A Novel Method for Load Flow Analysis of Unbalanced Three-Phase Radial Distribution Networks

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Abstract

This paper presents a novel method for load flow analysis in radially operated 3-phase distribution networks without solving the well-known conventional load flow equations. The method can be applied for distribution systems in which the loads are unbalanced. As the size of matrix used is very small compared to those in conventional methods, the amount of memory used is very small, the speed is very high, and the relative speed of calculation increases with the size of the system.

The method was applied to some practical networks and the results show the superiority of this method over the conventional ones. As this method is significantly faster than any other method developed to date, it has great potential for on-line operations.

Key Words: *Radial load flow, distribution, three-phase load-flow.*

1. Introduction

Load flow analysis forms an essential prerequisite for power system studies. Considerable research has already been carried out in the development of computer programs for load flow analysis of large power systems. However, these general purpose programs may encounter convergence difficulties when a radial distribution system with a large number of buses is to be solved and, hence, development of a special program for radial distribution studies becomes necessary.

There are many solution techniques for load flow analysis. The solution procedures and formulations can be precise or approximate, with values adjusted or unadjusted, intended for either on-line or off-line application, and designed for either single-case or multiple-case applications. It is noted, however, that an acceptable load flow analysis method should meet the following requirements [1]:

1. They should have high speed and low storage requirements, especially for real-time large system applications, as well as multiple case and interactive applications.

2. They should be highly reliable, especially for ill-conditioned problems, outage studies, and real-time applications.

3. They should have acceptable versatility and simplicity.

In fact, conventional load flow analysis methods, which were essentially developed to solve problems posed at the transmission network level, can encounter convergence problems when applied to distribution networks. The reason is that the R/X ratio is usually high for radial distribution systems; therefore, much effort has been made to develop modified versions of the conventional load flow methods to respond to the challenges posed by peculiar distribution system characteristics [2-4]. The methods presented are modifications to Newton-Raphson (N-R) and fast decoupled (FD) methods. They are simple in nature, having no mathematical approximations, and require almost no additional storage, but they are mainly devoted to solving transmission, not distribution networks, that have high R/X ratios and poorly initialized power systems.

In view of the topological specialty of distribution networks, and non-applicability of the simplifying assumptions of a decoupled Jacobian matrix normally applicable to transmission networks, researchers dealing with distribution systems have proposed several special load flow analysis techniques for distribution networks.

Kersting and Mendive [5], and Kersting [6] developed techniques for solving the load flow problem in radial distribution networks based on ladder-network theory in the iterative routine. This solution is complicated and has many assumptions for a typical distribution system, which is rarely a pure ladder network. In other words, the method is not designed to efficiently solve for meshed networks. Moreover, Stevens et al. [7] demonstrated that the ladder-based technique is very fast, but does not guarantee convergence.

Based on Kirchhoff's voltage and current laws, a method for solving radial distribution networks has been presented in [8]. In this method, a branch-numbering scheme was employed to enhance the computations. The method was then extended to apply to weakly meshed networks. In this method, they first break the interconnected grid at a number of points (breakpoints) in order to convert it into one simple radial network. Each breakpoint will open one simple loop. The radial network is solved by direct application of Kirchhoff's laws. Then they account for the flows at the breakpoints by injecting currents at their 2 end nodes. The numerical efficiency of this method, however, diminishes as the number of breakpoints required to convert the meshed network to a radial configuration increases. This restricts the practical application of the method to weakly meshed networks.

Baran and Wu [9,10] performed load flow analysis in a distribution system using an iterative solution of 3 fundamental equations representing real and reactive power, and voltage magnitude. The 3 equations are useful because they can be used in real systems rather than in other classically known forms. Here, they computed the system Jacobian matrix using a chain rule. The mismatches and the Jacobian matrix involve only evaluating simple algebraic expressions and no trigonometric functions. The formulation and evaluation of Jacobians are time consuming and require large amounts of computer memory storage.

Goswami and Basu [11] presented a direct method for solving radial and meshed distribution networks. Their method has the advantages of a no convergence problem, a guaranteed accurate solution for any realistic distribution system, and the ease with which composite loads can be represented. The disadvantages are difficulty numbering the nodes and branches, and that no node in the network is the junction of more than 3 branches.

