

# State-Space Synthesis of Current-Mode First-Order Log-Domain Filters

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## Abstract

*This paper proposes current-mode first-order log-domain filters, which are systematically derived using the state-space synthesis procedure. First-order low-pass, high-pass, and all-pass responses are obtained with different circuit types. The filter circuits have very simple structures, since they use only bipolar junction transistors (BJTs) and a grounded capacitor. They can be electronically tuned by changing an external current. The filters have a greater bandwidth due to inherently current-mode and log-domain operation. PSPICE simulations are given to confirm the theoretical analysis.*

**Key Words:** *Log-domain filters, current-mode circuits, state-space synthesis*

## 1. Introduction

Log-domain filters are of interest, mainly due to their suitability for low voltage, low power, large dynamic range, high frequency applications, and for being electronically tunable. The main concept is based on the exponential I-V characteristics of bipolar junction transistors (BJTs) or the MOS transistor operating in the sub-threshold region. Most interesting of all, they open the door to elegantly realizing a linear system with inherently nonlinear (logarithmic-exponential) circuit building blocks.

Adams introduced the idea of filtering in the log-domain in 1979 [1]. He proposed the first log-domain filter using log and anti-log techniques in conjunction with a combination of forward-biased diodes and a capacitor to obtain a distortionless first-order low-pass filter. A related concept using companding was introduced by Tsividis in 1990 [2]. At the same time, Seevinck independently reinvented the log-domain filter concept, which he denoted with the term current-mode companding [3]. In 1993, Frey showed that the synthesis of log-domain filters can be synthesized by state-space representation. For the state-space synthesis method, the BJT can be directly used to realize the log-domain filters by mapping from state-space linear differential equations. He introduced a systematic state-space synthesis method for designing log-domain filters [4,5]. Toumazou et al. published an implementation in weak inversion MOS, showing the potential for low-power operation [6]. The first experimental results were published by Perry and Roberts [7]. In addition, they proposed an alternative synthesis method based on the simulation of LC ladder filters. The first experimental results in sub-threshold MOS were presented by Ngarmnil et al. [8]. Punzenberger et al. demonstrated the suitability for low-voltage applications [9]. Since then, many other researchers have extensively investigated log-domain filters [10].

Due to the ongoing trends of lower supply voltages and low power operation, the area of analog integrated filters is facing serious challenges [1,3]. The maximal dynamic range achievable using conventional filter implementation techniques, such as Opamp-MOSFET-C, transconductance-C, and switched-capacitor, becomes severely restricted by the supply voltage. In ultra-low-power environments, linear resistors become too large for on-chip integration. Finally, the situation is complicated by high-frequency demands and the fact that the filter transfer function has to be tunable to compensate for process tolerances. In the area of continuous-time filters, a promising approach to meet these challenges is provided by the class of log-domain filters [5,11,12]. An important property of log-domain filtering is that it uses companding [2,13], whereby the signals are compressed logarithmically at the input stage before being processed, and then are expanded exponentially at the output stage. This makes it possible for log-domain circuits to operate with very low supply voltage, without sacrificing the dynamic range [14]. In addition, these filters contain low impedance nodes along the signal path, which can be exploited to achieve greater bandwidths.

Unlike conventional classes of filters, in which linear circuits are implemented using nonlinear devices, log-domain techniques directly exploit the nonlinear characteristic of the transistors to linearize the whole filter. Without the need for conventional circuit linearization techniques, log-domain filter circuits have a simple and elegant structure, and have the potential to run at high frequencies and operate with low-power supplies. Log-domain filters also possess many other attractive features, including the ability to be electronically tuned over a wide range of frequencies [15-17].

First-order filters are widely used in audio, and video, as well as in many applications in which simplicity and power consumption are important parameters. Currently, there are several first-order low-pass filters presented in the literature [1,3,4,12,16,18-20]. First-order all-pass filters using log-domain techniques have also been reported [21,22]. These authors introduced a systematic state-space synthesis method for designing first-order all-pass log-domain filters. Many biquadratic and high-order log-domain filters are presented in the literature [4,10,12,16,20,23].

The purpose of this paper is to propose first-order, current-mode, log-domain, low-pass, high-pass, and all-pass filters, which are systematically derived using the state-space synthesis procedure.

The paper is organized as follows. The principle of state-space synthesis of log-domain is first presented in Section II. These principles are then illustrated by the design of first-order, low-pass, high-pass, and all-pass filters. The simulation and performance results of these filters are presented in Section III. Finally, some conclusions are given in Section IV.

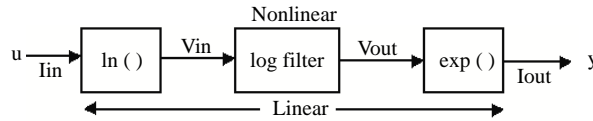
## 2. State-Space Synthesis of Log-Domain Filters

The state-space synthesis method is a very powerful and efficient approach for the synthesis of log-domain filters [4]. It provides a very general solution for realizing filter function. The key aspects of the use of state-space methods in this study are that it precisely relates internally nonlinear filters to equivalent linear systems. Linear state-space models can be used to realize any known externally linear filter [4,5]. A state-space formulation consists of a set of first-order differential equations. The state variables are equal to simple functions of exponentials of node voltages. There is a one-to-one correspondence between the mathematical formulation and the circuit realization. This facilitates systematic circuit implementation and makes debugging a simpler task. In addition, all of the capacitors in a current-mode implementation of a state-space filter have one terminal connected to ground. This makes the filter more suitable for IC implementation [16]. It was not until 1993 that a formal synthesis procedure, based on the state-space

description of the desired filter function, was introduced by Frey [4]. The state space synthesis method can be briefly reviewed and summarized for log-domain filters as follows [4,5,20]:

- Find an appropriate state-space description for the filter;
- Make an exponential mapping function to the input and state variables;
- Manipulate the equations to obtain a set of nodal equations;
- Design the circuit using transistors, grounded capacitors, and current sources.

