Turk J Elec Engin, VOL.14, NO.1 2006,

Secure Digital Communication using Chaotic Symbolic Dynamics

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Abstract

One of the major hurdles in implementing chaotic communication schemes is the synchronization of chaotic systems. For the last two decades, numerous contributions of varying successes have been made by researchers from different disciplines for the synchronization of chaotic systems. Symbolic dynamics based synchronization method is shown to be capable of providing high quality synchronization (HQS), which is essential for reliable communication. In this work, using this method, a secure digital communication system is proposed. The well known piece−wise linear 1−D map such as tent map and Bernoulli shift map are used for this study. Since the information is dynamically encoded to the system, higher level of encryption can be obtained. Using numerical simulations, the performance of the proposed scheme is compared with that of the binary phase shift keying (BPSK) and chaotic shift keying (CSK) schemes. The theoretical expression for the bit error rate is derived for the new system and it is numerically evaluated for AWGN and frequency selective channels. The Results indicate that the proposed scheme yield similar performances as that of the BPSK system at high signal to noise ratios (SNRs).

1. Introduction

Chaotic systems are dynamical systems which shows complex behavior. One of the defining attributes of a chaotic system is the sensitive dependence on its initial conditions. Time series generated from chaotic systems are wide band in nature and noise like in appearance [1]. Because of these special properties, chaotic systems are widely being studied for the secure communication applications [2]. Synchronization of chaotic systems is essential to implement chaotic communication schemes. It is believed that the synchronization of chaotic systems is an impossible task because of the sensitive dependence on the initial conditions. However, following the seminal paper of Pecora and Carrol [3], there is a multitude of results for the synchronization of chaotic systems. For a comprehensive treatment on this topic the reader may refer [4] and the references therein.

Synchronization means establishing a relationship between the states of two dynamical systems (one at the transmitter and the other at the receiver). In the drive−response system, proposed by Pecora and Carrol, a dynamical variable from the driving system is used for synchronizing the response system [3]. If all the transversal Lyapunov exponents of the response system are negative, it synchronizes with the drive system asymptotically. Alternatively, a coupling can be introduced between the states of the two chaotic systems (coupled synchronization). It is shown that if all the local transversal Lyapunov exponents are negative, a perfect synchronization is established [5]. Coupled synchronization is essentially a nonlinear observer design problem [6] and hence, methods from nonlinear control theory are used for the synchronization of chaotic systems [7]-[11]. Other forms of synchronization such as generalized synchronization [12], phase synchronization [13] etc. are also available in the literature.

Symbolic dynamics is defined as the coarse−grain description of the chaotic dynamics and has been used for the analysis of chaotic systems [14]. Recently, symbolic dynamics is being used for secure communication applications. In [15], chaotic communication by the feedback of symbolic dynamics is proposed. Application of symbolic dynamics for the differential chaotic shift keying (DCSK) is discussed in [16]. Symbolic dynamics based noise reduction and coding is proposed in [17], [18]. In [19], a high quality synchronization (HQS) is achieved using symbolic dynamics. The synchronization using symbolic dynamics is reformulated from an information theoretical point of view in [20].

In this paper, using the symbolic dynamics based synchronization, a dynamic encoding system is proposed for secure communication. The analytical and numerical bit error rate (BER) performance for the new system is obtained. This result is compared with that of the binary phase shift keying (BPSK) system and the chaotic shift keying (CSK) system. Results show that the proposed system has similar BER of that of the BPSK system at high signal to noise ratios (SNRs). Similar to the AWGN cannel, in band limited channels also the BER performance of the proposed system approaches that of the BPSK system at higher SNRs.

The remaining sections of the paper are organized as follows. In section 2, a brief overview of chaotic dynamics is provided. Synchronization of the chaotic systems is discussed in section 3. Chaotic shift keying based secure communication system is outlined in section 4. Symbolic dynamics is explained in section 5. In section 6, the proposed method is explained in detail and the theoretical expression for the BER is presented. Numerical results are presented in section 7 and the paper is concluded with some remarks in section 8.

2. Chaotic Dynamics

Chaotic systems are nonlinear dynamical systems with certain distinct characteristics. These systems can generate highly complex waveforms even though the number of interacting variables is minimal. For an iterated map a dynamical system with single variable can result in chaotic behaviour while for a continuous system, three coupled differential equations can result in a complicated dynamics [1] .Time series generated from chaotic dynamics have the following three interesting properties: (*i*) wide−band spectrum, (*ii*) noise−like appearance, and (*iii*) high complexity. In a chaotic system, trajectories starting from slightly different initial conditions diverge exponentially in time. This is called the sensitive dependence on the initial conditions.