Das et al. [12] presented a load flow method for radial distribution networks based on evaluating the total real and reactive power fed through any node. They created a unique node, branch, and lateral numbering scheme to enhance the evaluations of real and reactive loads fed through any node and receiving end voltages. This method has the advantage that all data can be stored in vector forms, thus saving an enormous amount of computer memory.

Ghosh and Das [13] presented a simple method for solving radial distribution networks by evaluating

only a simple algebraic expression of receiving end voltages. In this method, the authors assumed an initial flat voltage for all nodes. Then, by numbering the nodes beyond each branch, they calculated the loads and charging currents, followed by the branch currents. The modified nodal voltages were recalculated, as were the losses. Evaluating the difference between new and previous voltage values and then comparing it with an accepted tolerance verified the convergence for this method. The method is simple and has good and fast convergence, and can be used for composite load modeling, if the composition of the loads is known.

In this paper a novel technique, which is a developed form of the method described in [14] and is suitable for unbalanced 3-phase radial distribution systems with minimum memory requirements and faster convergence performance, is described.

2. The Load Flow Algorithm for Balanced Radial Distribution Networks

The new method explained in [14] was developed for balanced radial distribution feeders with laterals. Compared to conventional methods, the Y_{bus} matrix method has 2 advantages: relative insensitivity to the initial voltage estimate and programming simplicity. Its major weakness is a slow convergence rate, especially in radial systems. The Newton-Raphson algorithm is known for excellent convergence characteristics. Its major drawback is the requirement of the Jacobian matrix, which is approximately 4 times the size of the Y_{bus} matrix and needs to be recalculated for every iteration. In distribution studies, because of smaller X/R ratios associated with smaller conductors, the Jacobian matrix cannot be simplified. This fact makes the Newton-Raphson approach rather unattractive.

The new method does not use any large matrix and so the amount of computer memory used is much less compared to conventional ones. As the formulation is very simple, the method is very fast.

In this formulation the branches are represented by their single-line diagram and the loads are assumed to be balanced. In this method the analysis begins by assuming initial values for the bus voltages. The currents taken by different buses are calculated starting from the end buses to the source. The source bus current is updated and the branch currents are again calculated from the source to the end buses. The calculations are repeated until the difference between the losses calculated in 2 consecutive iterations becomes considerably low.

The method is completely different than conventional ones and has been applied to the study of some practical distribution networks, and the results given in [14] demonstrate its priority.

3. Proposed Method

The novel technique developed is suitable for unbalanced 3-phase radial distribution systems. The analysis proceeds from one branch to another in a systematic way until all the branches in the feeder have been traced. First, the voltages at all the buses, except the source bus, are assumed to be 1 p.u., with zero angle at phase a, $+120°$ on phase b, and $-120°$ on phase c. Based on these voltages and specified active and reactive power, simultaneously, the branch currents, starting from the end buses to the source, are calculated and saved. This, of course, requires a logical procedure to ensure that the branches of the system are correctly traced; therefore, the branch incidence table as described in [14] is used. Then, branch currents, including the return-conductor current, are computed in order to find the active and reactive power losses

in the system. The current at the source end is now calculated as follows:

$$I_a = \frac{(\sum\limits_a P + \sum\limits_a P_{LOSS}) + j(\sum\limits_a Q + \sum\limits_a Q_{Loss})}{V_{Sa}^*}$$

$$I_b = \frac{(\sum\limits_b P + \sum\limits_b P_{Loss}) + j(\sum\limits_b Q + \sum\limits_b Q_{Loss})}{V_{Sb}^*} \qquad (1)$$

$$I_b = \frac{(\sum\limits_b P + \sum\limits_b P_{Loss}) + j(\sum\limits_b Q + \sum\limits_b Q_{Loss})}{V_{Sc}^*}$$

