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# New Method for Investigation of Dynamic Parameters of Thermoelectric Modules

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**Abstract**

*Precise calculation of parameters of thermoelectric modules and thermoelectric devices under operating conditions by present methods is very difficult. In this study, a new method is developed to calculate all parameters of thermoelectric modules. This new method makes it possible to determine the dynamic parameters of a real thermoelectric module operated under different working regimes. Measurement of thermoemf created by an operating module is the basis of this new method. An unloaded thermoelectric module, whose dynamic output parameters are necessary in the design of a medical helmet for the cooling of brain, has been investigated using this new method. A special device has been designed and realized to be used in these investigations.*

**Key Words:** *Thermoelectric module, thermoelectric properties, method, measurement, dynamic parameter, test.*

## 1. Introduction

At the basis of thermoelectric devices, there is a module that consists of a thermoelement. The thermoelement consists of a positive (p) and negative (n) terminals (semiconductor's thermoelement) to which metallic conductors are attached. More than one pair of thermoelement are usually assembled together to form a thermoelectric module. A single-stage module consists of several thermoelements connected thermally in parallel and electrically in serial to increase the operating voltage of the module. All parameters of the modules that will be used to design thermoelectric devices must be calculated beforehand.

The microparameters of a semiconductor and the resistivity, Seebeck coefficient, thermal conductivity and the figure of merit of a module can be evaluated by many known methods [1-12]. The thermal parameters of a module can be obtained by virtue of these parameters. However these parameters are typically calculated while the module is not operated. These methods, while evaluating the thermal parameters of a working module, can lead to incorrect results. Therefore there is a need in a new approach for calculating the values of a working module.

To find the dynamic output parameters of a module, the microparameters of a semiconductor must be found while the module is working. Unfortunately it is very difficult to find the microparameters of semiconductors in a working module. To circumvent this problem, we proposed a new method that uses easily measurable macroparameters.

## 2. Method

Known equations from the literature are the basis of this newly developed method. But these equations transformed using equivalent changes.

The general form of a thermal balance equation of the cooling and warming surfaces of a thermoelectric module is given below in Eqs. (1), [13, 14]

$$Q_C = \bar{\alpha} I T_C - 0.5 I^2 R - K(T_H - T_C) - Q_L$$

$$Q_H = \bar{\alpha} I T_H + 0.5 I^2 R - K(T_H - T_C) \tag{1}$$

Here, $Q_C$ (W) and $Q_H$ (W) are the cooling rate of the cold side and heat rejection rate of the hot side of the module respectively and $R = \left(\frac{1}{\sigma_n} + \frac{1}{\sigma_p}\right)\frac{h}{a}$ and $K = (\lambda_n + \lambda_p)\frac{a}{h}$ are the electrical resistance and heat conductance of the module, $\alpha_n$, $\sigma_n$, $\lambda_n$ and $\alpha_p$, $\sigma_p$, $\lambda_p$ are the Seebeck coefficients, electrical and heat conductivity of the n type and p type thermoelement materials, h and a are the length and cross-sectional area of the thermoelements, $\bar{\alpha} = (|\alpha_p| + |\alpha_n|)$ is the total Seebeck coefficient of the thermoelements. Furthermore $(T_H - T_C) = \Delta T$ is the temperature difference between the hot and cold sides, $Q_L$ (W) = $Q_{Rad}$ (W) + $Q_{Conv}$ (W) is the total heat loads, $Q_{Rad}$ is the radiation heat load and $Q_{Conv}$ is the convection heat load.

P (W), the module's electrical input power, is described as

$$P = I^2 R + \bar{\alpha}\,(T_H - T_C)\,I \tag{2}$$

The COP (the coefficient of performance) of the module is similarly derived as

$$COP = \frac{Q_C}{P} \tag{3}$$

The figure of merit of the module can be written as (3):

$$Z = \frac{\bar{\alpha}^2}{RK} \tag{4}$$

These equations form the basis of the calculation of the thermal parameters of a thermoelectric module or the coolers that are made of the thermoelectric module. Calculations that are made by using this formulas lead to large inaccuracies [15, 16], because the obtained parameters are not the dynamic parameters of a module under real operating circumstances. Evaluation of output parameters of an ideal working module gives exaggerated values. The reason for this is that microparameters like $\alpha$, $\sigma$, $\lambda$, z change not only according to the heat load, but also according to the height of the thermoelements. Since, the structure and technological factors of a module does not taken into consideration in these formulas, they are insufficient [17, 21]. From the point of view of usability this method is not convenient, as it requires the heat values $T_H$ and $T_C$ of every thermoelements to be calculated separately. In this case two sensors have to be used for each thermoelement. Considering that each module consists of tens or hundreds of thermoelements, it is clear that the cost for suggested method will be too high and its application will be complicated. It is also difficult to calculate the total heat loads $Q_L$ that reaches the module by using the equations (1)-(4), because in most cases it is impossible to measure directly the cold side temperature ($T_C$) and the heat conductance (K) in a working module.

In the light of these remarks a method to determine the real thermal features of a thermoelectric module has both - low cost and practical application, which is important from theoretical and practical point of view. The new method is based on equations (1)-(4). But these equations are deformed by equal differences. As a result the thermal parameters of the thermoelectric modules can only be found by measuring the applied current, I and thermoelectric power, E that is produced. The current and the thermoelectric power can be measured with great sensitivity and ease.

