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Two-Variable Scattering Formulas to Describe Some Classes of Lossles Two-Ports with Mixed, Lumped Elements and Commensurate Stubs

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Abstract

Using the semi-analytic method based on the construction of two-variable scattering functions, which describe lossless two-ports with two kinds of elements, for some classes of ladder networks formed with lumped elements and commensurate stubs, the explicit descriptive formulas are produced up to six mixed-elements. To exhibit the efficiency of the explicit descriptive equations in the design of the broadband microwave circuits, a single matching design problem (UHF antenna matching) is solved by using the obtained two-variable scattering formulas.

Key Words: *Scattering parameters, two-variable description, mixed element networks.*

1. Introduction

Design of the lossless two-ports networks with lumped-distributed elements is one of the major concerns in the microwave and milimeterwave broadband network applications. More specifically, a microwave filter, a microwave amplifier or a matching network may include both the lumped elements and the commensurate transmission lines as two kinds of elements. In such a design, utilizing mixed, lumped-distributed elements in the circuit models would offer many advantages for the actual modelling of the interconnects and accurate simulation of MMIC layout in the implementation process.

As well known, the networks consisting mixed, lumped and distributed elements can be described in terms of the complex frequency variable p and the Richard variable $\lambda = \tan h(p\tau)$, τ being the equal delay lenght of transmission lines. In the earlier works, for the mixed elements network design, the network functions were expressed as transcendental functions of the complex frequency variable p, because of the hyperbolic dependence of p and λ. But, later, selecting the variables p and λ independently, the two-ports networks with mixed elements were described in two-variable formalism[1-4]. However, even for the simple mixed-elements design problems, an exact solution has not been obtained yet. Recently, based on the real frequency approach a novel semi-analytic technique has been proposed to describe the lossless two-ports with mixed elements. In the new approach[5-8], for the characterization of mixed elements networks, two-variable scattering functions are generated and for the restricted circuit topologies, especially, the low-pass LC ladders cascaded with commensurate transmission lines (Unit Elements), practical solutions are obtained.

In this paper, the semi-analytic approach defined in [8] has been applied to the constructing of the lossless mixed elements two-ports with commensurate stubs. Explicit descriptive formulas are given for some selected mixed elements topologies, up to 6 elements. Finally, a single matching problem, an UHF antenna matching network design problem is solved by using the new explicit formulas.

2. Two-Variable Scattering Descriptions for Lossless Two-Ports formed with simple lumped elements and commensurate stubs

Consider the generic form of the cascaded lossless two-ports with two-kinds of elements as shown in Figure 1. In the two-variable network, p variable impedance is related to the simple lumped L or C elements, whereas λ variable impedance is composed of the commensuratable open or short-circuited transmission lines (stubs).

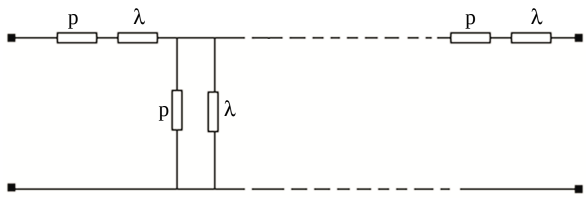


Figure 1. The generic form of the two-variable cascaded two-port.

Using the Belevitch canonical form, the two-variable scattering matrix describing the lumped-distributed elements two-port can be expressed as follows,

$$S(p,\lambda) = \frac{1}{g(p,\lambda)} \begin{pmatrix} h(p,\lambda) & \sigma f(-p,-\lambda) \\ f(p,\lambda) & -\sigma h(-p,-\lambda) \end{pmatrix} \tag{1}$$

