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My Collaboration with Raj Mittra: Contributions to the Theory of Perfectly Matched Layers

Mustafa KUZUOĞLU
Department of Electrical and Electronics Engineering, Middle East Technical University
06531, Ankara-TURKEY
e-mail: kuzuoglu@metu.edu.tr

It is impossible to imagine a young researcher in the vast field of applied electromagnetics to not have encountered the name of Raj Mittra during the first few months of his research activities. In 1980, I came across an article written by Professor Mittra when I was working on my M.Sc. thesis, on transient electromagnetics, long before I met him personally in October 1995, in Urbana-Champaign. During the interim 15 years, comprising my Master's and Ph.D. thesis and post-doctoral studies, I came to read and use several of Mittra's papers and books, as his contributions span nearly the entire field of electromagnetics, and distinguished by the elegance of his intuition, mastery of analytical and numerical methods and pertinence of problem selection to the real world.

I was thus quite excited on the evening of October 15, 1995 when, for the first time, I met Raj Mittra at Urbana-Champaign airport. This was the first day of my approximately one year visit to the University of Illinois as a NATO scholar. We talked about several things during the ride to my apartment but I remember that he was especially pleased to learn that my major research area is FEM (Finite Element Method), since it was also an area Prof. Mittra had worked in extensively.

The next day I visited the Electromagnetic Communications Laboratory and, as a lucky coincidence, happened to be a seminar on one of the year's hot topics. The speaker was one of Mittra's Ph.D. students, J. C. Veihl, and the subject was the utilization of PMLs (Perfectly Matched Layers) in FDTD applications. Though unable to recall the details of this talk (as I was under the spell of severe jet-lag) I do remember being intrigued by the idea of a layer capable of absorbing electromagnetic waves irrespective of frequency and direction of incidence. During the seminar, I did not know that this topic would be one of my favourite research subjects in the coming years.

In the first few months of my collaboration with Raj Mittra, we concentrated on the usage of PMLs in FEM applications. The PML concept was originally introduced by J. P. Berenger in his 1994 paper as a grid truncation approach in FDTD applications [1]. Berenger's approach was based on a field-splitting approach yielding non-Maxwellian fields within the PML. A formulation where the PML was realized as an anisotropic layer was achieved by Sacks et al. [2] and our main concern was to extend the idea of this layer sandwiched between planar surfaces to an arbitrary layer between curved surfaces. The main motivation behind this was the possibility of designing a PML conformal to the surface of a scatterer (or antenna).

I remember working on this exciting project during the winter of 1995–96. Our joint work spanned a few short months, but was extremely exciting for me since the subject was very interesting and our discussions were stimulating. Finally, we were able to obtain "non-planar" PMLs and verified their practical usage via several FEM simulations. Our results appeared in [3] and has been cited by several researchers in the following years.

Some interesting details of our research work are described below, after a short introduction to the PML concept.

Most boundary value problems of electromagnetic wave propagation are defined over spatially-unbounded domains. Asymptotically, at points far from the sources, the electromagnetic field behaves as an outgoing spherical (3D) or cylindrical (2D) wave in the form (where u denotes an arbitrary component of the electromagnetic field)

$$u(R,\theta,\varphi) \sim c(\theta,\varphi)\frac{\exp(-jkR)}{R}(3D) \tag{1}$$

$$u(r,\varphi) \sim c(\varphi)\frac{\exp(-jkr)}{\sqrt{r}}(2D). \tag{2}$$

The unbounded spatial domain can be converted to a bounded domain by choosing an artificial boundary far away from the sources by imposing boundary conditions to absorb outgoing waves. This "domain truncation" operation is illustrated in Figure 1. Such Absorbing Boundary Conditions (ABCs) have been obtained in the 1980's in wave propagation problems. It is easy to see that the following boundary conditions can be used as ABCs in 2D and 3D over the surfaces designated with $R = R_0$ and $r = r_0$:

$$\frac{\partial u}{\partial R} + \left(jk + \frac{1}{R}\right)u = 0 \text{ at } R = R_0(3D) \tag{3}$$

$$\frac{\partial u}{\partial r} + \left(jk + \frac{1}{2r}\right)u = 0 \text{ at } r = r_0(2D). \tag{4}$$

A major breakthrough occurred in 1994, when J. P. Berenger introduced the concept of a Perfectly Matched Layer (PML) for the purpose of grid truncation in FDTD applications. The PML is a layer which absorbs plane waves of arbitrary frequency and direction of incidence, without any reflection. Berenger's PML used the concept of field-splitting and the resulting fields are non-Maxwellian (i.e. the field quantities do not satisfy Maxwell's equations).