As it is known for an ideal thermoelectric module that works without load, when $Q_L = 0$ then $Q_C = 0$. Therefore the temperature difference between the hot and cold sides equals $\Delta T_{\max}$, and the temperature of the cold side equals $T_{C\min}$. In such a situation the applied current is $I_{\max}$ and voltage on the module is $V_{\max}$. Here from [14, 17] it is

$$V_{\max}= \bar{\alpha} T_H \tag{5}$$

Whereas the maximum voltage from [17] is

$$V_{\max}= I_{\max}R + \bar{\alpha}\Delta T_{\max}= I_{\max}R + E_{\max} \tag{6}$$

Here

$$\bar{\alpha}\Delta T_{\max}= \bar{\alpha}(T_H - T_{C\min}) = E_{\max} \tag{7}$$

As a result there will be

$$R = \frac{V_{\max} - E_{\max}}{I_{\max}} \tag{8}$$

Under these conditions the thermal balance will be:

$$\bar{\alpha} I_{\max} T_{C\min} - 0.5 I_{\max}^2 R - K\Delta T_{\max}= 0 \tag{9}$$

Also from the formula (7)

$$T_{C\min}= T_H - \frac{E_{\max}}{\bar{\alpha}} \tag{10}$$

is found. When we put these values (8) and (10) on their places in Eq. (9)

$$\left(V_{\max} - E_{\max}\right) I_{\max} - 0.5\left(V_{\max} - E_{\max}\right) I_{\max}= \left(\frac{KE_{\max}}{\bar{\alpha}}\right) \tag{11}$$

An equation will be obtained, from which K can be extracted as:

$$K = \frac{0.5\bar{\alpha}\left(V_{\max} - E_{\max}\right) I_{\max}}{E_{\max}} = \frac{0.5V_{\max}\left(V_{\max} - E_{\max}\right) I_{\max}}{T_H E_{\max}} \tag{12}$$

If equations (8), and (12) are used in their places in Eq.1 and if we assume $Q_L = 0$, then $Q_C$ of the module will be:

$$Q_C= \bar{\alpha} T_C I - \frac{0.5I^2\left(V_{\max} - E_{\max}\right)}{I_{\max}} - \frac{\bar{\alpha}\Delta T\left(V_{\max} - E_{\max}\right) I_{\max}}{E_{\max}} \tag{13}$$

and because the thermoelectric power of the module is:

$$E = \bar{\alpha}\Delta T = \bar{\alpha}(T_H - T_C) \tag{14}$$

and the temperature of the cold side will be:

$$T_C = T_H - \frac{E}{\bar{\alpha}} \tag{15}$$

and from (13) and (15) the Q$_C$ is found as:

$$Q_C = \bar{\alpha}(T_H - \frac{E}{\bar{\alpha}})I - \frac{0.5I^2\left(V_{\max} - E_{\max}\right)}{I_{\max}} - \frac{E\left(V_{\max} - E_{\max}\right)I_{\max}}{E_{\max}} \tag{16}$$

The hot side temperature of the module is T$_H$, which depends on the way of its cooling and generally it is held as constant. The hot side temperature of the working module is always nearly equal to the temperature of the material used in the heat transfer system as a heat transporter. The value changes of the electrical current intensity I of the thermal load affect the value T$_H$ very little. That is why we can use as the first approach formula (5) used for I$_{\max}$ in formula (16). Under this condition it can be written as:

$$\begin{aligned} Q_C &= V_{\max}I - \frac{0.5I^2\left(V_{\max} - E_{\max}\right)}{I_{\max}} - \left[I + \frac{0.5\left(V_{\max} - E_{\max}\right)I_{\max}}{E_{\max}}\right]E \\ Q_H &= V_{\max}I + \frac{0.5I^2\left(V_{\max} - E_{\max}\right)}{I_{\max}} - \frac{0.5\left(V_{\max} - E_{\max}\right)I_{\max}E}{E_{\max}} \end{aligned} \tag{17}$$

Also the electrical input power of the module can be calculated as:

$$P = \frac{I^2\left(V_{\max} - E_{\max}\right)}{I_{\max}} + EI \tag{18}$$

The COP of module will be:

$$COP = \frac{Q_C}{P} = \frac{V_{\max}I - \frac{0.5I^2\left(V_{\max} - E_{\max}\right)}{I_{\max}} - \left[I + \frac{0.5\left(V_{\max} - E_{\max}\right)I_{\max}}{E_{\max}}\right]E}{\frac{I^2\left(V_{\max} - E_{\max}\right)}{I_{\max}} + IE} \tag{19}$$

Additionally, the figure of merit of the module according to the Eqs (5), (8), and (12) can be written as:

$$Z = \frac{V_{\max}E_{\max}}{0.5\left(V_{\max} - E_{\max}\right)^2 T_H} \tag{20}$$

It is difficult to measure the side temperature of any surface of the module, especially in case, when the module is a part of any other thermoelectrical device. In this case one thermocouple should be placed on every side of a module and outlets of the thermocouple should be put from the outside of device. Nevertheless it is easier to measure the hot side temperature, T$_H$ of the module, because this surface is always outside of the device.

Following from the equations (5) and (10) relation between the cold and hot side temperatures can be written as:

$$T_C = T_H\left(1 - \frac{E}{V_{\max}}\right). \tag{21}$$

According to this formula we can find the cooling surface temperature T$_C$, eliminating the need for measuring it directly, but by measuring at any time only T$_H$ and E.