State-space synthesis methods are based on the setup depicted in Figure 1, consisting of 3 essential parts. At the input, a single transistor is used to compress the input current,  $I_{in}$ , resulting in a logarithmically related voltage,  $V_{in}$ . Next, this voltage is filtered by means of a so-called “log-filter”. The resulting output voltage,  $V_{out}$ , is expanded exponentially, again, by a single transistor, into the output current,  $I_{out}$ .



**Figure 1.** Log-domain filter set-up.

The first step used to synthesize a mapped state-space filter is to obtain appropriate system equations [4,5,20].

$$\frac{d}{dt} \bar{x} = A\bar{x} + \bar{b}u \tag{1}$$

$$y = \bar{p}^T \bar{x} + du \tag{2}$$

The global input, output, and state vector are denoted by  $u$ ,  $y$ , and  $\bar{x}$ , respectively, where a bar above a quantity denotes a vector. A single input-single output case is chosen for the sake of clarity; however, it can be easily extended to multiple input and output cases. Eq. (1) and Eq. (2) give the transfer function of  $H(s)$  as follows [4,5,20]:

$$H(s) = \frac{Y(s)}{U(s)} = \bar{p}^T (sI - A)^{-1} \bar{b} + d \tag{3}$$

The second setup is the mapping. Apply the following mapping to the input and each state,

$$u = f(v_0)$$

$$x_i = f(v_i)$$

$$\text{where } i = 1, 2, \dots, N \tag{4}$$

Let us substitute these mapping functions with Eq. (4) into the first line of Eq. (1) and Eq. (2), and scale each line with

$$\frac{C_i}{\frac{d}{dv_i}f(v_i)} \tag{5}$$

where the  $C_i$ 's are arbitrary constants. We obtain the following equations:

$$C_i \dot{v}_i = \left[ \sum_{j=1}^N \frac{C_i A_{ij}}{\frac{d}{dv_i}f(v_i)} f(v_j) \right] + \frac{C_i b_i}{\frac{d}{dv_i}f(v_i)} f(v_0) \tag{6}$$

Eq. (6) may be interpreted as a set of nodal equations. In fact, if we let  $v_i$  denote the  $i$ -th node voltage in a circuit, then the left-hand side of Eq. (6) represents the current flowing into a grounded capacitor, which is tied to the  $i$ -th node. In the same manner, the right-hand side of Eq. (6) may be considered the sum of some currents flowing into this capacitor. These currents are a form of voltage-controlled current sources, or transconductances. If these transconductances can be realized by using some practical devices, then this set of nodal equations gives a real circuit [4,5,20]. The output can be realized using the same procedure. From Eq. (2) the output is the current flowing into the source. The node equation can be written as follows [4,5,20]:

$$y = \left[ \sum_{j=1}^N \frac{\widehat{p}_j}{\frac{d}{dv_{N+1}}f(v_{N+1})} f(v_j) \right] + \frac{\widehat{d}}{\frac{d}{dv_{N+1}}f(v_{N+1})} f(v_0) \tag{7}$$

$$\widehat{p}_j = p_j \frac{d}{dv_{N+1}}f(v_{N+1}) \tag{8}$$

$$\widehat{d} = d \frac{d}{dv_{N+1}}f(v_{N+1}) \tag{9}$$

The voltage,  $v_{N+1}$ , is introduced only for uniformity. It can be zero for the sake of simplicity. Also note that the elements of the  $p$  vector and  $d$  is also modified only to make the output equation similar to the rest of  $N$  equations. All we need to do to realize these  $N + 1$  nodal equations is find a nonlinear transconductance whose output current is given by [4,5,20]

$$i_{OUT} = K_{ij} \frac{f(v_j)}{\frac{d}{dv_i}f(v_i)} \tag{10}$$

where  $K_{ij}$  is a constant. The corresponding transconductances can be realized by using BJTs or FETs. If the function is an exponential function, the corresponding transconductances can be realized by BJTs since their voltage-current relationship is exponential.

### 2.1. State-Space Synthesis of a First-Order Log-Domain Low-Pass Filter

Several first-order low-pass filters have been proposed in the literature [1,3,4,12,16,18-20]. Realization of first-order log-domain low-pass filters using state-space synthesis techniques has been reported [20]. It introduced

a systematic state-space synthesis method for designing first-order low-pass log-domain filters, but can be rearranged and summarized for log-domain low-pass filters. We propose the design of a first-order low-pass filter with current-mode circuit realization that is systematically derived using the state-space synthesis procedure. Two different external currents,  $I_{f1}$  and  $I_{f2}$ , were used to realize filter function. The proposed filter has advantages with respect to other filter structures in that minimum components are used to realize filter function. Gain and cut-off frequency of the filter can be tuned electronically.