Because of these distinctive properties, chaotic systems are widely being studied for secure communication applications. Basically there are two ways by which chaos can be used in communication system: (*i*) to use chaotic time series as wide-band carrier so that coding and modulation can be accomplished together (eg., CSK, frequency modulated differential chaotic shift keying (FM−DCSK) *etc* [21]), and (*ii*) to use chaotic sequences as an alternative source for spreading sequences in direct sequence spread spectrum (DS/SS) communication systems [22, 23, 24]. In this paper, these two approaches are combined to develop a new secure communication scheme so that the dynamic encoding of the first method is combined with the better bandwidth utilization offered by the second method.

3. Synchronization of Chaotic Systems

The objective of synchronization in chaotic communication is to synchronize two chaotic systems: one at the transmitter and the other at the receiver. Since the chaotic signals are used as the carrier waveforms to transmit the information from the transmitter to the receiver, to retrieve the information at the receiver, the two systems must be synchronized. By applying a suitable mechanism, a relationship between the trajectories of the transmitter and the receiver systems can be established. This procedure is known as synchronization. Consider two chaotic systems given by the following set of equations:

$$\dot{\mathbf{x}}_{k+1} = \mathbf{f}(\mathbf{x}_k) \quad (1a)$$

$$\dot{\hat{\mathbf{x}}}_{k+1} = \mathbf{f}(\hat{\mathbf{x}}_k) \quad (1b)$$

where, $\mathbf{x}_k = [x_k^1, \ldots, x_k^n]^T$ and $\hat{\mathbf{x}}_k = [\hat{x}_k^1, \ldots, \hat{x}_k^n]^T$ are the $n-$dimensional state vectors of the transmitter and the receiver systems, respectively. $\mathbf{f} = [f_1(.), \ldots, f_n(.)]^T$ is a smooth nonlinear vector valued function. These two systems can be synchronized if

$$\lim_{k \to \infty} ||\mathbf{x}_k - \hat{\mathbf{x}}_k|| = 0. \quad (2)$$

At the receiver, only a partial information about the transmitter state is available which is corrupted by the channel noise, $\mathbf{v}_k$. The received signal is given by,

$$\mathbf{y}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k, \quad (3)$$

where, $\mathbf{h}(.) = [h_1(.), \ldots, h_m(.)]^T$ is an $m-$dimensional linear/nonlinear output function and $\mathbf{y}_k = [y_k^1, \ldots, y_k^m]^T$, with $m \leq n$. The objective of the synchronizing scheme is to estimate $\mathbf{x}_k$ from the available information $\mathbf{y}_k$.

4. Chaotic Shift Keying

Figure 1 shows the CSK scheme. It is one of the earliest chaotic communication methods [25]. Most widely studied CSK system is the binary CSK where two identical chaotic systems are present at the transmitter and the receiver. Depending on the information bit ($b_k = \pm 1$), one of these chaotic systems are selected and the state variable corresponding to that system is transmitted. This transmitted state corrupted by the channel noise ν_n is available at the receiver. The classical CSK receiver works on the assumption that chaotic systems can typically synchronize an identically driven version of themselves through a suitable coupling. In that case, the synchronized system results in a lower mean square error with the received signal. But due the inability of the other system to synchronize with the driving system, the mean square error will be large. Hence, it is easy to decode the information by simply looking at the magnitude of the mean square error. From Figure 1, if $f(x_n)$ is selected (say bit $+1$ is transmitted) the mean square error $\mathbb{E}[e_1^2(n)]$ should be less than $\mathbb{E}[e_2^2(n)]$ to decode the message correctly (here ergodicity of the chaotic dynamics is assumed). Otherwise, the bit is detected incorrectly.

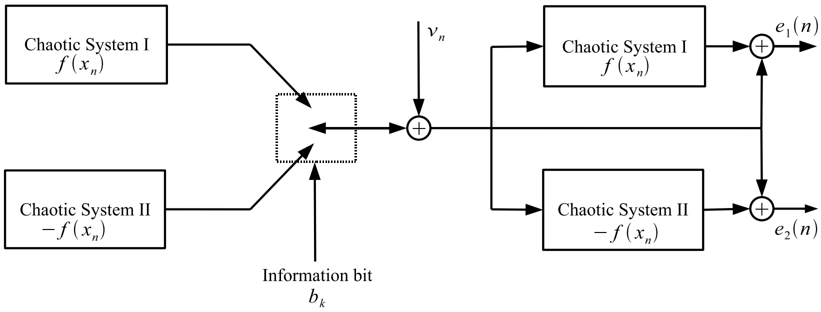


Figure 1. Chaotic shift keying scheme.