From the same equation T$_H$ can be found by using the values T$_C$ and E.

$$T_H = \frac{T_C}{\left(1 - \dfrac{E}{V_{\max}}\right)} \tag{22}$$

The dependence of thermoelectric power E of current intensity and Q$_C$ is expressed by the formula written below:

$$E = \frac{V_{\max} I - \dfrac{0.5 I^2 \left(V_{\max} - E_{\max}\right)}{I_{\max}} - Q_C}{I + \dfrac{0.5 \left(V_{\max} - E_{\max}\right) I_{\max}}{E_{\max}}} \tag{23}$$

The heat equations (8), (12), (13), (17) and (23) give all the parameters any time during the operation of the module goes on. In order to use these equations the values I$_{\max}$, V$_{\max}$ and E$_{\max}$ have to be found. These values are different for each module and can be named the experimental parameters of the module. Also to use these equations we have to measure one of the temperature values -T$_C$ or T$_H$- directly. Here V$_{\max}$ and E$_{\max}$ characterize the semiconductor materials of the thermoelement that are used to build the module. They are not related to the geometric factor (G, of the element as the cross-sectional area divided by the length G = A/h) of thermoelement and they form the macro size of the module. These values can be easily measured with a large sensitivity.

The formulas (17) – (21) express the thermal output parameters of a working thermoelement or a thermoelectric module. The analytic expressions Q$_C$ = Q$_C$ (I,E), Q$_H$ = Q$_C$ (I,E), P = P(I,E), COP = COP(I,E), T$_C$ = T$_C$(I,E) of the functions will change according to the applied current and the working regime of the module. For example, if the current is I = I$_{\max}$ and T$_C$ = T$_{C\min}$, and the working regime and output parameters of the module are Q$_C$ = 0, E = E$_{\max}$, then:

$$\begin{aligned}
&Q_C = 0\\
&Q_H = V_{\max} I_{\max}\\
&P = V_{\max} I_{\max}\\
&COP = 0\\
&T_{C\min} = T_H \left(1 - \frac{E_{\max}}{V_{\max}}\right)\\
&Z = \frac{V_{\max} E_{\max}}{0.5 \left(V_{\max} - E_{\max}\right)^2 T_H}
\end{aligned} \tag{24}$$

According to the module's Q$_{C\max}$ working regime, T$_C$ = T$_H$ and $\Delta T = 0$ and because $E = \bar{\alpha}\Delta T$ = 0, the formula will be like this:

$$
\begin{aligned}
&Q_{C\max}= 0.5I_{\max}\left(V_{\max}+E_{\max}\right)\\
&Q_{Hmax}= 0.5I_{\max}(3V_{\max}-E_{\max})\\
&P_{\min}= I_{\max}(V_{\max}-E_{\max})\\
&COP_{\max}= 0.5+\frac{E_{\max}}{V_{\max}-E_{\max}}=\frac{V_{\max}}{V_{\max}-E_{\max}}-0.5\\
&T_C= T_H\\
&Z=\frac{V_{\max}E_{\max}}{0.5\left(V_{\max}-E_{\max}\right)^2 T_H}
\end{aligned}
\tag{25}
$$

If the applied current is I< I$_{\max}$, the equations for these two regimes are as follows: When module T$_{C\min}$ is being operated these formulas are:

$$
\begin{aligned}
&Q_C= 0\\
&Q_H= V_{\max}I+\frac{0.5I^2\left(V_{\max}-E_{\max}\right)}{I_{\max}}-\frac{0.5\left(V_{\max}-E_{\max}\right)I_{\max}E^I_{\max}}{E_{\max}}\\
&P=\frac{I^2\left(V_{\max}-E_{\max}\right)}{I_{\max}}+IE^I_{\max}\\
&COP=0\\
&T^I_{C\min}=T_H\left(1-\frac{E^I_{\max}}{V_{\max}}\right)\\
&Z=\frac{V_{\max}E_{\max}}{0.5\left(V_{\max}-E_{\max}\right)^2 T_H}=\frac{E_{\max}E^I_{\max}}{0.5\left(V_{\max}-E_{\max}\right)^2\Delta T}\\
&E^I_{\max}=\frac{V_{\max}I-\dfrac{0.5I^2\left(V_{\max}-E_{\max}\right)}{I_{\max}}}{I+\dfrac{0.5\left(V_{\max}-E_{\max}\right)I_{\max}}{E_{\max}}}
\end{aligned}
\tag{26}
$$

When the module provides T$_{C\min}$ for every current that is smaller than I$_{\max}$, the maximum values that it is going to produce are shown as a thermoelectric power, $E^I_{\max}$. In the working regime of Q$_{C\max}$, the thermal parameters of the module are calculated as follows:

$$
\begin{aligned}
&Q^{I}_{C\max}= V_{\max}I-\frac{0.5I^2\,(V_{\max}-E_{\max})}{I_{\max}}\\
&Q_H= V_{\max}I+\frac{0.5I^2\,(V_{\max}-E_{\max})}{I_{\max}}\\
&P =\frac{I^2\,(V_{\max}-E_{\max})}{I_{\max}}\\
&COP =\frac{V_{\max}I_{\max}}{I(V_{\max}-E_{\max})}-0.5\\
&T_C= T_H\\
&Z = \frac{V_{\max}E_{\max}}{0.5\,(V_{\max}-E_{\max})^2\,T_H}
\end{aligned}
\tag{27}
$$

When I< $\mathrm{I}_{\max}$ the COP value of $\mathrm{Q}_{C\max}$ is bigger than I $=\mathrm{I}_{\max}$ and theoretically I→ 0 then COP→ ∞. Also in this working regime for each current we can talk about the value$Q^{I}_{C\max}$. As can be seen from the equations (24) – (27) the value of Z stays the same as in the both regimes.