Where, the real polynomials h(p,λ) and g(p,λ) are given in the matrix form, g(p,λ)= $\mathrm{p}^T \quad \Lambda_g \quad \lambda$, h(p,λ)= $\mathrm{p}^T \quad \Lambda_h \quad \lambda$, $\{p^T = \begin{bmatrix} 1 & p & p^2 & \cdots & p^{n_p} \end{bmatrix},\ \lambda^T = \begin{bmatrix} 1 & \lambda & \lambda^2 & \cdots & \lambda^{n_\lambda} \end{bmatrix}\}$

$$\Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0n_\lambda} \\ g_{10} & g_{11} & \cdots & g_{1n_\lambda} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n_p0} & g_{n_p1} & \cdots & g_{n_pn_\lambda} \end{bmatrix} \quad , \quad \Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0n_\lambda} \\ h_{10} & h_{11} & \cdots & h_{1n_\lambda} \\ \vdots & \vdots & \ddots & \vdots \\ h_{n_p0} & h_{n_p1} & \cdots & h_{n_pn_\lambda} \end{bmatrix} \tag{2}$$

Here, n_p and n_λ designate the total number of lumped circuit elements and commensurate stubs in the two-port respectively, $g(p,\lambda)$ is a Scattering Hurwitz polynomial, $f(p,\lambda)$ is monic polynomial that depends on the topology of the mixed structures under consideration, σ is unimodular constant, Λ_h and Λ_g are called connectivity matrices[5]; and the canonic polynomials $h(p,\lambda)$, $f(p,\lambda)$ and $g(p,\lambda)$ are related by the losslessness condition of the two-port as given in (3):

$$g(p,\lambda)g(-p,-\lambda) = h(p,\lambda)h(-p,-\lambda) + f(p,\lambda)f(-p,-\lambda) \tag{3}$$

To ensure the realizabilty as a passive lossless cascade structure, some additional conditions on the canonic polynomials should be satisfied[5]. The recent paper [8] has shown that for the canonic ladder structures with mixed, lumped and distributed elements, by using one-variable boundary conditions and the topologic properties, the additional conditions could be obtained. It has also presented that for the synthesis of the mixed elements two-port, the above conditions could be sufficient. In this regard, the following properties may be given for the canonic, restricted ladder form shown in Figure 1.

• The polynomial $f(p,\lambda)$ defines the transmission zeros of the cascade, mixed elements two-port under consideration and is given as follows.

$$f(p,\lambda) = f_L(p) f_D(\lambda) \tag{4}$$

Where, $f_L(p)$ and $f_D(\lambda)$ contain the transmission zeros (only at origin or infinity) of the lumped and distributed subsections in the cascade ladder structure, respectively. For the mixed elements network in Figure 1, it is appropriate to choose both $f_L(p)$ and $f_D(\lambda)$ as even/odd real polynomials. If the mixed structure contains only the lumped and distributed elements having transmission zeros at infinity, $f_L(p) = 1$, $f_D(\lambda) = 1$ (Figure 2a); or both at origin, $f_L(p) = p^{np}$, $f_D(\lambda) = \lambda^{n_\lambda}$ (Figure 2b); only the lumped elements at infinity, but, the distributed elements at the origin, $f_L(p) = 1$, $f_D(\lambda) = \lambda^{n_\lambda}$ (Figure 2c); and only the lumped elements at the origin, but, the distributed elements at infinity, $f_L(p) = p^{np}$, $f_D(\lambda) = 1$ (Figure 2d), some proposed types of the ladder networks with two kinds of elements can be generated as shown in Figure 2.

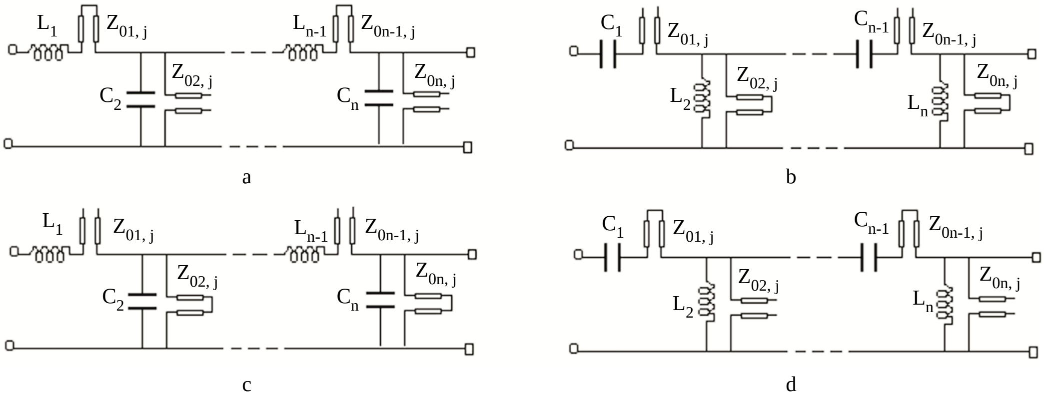


Figure 2. The generic forms of (a) First type, (b) Second type, (c) Third type, (d) Fourth type of ladder networks with simple lumped elements and commensurate stubs.