A first-order low-pass filter transfer function can be written as follows:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{I_{out}(s)}{I_{in}(s)} = a_1 \frac{\omega_0}{s + \omega_0}, \quad a_1 > 0 \quad (11)$$

where  $\omega_0$  is the cut-off frequency of the filter and  $a_1$  is the gain of the filter. Transfer function was transformed to the following equation:

$$\dot{y} = -\omega_0 y + a_1 \omega_0 u \quad (12)$$

Its state variables are determined by using the Companion-I form. State variable  $x$  is chosen as

$$x = y \quad (13)$$

Eq. (12) is arranged to form the following equation:

$$\dot{x} = -\omega_0 x + a_1 \omega_0 u \quad (14)$$

The output equation is

$$y = x \quad (15)$$

where  $u$  is the input,  $y$  is the output, and  $x$  is the state variable. Eq. (14) can be transformed into a set of nodal equations by using exponential mappings on the input and state variables. The following mappings can therefore be applied to quantities in the equation [4,5,20] (infinite  $\beta$  condition),

$$x = I_s e^{V_1/V_t}, \quad u = I_s e^{V_0/V_t} \quad (16)$$

where  $I_s$  is the saturation current and  $V_t$  is the thermal voltage,  $V_t = kT/q$ . The derivative of  $u$  and  $x$ ,

$$\dot{x} = I_s \frac{1}{V_t} \dot{V}_1 e^{V_1/V_t}; \quad \dot{u} = I_s \frac{1}{V_t} \dot{V}_0 e^{V_0/V_t} \quad (17)$$

The above relationship was applied to Eq. (14) and scaling factors are multiplied through the equation with  $CV_t/I_s e^{V_1/V_t}$ , and then it is arranged to form the following nodal equations:

$$C\dot{V}_1 = -\omega_0 CV_t + a_1 \omega_0 CV_t e^{\frac{V_0 - V_1}{V_t}} \quad (18)$$

where  $I_{f1}$  and  $I_{f2}$  are positive constants, which are defined by the following equations:

$$I_{f1} = \omega_0 CV_t, I_{f2} = a_1 \omega_0 CV_t \tag{19}$$

$$C\dot{V}_1 = -I_{f1} + I_{f2} e^{\frac{V_0 - V_1}{V_t}} \tag{20}$$

If  $I_{f2}$  is equal to  $I_s e^{V_{f2}/V_t}$ , Eq. (20) can be arranged as

$$C\dot{V}_1 = -I_{f1} + I_s e^{\frac{V_0 + V_{f2} - V_1}{V_t}} \tag{21}$$

The realization of first-order log-domain low-pass filter circuits using Eq. (21) is shown in Figure 2.

The cut-off frequency and gain of the filter are

$$\begin{aligned} \omega_0 &= I_{f1}/CV_t \\ a_1 &= I_{f2}/I_{f1} \end{aligned} \tag{22}$$

It should be noted that  $\omega_0$  can be electronically tuned by changing  $I_{f1}$  and  $a_1$  can also be electronically tuned by changing  $I_{f2}$ .

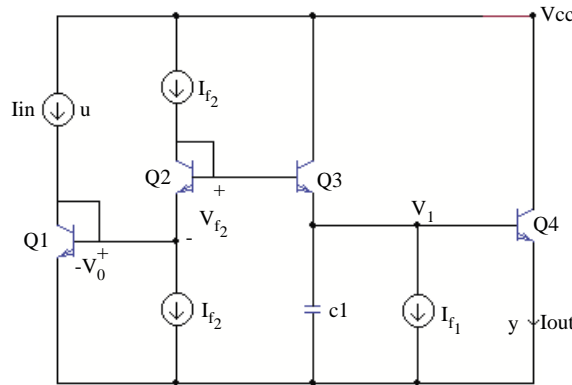


Figure 2. A first-order log-domain low-pass filter.

## 2.2. State-Space Synthesis of a First-Order Log-Domain High-Pass Filter

A literature survey shows that no log-domain first-order high-pass class-A filters realized with the state-space synthesis method exist. Realization of a first-order differential class-AB high-pass filter using log-domain techniques is reported [24]. In this study, gain and cut-off frequency of the filter can be tuned electronically. A first-order high-pass filter transfer function can be written as follows:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{I_{out}(s)}{I_{in}(s)} = -a_1 \frac{s}{s + \omega_0} \quad , \quad a_1 > 0 \tag{23}$$

where  $\omega_0$  is the cut-off frequency of the filter. Here,  $a_1$  must be greater than zero, otherwise this transfer function can not be realized with a class-A filter type. Transfer function is transformed to the following equation:

$$\dot{y} = -\omega_0 y - a_1 \dot{u} \quad (24)$$

Its state variables determined by using the companion form. In Eq. (24), the derivative of input  $u$  is a drawback for realizing the filter. The derivative of  $u$  must be eliminated from Eq. (24). State variable  $x$  is chosen as:

$$x = y + k_1 u \quad (25)$$

Eq. (24) is arranged to form the following equation:

$$\dot{x} = -\omega_0(x - k_1 u) + (k_1 - a_1)\dot{u} \quad (26)$$

where  $k_1$  is determined from Eq. (26),

$$k_1 = a_1$$

When Eq. (24) is simplified and the derivative of  $u$  is eliminated, it can be derived that

$$\dot{x} = -\omega_0 x + a_1 \omega_0 u \quad (27)$$

The output equation is

$$y = x - a_1 u \quad (28)$$

where  $u$  is the input,  $y$  is the output, and  $x$  is the state variable. Eq. (27) can be transformed into a set of nodal equations by using exponential mappings on the input and state variables. The following mappings can therefore be applied to quantities in Eq. (27), [4,5,20] (infinite  $\beta$  condition).