To prove that the formulas (17) – (21) are equivalent to the equations (1) - (4) we have each of thermal parameters of an ideal isolated module to be calculated with these formulas and the results should be compared. To achieve this, a thermoelement can be taken where K = $36.10^{-3}$ W / °C, Z = 1. 8. $10^{-3}$/ °C, $\bar{\alpha}$ = 380 $\mu$V / °C, and which is made of semiconductors that have equal resistivities with $\rho$ = (1/900) cm$\Omega$ and a size of h = 0. 4 cm, a = 0, 5 cm$^2$ and R = 2$\rho$h/A = 0.00222$\Omega$. Before calculating the parameters of the module, we obtain the output parameters like $\mathrm{V}_{\max}$, $\mathrm{E}_{\max}$ and $\mathrm{I}_{\max}$ of the module in the $\mathrm{T}_{C\min}$ working regime. For an ideal module, if $\mathrm{Q}_C$ = 0 and I = $\mathrm{I}_{\max}$, then $\mathrm{T}_C = \mathrm{T}_{Cmin.}$ Also according to [17]:

$$T_{C\min}=\frac{\sqrt{1+2ZT_H}-1}{Z}\tag{28}$$

Because the hot side temperature of the modules is 16 °C or $\mathrm{T}_H$ = 273 + 16 = 289K, $\mathrm{T}_{C\min}$ will be $\mathrm{T}_{C\min}$ = 238K or be –35 °C. From here $\Delta T_{\max}$ = 51K will be found. At this point according to (5), (7), (8), (12) and (20) values such as $\mathrm{V}_{\max} = \bar{\alpha}\mathrm{T}_H$ = 0,10982V, $\mathrm{E}_{\max} = \bar{\alpha}\Delta T_{\max}$ = 0.01938V, $\mathrm{I}_{\max}$ = ($\mathrm{V}_{\max}$ – $\mathrm{E}_{\max}$) / R = 40.7 A , K = $36.10^{-3}$ W / °C and Z = $1.8.10^{-3}$/ °C are gotten. Also according to (24)

$$T_{C\min}=T_H\left(1-\frac{E_{\max}}{V_{\max}}\right)\tag{29}$$

and

$$\Delta T_{\max}= (T_H-T_{C\min}) =\frac{T_H E_{\max}}{V_{\max}}\tag{30}$$

When $\mathrm{T}_{C\min}$ = 238K or –35 °C $\Delta T_{\max}$ = 51 will be obtained and from formula (25) $\mathrm{Q}_{C\max}$ = 2,63W will be found. It is seen that the values K and Z of the used semiconductors have been same, as obtained by calculation and two methods give the same results for both - $\mathrm{T}_{C\min}$ and $\Delta T_{\max}$ as well. First we have to calculate all parameters of an ideal module $\mathrm{Q}_C$ = 0, I = $\mathrm{I}_{\max}$, $\mathrm{T}_C = \mathrm{T}_{C\min}$ and $\mathrm{T}_C = \mathrm{T}_H$, $\Delta T$ = 0,$E = \bar{\alpha}\Delta T$ = 0, $\mathrm{Q}_C = \mathrm{Q}_{C\max}$ in two different working regimes by using the formulas (1) - (4) and (24), (25). The results of these calculations are shown in Table 1.

**Table 1**

| Working Regime | According to Basic (1) - (4) Equations | | | | | | |
|---|---|---|---|---|---|---|---|
| | Qc,W | $Q_H$,W | P,W | COP | $T_C$,$^0$C | ΔT,$^0$C | $10^{-3}$Z / $^0$C |
| $T_{Cmin}$,E = $E_{max}$, $Q_C = 0$ | 0 | 4,47 | 4,47 | 0 | -35 | 51 | 1,8 |
| Qcmax, $T_{C=}T_H$, E = 0 | 2,63 | 6,31 | 3,68 | 0,7 | 16 | 0 | 1,8 |
| | According to Obtained Equations (1) - (4) | | | | | | |
| | Qc,W | $Q_H$,W | P,W | COP | $T_C$,$^0$C | E,V | $10^{-3}$Z / $^0$C |
| $T_{Cmin}$,E = $E_{max}$, $Q_C = 0$ | 0 | 4,47 | 4,47 | 0 | -35 | 0,01938 | 1,8 |
| Qcmax, $T_{C=}T_H$, E = 0 | 2,63 | 6,31 | 3,68 | 0,7 | 16 | 0 | 1,8 |

In Table 2, the parameter calculation results of an ideal module working in two different regimes for different currents are given according to the formulas (1) - (4).