For the proposed ladder structures depicted in Figure 2, the single variable boundary conditions are established as follows:

When the short-circuited and open-circuited transmission lines are removed from the mixed elements ladder structures, the resulting lumped networks whose transmission zeros are defined by $f_L(p)$ can be fully described by means of h(p,0), g(p,0) for the mixed elements networks in Figure 2a,2d and

$$g(p,0)g(-p,0) = h(p,0)h(-p,0) + f_L(p)f_L(-p) \tag{5}$$

h(p,∞), g(p,∞) for the networks with two-kinds of elements in Figure 2b,2c and

$$g(p,\infty)g(-p,\infty) = h(p,\infty)h(-p,\infty) + f_L(p)f_L(-p) \tag{6}$$

where g(p,0) and g(p,∞) are strictly Hurwitz.

When the lumped elements are removed from the ladder structures, the short-circuited and open-circuited transmission lines whose transmisson zeros are defined by $f_D(\lambda)$, are obtained and fully described by means of $h(0,\lambda)$, $g(0,\lambda)$ for the ladder sructures in Figure 2a,2c and

$$g(0,\lambda)g(0,-\lambda) = h(0,\lambda)h(0,-\lambda) + f_D(\lambda)f_D(-\lambda) \tag{7}$$

$h(\infty,\lambda)$, $g(\infty,\ \lambda)$ for the cascade structures in Figure 2b,2d and

$$g(\infty,\lambda)g(\infty,-\lambda) = h(\infty,\lambda)h(\infty,-\lambda) + f_D(\lambda)f_D(-\lambda) \tag{8}$$

where $g(0,\lambda)$ and $g(\infty,\ \lambda)$ are strictly Hurwitz.

Finally, the boundary conditions for the ladder networks into consideration can be established as in Table 1.

Table 1. Boundary conditions for the selected ladder topologies.

The Ladder Types	First Type	Second Type	Third Type	Fourth Type
Setting Variables	$p = 0/\lambda = 0$	$p = \infty/\lambda = \infty$	$p = 0/\lambda = \infty$	$p = \infty/\lambda = 0$

Preselecting the complexity and the transmission zeros of the lumped and distributed parts of the mixed element networks shown in Figure 2a-d, the lumped and distributed prototypes are fully described by the boundary conditions (5-8). The single variable boundary polynomials {h(p,0), g(p,0)}, {h(p,∞), $g(p,\infty)$}, $\{h(0,\lambda),\ g(0,\lambda)\}$ and $\{h(\infty,\ \lambda),\ g(\infty,\lambda)\}$ define the first column, the last column, the first row and the last row entries of Λ_h and Λ_g matrices, respectively. Now, the problem is to compute the remaining unknown entries related to the cascade connectivity information of the mixed elements. To generate the explicit coefficient relations carrying on the connectivity information, it is essential to establish the paraunitary relation of (3). In order to construct the two-variable real polynomials defining the scattering matrices of the two-port network, by utilizing (3) and equating the coefficients of the same powers of p and λ variables, a set of equations called Fundamental Equation Set (FES) are obtained as follow[5,8]:

