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Analytical regularization method for electromagnetic wave diffraction by axially symmetrical thin annular strips

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Abstract

A new mathematically rigorous and numerically efficient method based on the combination of Orthogonal Polynomials Method, and Analytical Regularization Method, for electromagnetic wave diffraction by a model structure for various antennae such as Fresnel zone plates is proposed. It can be used as validation tool for the other (more general or less accurate) numerical methods and physical approaches. The initial boundary value problem is equivalently reduced to the infinite system of the linear algebraic equations of the second kind, i.e. to an equation of the type $(I + H)x = b$ *in the space* l_2 *with compact operator* H*. This equation can be solved numerically by means of truncation method with, in principle, any required accuracy. Numerical results show that physical optics approximation may not be valid, especially in diffraction by sources other than plane wave, as it does not take into account the contribution of strip edges and interaction of strips by means of traveling slow waves.*

Key Words: *Analytical Regularization, Annular Strips, Antenna, Fresnel Zone Plates, Integral Equations.*

1. Introduction

Electromagnetic wave diffraction by flat and perfectly conductive annular strips (see Figure 1) is a canonic diffraction problem which occurs, in practice, during the design and modeling of various antennas including Fresnel zone plate antennas (FZPA) [1, 2]. Essential part of the methods used in such investigations concerns boundary integral equations of the first kind. Reduction of such integral equations lead to linear algebraic systems of the first kind, the numerical instability and inaccuracy of which have already been subject of attention [3–5]. Mathematical reason for the instability and inaccuracy is fast and unbounded growth of condition numbers of finite dimensional (truncated) systems as system size increases [6]. In this paper we suggest a new method free of such a drawback for the canonic problem mentioned above. The method is based on successive application of Grinberg method [7–9], conventional Orthogonal Polynomial Method (OPM) [10] (it is noteworthy that many authors have used various methods that are very similar to OPM; see [11] for example) and Analytical Regularization Method (ARM) [12–17] aimed at final infinite algebraic system to be of the second kind. Equations of the second kind have their well-known benefits in numerical stability and reliability; see [3–5]. ARM is the principal stage of the method suggested.

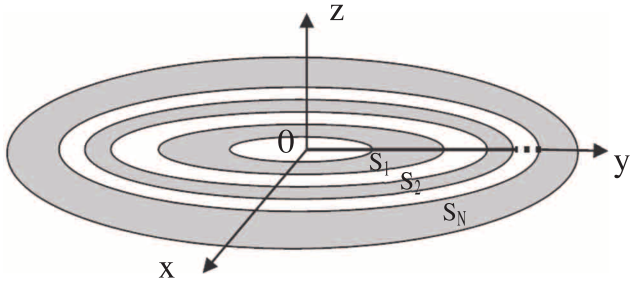

Figure 1. Geometry of the problem.

It clearly follows from our description that the technique proposed is rather universal. It can be used for much more complicated structures without any loss of efficiency and accuracy. Consider, for example, the case of arbitrary shaped, axially symmetrical toroidal screens (ARM approach for scalar wave diffraction by arbitrary shaped toroidal screen can be found in [17]). We use Analytical Regularization in the sense of [12, 14]. The historical and ideological background of ARM is a semi-inversion procedure which was invented almost half a century ago (see [12, 14] for historical references). It consists of nothing but analytical construction and application of either left-hand or right-hand regularizers for the operator of the corresponding boundary value problem. Regularizers (together with the same terminology) are widely used in functional analysis, including theory of integral equations (see [21] and Chapter 5 in [18], for example), but for the case, when the regularizer is already known.

It is necessary to outline construction of the second kind of equation of good quality (i.e. efficiently solvable) by means of the one-sided regularizer; this is possible for the simplest diffraction problems only (see [12] for details). For more complicated diffraction problems, two sided regularization of their boundary value problem operators is much more flexible and powerful (see Appendix A), and it is always a challenge to construct such regularizers in close analytical form for a new class of diffraction problems. In particular, for the diffraction problem considered herein we have no other choice but to use the corresponding two-sided regularization that we constructed analytically. In this paper, implementation of ARM is based on our method [15, 16] of solving of conventional OPM integral equation and on Analytical Regularization procedure for new class of integral equations involving so-called "free constants;" see Appendix C.