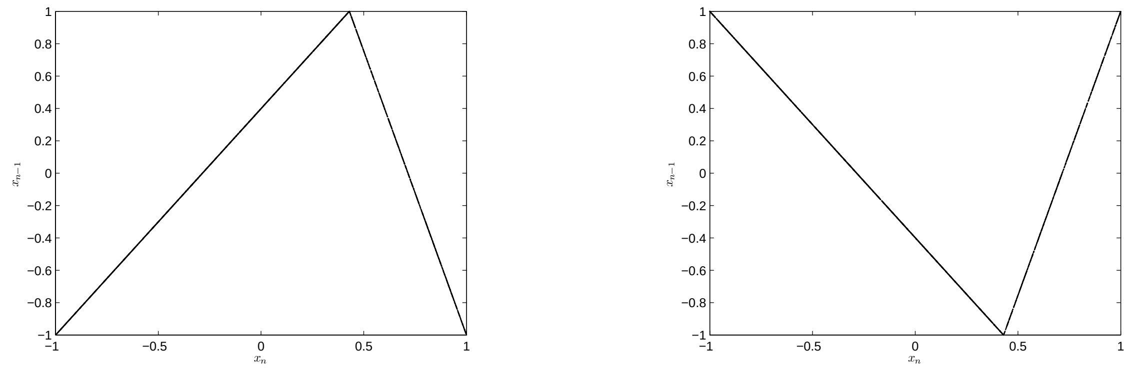
In this experiment, a skewed tent map and an inverted skewed tent map are used to obtain the two chaotic systems at the transmitter [26]. The skewed Tent map is given by,

$$x_{n+1} = f(x_n) = \begin{cases} \frac{2x_n+1-a}{a+1} & \text{for} \quad -1 \leq x_n \leq a \\ \frac{2x_n-1+a}{a-1} & \text{for} \quad a \leq x_n \leq 1 \end{cases} . \tag{4}$$

The inverted skewed tent map is given by,

$$x_{n+1} = -f(x_n) = \begin{cases} \frac{a-2x_n-1}{a+1} & \text{for} \quad -1 \leq x_n \leq a \\ \frac{1-2x_n-a}{a-1} & \text{for} \quad a \leq x_n \leq 1 \end{cases} . \tag{5}$$

These maps are defined in the intervals $[-1+1]$. The state spaces of the above two maps are shown in Figure 2. Exact copy of these maps are used at the receiver. However, the initial conditions are uncertain. To keep the phase continuity, at the transmitter the last state of the map currently selected is used as the initial condition for the next bit duration.



(a) (b)

Figure 2. State space of the skewed tent maps ($a = 0.43$): (a) Skewed tent map (b) Inverted skewed tent map.

5. Symbolic Dynamics

Symbolic dynamics is the coarse-grain description of the actual system [14]. It is being widely applied for the analysis of chaotic systems. By partitioning a chaotic phase space to arbitrary regions, and labeling each region with a specific symbol, the trajectories can be converted to a symbolic sequence. This coarse grain formulation of the system makes the deterministic nature of the dynamical system to a stochastic nature. Hence, such systems can be treated as Markov systems.

Let the state space ($\mathcal{S}$) of the chaotic system is partitioned to m disjoint regions, $\beta = \{\mathcal{C}\}_{i=1}^{m}$, such that $\mathcal{C}_i \cap \mathcal{C}_j = \emptyset$ for $i \neq j$ and $\cup_{i=1}^{m} \mathcal{C}_i = \mathcal{S}$. If one can assign m alphabets ($\mathbf{X} = [X_1, \ldots, X_m]$) to each of the disjoint region, the dynamics of the system can be represented by a sequence of finite alphabet $\mathbf{X}$. This sequence is called the symbolic dynamics of the system. The entropy of the new information source is given by,

$$H_n^{\beta} = -\sum_{\mathbf{Y}_n} P(\mathbf{Y}_n^i) \log P(\mathbf{Y}_n^i) \tag{6}$$

where $P(\mathbf{Y}_n^i)$ is the probability to find a code word $\mathbf{Y}_n^i$ of length n. The superscript i in the above equation represents a specific combination of symbolic sequence. The summation is taken over all such possible sequences. The source entropy of a dynamical system is given as:

$$h^{\beta} = \lim_{n \to \infty} h_n^{\beta} = \lim_{n \to \infty} \frac{1}{n} H_n^{\beta} \tag{7}$$

The Kolmogorov Sinai entropy of the system is defined as,

$$h_{KS} = \sup_{\beta} h^{\beta} \tag{8}$$

From the above discussions, it is clear that the iterated chaotic map is an information source with entropy h_{KS}.