$$x = I_s e^{V_1/V_t}, \quad u = I_s e^{V_0/V_t} \quad (29)$$

The derivative of  $u$  and  $x$ ,

$$\dot{x} = I_s \frac{1}{V_t} \dot{V}_1 e^{V_1/V_t}; \quad \dot{u} = I_s \frac{1}{V_t} \dot{V}_0 e^{V_0/V_t} \quad (30)$$

The above relationship is applied to Eq. (27) and scaling factors are multiplied through the equation with  $CV_t/I_s e^{V_1/V_t}$ , and then it is arranged to form the following nodal equation:

$$C\dot{V}_1 = -\omega_0 CV_t + a_1 \omega_0 CV_t e^{\frac{V_0 - V_1}{V_t}} \quad (31)$$

where  $I_{f1}$  and  $I_{f2}$  are positive constants, which are defined by the following equations:

$$I_{f1} = \omega_0 CV_t, \quad I_{f2} = a_1 \omega_0 CV_t \tag{32}$$

$$C\dot{V}_1 = -I_{f1} + I_{f2} e^{\frac{V_0 - V_1}{V_t}} \tag{33}$$

If  $I_{f2}$  is equal to  $I_s e^{V_{f2}/V_t}$ , Eq. (46) can be arranged as

$$C\dot{V}_1 = -I_{f1} + I_s e^{\frac{V_0 + V_{f2} - V_1}{V_t}} \tag{34}$$

The cut-off frequency and the gain of the filter are:

$$\omega_0 = I_{f1}/CV_t \quad a_1 = I_{f2}/I_{f1} \tag{35}$$

It should be noted that  $\omega_0$  can be electronically tuned by changing  $I_{f1}$  and  $a_1$  can also be electronically tuned by changing  $I_{f2}$ .

If  $a_1$  is 1,  $I_{f1}$  is equal to  $I_{f2}$ . The realization of the first-order log-domain high-pass filter circuit using Eq. (34) is shown in Figure 3.

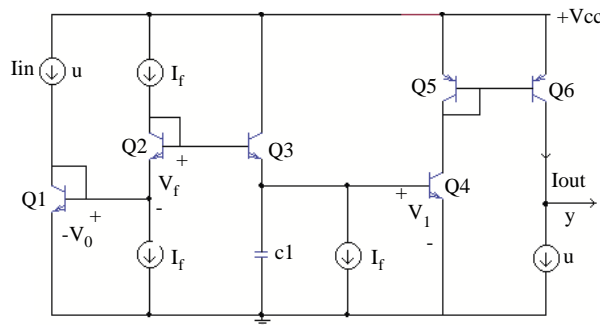


Figure 3. A first-order log-domain high-pass filter.

### 2.3. State-Space Synthesis of a First-Order Log-Domain All-Pass Filter

All-pass filters are among the most important building blocks of many analog signal processing applications, and have, therefore, received much attention. They are generally used for introducing a frequency-dependent delay while keeping the amplitude of the input signal constant over the desired frequency range. A first-order all-pass filter transfer function can be written as follows:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{I_{out}(s)}{I_{in}(s)} = -a_1 \frac{s - \omega_0}{s + \omega_0}, \quad a_1 > 0 \tag{36}$$

where  $\omega_0$  is the cut-off frequency of the filter. Here,  $a_1$  must be greater than zero, otherwise this transfer function cannot be realized with a class-A type filter. Transfer function is transformed to the following equation:



$$\dot{y} = -\omega_0 y + a_1 \omega_0 u - a_1 \dot{u} \quad (37)$$

Its state variables are determined by using the companion form. In Eq. (37), the derivative of input  $u$  is a drawback for realizing the filter. The derivative of  $u$  must be eliminated from Eq. (37). State variable  $x$  is chosen as

$$x = y + k_1 u \quad (38)$$

Eq. (37) is arranged to form the following equation:

$$\dot{x} = -\omega_0(x - k_1 u) + a_1 \omega_0 u + (k_1 - a_1) \dot{u} \quad (39)$$

where  $k_1$  is determined from Eq. (39),

$$k_1 = a_1$$

When Eq. (37) is simplified and the derivative of  $u$  is eliminated, it can be derived that

$$\dot{x} = -\omega_0 x + 2a_1 \omega_0 u \quad (40)$$

The output equation is

$$y = x - a_1 u \quad (41)$$

where  $u$  is the input,  $y$  is the output, and  $x$  is the state variable. Eq. (40) can be transformed into a set of nodal equations by using exponential mappings on the input and state variables. The following mappings, therefore, can be applied to quantities in Eq. (40), [4,5], (infinite  $\beta$  condition)

$$x = I_s e^{V_1/V_t}, \quad u = I_s e^{V_0/V_t} \quad (42)$$

The derivative of  $u$  and  $x$ ,

$$\dot{x} = I_s \frac{1}{V_t} \dot{V}_1 e^{V_1/V_t}; \quad \dot{u} = I_s \frac{1}{V_t} \dot{V}_0 e^{V_0/V_t} \quad (43)$$