To outline the work here, initially, the electromagnetic wave diffraction boundary value problem (BVP) is reduced to scalar integral equations of the first kind by means of standard magnetic vector and scalar potentials using Grinberg's method [8, 9]. For such integral equations of the first kind, analytical construction of a two-sided regularizer is not straightforward. As such, we need to transform the integral equation at hand to the canonic integral equation of OPM (but with free constants; see Appendix C), the regularizers of which are already known [15, 16]. This transformation, invented in [17], is made by scaling the unknown functions and the kernel of the integral equation. The functional equations constructed in this way are of the second kind, but bear the assumption that the "free constants" are known (when actually they are not yet known). Condition of the electromagnetic field energy finiteness in any bounded space volume leads to the Meixner edge conditions, and both of them can be treated as restrictions on functional classes that the surface current must belong to. The idea of eliminating such "free constants" by means of the energy finiteness condition is given in [12]. A similar procedure to determine the "free constants" by means of the Meixner condition is considered in [7], but, unfortunately, not in a form where one can immediately obtain an equation of the second kind. That is why we used both restrictions mentioned above for making the "secondary regularization" that has been applied for elimination of such drawback as the last approach; see Appendix C. As a result of these efforts, we obtained the algebraic system of the second kind of high quality that allows its efficient solution.

FZPA characteristics with parameters of practical importance are calculated to compare the exact results for FZPA with the Physical Optics (PO) approach. It is shown that PO approach is not always relevant for design of FZPA, and the design needs rather strong mathematical modeling of the kind suggested herein.

In appendix A, the abstract scheme of the ARM is given in brief. In appendix B, the canonic scalar integral equation of OPM is equivalently reduced to infinite linear algebraic system of the first kind. Thereafter, ARM is applied to transform this system to the one of the second kind. In appendix C, presented is an extension of appendix B, considers the case of the canonical scalar integral equations of OPM with "free constants." Actually, this equation with "free constants" is the target for the electromagnetic wave diffraction problem mentioned above. The idea (the technique) of using "free constants" to narrow the class of solution can be found in [7].

2. Electromagnetic wave diffraction by thin perfectly conductive flat annular strips

We consider here the problem of wave diffraction of an incident electromagnetic wave $(E^i,\ H^i)$ by perfectly conductive and infinitely thin screen defined with the surface $S = \bigcup S_j$, $S_j \bigcap S_l = \emptyset$ for $l \neq j$ and $S_j=\{(z,\ \rho,\ \varphi)\colon\ z = z_j,\ \rho \in [a_j, b_j],\ \varphi \in [-\pi, \pi]\}$, $j = 1, 2, \ldots, N$ (see Figure 1). The scattering field $(E^s,\ H^s)$ is what we seek. The time dependence of the fields is chosen as $\mathrm{e}^{-i\omega t}$, and is omitted below. The posing of the diffraction BVP is traditional in radio science [19]. Our nearest purpose is to reduce this problem into special vector integral equation. That can be done by Grinberg's method [7–9], which is essentially based on flatness of an obstacle and, therefore, on identity $A_z = 0$, $z \in S$, where A_z is z-component of magnetic vector potential A. Taking into account the standard boundary conditions $E^{\mathrm{total}}_{\mathrm{tangental}}(q) = 0$ on surface S and absence of sources on S, one can show that scalar Lorentz potential ψ^s is a solution of the following inhomogeneous Helmholtz equation on the surface S [19] (indices i and s denote "incident" and "scattering," respectively):

$$\left[\Delta_t + k^2\right]\ \psi^s = -\frac{\partial E_z^i}{\partial z},\ \text{on } S \tag{1}$$

Using Green's formula technique similar to [19], but for unclosed screens (see [14]), one obtains the integral representation of magnetic vector potential $A(q) = \int J(p) G(q,\ p)\, dS$ on the surface S (p and q are integration and observation points, respectively, and $R(q, p)$ is the distance between them), where $G(q,\ p) = [\text{-}4\pi R(q,$