$$g_{0,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}g_{0,l}g_{0,2k-l} = h_{0,k}^2 + f_{0,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}(h_{0,l}h_{0,2k-l} + f_{0,l}f_{0,2k-l})$$
$$\vdots \quad (k = 0, 1, ..., n_\lambda))$$
$$\sum_{j=0}^{i}\sum_{l=0}^{k}(-1)^{i-j-l}g_{j,l}g_{i-j,2k-1-l} = \sum_{j=0}^{i}\sum_{l=0}^{k}(-1)^{i-j-l}\left[h_{j,l}h_{i-j,2k-1-l}+f_{j,l}f_{i-j,2k-1-l}\right]$$
$$\vdots \quad (i = 1, 3, ..., 2n_p - 1, \qquad k = 0, 1, ..., n_\lambda - 1)$$
$$\sum_{j=0}^{i}(-1)^{i-j}(g_{j,k}g_{i-j,k} + 2\sum_{l=0}^{k-1}(-1)^{k-l}g_{j,l}g_{i-j,2k-l}) =$$
$$\sum_{j=0}^{i}(-1)^{i-j}\left(h_{j,k}h_{i-j,k} + f_{j,k}f_{i-j,k} + 2\sum_{l=0}^{k-1}(-1)^{k-l}[h_{j,l}h_{i-j,2k-l} + f_{j,l}f_{i-j,2k-l}]\right)$$
$$\vdots \quad (i = 2, 4, ..., 2n_p - 2, \qquad k = 0, 1, ..., n_\lambda)$$
$$g_{n_p,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}g_{n_p,l}g_{n_p,2k-l} = h_{n_p,k}^2 + f_{n_p,k}^2 + 2\sum_{l=0}^{k-1}(-1)^{k-l}(h_{n_p,l}h_{n_p,2k-l} + f_{n_p,l}f_{n_p,2k-l})$$
$$\vdots \quad (k = 0, 1, ..., n_\lambda) \tag{9}$$

To end up an acceptable solution of FES (3) that ensures the realizibilty as the passive lossless ladder structures, the scattering matrices and the two-variable canonical polynomials $g(p,\lambda)$, $h(p,\lambda)$ and $f(p,\lambda)$ have to satisfy some independent conditions. The simplest way of finding the independent conditions is to use the topological properties of the mixed element two-ports. Thus, by using one-variable boundary conditions and the obtained topologic properties in Table 2, for the mixed structures depicted in Figure 2, the coefficient constraints leading to an acceptable solution of FES are provided.

These constraints reflecting the connectivity information for the proposed cascade topologies are utilized in FES (3) properly. Then, by solving the FES explicit formulas yielding the coefficients of $g(p,\lambda)$ and $h(p,\lambda)$ are obtained for low-order mixed structures, up to 6 mixed element, as given in Table 3.

3. Application

In this section, to show the application of the obtained two-variable scattering formulas defining the mixed networks with lumped elements and commensurate stubs under consideration, a broadband UHF antenna matching network is designed over the normalized matched frequency band of (0.6 to 1.4).

In Section 2, it is shown that description of the mixed element network is done by the real scattering parameters, which are freely chosen coefficients of polynomials of $h(p,0/\infty)$ and $h(0/\infty,\lambda)$. By utilizing the semi-analytic procedure, these unconstrained coefficients are used to determine of the unknown coefficients of $h(p,\lambda)$ and $g(p,\lambda)$, if the complexity of the network topology is set in advance by the designer. In this sense, for the proposed single matching network, the mixed element network topology consisting 3 sections shown in Figure 2c is selected, i.e. $n_\lambda = 3$ and $n_p = 3$. Initializing the unknown independent parameters

($h_{00} = -1$, $h_{01} = 1$, $h_{02} = -1$, $h_{13} = 1$, $h_{23} = -1$, $h_{33} = -1$) by ad-hoc choices as +1 or -1 and ($\tau = 0.529$) the transducer power gain of the matching network is optimized employing the Levenberg-Marquartd technique over normalized frequency band (0.6 to1.4) in a similiar way to that of [5-8]. In the optimization scheme, an ideal form of the transducer power gain is approximated in the least square sense, and so the Levenberg-Marquartd method providing a good solution to the approximation is used.

Table 2. The generic forms of coefficient matrices and topologic properties for the ladder structures.