The above relationship is applied to Eq. (40) and scaling factors are multiplied through the equation with  $CV_t/I_s e^{V_1/V_t}$ , and then it is arranged to form the following nodal equation:

$$C\dot{V}_1 = -\omega_0 CV_t + 2a_1 \omega_0 CV_t e^{\frac{V_0 - V_1}{V_t}} \quad (44)$$

where  $I_{f1}$  and  $I_{f2}$  are positive constants, which are defined by the following equations:

$$I_{f1} = \omega_0 CV_t, \quad I_{f2} = 2a_1 \omega_0 CV_t \quad (45)$$

$$C\dot{V}_1 = -I_{f1} + I_{f2}e^{\frac{V_0 - V_1}{V_t}} \quad (46)$$

If  $I_{f2}$  is equal to  $I_s e^{V_{f2}/V_t}$ , Eq. (46) can be arranged as,

$$C\dot{V}_1 = -I_{f1} + I_s e^{\frac{V_0 + V_{f2} - V_1}{V_t}} \quad (47)$$

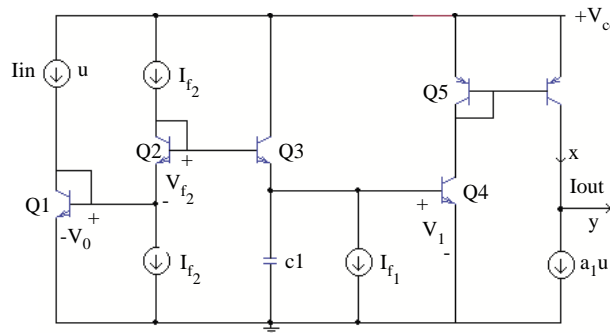
The realization of the first-order log-domain all-pass filter circuit using Eq. (47) is shown in Figure 4.

The resonant frequency and the gain of the filter are:

$$\omega_0 = I_{f1}/CV_t \quad (48)$$

$$a_1 = I_{f2}/2\omega_0 CV_t \quad (49)$$

It should be noted that  $\omega_0$  can be electronically tuned by changing  $I_{f1}$  and  $a_1$  can also be electronically tuned by changing  $I_{f2}$ .



**Figure 4.** A first-order log-domain all-pass filter.

The proposed all-pass filter has the following phase responses:

$$\varphi(\omega) = -2 \arctan\left(\frac{\omega}{\omega_0}\right) \quad (50)$$

Thus, the phase can be tuned by changing external currents.

### 3. Simulation Results

#### 3.1. Low-Pass Filter Simulation Results

The proposed low-pass filter was simulated by using both ideal and AT& T CBIC-R (NR200N-2X NPN), (PR200N-2X PNP) transistors. The circuit parameters are chosen as  $a_1 = 1$ ,  $V_{CC} = 3V$ ,  $I_{f1} = 100\mu A$ ,  $I_{f2} = 100\mu A$ , and  $C = 200pF$ . The cut-off frequency of the filter is  $f_0 = 3mHz$ . The gain and phase responses of a first-order log-domain low-pass filter are shown in Figures 5 and 6.

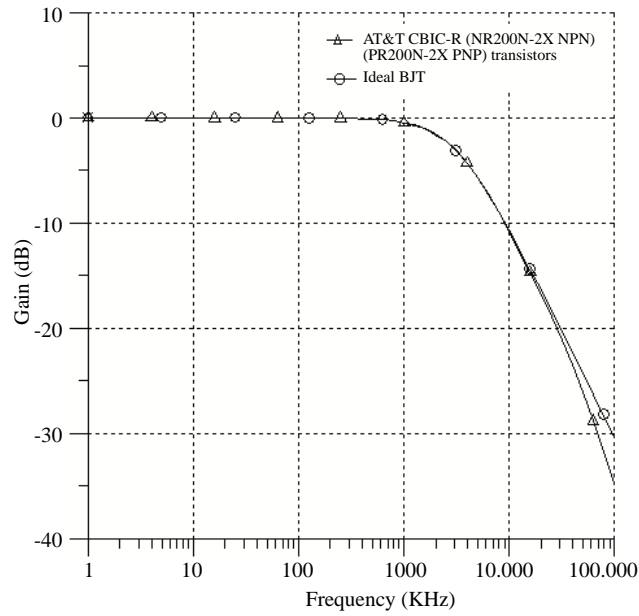


Figure 5. Gain response of a first order log-domain low-pass filter.

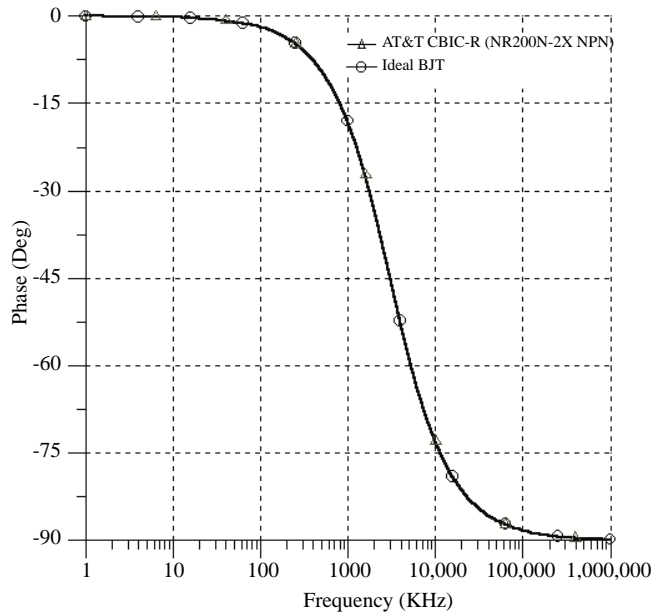


Figure 6. Phase response of a first order log-domain low-pass filter.