A PML design yielding Maxwellian fields was proposed by Z. S. Sacks et al. Their main contribution was to design the PML medium as an anisotropic layer with suitably-defined permittivity and permeability tensors. Their approach is equivalent to a transformed coordinate system with complex coordinates. As shown in Figure 2, consider a PML half-space $\Omega_{PML} = \left\{(x,y,z) \in \mathrm{R}^3 \,|x > 0\right\}$, and assume that the free-space region is $\Omega_{FS} = \left\{(x,y,z) \in \mathrm{R}^3 \,|x < 0\right\}$. Let a TM_z plane wave be incident on the free-space/PML interface:

$$E_z = \exp\left[-jk\left(\cos\theta x + \sin\theta y\right)\right] \tag{5}$$

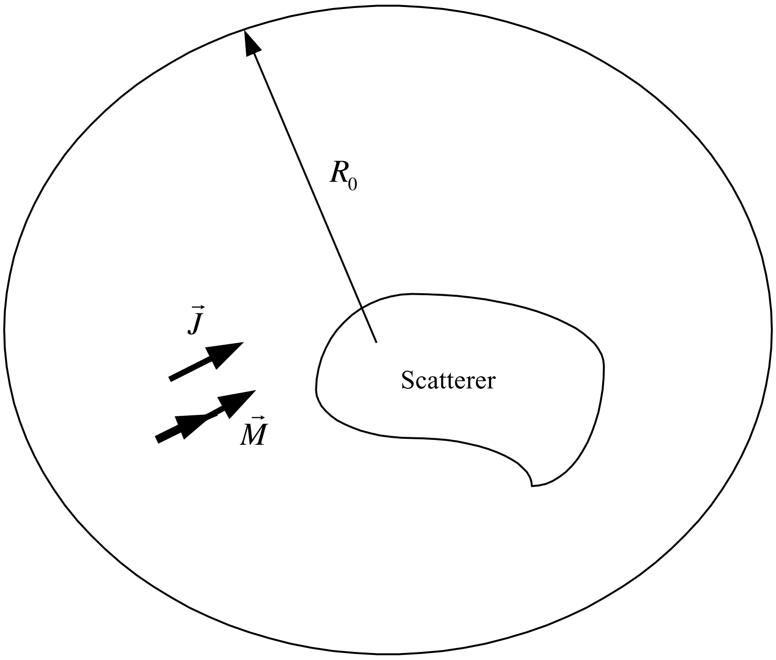

Figure 1. Domain truncation.

Replace the real x coordinate variable by the complex coordinate variable x', defined as

$$x' = \left(1 + \frac{\alpha}{jk}\right) x \tag{6}$$

where $\alpha > 0$ is a real-valued parameter. Via the form-invariance of Maxwell's equations under coordinate transformations [4], it can be asserted that the PML region Ω_{PML} is effectively a medium with constitutive parameters $\varepsilon = \varepsilon_0 \Lambda$ and $\mu = \mu_0 \Lambda$, where

$$\Lambda = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \tag{7}$$

and $a = 1 + \frac{\alpha}{jk}$. The field in Ω_{PML} can be found easily by replacing the real x coordinate variable by its complex counterpart x' as follows:

$$E_z = \exp\left[-jk\left(\cos\theta x' + \sin\theta y\right)\right] \qquad = \exp\left[-\alpha\cos\theta x\right]\exp\left[-jk\left(\cos\theta x + \sin\theta y\right)\right] \tag{8}$$

It is clear that the plane wave is transmitted into the PML region without any reflection, and it decays exponentially within the region as it propagates.