$$I_d = -I_a - I_b - I_c$$

where $\sum\limits_m P$ and $\sum\limits_m Q$ are the sum of loads, on phase m, $\sum\limits_m P_{Loss}$ and $\sum\limits_m Q_{Loss}$ are the total losses in phase m, and V_{Sm}^* is the conjugate of source voltage on phase m. The computation then proceeds from the source to the end of the feeder to find the voltage drop, current, and loss in each branch in each phase of the feeder, including the return conductor, in a systematic manner. The branch incidence table is again used to facilitate proper retracting of the network branches. Once this process is completed, the total losses are calculated and compared to the values initially obtained by assuming one per unit voltage at all the buses. If the difference is outside the specified tolerance limits, the source current is re-computed using eq. (1), in terms of the newly obtained values for losses, and the path retracting operation is repeated. The process is repeated until the difference in losses between 2 successive values of the source current is within the specified tolerance limits. The feeder is represented by its unbalanced 4-line representation [14] shown in the i-th branch in Figure 1.
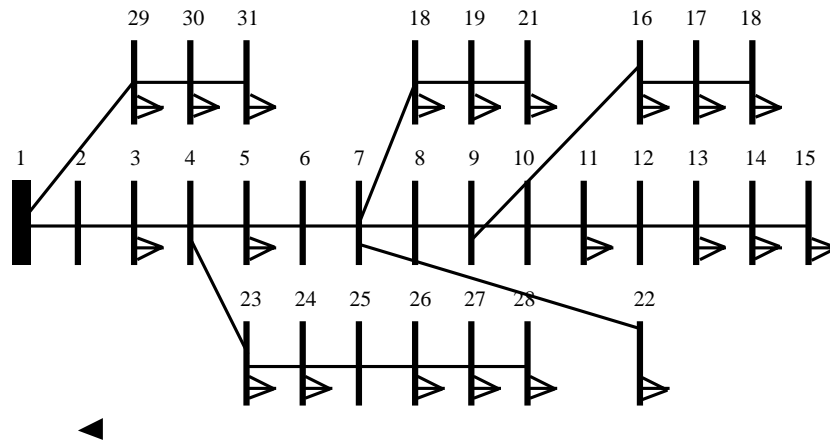
Figure 1. Four-line representation of each branch in an unbalanced distribution feeder.

The voltage drop in each branch is then calculated by

$$\begin{bmatrix} V_a - V_a^\bullet \\ V_b - V_b^\bullet \\ V_c - V_c^\bullet \\ V_d - V_d^\bullet \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{ad} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bd} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cd} \\ Z_{da} & Z_{db} & Z_{dc} & Z_{dd} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} \qquad (2)$$

and the losses in each branch are computed by

$$
\begin{bmatrix} (S_{LOSS})_a \\ (S_{LOSS})_b \\ (S_{LOSS})_c \\ (S_{LOSS})_d \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix}^t \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{ad} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bd} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cd} \\ Z_{da} & Z_{db} & Z_{dc} & Z_{dd} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix}^* \tag{3}
$$

The computer program flow-chart developed for the calculations is given in Figure 2.

Figure 2. Flow-chart for the proposed 3-phase load flow method.