THE FIRST TYPE OF LADDER NETWORK

$$\Lambda_h = \begin{bmatrix} 0 & h_{01} & \cdots & h_{0,n-1} & h_{0,n} \\ h_{10} & h_{11} & \cdots & h_{1,n-1} & 0 \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ h_{n-1,0} & h_{n-1,1} & \cdots & 0 & 0 \\ h_{n,0} & 0 & \cdots & 0 & 0 \end{bmatrix} \quad \Lambda_g = \begin{bmatrix} 1 & g_{01} & \cdots & g_{0,n-1} & g_{0,n} \\ g_{10} & g_{11} & \cdots & g_{1,n-1} & 0 \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ g_{n-1,0} & g_{n-1,1} & \cdots & 0 & 0 \\ g_{n,0} & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} &h_{k,n-m} = g_{k,n-m} = 0, \quad k > m, \\ &h_{k,n-m} = \mu g_{k,n-m}, \quad k = m, \\ &\mu = \mathrm{sgn}(h_{0,n}) = \mathrm{sgn}(h_{n,0}) = \pm 1, \\ &g_{11} = g_{01}g_{10} - h_{01}h_{10}. \end{aligned}$$

THE SECOND TYPE OF LADDER NETWORK

$$\Lambda_h = \begin{bmatrix} 0 & 0 & \cdots & 0 & h_{0,n} \\ 0 & 0 & \cdots & h_{1,n-1} & h_{1,n} \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & h_{n-1,1} & \cdots & h_{n-1,n-1} & h_{n-1,n} \\ h_{n,0} & h_{n1} & \cdots & h_{n,n-1} & 0 \end{bmatrix}, \Lambda_g = \begin{bmatrix} 0 & 0 & \cdots & 0 & g_{0,n} \\ 0 & 0 & \cdots & g_{1,n-1} & g_{1,n} \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & g_{n-1,1} & \cdots & g_{n-1,n-1} & g_{n-1,n} \\ g_{n,0} & g_{n,1} & \cdots & g_{n,n-1} & 1 \end{bmatrix}$$

$$\begin{aligned} &h_{n-k,m} = g_{n-k,m} = 0, \quad k > m, \\ &h_{n-k,m} = \mu g_{n-k,m}, \quad k = m, \\ &\mu = \mathrm{sgn}(h_{0,n}) = \mathrm{sgn}(h_{n,0}) = \pm 1, \\ &g_{n-1,n-1} = g_{n,n-1}g_{n-1,n} - h_{n,n-1}h_{n-1,n}. \end{aligned}$$

THE THIRD TYPE OF LADDER NETWORK

$$\Lambda_h = \begin{bmatrix} h_{00} & h_{01} & \cdots & h_{0,n-1} & 0 \\ 0 & h_{11} & \cdots & h_{1,n-1} & h_{1,n} \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & 0 & \cdots & h_{n-1,n-1} & h_{n-1,n} \\ 0 & 0 & \cdots & 0 & h_{n,n} \end{bmatrix}, \Lambda_g = \begin{bmatrix} g_{00} & g_{01} & \cdots & g_{0,n-1} & 1 \\ 0 & g_{11} & \cdots & g_{1,n-1} & g_{1,n} \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ 0 & 0 & \cdots & g_{n-1,n-1} & g_{n-1,n} \\ 0 & 0 & \cdots & 0 & g_{n,n} \end{bmatrix}$$

$$\begin{aligned} &h_{k,m} = g_{k,m} = 0, \quad k > m, \\ &h_{k,m} = \mu g_{k,m}, \quad k = m, \\ &\mu = \mathrm{sgn}(h_{00}) = \mathrm{sgn}(h_{n,n}) = \pm 1, \\ &g_{1,n-1} = g_{0,n-1}g_{1,n} - h_{0,n-1}h_{1,n}. \end{aligned}$$