### 3.2. High-Pass Filter Simulation Results

The proposed high-pass filter was simulated by using both ideal and AT& T CBIC-R (NR200N-2X NPN), (PR200N-2X PNP) transistors. The circuit parameters are chosen as  $a_1 = 1$ ,  $V_{CC} = 3V$ ,  $I_{f1} = 100\mu A$ ,  $I_{f2} = 100\mu A$ ,  $C = 200pF$ . The cut-off frequency of the filter is  $f_0 = 3mHz$ . The gain and phase responses of a first-order log-domain high-pass filter are shown in Figures 7 and 8.

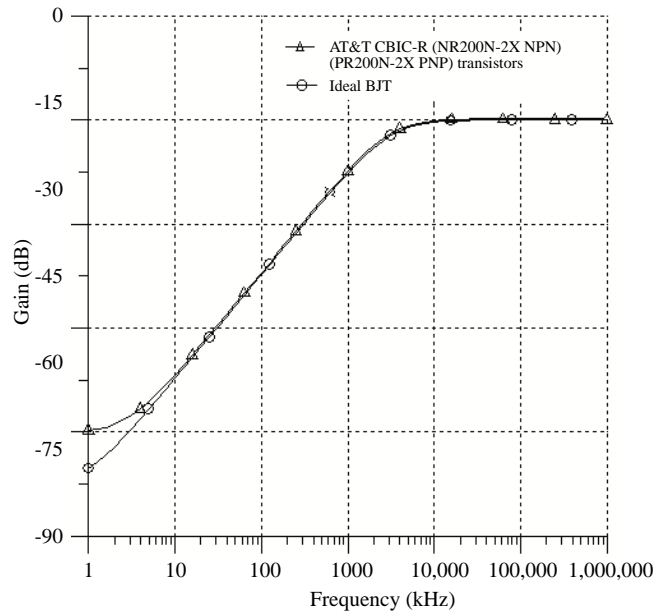


Figure 7. Gain response of a first order log-domain high-pass filter.

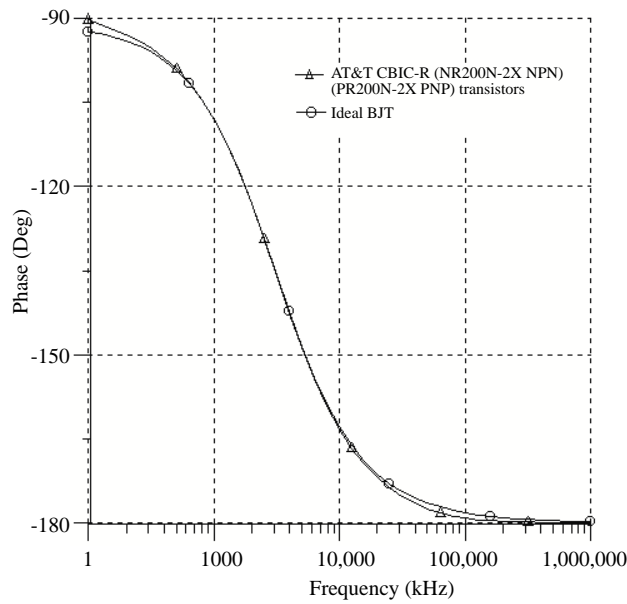
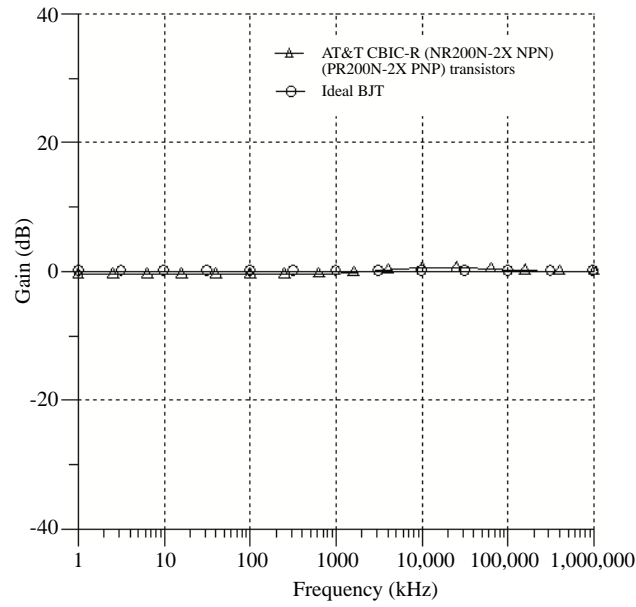


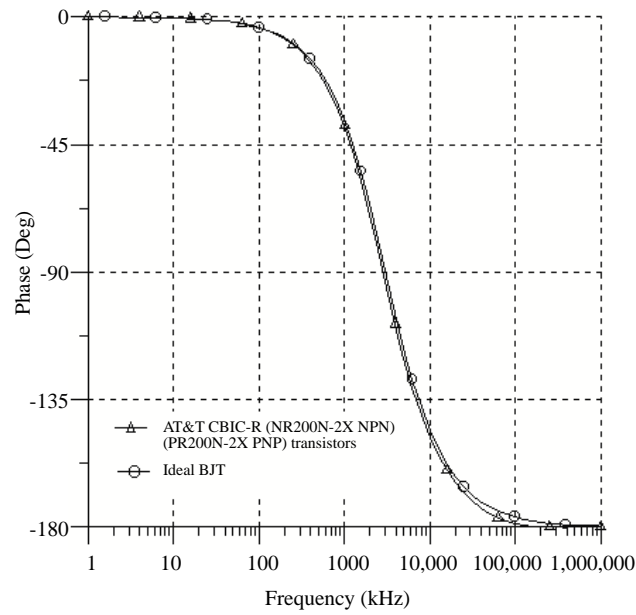
Figure 8. Phase response of a first order log-domain high-pass filter.