5.1. Symbolic dynamics of the Tent map

One of the simple maps with complex dynamics is the Tent map. It is a piecewise linear $1-$D map. The dynamics of the tent map is given by [27],

$$x_{n+1} = \begin{cases} 2x_n & 0 < x_n \leq 0.5 \\ 2 - 2x_n & 0.5 \leq x_n < 1 \end{cases} . \tag{9}$$

This equation has a phase space which spans in the unit interval. Binary partition (0 or 1) for generating symbolic dynamics is shown in Figure 3. Here, $'0'$ is assigned if $0 < x_n \leq 0.5$ and $'1'$ is assigned if $0.5 \leq x_n < 1$. This way the binary sequence for the entire trajectory can be obtained.

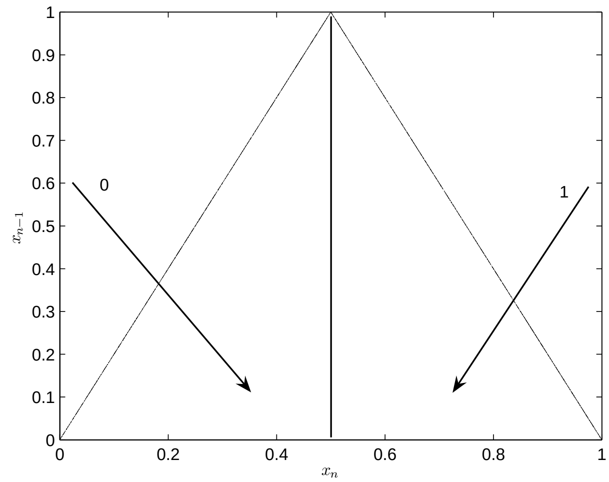


Figure 3. Generating partition of the Tent map.

5.2. Symbolic dynamics of Bernoulli shift map

Another widely studied piecewise linear $1-\mathrm{D}$ map is the Bernoulli shift map. The dynamics of this map is given by,

$$x_{n+1} = \begin{cases} 2x_n + 1 & -1 \leq x_n < 0 \\ 2x_n - 1 & 0 \leq x_n < 1 \end{cases}. \tag{10}$$

The binary partition for generating the symbolic dynamics is shown in Figure 4. In this case, $'0'$ is assigned if $-1 \leq x_n < 0$ and $'1'$ is assigned if $0 \leq x_n < 1$.

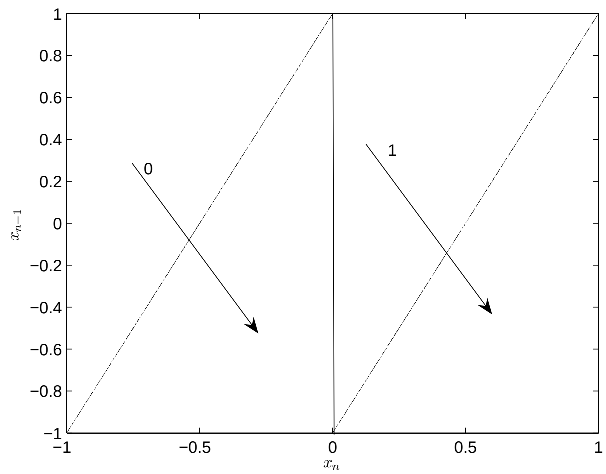


Figure 4. Generating partition of the Bernoulli shift map.

5.3. Synchronization using symbolic dynamics

Using the Tent map with binary partition, the symbolic dynamics based synchronization is explained [20]. Consider the chaotic system described by Eq. 9. Assume that there is no message transmitted and there is no channel noise. For an initial condition x_0, let $\mathcal{X} = [x_0, \ldots, x_{m-1}]$ be the trajectory generated by Eq. 9. Here a finite length trajectory is considered for simplicity. Let the corresponding binary trajectory be $\mathcal{X}_b$

of the same length m which is transmitted from the transmitter to the receiver. At the receiver, an exact copy of the same map is available. However, the initial condition is unknown. The task is then to estimate the initial condition, x_0, using $\mathcal{X}_b$.