4. Case Study and Results

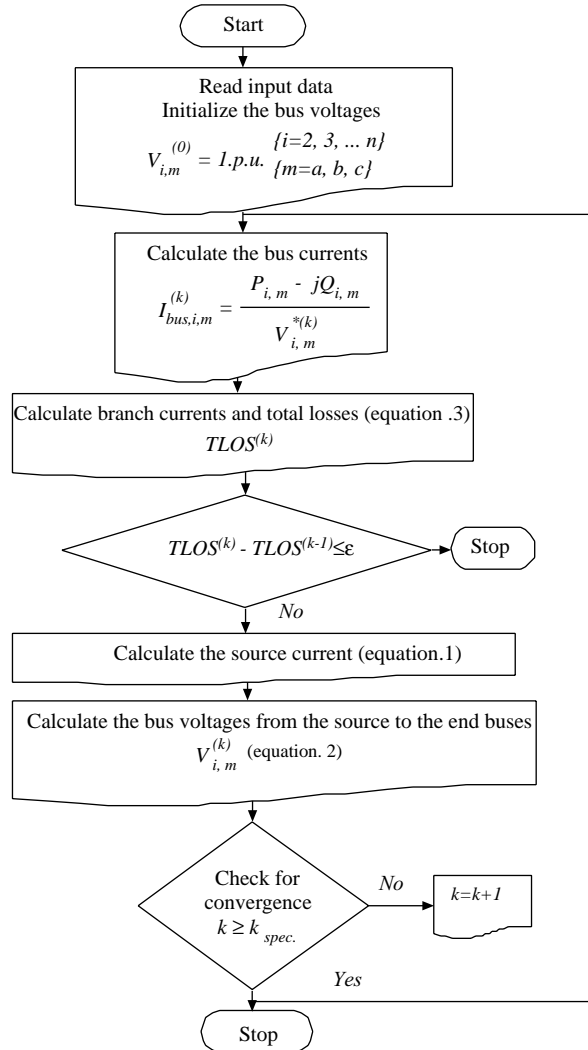The iterative design procedure presented in this paper was applied to the elaborate numerical example shown in Figure 3. Chosen was a realistic multi-grounded, 3-phase feeder with lateral branches and 5 wire sizes (Table 1). This feeder had 3-phase, 2-phase, and single phase branches, and was almost 35 miles in overall length.
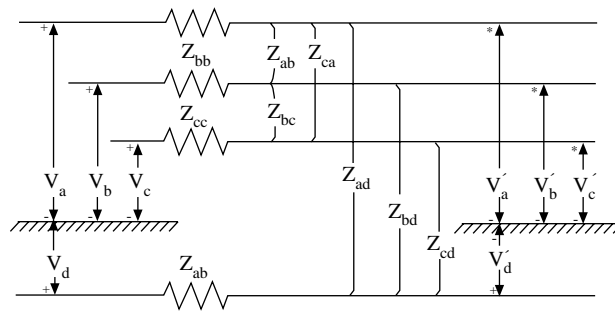
Figure 3. An unbalanced distribution feeder with unbalanced load (ground conductor is not shown).

Table 1. Physical characteristics of the phase conductors.

Branch	Phases	Spacing-type	Phase Conductor	Ground Conductor	Length (Mile)
1-2	A,B,C	1	300 Cu	1/0 Cu	3.01
2-3	A,B,C	4	336 Al	1/0 Cu	0.85
3-4	A,B,C	4	336 Al	1/0 Cu	0.86
4-5	A,B,C	4	2/0 Cu	4 Cu	1.51
5-6	A,B,C	4	2/0 Cu	4 Cu	0.70
6-7	A,B,C	2	2 Cu	4 Cu	1.29
7-8	A,B,C	2	2 Cu	4 Cu	1.06
8-9	A,B,C	2	2 Cu	4 Cu	1.13
9-10	A,B,C	3	4 Cu	6 Cu	1.13
10-11	A,B,C	3	4 Cu	6 Cu	1.13
11-12	A,B,C	3	4 Cu	6 Cu	3.21
12-13	C	3	4 Cu	6 Cu	1.48
13-14	C	3	4 Cu	6 Cu	0.60
14-15	C	2	4 Cu	6 Cu	1.24
9-16	A,C	3	4 Cu	6 Cu	0.56
16-17	A,C	3	4 Cu	6 Cu	0.54
17-18	A,C	2	2 Cu	4 Cu	0.98
7-19	B,C	2	2 Cu	4 Cu	0.63
19-20	B,C	2	2 Cu	4 Cu	2.14
20-21	B,C	2	2 Cu	4 Cu	1.66
7-22	A,B,C	2	2 Cu	4 Cu	0.61
4-23	A,B,C	4	2/0 Cu	1/0 Cu	1.27
23-24	A,B,C	4	2/0 Cu	4 Cu	0.77
24-25	B	2	2 Cu	4 Cu	0.68
25-26	B	2	2 Cu	4 Cu	1.68
26-27	B	2	2 Cu	4 Cu	1.30
27-28	B	3	4 Cu	6 Cu	1.31
2-29	C	4	336 Al	1/0 Cu	1.68
29-30	C	3	2/0 Cu	4 Cu	1.22
30-31	C	3	4 Cu	6 Cu	0.81

The self and mutual impedances were calculated for different spatial configurations (Table 2).

Table 2. Physical configuration of the phase conductors (Deq is the equivalent equilateral spacing between phases).