$p)]^{-1} exp\{ikR(q,\ p)\}$ is the Green's function of free space for Helmholtz equation, and $J(p)$ is the current density on S, which actually is the jump of normal derivatives of A on both sides of S. By means of A and ψ^s, the boundary condition gives the following vector integral equation of the first kind:

$$\int_S J(p)\, G(q,p)\, dS = (ik)^{-1}[grad_2\psi^s(q) - E^i_{\text{tangential}}(q)]\,,\ \ q \in S \tag{2}$$

with unknowns $J(p)$ and $\psi^s(q)$. Here, the operator grad_2 denotes a surface gradient. The general form (solution) of ψ^s can be obtained from (1); see below. Simultaneously with (2), the $J(p)$ can be sought. Axial symmetry of the obstacle lets one use Fourier expansions of the terms in (1) and (2). Performing this leads us from (1) to the following general solution for Fourier coefficient of ψ^s for S_j:

$$\psi^j_m(\rho_j) = A^j_m J_m(k\rho_j) + B^j_m Y_m(k\rho_j) + Z^j_m(k\rho_j)\,, \tag{3}$$

$$Z^j_m(k\rho) = \frac{\pi}{2}\left\{ Y_m(k\rho_j)\int_{a_j}^{\rho_j} f^j_m(\xi)\ J_m(k\xi)\, d\xi\ + J_m(k\rho_j)\int_{\rho_j}^{b_j} f^j_m(\xi)\ Y_m(k\xi)\, d\xi \right\}, \tag{4}$$

where A^j_m and B^j_m are some arbitrary (unknown in the moment) constants, $f^j_m(\xi)$ is Fourier coefficient of right-hand side of (1), and $J_m(k\rho_j), Y_m(k\rho_j)$ are Bessel and Neumann functions of order m respectively.

Equation (2) is considered in cylindrical coordinates. Because of axial symmetry, it is convenient to apply discrete Fourier transform to both sides of (2). Doing this, one arrives to the two scalar and coupled integral equations of the first kind ($m = 0, \pm1, \pm2, \pm3, ...$):

$$\begin{gathered}
2\pi\sum_{j=1}^{N}\int_{a_j}^{b_j}\left\{\begin{array}{l} {}^j j^\rho_m(z_p,\rho_p)\cdot {}^{j,\,l}G^c_m(z_q-z_p;\rho_q,\rho_p)+ \\ +{}^j j^\varphi_m(z_p,\rho_p)\cdot {}^{j,\,l}G^s_m(z_q-z_p;\rho_q,\rho_p) \end{array}\right\}\rho^p_j d\rho^p_j = {}^l\lambda^\rho_m(z_q,\rho_q) + \tfrac{i}{k}\tfrac{\partial}{\partial\rho_q}\psi^l_m(\rho_q) \\
2\pi\sum_{j=1}^{N}\int_{a_j}^{b_j}\left\{\begin{array}{l} -{}^j j^\rho_m(z_p,\rho_p)\cdot {}^{j,\,l}G^s_m(z_q-z_p;\rho_q,\rho_p)+ \\ +{}^j j^\varphi_m(z_p,\rho_p)\cdot {}^{j,\,l}G^c_m(z_p-z_p;\rho_q,\rho_p) \end{array}\right\}\rho_j d\rho_j = {}^l\lambda^\varphi_m(\rho_q) + \tfrac{im}{\rho_q}\psi^l_m(\rho_q) \\
\rho_l \in [a_l, b_l],\ \ l = 1,2,3,...,N.
\end{gathered} \tag{5}$$

Scalar functions ${}^l\lambda^{\rho;\varphi}_m$ are Fourier coefficients of right-hand side of (2), concerning the tangential electric field on the S. The unknown Fourier coefficients of the current density in (5) are written in integration point coordinates, which leads the kernel to the form ($z_q = z_p$, $\varphi = \varphi_q$-φ_p)

$$\begin{gathered}
{}^{j,\,l}G(0,\rho_q,\rho_p,\varphi)\left\{\begin{array}{c}\cos\varphi \\ \sin\varphi\end{array}\right\} = \sum_{m=-\infty}^{\infty} {}^{j,\,l}G^{c,s}_m(\rho_q,\rho_p)e^{im\varphi} = \\
= \frac{1}{2(i)^t}\left[\sum_{m=-\infty}^{\infty} {}^{j,\,l}G_{m-1}(\rho_q,\rho_p)e^{im\varphi} \pm \sum_{m=-\infty}^{\infty} {}^{j,\,l}G_{m+1}(\rho_q,\rho_p)e^{im\varphi}\right],\quad t = \left\{\begin{array}{l} 0,\ \ for\ \ c \\ 1,\ \ for\ \ s\end{array}\right. .
\end{gathered} \tag{6}$$