If the first element of $\mathcal{X}_b$, $\mathcal{X}_b(0)$, is available and if it is 0, then the initial condition x_0 is in between 0 and 0.5. If $\mathcal{X}_b(0) = 1$, then x_0 is in between 0.5 and 1. By considering the next symbol of the sequence, $\mathcal{X}_b(1)$, a more accurate estimate of the initial condition can be obtained by taking the pre−image of the map. If this procedure is continued, the initial condition can be estimated with an accuracy of the order of $1/2^m$, if m symbols are considered. It has been reported in the literature that the conventional synchronization methods suffer from the intermittent desynchronization issues near to the unstable periodic points [28]. However, symbolic dynamics based synchronization scheme is immune to such desynchronization phenomenon and an HQS is guaranteed to be obtained by such methods.

6. Dynamic Encoding

Using the synchronization method described in subsection 5.3, a new dynamic encoding is proposed. It is worth noting that chaotic systems are capable of generating independent and identically distributed (iid) binary sequences [29]. The baseband representation of the proposed method is shown in Figure 5. The trajectories of the chaotic generator (Tent map), $\mathcal{X}$, is converted to a symbolic sequence ($\mathcal{X}_b$) of length N. This symbolic sequences are then sent to an encoder. The format of the transmitted sequence is shown in Figure 6. At the encoder, the first m bits are used to provide the information about the initial condition. The last $N-m$ bits are used to carry the information. If the message bit (b_k) is $+1$, then the corresponding bit of the symbolic sequence is transmitted as it is. If the message bit is 0, then the symbolic sequence is negated and transmitted.

This sequence is transmitted using conventional digital communication techniques such as BPSK or QPSK. At the receiver, signal corrupted by AWGN (ν_n) is available. Using conventional matched filter receiver, the transmitted sequence can be detected. Sequence thus obtained is used to estimate the initial condition, $\hat{x}_0$, using the synchronization scheme discussed in subsection 5.3. Once the initial condition is estimated correctly, a complete trajectory ($\hat{x}_n$) is reconstructed at the receiver. An approximate symbolic sequence $\hat{\mathcal{X}}_b$ can be generated at the receiver corresponding to $\mathcal{X}_b$ using $\hat{x}_n$. The message bits are then retrieved using the following steps. The last $N-m$ bits of the estimated symbolic sequence is multiplied bit by bit with the corresponding received signal. Using threshold detection, the information bits can be retrieved. If the channel is bandlimited and frequency selective, already existing tools from digital communication systems such as the maximum likelihood estimation, decision feed back equalization etc. can be used.

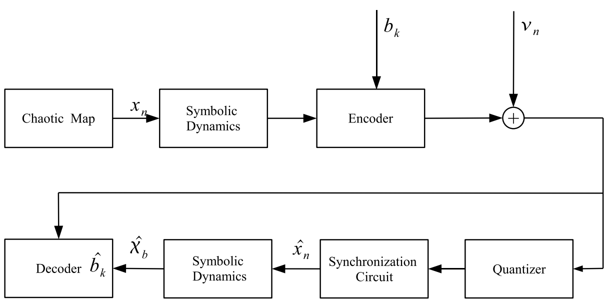


Figure 5. Proposed communication system.

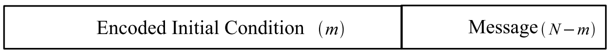


Figure 6. Transmission sequence format.

6.1. Theoretical BER

It is clear from Figure 6 that there are two possibilities for the bit error to occur: (i) The decoding information may be wrong and it causes a wrong estimation of the initial condition, and (ii) the detection of the message itself is wrong due to the noise.

To decode the message completely, all the m bits should be detected correctly. Let the probability of bit error (BER), p_b, be

$$p_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{11}$$

where N_0 is the noise power and E_b is the bit energy. Hence, the probability of wrongly detecting the sequence, p_s, is

$$p_s = 1 - (1 - p_b)^m \tag{12}$$

Here, it is assumed that the symbolic alphabets are equiprobable (0.5). If a sequence is wrongly detected, then the probability of wrong decision about the transmitted message, p_d, which is given by,

$$p_d = 0.5(1 - p_b) \tag{13}$$

Then the probability of error when the decoding information is wrong is given by,

$$p_1 = p_d p_s = \left(1 - (1 - p_b)^m\right) p_d \tag{14}$$

Considering the second situation, where the first m bits are decoded correctly and the message decoding is incorrect, the bit error probability is given by

$$p_2 = (1 - p_s)p_b \tag{15}$$

Hence the total probability of error (BER) is given by:

$$\begin{aligned} p &= p_1 + p_2 = \left(1 - (1 - p_b)^m\right) p_d + (1 - p_s)p_b \\ &= \left(1 - (1 - p_b)^m\right) 0.5(1 - p_b) + (1 - p_b)^m p_b \\ &= 0.5(1 - p_b) + (1 - p_b)^m (1.5p_b - 0.5) \end{aligned} \tag{16}$$

When the SNR is high, p_b is close to zero and hence $(1 - p_b)^m \approx 1$. Then from Eq. 16, it can be clearly seen that the proposed system has a BER performance similar to that of the BPSK communication system.