THE FOURTH TYPE OF LADDER NETWORK

$$\Lambda_h = \begin{bmatrix} h_{00} & 0 & \cdots & 0 & 0 \\ h_{10} & h_{11} & \cdots & 0 & 0 \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ h_{n-1,0} & h_{n-1,1} & \cdots & h_{n-1,n-1} & 0 \\ 0 & h_{n,1} & \cdots & h_{n,n-1} & h_{n,n} \end{bmatrix}, \Lambda_g = \begin{bmatrix} g_{00} & 0 & \cdots & 0 & 0 \\ g_{10} & g_{11} & \cdots & 0 & 0 \\ \vdots & \vdots & \iddots & \vdots & \vdots \\ g_{n-1,0} & g_{n-1,1} & \cdots & g_{n-1,n-1} & 0 \\ 1 & g_{n,1} & \cdots & g_{n,n-1} & g_{n,n} \end{bmatrix}$$

$$\begin{aligned} &h_{n-k,n-m} = g_{n-k,n-m} = 0, \quad k > m, \\ &h_{n-k,n-m} = \mu g_{n-k,n-m}, k = m, \\ &\mu = \mathrm{sgn}(h_{00}) = \mathrm{sgn}(h_{n,n}) = \pm 1, \\ &g_{n-1,1} = g_{n,1}g_{n-1,0} - h_{n,1}h_{n-1,0}. \end{aligned}$$

Where, the nonzero entries of the all Λ_g are nonnegative real numbers, k,m =(0,1,.....,n)

Table 3. The explicit formulas for low-order mixed structures.