Type	1	2	3	4
	$a : \bullet$ $b : \bullet$ $c : \bullet$	$b : \bullet$	$b : \bullet$	$\bullet \bullet \bullet$
		$a : \bullet$ $c : \bullet$	$a : \bullet$ $c : \bullet$	b_c
	$w : \bullet$	$w : \bullet$	$w : \bullet$	$w : \bullet$
Spacing				
Dab	42	32	28	84
Dbc	42	32	28	42
Dca	84	36	36	42
Daw	142	60	43	62
Dbw	100	84	53	50
Dcw	58	60	43	62
Deq	53	33	30	52

The parameter calculation was performed as described in [15]. The loading at each bus of the feeder is shown in Table 3.

Table 3. Phase loads in kW(blank spaces indicate that the corresponding buses do not physically exist).

Bus No.	Phase A	Phase B	Phase C
3	155	0	58
5	0	0	104
6	63	0	0
7	13	0	0
10	0	11	21
12	29	12	34
13			73
14			37
15			81
16	53		53
17	61		61
18	53		53
19		48	48
20		58	0
21		55	55
22	58	46	23
23	109	29	58
24	0	0	213
26		124	
27		61	
28		88	
29			98
30			98
31			98

The voltage profile obtained using the proposed method is presented in Table 4, which is the same as the results given by conventional methods.

Table 4. Voltage profile of the feeders.

Bus No.	Phase A (p.u.)	Phase B (p.u.)	Phase C (p.u.)	Phase N (p.u.)
1	1.0000	1.0000	1.0000	0.000
2	0.9731	0.9759	0.9693	0.0083
3	0.9710	0.9738	0.9662	0.0106
4	0.9695	0.9717	0.9633	0.0129
5	0.9671	0.9696	0.9582	0.0191
6	0.9660	0.9686	0.9562	0.0213
7	0.9630	0.9665	0.9507	0.0256
8	0.9613	0.9665	0.9473	0.0297
9	0.9594	0.9665	0.9436	0.0327
10	0.9590	0.9663	0.9405	0.0364
11	0.9586	0.9663	0.9376	0.0399
12	0.9574	0.9661	0.9294	0.0503
13	0.9574	0.9661	0.9261	0.0547
14	0.9574	0.9661	0.9253	0.0559
15	0.9574	0.9661	0.9242	0.0574
16	0.9583	0.9665	0.9426	0.0341
17	0.9576	0.9665	0.9419	0.0351
18	0.9572	0.9665	0.9415	0.0355
19	0.9630	0.9658	0.9501	0.0254
20	0.9630	0.9639	0.9492	0.0248
21	0.9630	0.9633	0.9484	0.0252
22	0.9627	0.9663	0.9506	0.0254
23	0.9687	0.9697	0.9617	0.0134
24	0.9687	0.9686	0.9609	0.0136
25	0.9687	0.9671	0.9609	0.0122
26	0.9687	0.9635	0.9609	0.0114
27	0.9687	0.9619	0.9609	0.0122
28	0.9687	0.9607	0.9609	0.0126
29	0.9731	0.9659	0.9687	0.0094
30	0.9731	0.9659	0.9687	0.0094
31	0.96	0.9659	0.9687	0.0094

The above methods were used to study the 32-feeder on a cyber 180-855 CDC computer, and the computing times, using the Gauss-Seidel, the Newton Raphson, and the new methods, were 70, 30, and 0.2 s, respectively.

Comparing the accuracy and the speed of the method presented with the conventional methods, the same results with the same accuracy were calculated, but the speed was significantly increased and the computational memory requirement was greatly reduced.

5.   Conclusions

In this paper a novel technique was presented for the load flow analysis of unbalanced 3-phase radial distribution feeders. The method does not use large matrices so that the computational memory requirement is smaller compared to those in conventional approaches.

The method is completely different from conventional approaches. It was applied to the study of similar practical networks, and the results demonstrated its computational superiority.

This method is based on the formation of a constant matrix based on the network topology, which is required to be formed only once. This method does not involve any complex mathematical routines, nor does it require any matrix inversion. Additionally, the loads can be represented by any suitable model. It is considered that other applications can be developed as a consequence of the proposed method, opening new research possibilities in the field of distribution networks, which perhaps has not received much attention in the past.

Implementation and results show that this development is simple, accurate, fast, reliable, and has low storage requirements. It can also be used for on-line control applications.

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