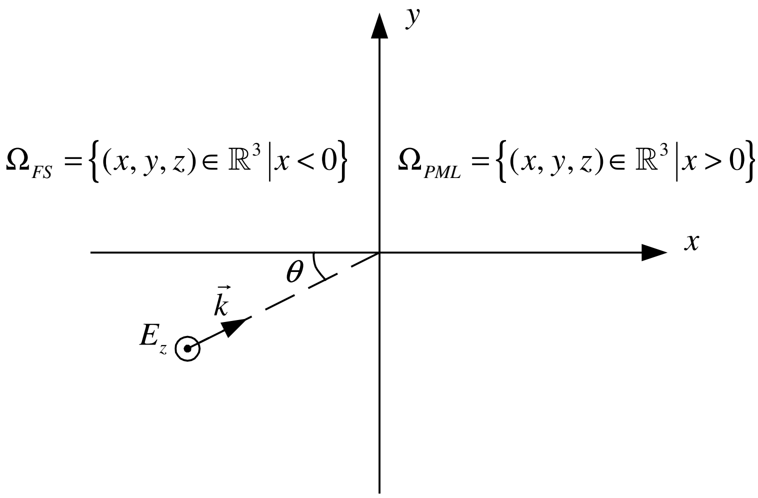

Figure 2. A TM_z plane wave incident to a PML half-space.

It is possible to use this idea as a mesh truncation technique in finite methods such as the finite difference method, finite element method, finite volume method, etc. The unbounded domain is converted to a bounded domain by introducing a PML enclosing the convex hull of the scatterer (or scatterers) and sources, as can be seen in Figure 3. The PML interfaces may be chosen as constant Cartesian coordinate surfaces and the resulting PML is called a Cartesian PML.

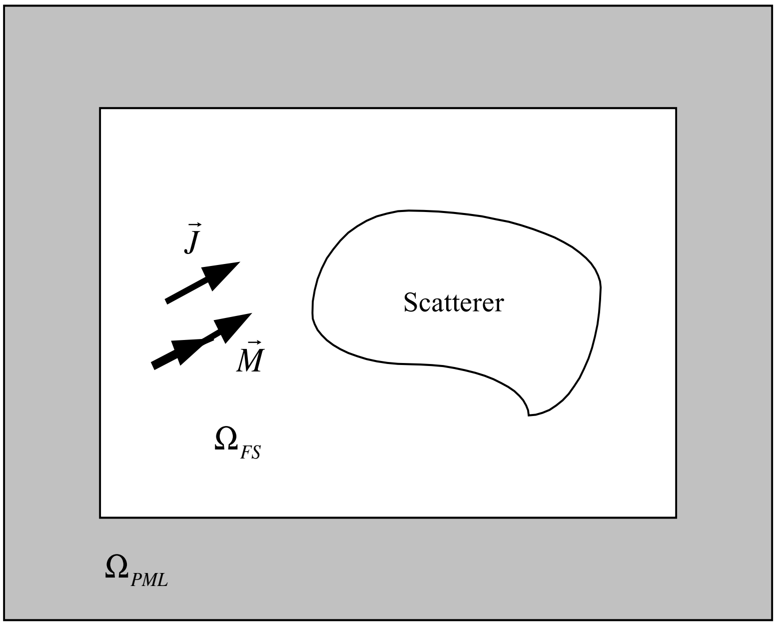

Figure 3. A Cartesian PML configuration.

This was the state-of-the-art in the PML approach, when my one-year visit to the University of Illinois at Urbana-Champaign started in the autumn of 1995. There were several open problems related to the theory of the recently-introduced PML concept. One of the major unanswered questions was about the applicability of the PML approach in coordinate systems other than the Cartesian system. This case was not crucial in the context of the FDTD (where the grid is mostly Cartesian), but was of extreme importance in FEM applications. We started to work in this direction, and generalized the PML concept to cylindrical and spherical coordinate systems, by introducing the PML constitutive relations in these coordinate systems and obtained the analytical solutions by separation of variables. We have used the isoparametric mapping idea, so often used in the context of FEM, for the generation of perfectly matched lossy elements for mesh truncation in FEM. We were able to obtain these results by using the PML constitutive relations in a local coordinate system, and transforming these equations to the global coordinates via appropriately-designed rotation matrices. This approach provided the means for conformal PMLs (see Figure 4).