Section	**Coefficient Relations for the FIRST TYPE of Ladder Network**
2	*independent coefficients* : $[h_{01}, h_{02}, h_{10}, h_{20}]$ $g_{10}=(h_{10}^2+2g_{00}g_{20})^{1/2}$, $g_{20}=\lvert h_{20}\rvert$, $g_{01}=(h_{01}^2+2g_{00}g_{02})^{1/2}$, $g_{02}=\lvert h_{02}\rvert$, $g_{11}=(1/g_{00})(g_{01}g_{10}-h_{01}h_{10})$, $h_{11}=\mu g_{11}$, $\mu=\pm 1$
3	*independent coefficients* : $[h_{01}, h_{02}, h_{03}, h_{10}, h_{20}, h_{30}]$ $g_{10}=(h_{10}^2+2g_{00}g_{20})^{1/2}$, $g_{20}=(h_{20}^2+2g_{30}\beta)^{1/2}$, $g_{30}=\lvert h_{30}\rvert$, $g_{01}=(h_{01}^2+2g_{00}g_{02})^{1/2}$, $g_{02}=(h_{02}^2+2g_{03}\alpha)^{1/2}$ $g_{03}=\lvert h_{03}\rvert$ $g_{11}=(1/g_{00})(g_{01}g_{10}-h_{01}h_{10})$, $h_{11}=(\beta/\alpha)h_{02}+(\alpha/\beta)h_{20}$ $g_{21}=(1/\beta)[g_{20}g_{11}-h_{20}h_{11}-\alpha g_{30}]$ $h_{21}=\mu g_{21}$, $\alpha=g_{01}-\mu h_{01}$, $\mu=\pm 1$ $g_{12}=(1/\alpha)[g_{02}g_{11}-h_{02}h_{11}-\beta g_{03}]$, $h_{12}=\mu g_{12}$ $\beta=g_{10}-\mu h_{10}$, $[f=1]$
	Coefficient Relations for the SECOND TYPE of Ladder Network
2	*independent coefficients* : $[h_{20}, h_{21}, h_{02}, h_{12}]$ $g_{12}=(h_{12}^2+2g_{22}g_{02})^{1/2}$, $g_{02}=\lvert h_{02}\rvert$, $g_{21}=(h_{21}^2+2g_{22}g_{20})^{1/2}$, $g_{20}=\lvert h_{20}\rvert$, $g_{11}=(1/g_{22})(g_{21}g_{12}-h_{21}h_{12})$, $h_{11}=\mu g_{11}$, $\mu=\pm 1$
3	*independent coefficients*: $[h_{30}, h_{31}, h_{32}, h_{23}, h_{13}, h_{03}]$ $g_{23}=(h_{23}^2+2g_{33}g_{13})^{1/2}$, $g_{13}=(h_{13}^2+2g_{03}\beta)^{1/2}$, $g_{03}=\lvert h_{03}\rvert$, $g_{31}=(h_{31}^2+2g_{30}\alpha)^{1/2}$ $g_{32}=(h_{32}^2+2g_{33}g_{31})^{1/2}$, $g_{30}=\lvert h_{30}\rvert$, $g_{22}=(1/g_{33})(g_{32}g_{23}-h_{32}h_{23})$, $h_{22}=(\beta/\alpha)h_{31}+(\alpha/\beta)h_{13}$ $g_{12}=(1/\beta)[g_{13}g_{22}-h_{13}h_{22}-\alpha g_{03}]$ $h_{12}=\mu g_{12}$, $\alpha=g_{32}-\mu h_{32}$, $g_{21}=(1/\alpha)[g_{31}g_{22}-h_{31}h_{22}-\beta g_{30}]$, $h_{21}=\mu g_{21}$ $\beta=g_{23}-\mu h_{23}$, $[f=p^3\lambda^3]$
	Coefficient Relations for the THIRD TYPE of Ladder Network
2	*independent coefficients* : $[h_{00}, h_{01}, h_{12}, h_{22}]$ $g_{12}=(h_{12}^2+2g_{02}g_{22})^{1/2}$, $g_{22}=\lvert h_{22}\rvert$, $g_{01}=(h_{01}^2+2g_{02}g_{00})^{1/2}$, $g_{00}=\lvert h_{00}\rvert$, $g_{11}=(1/g_{02})(g_{01}g_{12}-h_{01}h_{12})$, $h_{11}=\mu g_{11}$, $\mu=\pm 1$
3	*independent coefficients*: $[h_{00}, h_{01}, h_{02}, h_{13}, h_{23}, h_{33}]$ $g_{13}=(h_{13}^2+2g_{03}g_{23})^{1/2}$, $g_{23}=(h_{23}^2+2g_{33}\beta)^{1/2}$, $g_{33}=\lvert h_{33}\rvert$, $g_{00}=\lvert h_{00}\rvert$, $g_{01}=(h_{01}^2+2g_{00}\alpha)^{1/2}$ $g_{12}=(1/g_{03})(g_{02}g_{13}-h_{02}h_{13})$, $h_{12}=(\beta/\alpha)h_{01}+(\alpha/\beta)h_{23}$ $g_{02}=(h_{02}^2+2g_{03}g_{01})^{1/2}$, $g_{22}=(1/\beta)[g_{23}g_{12}-h_{23}h_{12}-\alpha g_{33}]$, $h_{22}=\mu g_{22}$, $\alpha=g_{02}-\mu h_{02}$, $g_{11}=(1/\alpha)[g_{01}g_{12}-h_{01}h_{12}-\beta g_{00}]$, $h_{11}=\mu g_{11}$ $\beta=g_{13}-\mu h_{13}$, $[f=\lambda^3]$
	Coefficient Relations for the FOURTH TYPE of Ladder Network
3	*independent coefficients*: $[h_{00}, h_{10}, h_{20}, h_{31}, h_{32}, h_{33}]$ $g_{20}=(h_{20}^2+2g_{30}g_{10})^{1/2}$, $g_{10}=(h_{10}^2+2g_{00}\beta)^{1/2}$, $g_{00}=\lvert h_{00}\rvert$, $g_{31}=(h_{31}^2+2g_{30}g_{32})^{1/2}$, $g_{32}=(h_{32}^2+2g_{33}\alpha)^{1/2}$ $g_{33}=\lvert h_{33}\rvert$, $g_{21}=(1/g_{30})(g_{31}g_{20}-h_{31}h_{20})$, $h_{21}=(\beta/\alpha)h_{32}+(\alpha/\beta)h_{10}$ $\mu=\pm 1$ $g_{11}=(1/\beta)[g_{10}g_{21}-h_{10}h_{21}-\alpha g_{00}]$ $h_{11}=\mu g_{11}$, $\alpha=g_{31}-\mu h_{31}$ $g_{22}=(1/\alpha)[g_{32}g_{21}-h_{32}h_{21}-\beta g_{33}]$, $h_{22}=\mu g_{22}$, $\beta=g_{20}-\mu h_{20}$, $[f=p^3]$