7. Results and Discussions

Extensive numerical simulations are carried out to assess the performance of the proposed secure communication system. Tent map and Bernoulli shift map are used for the generation of chaotic sequences. 10^5 bits are transmitted for each SNR values and corresponding BER is calculated. The experiments are carried out for simple AWGN and frequency selective channels. The BER performance of the proposed system for the AWGN channel is presented in Figure 7. This performance is compared to that of the CSK and conventional BPSK schemes. As it is expected, at lower SNR, the BER of the proposed scheme is relatively high. For instance, at an SNR value of 4dB, the proposed method has a BER of 0.31 while CSK has a BER of 0.2 and BPSK has BER of 0.06. In order to estimate the initial condition accurately, all the m symbols should be detected correctly which is very unlikely at lower SNR values. However, when the BPSK achieves a BER of 10^{-3}, the BER performance of the proposed system starts following that of the BPSK. For example, at 12dB SNR, the proposed system has a BER of 4×10^{-4}. The corresponding BER values of BPSK and CSK systems are 10^{-4} and 8.2×10^{-3}, respectively. It is also interesting to note that even though the CSK based communication scheme has a slight performance advantage over the proposed system at low SNR regions, at high SNR values this method is unable to provide fast BER decay. To see the BER performances clearly, the theoretical results (derived in Eq. 16) are plotted in Figure 8 along with that of the BPSK system. It can be seen that when the SNR increases, the BER performance of the proposed method closely follows that of the BPSK system.

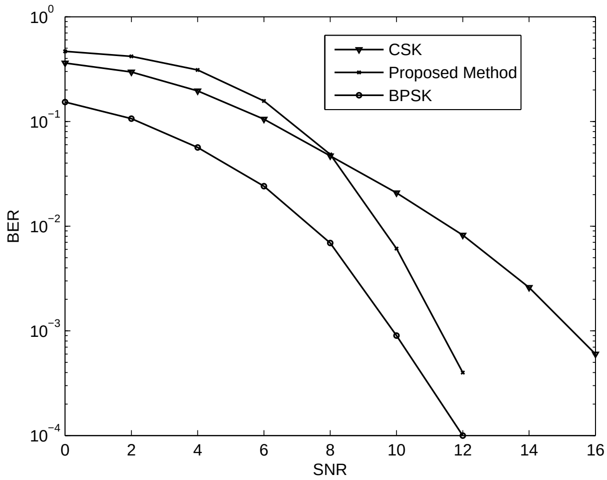


Figure 7. BER performance under AWGN channel (Tent map).

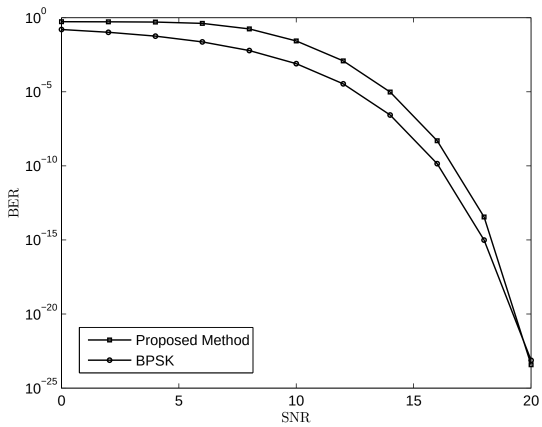


Figure 8. Theoretical BER results of BPSK and the proposed method under AWGN channel.

The same experiment is repeated for the Bernoulli shift map also and the results are compared to that of the BPSK system (Figure 9). Here also it can be seen that the BER characteristics are similar to that of the Tent map based system; at low SNR values the new system has a high BER performance and as the SNR increases the BER curve of the proposed system closely follows that of the BPSK system.

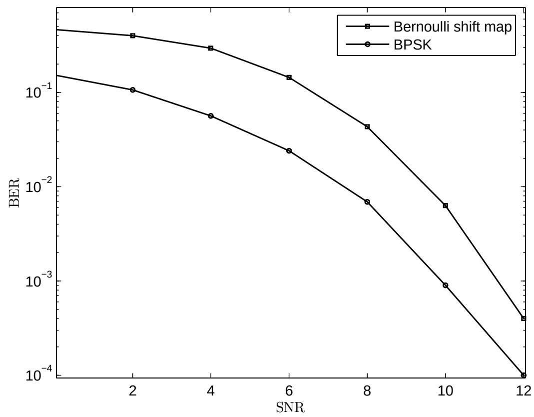


Figure 9. BER performance under AWGN channel (Bernoulli shift map).