Here, ${}^{j,l}G(z_q - z_p, \rho_q,\ \rho_p,\ \varphi_q$-$\varphi_p)$ is free-space Green's function for Helmholtz equation, written in cylindrical coordinates system for points q=$(z_q,\ \rho_q,\ \varphi_q)$ and p=$(z_p,\ \rho_p,\ \varphi_p)$. Well known edge condition dictates the following representation of unknown Fourier coefficients of the current density:

$$ {}^{j}j_m^{\rho,\varphi}(\rho_j) = [(\rho_j - a_j)(b_j - \rho_j)]^r \, {}^{j}h_m^{\rho,\varphi}(\rho_j), \quad r = \begin{cases} 1/2, \ for\,\rho \\ -1/2, \ for\,\varphi \end{cases}, \tag{7} $$

where ${}^{j}h_m^{\rho,\varphi}(\rho)$ are some smooth functions. Conditions (7) determine the classes of functions in which the solutions of equations in (5) must be found. In particular, condition for radial component of the unknown vector enables us to determine constants A_m^j and B_m^j of equation (3); see Appendix C.

3. Reduction of scalar integral equations to the canonic OPM form

Let smooth function $\eta^{\,j}(t),\ t \in$[-1,1] with $d\eta^{\,j}(t)/dt > 0$ be given, and it is a parameterization of interval $[a_j,\ b_j]$ by means of points of interval [-1,1]. With substitution of $\rho_q = \eta^{\,l}(u),\ \rho_p = \eta^{\,j}(v)$, both equations in (5) can be rewritten as

$$ \begin{gathered} 2\pi \sum_{j=1}^{N} \int_{-1}^{1} \left\{ {}^{j}\tilde{Z}_m^{\rho}(v)\, {}^{j,l}\tilde{G}_m^{c}(u,v) + {}^{j}\tilde{Z}_m^{\varphi}(v)\, {}^{j,l}\tilde{G}_m^{s}(u,v) \right\} \eta^j(v)\, \eta^{j'}(v)\, d\eta^j(v) = {}^{l}\tilde{g}_m^{\rho}(u); \\ 2\pi \sum_{j=1}^{N} \int_{-1}^{1} \left\{ -{}^{j}\tilde{Z}_m^{\rho}(v)\, \tilde{G}_m^{s}(u,v) + {}^{j}\tilde{Z}_m^{\varphi}(v)\, \tilde{G}_m^{c}(u,v) \right\} \eta^j(v)\, \eta^{j'}(v)\, d\eta^j(v) = {}^{l}\tilde{g}_m^{\varphi}(u), \\ \rho_l \in [a_l, b_l],\ l = 1,2,3,...,N \end{gathered} \tag{8} $$

where ${}^{j,l}\tilde{G}_m^{c,s}(u,v) = {}^{j,l}G_m^{c,s}\left(\eta^l(u), \eta^j(v)\right)$, ${}^{j}\tilde{Z}_m^{\rho,\varphi}(v) = {}^{j}j_m^{\rho,\varphi}\left(\eta^j(v)\right)$, and ${}^{l}\tilde{g}_m^{\rho,\varphi}(u)$ are right hand sides of equations in (5) parameterized by $\eta^{\,l}(u)$, that includes constants A_m^j and B_m^j (which are yet unknown).

Before attempting to solve equation (8), it is necessary to understand the singular structure and properties of smoothness of the kernels ${}^{j,l}\tilde{G}_m^{c,s}(u,v)$. Functions $\{{}^{j,l}G_m^{c,s}\}$, j =1,2,. . . ,N in (8) are infinitely smooth, if $j \neq l$. For $j = l$ it is clear that ${}^{j,j}\tilde{G}_m^{c,s}(u,v)$ are infinitely smooth function in any point $u \neq v$, and only a vicinity of the points $u = v$ requires more detailed investigation. Omitting left superscripts for this case, the analysis we have made shows that functions $\tilde{G}_m^{c,s}(u,v)$ have the following representation:

$$ \tilde{G}_m^{c,s}(u,v) = -\frac{1}{4\pi\sqrt{\eta(u)\eta(v)}} \left[\ln|u-v| \left\{ \begin{pmatrix} 1, \text{ for } c \\ 0, \text{ for } s \end{pmatrix} + \sum_{n=2}^{3} {}^{c,s}A_n^m(u)|u-v| \right\} + H_{3,(c,s)}^m(u,v) \right] \tag{9} $$