**Table 2**

| Working Regime | According to Basic (1) - (4) Equations | | | | | | |
|---|---|---|---|---|---|---|---|
| Current, A | Qc,W | $Q_H$,W | P,W | COP | $T_C$, °C | ΔT, °C | $10^{-3}$Z / °C |
| I = 10A, $T_{C\min}$ | 0 | 0,32 | 0,32 | 0 | -9 | 25 | 1,8 |
| I=10A, Qcmax | 0,99 | 1,21 | 0,22 | 4,4 | 16 | 0 | 1,8 |
| I = 15A, $T_{C\min}$ | 0 | 0,69 | 0,69 | 0 | -17 | 33 | 1,8 |
| I=15A, Qcmax | 1,40 | 1,90 | 0,50 | 2,8 | 16 | 0 | 1,8 |
| I = 20A, $T_{C\min}$ | 0 | 1,19 | 1,19 | 0 | -24 | 40 | 1,8 |
| I=20A, Qcmax | 1,75 | 2,64 | 0,89 | 2,0 | 16 | 0 | 1,8 |
| I = 25A, $T_{C\min}$ | 0 | 1,82 | 1,82 | 0 | -29 | 45 | 1,8 |
| I=25A, Qcmax | 2,05 | 3,44 | 1,39 | 1,5 | 16 | 0 | 1,8 |
| I = 30A, $T_{C\min}$ | 0 | 2,55 | 2,55 | 0 | -32 | 48 | 1,8 |
| I=30A, Qcmax | 2,3 | 4,29 | 2,00 | 1,1 | 16 | 0 | 1,8 |
| I = 35A, $T_{C\min}$ | 0 | 3,39 | 3,39 | 0 | -34 | 50 | 1,8 |
| I=35A, Qcmax | 2,5 | 5,20 | 2,72 | 0,9 | 16 | 0 | 1,8 |

In Table 3, the parameter calculation results of an ideal module working in two different working regimes for different currents are given according to the formulas (26), (27).

**Table 3**

| Working Regime | According to Basic Equations (26), (27) | | | | | | |
|---|---|---|---|---|---|---|---|
| Current, A | Qc,W | $Q_H$,W | P,W | COP | $T_C$, °C | E,V | $10^{-3}$Z / °C |
| I = 10A, $T_{C\min}$ | 0 | 0,32 | 0,32 | 0 | -9 | 0,00940 | 1,8 |
| I=10A, Qcmax | 0,99 | 1,21 | 0,22 | 4,4 | 16 | 0 | 1,8 |
| I = 15A, $T_{C\min}$ | 0 | 0,69 | 0,69 | 0 | -17 | 0,01271 | 1,8 |
| I=15A, Qcmax | 1,40 | 1,90 | 0,50 | 2,8 | 16 | 0 | 1,8 |
| I = 20A, $T_{C\min}$ | 0 | 1,19 | 1,19 | 0 | -24 | 0,01524 | 1,8 |
| I=20A, Qcmax | 1,75 | 2,64 | 0,89 | 2,0 | 16 | 0 | 1,8 |
| I = 25A, $T_{C\min}$ | 0 | 1,82 | 1,82 | 0 | -29 | 0,01710 | 1,8 |
| I=25A, Qcmax | 2,05 | 3,44 | 1,39 | 1,5 | 16 | 0 | 1,8 |
| I = 30A, $T_{C\min}$ | 0 | 2,55 | 2,55 | 0 | -32 | 0,01836 | 1,8 |
| I=30A, Qcmax | 2,3 | 4,29 | 2,00 | 1,1 | 16 | 0 | 1,8 |
| I = 35A, $T_{C\min}$ | 0 | 3,39 | 3,39 | 0 | -34 | 0,01910 | 1,8 |
| I=35A, Qcmax | 2,5 | 5,20 | 2,72 | 0,9 | 16 | 0 | 1,8 |

From these three tables, we can see that the results obtained from the equations (1) - (4) are same as the results of the formulas (24) - (27). In other words, (1) - (4) and (24) - (27) are identical. So, by using the expressions of (24) - (27) and by measuring only the entry parameters I and $T_H$ and the output parameter E of the module, all electrical and thermal parameters can be measured.

In Figure 1 and Figure 2 we can see graphics of dependence to thermal parameters, obtained by formulas (24) – (27) for an ideal thermoelectric module working in two different regimes and different currents from thermoelectric power.

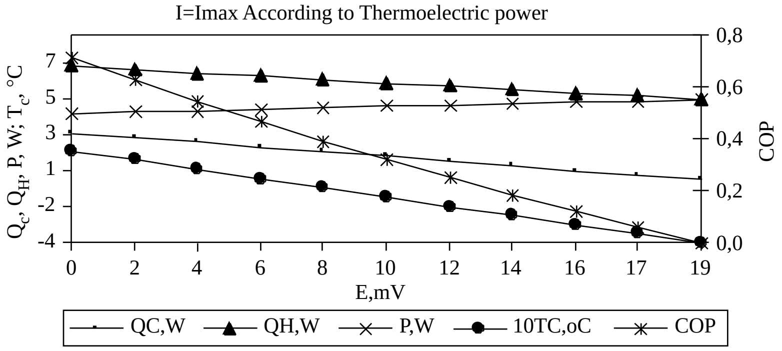

**Figure 1.** The Change of the Thermal Parameters for I = $I_{\max}$ According to Thermoelectric Power.

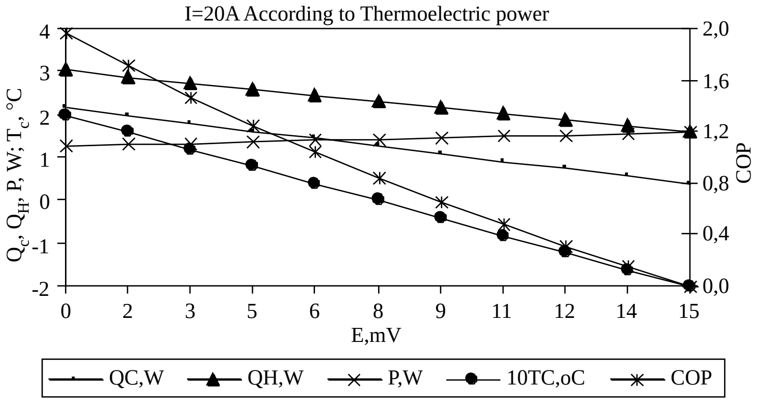

**Figure 2.** The Change of the Thermal Parameters for I=20A According to Thermoelectric Power.

Here, as an example, graphics are shown for I = $I_{\max}$ = 40.7A and I = 20A.