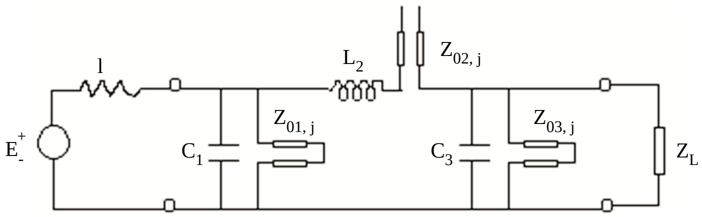
As a result of optimization, the coefficient matrices describing the antenna matching network are as follows:

$$\Lambda_g = \begin{bmatrix} 0.212 & 0.912 & 1.513 & 1 \\ 0 & 1.467 & 1.607 & 1.423 \\ 0 & 0 & 1.419 & 0.693 \\ 0 & 0 & 0 & 0.329 \end{bmatrix} \quad \Lambda_h = \begin{bmatrix} -0.212 & 0.692 & -0.682 & 0 \\ 0 & -1.467 & 0.166 & -0.8 \\ 0 & 0 & -1.419 & -0.265 \\ 0 & 0 & 0 & -0.329 \end{bmatrix}$$

The final matching network with the normalized element values and the gain performance of the system are depicted in Figure 3 and in Figure 4, respectively.

For comparison, the used elements number by this technique is listed in Table 4 together with some different solutions in the literature for the same example; the lumped elements solution of Hatley[10], Linear Least Square Approximation-LLSQA[5], the semi-analytic solution for BPLU (Band-Pass Ladder with Unit Elements)[6] and the CAD technique [9]. Also, for the sake of comparing the different methods the relative convergence rates are given at the same table.

On the other hand, all the methods except 'CAD' require selecting of the network topologies in the beginning of design process. While the new mixed element networks need no transformer caused the difficulties in implementation, the performance of the power gain characteristic is very close to that of the BPLU structure.



[C_1=1.537, L_2=0.623, C_3=0.687, Z_{01}=3.777, Z_{02}=0.83, Z_{03}= 0.518, τ=1.15].

Figure 3. Mixed-element matching network for an UHF antenna.

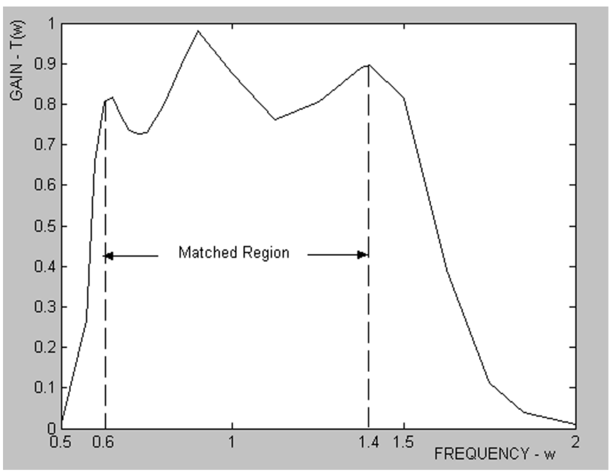


Figure 4. Gain performance of the antenna matching example.

Table 4. Comparison of the alternative solutions.

Method	Lumped Elements	Commensurate Trans. Lines	Transformers	Convergence Rate
Hatley (only lumped)	5	0	0	Fast
LLSQA (replacement)	3	2	2	Slow
CAD (numeric)	4	4	0	Slow
BPLU (semi-anly)	4	2	1	Fast
New struct. (semi-anly)	3	3 (stubs)	0	Fast

4. Conclusion

In order to describe explicitly the new mixed elements network topologies formed with lumped elements and commensurate stubs, two-variable scattering functions are obtained. The construction of the broadband matching networks with mixed elements is demonstrated via the UHF antenna matching design example. The example show that using these mixed element structures, the broadband microwave circuits can be implemented practically by providing the physical connections between lumped elements and the parasitic effects naturally embedded in the design process. Hence, the new regular mixed element networks will be useful in the design and the implementation of microwave integrated circuits and MMIC.

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