### 3.3. All-Pass Filter Simulation Results

The proposed all-pass filter was simulated by using both ideal and AT& T CBIC-R (NR200N-2X NPN), (PR200N-2X PNP) transistors. The circuit parameters are chosen as  $a_1 = 1$ ,  $V_{CC} = 3V$ ,  $I_{f1} = 100\mu A$ ,  $I_{f2} = 200\mu A$ ,  $C = 200pF$ . The cut-off frequency of the filter is  $f_0 = 3mHz$ . The gain and phase responses of a first-order log-domain all-pass filter are shown in Figures 9 and 10, respectively.



**Figure 9.** Gain response of first order log-domain all-pass filter.

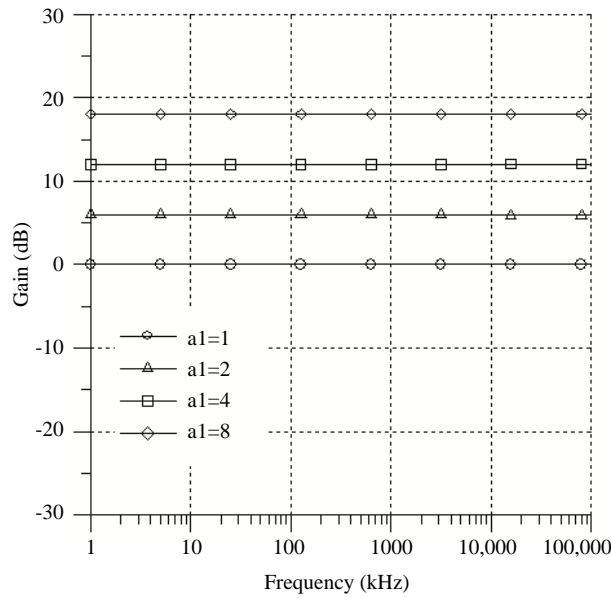


**Figure 10.** Phase response of a first order log-domain all-pass filter.

DC gain,  $a_1$ , tuning characteristics were observed by changing the external current as shown in Table 1 and Figure 11. For this property, the external currents were changed from  $I_{f2} = 200\mu A$  to  $I_{f2} = 1600\mu A$  and the gain was tuned from  $a_1 = 1$  to  $a_1 = 8$ .

**Table 1.** DC gain,  $a_1$ , tuning with varying the values of external current.

$a_1$	$f_0$	$I_{f1}$	$I_{f2}$	$C$
1	3 mHz	100 $\mu A$	200 $\mu A$	200 pF
2	3 mHz	100 $\mu A$	400 $\mu A$	200 pF
4	3 mHz	100 $\mu A$	800 $\mu A$	200 pF
8	3 mHz	100 $\mu A$	1600 $\mu A$	200 pF

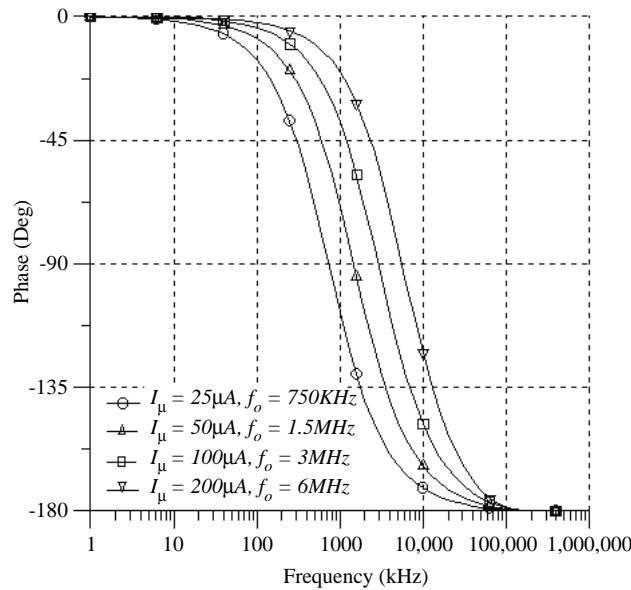


**Figure 11.** DC gain tuning characteristics of the proposed all-pass filter by changing the external current.

The center frequency characteristics were observed by changing the external current as shown in Table 2 and Figure 12. For this property, the external currents were changed from  $25\mu A$  to  $200\mu A$  and the filter cut-off frequency was tuned from  $750\text{kHz}$  to  $6\text{MHz}$ .