The values that are on the E = 0 axis express the working regime of $Q_{C\max}$ for the thermoelement, the values that are on the E = $E_{\max}$ axis show the values of the working regime of $T_C = T_{C\min}$. As it can be seen, in the working regime $Q_{C\max}$: E = 0, $Q_C = Q_{C\max}$, $Q_H = Q_{Hmax}$, P = $P_{\min}$, COP = $COP_{\max}$ and $T_C = T_H$, but in the working regime $T_{C\min}$: E = $E_{\max}$, $Q_C$ = 0, $Q_H = Q_{Hmin}$, P = $P_{\max}$, COP = 0, $T_C = T_{C\min}$ and $Q_{Hmin} = P_{\max}$. Also as it is seen in the graphics, when the current intensity gets weak, the value of COP increases. Of course the results are valid not only for the given example but also for all currents.

In Figure 3 the curves of the functions I = 10A, 20A, 30A and I = $I_{\max}$ = 40.7A for $Q_C = Q_C$ (E), $T_C = T_C$ (E) are shown.

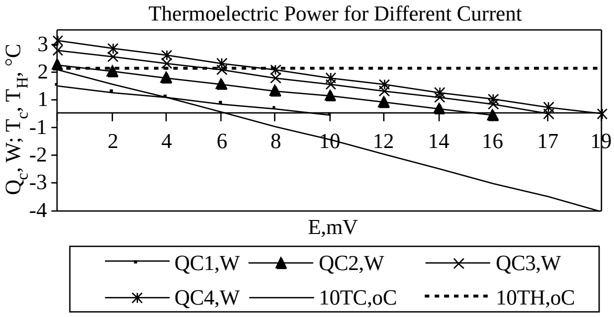

**Figure 3.** The Thermal Load for an Ideal Thermoelement and the Change of $T_C$ According to the Thermoelectric power.

As it is seen in Figure 3 in the $T_{C\min}$ working regime ($Q_C = 0$ and when $E = E^I_{\max}$) $Q_C$ (E) the points of the curves that intersect with the E axis give the $E^I_{\max}$ value of the module of 10A, 20A, 30A and 40.7A. Also in the $Q_{C\max}$ working regime (when $E = 0$, $T_C = T_H$) $Q_C$ (E) the points of the intersection with $Q_C$ show the $Q^I_{C\max}$ value that has a maximum cooling power of the module at 10A, 20A, 30A and 40.7A.

Also because the curves $Q_C$ (E) and $T_C$ (E) in Figure 3 are linear, making use of these curves or of these analytical formulas and measuring the thermal load and the temperature of cooling surface by measuring the thermoelectric power of a module that works under any working regimes. Similarly, the linear functions $Q_H = Q_H$ (E), P = P (E) and COP = COP (E) can be used to obtain all of the parameters of a working module.

By this way, the Eqs (17) - (27) which have become the basis of new method, came out of the Eqs. (1) - (4), which were deformed by equivalent changes. These changes were done so that calculated and experimentally measured parameters of a real module could be as close to each other as possible. The reason for making these changes was that the results that come out of the Eqs. (1) - (4) differed very much from the experimental results. The problem here is that: The Eqs. (1) - (4) do not contain geometrical or constructional factors that affect the parameters of a real thermoelement or module, especially in working conditions.

In order to solve this problem a new method has been developed. Instead of the static parameters of the semiconductors, its approach is based on output parameters like $V_{\max}$, $E_{\max}$, $I_{\max}$ of the thermoelement that reflect these static parameters and which can be easily measured. As a result of this new approach a series of formulas that are able to calculate all of electrical and thermal parameters of a real module are obtained. The classic approach is based on using the static parameters of the semiconductors and the new one is based on using the dynamic parameters of the thermoelement and both of them are based on the same principles. For an ideal thermoelement, the theoretic formulas that were obtained according to both approaches lead to absolutely identical results that prove their total equivalence. But the situation changes, if we talk of a working module. The results obtained by using the early approach are much more exaggerated, while the values that are calculated with the new formulas are closer to real values. A new method is based on the use of new formulas, and its main means are: entry parameters of the applied current, output parameters of the thermoelectric power and hot side temperature value of a working thermoelement. Also the new method makes it possible to investigate the factors which affect the work of thermoelement.