Most of the communication channels encountered in practice are band limited and frequency selective. To study the performance of the proposed system in such channels, another set of simulations are carried out. Two different channel models discussed in [30, Chapter 10] are considered. The first channel is a three ray channel model with tap weights $[0.474, 0.815, 0.474]$ and the second channel is a six ray channel with tap weights $[0.227, 0.460, 0.688, 0.460, 0.227]$, respectively.

At the receiver end, the maximum likelihood sequence estimation is used to remove the inter symbol interference caused by the channel. Simulation results are presented in Figures 10 and 11. The proposed system behaves exactly as in the previous situation; at low SNR it exhibit a high BER and as the SNR increases the BER closely follows the BER curve of the BPSK system. In the second channel condition as well, the proposed system has a fast BER decay which can be observed from the Figure 11.

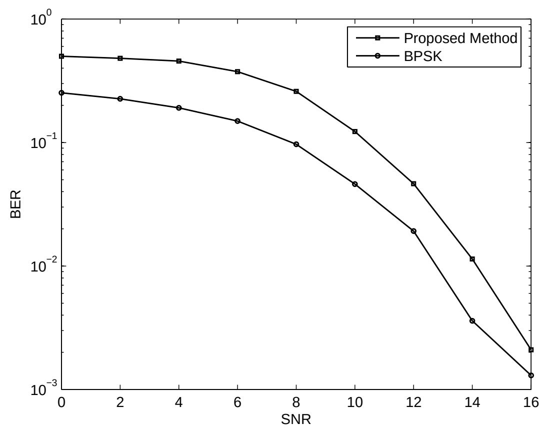


Figure 10. BER performance under bandlimited channel.

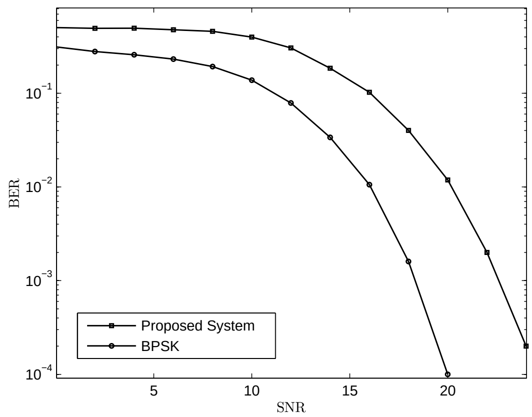


Figure 11. BER performance under bandlimited channel.

8. Conclusion

Synchronization of chaotic systems is an important step to implement chaotic communication schemes. Especially in noisy environments, the application of symbolic dynamics to synchronize chaotic systems proves to be a good choice. symbolic dynamics of the chaotic system, a new scheme for secure communication is proposed in this work. The information is dynamically encoded using piece − wise linear iterative chaotic maps. The proposed method is tested for different maps like tent map and Bernoulli shift map. BER performance of the proposed scheme is analyzed analytically and numerically. It is found that, at high SNR, the proposed system has similar BER performance of that of the conventional BPSK system. The BER performance of the new scheme is superior to the BER offered by the CSK communication scheme.

References

[1] S. H. Strogatz, Nonlinear Dynamics and Chaos: with Applications to Physics, Biology, Chemistry, and Engineering, Perseus Books, Cambridge MA, 2001.

[2] K. Cuomo, A. V. Oppenheim, and S. H. Strogatz, "Synchronization of Lorenz–based chaotic circuits with applications to communications," IEEE Trans. on Circuits and Sys.–II, Vol.40 , 10, pp. 626–633, 1993.

[3] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," Phys. Rev. Lett., Vol. 64, pp. 821–824, 1990.

[4] L. M. Pecora, T. L. Carroll, G. A. Johnson, D. J. Mar, and J. F. Heagy, "Fundamentals of synchronization in chaotic systems, concepts, and applications," Chaos, Vol. 7, pp. 520–543, 1997.

[5] M. J. Ogorzalek, "Taming chaos–Part I: Synchronization," IEEE Trans. on Circuits and Sys.–I, Vol.40, 10, pp. 693–699, 1993.

[6] H. Nijmeijer and I. M. Y. Mareels, "An observer looks at synchronization," IEEE Trans. on Circuits and Sys.–I, Vol. 44, pp. 882–890, 1997.