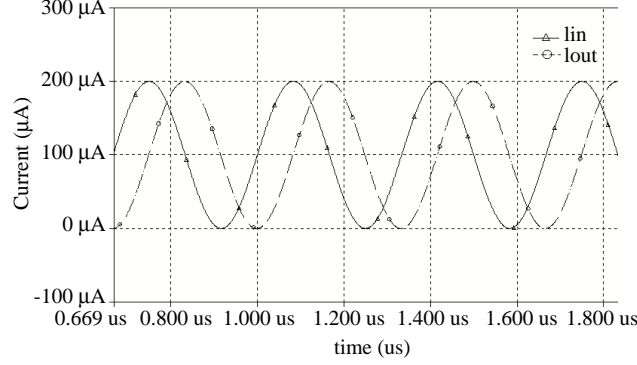
**Table 2.** The cut-off frequency tuning with varying the values of external currents.

$a_2$	$f_0$	$I_{f1}$	$I_{f2}$	$C$
1	750 kHz	25 $\mu A$	50 $\mu A$	200 pF
1	1.5 MHz	50 $\mu A$	100 $\mu A$	200 pF
1	3 MHz	100 $\mu A$	200 $\mu A$	200 pF
1	6 MHz	200 $\mu A$	400 $\mu A$	200 pF



**Figure 12.** Phase tuning of the proposed all-pass filter by changing the external currents.

Figure 13 shows the time-domain response of the filter. A sine-wave input at a frequency of 3 mHz was applied to the filter. This causes a 86 ns time delay at the output of the filter corresponding to 93° phase difference, which is close to the theoretical value (90°).



**Figure 13.** Time-domain response of the proposed all-pass filter.

The all-pass output signal's THD (total harmonic distortion, %) was measured with different input current values by using both ideal and AT& T CBIC-R (NR200N-2X NPN and PR200N-2X PNP) transistors. The filter was set to 3MHz cut-off frequency with  $I_{f1} = 100\mu A$  and the input frequency was also set to this value. Then, a sinusoidal signal was applied to the filter with different input currents, 5uA, 10uA, 20uA, 40uA, and 100uA. The results of THD (%) obtained for ideal BJTs and real BJTs are shown in Table 3. THD results indicate a good linearity response of this type of filter structure.

**Table 3.** Total harmonic distortion (%).

Input	Ideal BJT	NR200N-2X
5 uA	0.0004	0.117
10 uA	0.0004	0.139
20 uA	0.0008	0.298
40 uA	0.0016	0.614
100 uA	0.0041	1.584

Log-domain filters suffer directly from transistor-level non-idealities, such as parasitic emitter and base resistance, finite  $\beta$ , Early voltages, and area mismatches. Currently, there are several simple electronic compensation schemes, which have been proposed to correct these transistor nonidealities [12,16,24]. The tolerable differences observed indicate that realization of these filters in simulations has provided satisfactory results.

## Conclusion

First-order current-mode log-domain low-pass, high-pass, and all-pass filter structures are presented. A systematic synthesis procedure to derive the filter circuit is also given. PSPICE simulations are provided to confirm the theoretical analysis. The presented filters have the following advantages:

Realizing a linear system with inherently nonlinear (logarithmic-exponential)

- i) circuit building blocks;

- ii) Can be electronically tuned;
- iii) A wide bandwidth;
- iv) A large dynamic range;
- v) Employ only BJTs and a grounded capacitor;
- vi) A very simple structure;
- vi) Suitable for VLSI (very large-scale integration) technologies;
- vii) Suitable for low voltage/power applications.

The small deviations in the gain and frequency response from theoretical values are caused by the nonidealities of the BJT, such as finite-beta, nonzero ohmic junction resistances, and Early voltages. It is expected that the proposed current-mode log-domain first-order filters will be useful in the design of analog signal processing applications.

## Appendix

### The SPICE model used for the AT& T CBIC-R transistors:

\*NR200N-2X NPN TRANSISTOR

```
.MODEL NX2 NPN (Rb=262.5 Irb=0 Rbm=12.5 Rc=25 Re=0.5 Bf=137.5
+ Is=242E-18 Eg=1.206 Xti=2 Xtb=1.538 Ikf=13.94E-3 Br=0.7258 Nf=1
+ Vaf=159.4 Ise=72E-16 Ne=1.713 Ikr=4.396E-3 Nr=1.0 Var=10.73 Isc=0 Nc=2
+ Tf=0.43E-9 Tr=0.43E-8 Cje=0.428E-12 Vje=0.5 Mje=0.28 Cjc=1.97E-13
+ Vjc=0.5 Mjc=0.3 Xcjc=0.065 Cjs=1.17E-12 Vjs=0.64 Mjs=0.4 Fc=0.5)
```

\*PR200N-2X PNP TRANSISTOR

```
.MODEL PX2 PNP (Rb=163.5 Irb=0 Rbm=12.27 Rc=25 Re=1.5 Bf=110
+ Is=147E-18 Eg=1.206 Xti=1.7 Xtb=1.866 Ikf=4.718E-3 Br=0.04745 Nf=1
+ Vaf=51.8 Ise=50.2E-16 Ne=1.65 Ikr=12.96E-3 Nr=1.0 Var=9.96 Isc=0 Nc=2
+ Tf=0.610E-9 Tr=0.610E-8 Cje=0.36E-12 Vje=0.5 Mje=0.28 Cjc=0.328E-13
+ Vjc=0.8 Mjc=0.4 Xcjc=0.074 Cjs=1.39E-12 Vjs=0.55 Mjs=0.35 Fc=0.5)
```



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