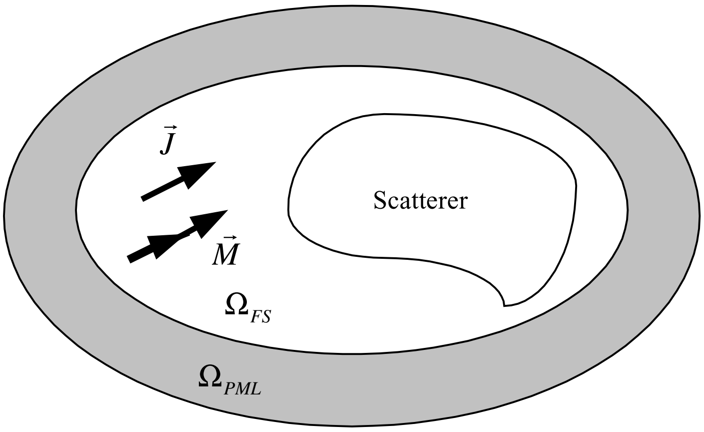

Figure 4. A conformal PML configuration.

We have obtained another interesting result as a result of one of our discussions about the low frequency behaviour of PMLs. Right after Berenger's 1994 paper, there appeared some results showing the deterioration of PML performance for low frequency applications [5]. Professor Mittra told me about this problem and I tried to figure out a possible solution. One day, it suddenly appeared to me that the problem lies in the definition of the PML parameter $a = 1 + \frac{\alpha}{jk}$ in (3). The parameter clearly becomes singular as $k \to 0$. In order to resolve this difficulty, we modified the parameter as follows [6]:

$$a = 1 + \frac{\alpha}{\beta + jk} \tag{9}$$

Initially, our approach was not popular among the users of FDTD+PML, because such a modification in the PML parameter resulted in the necessity to introduce additional variables in the FDTD implementation. This gave rise to an expensive algorithm involving convolution operations. However, J. A. Roden and S. D. Gedney found a solution to this difficulty by implementing the convolutions iteratively [7]. Later on, our approach has been used and developed by a large number of researchers and is now known as the complex frequency shifted PML (CFS-PML).

I have tried to give a summary of my collaboration with Raj Mittra during the winter of 1995–96. Our joint work lasted for a relatively short period, but his impact on my career choice and research was and still is felt strongly and positively. He inspired me to think broadly and deeply, and for that I am grateful. Since then I have met him again and again (usually during summer vacations), but for short spells only. Our mutual encounters proved to be vitally important for me, which is not at all surprising because his brilliant career, spanning more than 40 years, impacted many of the lives of those whom he taught, mentored and with whom he collaborated.

Professor Mittra is admired by many people for his achievements but I also admire him as a human being. There are several instances when I came across this side of Raj Mittra:

- One midnight in February 1996, I had severe abdominal pain and I had to give a phone call to Raj early in the morning to help me go to the hospital. He came to my apartment abruptly, and I was hospitalized for a few days. The cause of my pain was a kidney stone which I luckily passed after approximately 36 hours, just before a possible operation. I cannot forget his warm support during my illness.
- On several occasions I witnessed the intimacy between Raj Mittra and his former students and post-docs. During a private conversation, I remember him saying that he always paid attention to establish a life-long friendship with his students. I believe that this mutual friendship provided the means for the evolution and continuation of Raj Mittra's work in the research and practice of his former students and colleagues.
- In 1996, Professor Mittra retired and moved to Pennsylvania State University. I had the privilege of visiting him in each of the years 1997–2001 and 2003. During my visits I had the opportunity to see how organized he is and how he manages the research activities of the members of his laboratory. It was a real pleasure to be present in the Friday afternoon meetings, where each researcher summarizes his/her work of the previous week. This was a model for an effective, dynamic, stimulating group making real scientific progress. During these meetings, I was really amazed by the width and depth of his knowledge in different areas of electromagnetics.
- Professor Mittra has a special talent in motivating people in research activities. It is not possible to become pessimistic about the outcomes after discussing the results with him. Almost immediately, he points out a way to get around the difficulties. Most probably that is why his research has resulted in several important and first-rate results.

I feel myself extremely honoured by the opportunity to work with Raj Mittra, a leading figure of the electromagnetics community.

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