## 3. An Example Application

This method was practically applied with the aim of studying dynamic parameters of the thermoelectric modules from which Thermohypoterm medical apparatus consists. A European patent has been got for it in Turkey [22, 23]. The Thermohypoterm Medical Apparatus is designed for treatment of different diseases by cooling the brain of the patient. This apparatus can be used not only for medical treatment; but also in medical investigations concerning human brain, for measuring the heat amount of it. The Thermohypoterm Medical Apparatus consists of the thermoelectric helmet which works as a single-stage module. The thermoelectric helmet consists of 120 thermoelements connected thermally in parallel and electrically in series to increase the operating voltage of the module. It is difficult to measure the thermal load, coming from the brain to the helmet, because of the construction of the helmet; the way of the difficulty of using it outside and the physiological features of the living organism. In order to be able to measure the thermal load it is necessary to measure the thermal load, coming to each module. But as far as the contacts of the thermoelectric modules with the head differ and because of the reaction of the blood vein to the cold, thermal load that is coming to each module will also differ [24]. This makes the task more complicated. To solve this problem and to study the thermoelectric features of the helmet, we use a new approach as described above. Using this method for obtaining the total thermal load coming to the helmet, only the applied current and produced thermoelectric power should be measured. Before studying the features of the Thermohypoterm apparatus with the new method, the features of the modules used in this apparatus were studied first. In order to do this a special measuring set consisting of four modules was developed. It is shown in Figure 4. The four modules that form the experimental set are connected electrically in series and thermally in parallel with each other. The cool surfaces of the modules are face to face and there is also a heater between them forming a thermal load. Anodized and so insulating electricity panels are put between the cool surfaces of the modules and the heater. These panels insulate the modules from the heater electrically. Also in order to splutter the heat homogenously, thin copper wires were used, and to measure the cold and warm surface temperatures directly, a CIE 307 model thermometer with two digital thermocouples was used. The current applied to the thermoelectric modules was measured with a Fluke 380 all-purpose meter. On the same model the second all-purpose meter was used to measure the voltage and the thermoelectric power that those modules produce. A power supply of DC 0-50A was used for the module and a variac of AC 0-50V was used for the heater. In order to cool the warm surfaces of the modules the water passes through pipes which are connected in parallel and serial to each other. The water enters parallel through the pipes of the above modules and exits parallel through the pipes below. The mechanism in Figure 4 is used for studying the dynamic characteristics of the module in both conditions: with no load and loaded.

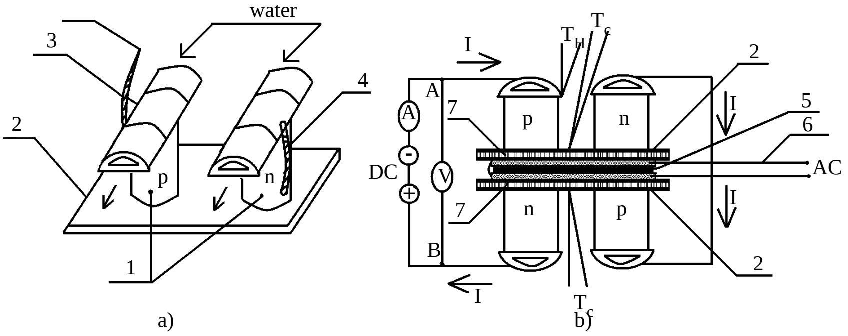

**Figure 4.** A Section of the Module Study Set.

Here in a) the structure of a module is shown; in b) an experimental set that consists of four modules. Also 1 – p- and n-type semiconductors, 2 – the cool surfaces of the modules, made of copper panels, 3 – the hot surfaces of the modules, made of copper panels, 4 – the multi-wired copper wire, used for the voltage entry, 5 – the anodized aluminium panel on which a warming wire is wrapped, 6 – the outputs of the heater producing a thermal load, 7 – the thin copper panel that helps to distribute the heat homogenously, $T_C$ and $T_H$ –show the K typed thermocouples, soldered to the cold and hot surfaces and used for measuring their temperatures, A and V values show the ampermeter and the voltmeter respectively.

# 4. Results and Discussions

Our study was made in a room temperature of 21 °C. The experiments were made, while the module had no load. Therefore cotton was put on the modules and so a full insulation was ensured. The features of the module were only studied for the input $T_H = 16$ °C. Temperature on the hot surface of the module was held fixed by 16 °C or $T_H = 273 + 16 = 289$K. For this reason, water from the faucet was flown continuously through the pipes that are shown in Figure 4. Their heat was controlled with the thermocouple $T_H$ In order to find the values $I_{\max}$, $V_{\max}$ and $E_{\max}$ of the module different currents have been applied and the values of the cooling surface $T_{C\min}$ were found. These temperatures were measured with the $T_C$ thermocouple that is shown in Figure 4. According to the obtained results, $T_{C\min} = -32$ °C, for one module the related values were measured as: $I_{\max} = 40$A, $V_{\max} = 0.107$V and $E_{\max} = 0.018$V. If we calculate $T_{C\min}$ according to (26), then $T_{C\min}$ is as $T_{C\min} = -33$ °C. As it is seen the value that is calculated according to (26) is only 2 °C more than the value that is calculated according to (25) and only 1 °C lower than the value that is measured directly. Those kinds of results are related to sensitivity of the measurements and incomplete insulation of the module. But the temperature differences are acceptable and it can be eliminated by more sensitive measurements and better insulation. In order to compare the output parameters like E and $T_C$ that are measured directly under different currents, with the calculated values of the developed model, $Q_C$ is assumed as $Q_C = 0$ and the formulas (24) and (21) are used. The results are shown in Figure 5.

