Murtagh, "Validity-Guided (Re)Clustering with Application to Image Segmentation", IEEE Trans.on Fuzzy Systems, Vol. 4, pp. 112-123, 1996.
- [26] Duda. R. and Hart. P., Pattern Classification and Scene Analysis, New York, Wiley, 1973.
- [27] Oh-Jin Kwon, Rama Chellappa, "Segmentation-Based Image Compression", Optical Engineering, Vol. 32 (7), pp. 1581-1586, 1993.
- [28] Sankar K.Pal and Robert A.King, "Image Enhancement Using Fuzzy Set", Electronic Letter, Vol. 16 (10), pp. 376-378, 1980.
- [29] Sankar K.Pal and Robert A.King, "Image Enhancement Using Smoothing with Fuzzy Sets", IEEE Transactions on Systems, Man and Cybernetics, Vol. Smc-11(7), pp. 494-501, 1981.
- [30] Grosse, HJ., Varley MR., Terreli, TJ., Chan, YK., "Improved Coding of Transform Coefficients in JPEG-Like Image Compression Schemes", Pattern Recognit. Lett., Vol. 21, pp.1061-1069, 2000.
- [31] Mu-King Tsay, Jen-Fa Huang and Wei-Ping Chang-Chunago, "Image Compression Using VQ and Fuzzy Classified Algorithm" IEEE International Conference on Systems, Man and Cybernatics, Beijing-China, Vol.1, pp. 466-470, 1996.
- [32] M. Kaya, "A New Image Clustering and Compression Method Based on Fuzzy Logic and Discrete Cosine Transform", International Conference on Electrical and Electronics Engineering, pp. 153-157, Bursa-Turkiye, December 2003.