[7] T. B. Flower, "Application of stochastic control techniques to chaotic nonlinear systems," *IEEE Trans. on Auto. Control*, 34, pp. 201–205, 1989.

[8] D. J. Sobiski and J. S. Thorp, "PDMA-1: Chaotic communication via the extended Kalman filter," IEEE Trans. on Circuits and Sys.–I, Vol.45, pp. 194–197, 1998.

[9] C. Cruz and H. Nijmeijer, "Synchronization through filtering," Int. J. of Bifurc. and Chaos, 10, pp. 763–775, 2000.

[10] H. Leung and Z. Zhu, "Performance evaluation of EKF based chaotic synchronization," IEEE Trans. Circuits and Syst.–I, Vol.48, pp. 1118–1125, 2001.

[11] H. Leung and Z. Zhu, "Time varying synchronization of chaotic systems in the presence of system mismatch," Phys. Rev. E, Vol. 69, pp. 026201.(1–5), 2004.

[12] N. F. Rulkov, M. M. Sushchik, L. S. Tsimring, and H. D. I. Abarbanel, "Generalized synchronization of chaos in directionally coupled chaotic systems," Phys. Rev. E, Vol. 51, pp.980-994, 1995.

[13] M. G. Rosenblum, A. S. Pikovsky, and J. Kurths, "Phase synchronization of chaotic oscillators," Phys. Rev. E, 76, pp. 1804-1807, 1996.

[14] H. Bai-lin, Elementary Symbolic Dynamics and Chaos in Dissipative Systems, World Scientific, Singapore, 1989.

[15] E. R. Bollt, "Review of chaos communication by feedback control of symbolic dynamics," Intl. J. Bifurc. and Chaos, Vol. 13, pp. 269–285, 2003.

[16] G. M Maggio and G. Galias, "Application of symbolic dynamics to differential chaotic shift keying," IEEE Trans. Circuits and Syst.–I, Vol. 49, pp. 1729–1735, 2002.

[17] J. Schweizer and T Schimming, "Symbolic dynamics for processing chaotic signals-I: Noise reduction of chaotic sequences," IEEE Trans. Circuits and Syst.–I, Vol. 48, pp. 1269–1282, 2001.

[18] J. Schweizer and T Schimming, "Symbolic dynamics for processing chaotic signals-II: Communication and coding" IEEE Trans. Circuits and Syst.–I, Vol. 48, pp. 1283–1295, 2001.

[19] T. Stojanovski, L. Kocarev, and R. Harris, " Application of symbolic dynamics in chaos synchronization," IEEE Trans. Circuits and Syst.–I, Vol. 44, pp. 1014–1018, 1997.

[20] A. S. Dimitriev, G. A. Kassian and A. D. Khilinsky, "Chaotic synchronization. Information viewpoint," Intl. J. Bifurc. and Chaos, Vol. 10, pp. 749–761, 2000.

[21] M. P. Kennedy and G. Kolumban, "Digital communications chaos," Signal Processing, Vol. 80, pp. 1307–1320, 2000.

[22] G. Heidari-Bateni, "A chaotic direct-sequence spread-spectrum communication system," IEEE Trans. on Comm. Syst., Vol.42, pp. 1524–1527, 1994.

[23] —–, "Chaotic sequences for spread spectrum: an alternative to PN-sequences," Proc. of the IEEE Intl. Conf. on Selec. Topics in Wireless Comm., Vancouver, B.C., Canada, pp. 437–440, 1992.

[24] Q. Zhang and J. Zheng, "Choice of chaotic spreading sequences for asynchronous DS-CDMA communication," Proc. of IEEE Asia-Pacific Conf. on CAS., pp. 642–645, 2000.

[25] Y. L Bel'skii, and A. S. Dimitriev,"Information transmission deterministic chaos (*in Russian*)," Radiotekh Electron, Vol. 38, pp. 1310–1215,1993.

[26] T. Schimming, "Statistical analysis and optimization of the chaos based broadband communications," PhD thesis, Ècole Polytechnique fèdèrale de Lausanne, 2002.

[27] R. L. Devaney, An introduction to chaotic dynamical system, The Benjamin/Cummings Publishing Company Inc., 1985.

[28] J. F. Heagy, T. L. Carroll, and L. M. Pecora, "Desynchronization by periodic orbits," Phys. Rev. E, Vol. 52, pp. R1253–R1256, 1995.

[29] T. Kohda, "Sequence of IID random variables from chaotic dynamics," Proc. of SETA, 1998.

[30] J. G. Proakis, Digital Communications, 4^{th} Ed, McGraw Hill, 2001.