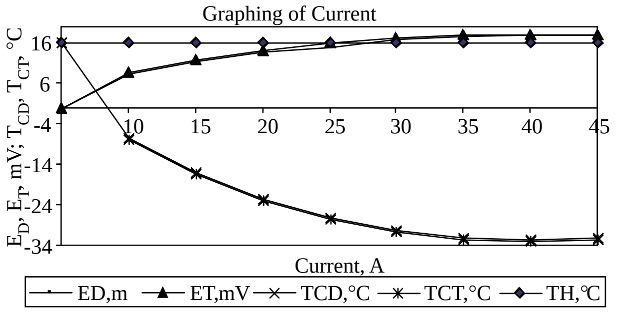

**Figure 5.** The Change of Experimental andTheoretical Output Parameters According to Currents.

The experimental (values indexed as D) and theoretical results (values indexed as T) of Figure 5 are close to each other. This made it possible to study characteristics of the module with the developed model. The reason of the experimental results being lower than the theoretical results can be explained by not complete insulation of the module during the experiment. In other words, despite the insulation, the total thermal load coming to the module becomes different than zero. Still the new method can ensure measuring this load as well.

By using the values $I_{\max} = 40$A, $V_{\max} = 0.107$V and $E_{\max} = 0.018$V that came out experimentally as in Eqs. (28) – (30), first the necessary formulas were formed and later the output parameters were calculated.

In Table 4, the below formulas of calculations and Table 4 with the result are shown below:

$$\begin{aligned} Q_C &= 0.107I - 0.0011I^2 - (I + 98.89)E \\ Q_H &= 0.107I + 0.0011I^2 - 98.89E \end{aligned} \tag{31}$$

$$P = 0.0022I^2 + EI \tag{32}$$

$$COP = \frac{Q_C}{P} = \frac{0.107I - 0.0011I^2 - (I + 98.89)E}{0.0022I^2 + IE} \tag{33}$$

$$T_C = 289(1 - 9.35E). \tag{34}$$

$$E = \frac{0.107I - 00011I^2}{(I + 98.89)} \tag{35}$$

Normally Q$_C$ has to be 0 for all currents, but as is seen in Table 4 Q$_C \neq 0$. The reason for this can be rounding of the calculations or the thermal load, despite the insulation of the module. The value of this load under different currents is also shown in Table 4. Therefore, by using the Eqs. (30) – (35) obtained for the thermoelectric module used in the experiment, all of its parameters for any current can be found easily.

**Table 4**

| I, A | E$_D$, mV | Q$_C$, W | P, W | COP | Q$_H$, W |
|---|---|---|---|---|---|
| 0 | 0,0 | 0 | 0 | 0 | 0 |
| 10 | 8,5 | 0,03 | 0,3 | 0,096 | 0,3 |
| 15 | 11,6 | 0,04 | 0,7 | 0,054 | 0,7 |
| 20 | 13,8 | 0,06 | 1,2 | 0,051 | 1,2 |
| 25 | 15,0 | 0,13 | 1,8 | 0,074 | 1,9 |
| 30 | 16,8 | 0,05 | 2,5 | 0,022 | 2,5 |
| 35 | 17,7 | 0,03 | 3,3 | 0,008 | 3,3 |
| 40 | 18,0 | 0,02 | 4,2 | 0,005 | 4,3 |
| 45 | 17,9 | 0,01 | 5,3 | 0,002 | 5,3 |

Also by using Eqs (8), (12), (20) and (23) in which experimentally measured values I$_{\max}$, E$_{\max}$ and V$_{\max}$ are present, R, K, Z and Q$_{C\max}$ parameters of the module were found. According to the results: R = 0.0022$\Omega$, K = 36.6.10$^{-3}$ W/C, Z = 1.68.10$^{-3}$ / °C and Q$_{C\max}$ = 1.78W.

While stepping from an ideal thermoelectric module to real one, or to battery, and also to thermoelectric systems and devices, various factors like soldering and structural features lower the expected output parameters. In addition to this, in a working module the exposure of inner mechanisms will also decrease the parameters. The most important parameter that acts here is Z. That value of Z that is obtained during the experiment is lower than its theoretical value as it was explained above. But the experimental and theoretical values are close to each other; indicating the high quality of the module used in the construction of the thermoelectric module.

## 5. Conclusions

The microparameters of a semiconductor and the resistivity, Seebeck coefficient, thermal conductivity and the figure of merit of a module can be evaluated with many known methods. Using these parameters we can get when necessary, the thermal parameters of a module. But they are calculated while the module is not operating. These methods, while evaluating the thermal parameters of a working module, can lead to inaccuracies. Therefore there was a need for a new approach in order to be able to measure the values of the working module.

In order to find the dynamic output parameters of a module, the microparameters of a semiconductor must be found while the module is operating. But it is very difficult to find the microparameters of the semiconductors in a working module. In order to solve this problem, a method has been developed in this work, which is based on easily measurable macroparameters. With the theoretical model developed in this work, all of the dynamic parameters of the thermoelectric module can be measured and also the inner and outer factors that affect these parameters can be studied. It is known that this new ability has a great importance from both points of view: scientific studies and design of new thermoelectric systems. The newly developed method can be a very practical and affective solution, because the basis of the method is formed by the current parameter that it pulls, the thermoelectric power and the measurement of the hot side temperature. As a result in order to investigate the dynamic thermal and electric features of a module, it is enough to measure the supplied current, I, and the voltage of a working module, V, the thermoelectric power, E, that the module produces and the temperature of any surface of the module. These measures are very simple and can be done with large accuracy. The cost of this method is very low. Also the obtained formulas characterize the dynamic parameters of a working module and they can be used for evaluating the constructional methods, applied in production of thermoelectric devices.

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