with ${}^{c,s}A_n^m = \left(2\,(i)^t\right)^{-1} \left\{A_n^{m-1} \pm A_n^{m+1}\right\}$, t is defined in (6), and

$$ \mathrm{A}_2^m(u) = \frac{[k\eta'(u)]^2}{4} - \frac{m^2 - 1/4}{4}\left(\frac{\eta'(u)}{\eta(u)}\right)^2, \ \ \mathrm{A}_3^m(u) = \frac{m^2 - 1/4}{4}\left(\frac{\eta'(u)}{\eta(u)}\right)^3 \tag{10} $$

are smooth functions. Functions $H_{3,(c,s)}^m(u,v)$ are continuous with all its first partial derivatives up to the third order, and all its partial fourth derivatives have at most logarithmic singularity only. We have used the simplest parameterization:

$\eta^{\,j}(t)$=0.5[-$(a_j - b_j)t$+$(a_j + b_j)$]; $a_j < b_j, t \in$[-1,1]

for numerical results presented below (see the discussion about choice and construction of the most efficient parameterization in [14]).

Representations given by (9) complete the description of the singular behavior of function $\tilde{G}^{c,s}_m(u,v)$ for our purposes. Defining $\hat{G}^{c,s}_m(u,v) = 4\pi^2\sqrt{\eta(u)\eta(v)}\,\tilde{G}^{c,s}_m(u,v)$ as new kernel and function $K_m(u,v)$ by means of the equality

$$-\pi^{-1}\hat{G}^c_m(u,v) = -\pi^{-1}ln|u-v| + K_m(u,v), m = 0, \pm1, \pm2, \pm3, \ldots \tag{11}$$

one can see from formulae (9)–(11) that function $K_m(u,v)$ has the representation as

$$K_m(u,v) = -\frac{1}{\pi}\left\{\ln|u-v|\sum_{n=2}^{3}{}^{c}A^m_n(u)\,|u-v|^n\right\} - \frac{1}{\pi}H^m_{3,c}(u,v)\,. \tag{12}$$

Thus function $K_m(u,v)$ is much smoother than $\ln|u-v|$ (like $\delta^2\ln|\delta|$, where δ=$u-v$); for after closer investigation, we used $\hat{G}^{c,s}_m(u,v)$ below instead of $\tilde{G}^{c,s}_m(u,v)$. Otherwise, the main singularity in (12) will be proportional to $\delta\,\ln|\,\delta\,|$, that to say, the kernel $K_m(u,v)$ would be much more singular that immediately reduces the quality of the final algebraic system (making inequality (B15) invalid, for example). It is worth noting that the singularity structure of $K_m(u,v)$ is the same as one of $\hat{G}^s_m(u,v)$.

Let us denote by M the operator $M : \Lambda^2 \to \Lambda^2$ of integral equation (8), where $\Lambda^2 = \Lambda \times \Lambda$ and $\Lambda = L_2([-1,1])$ is the space of square integrable on $[-1,1]$ functions. Space Λ^2 would be standard choice of a functional space for an implementation of classic Method of Moment scheme, and, definitely, it is the space of choice of any real world computer; see [12, 14] for explanation of this fact.

The singularities of kernels in (8) are at most proportional to $\ln|u-v|$ and, therefore, the kernels are square integrable. Consequently, M is compact operator in Λ^2, and (8) is an equation of the first kind in Λ^2. It is noteworthy that the proper physical solution of (8) does not belong to Λ^2 (because current function $\tilde{Z}^{\varphi}_m(v)$ has in general the representation of the kind (B2) and, evidently, is not square integrable). All these mean that (8) is ill-conditioned equation in Tikhonov sense (see Chapter 15 in [18] and [20], for example). Thus, any direct method of equation (8) solving is numerically unstable.

A similar though simpler integral equation is considered in [4], where, in particular, the authors point out two principal origins of Method of Moment instability: coordinate-free instability (or stability) and ill-conditioning (or well-conditioning) of the coordinate family, i.e. the basis functions chosen for the implementation of the numerical method. The second origin depends on the details of the method implementation.

One can conclude by arguments in [4] that condition number ν(M)=||M||||M^{-1}|| of the truncated system must grow at least linearly as ν_N(M)$\geq C|\lambda_{max}$(M)$|N$, where N is the algebraic system size, λ_{max}(M) is eigenvalue of M with maximum modulus, and C is some constant. According to the observation made in [23] for a strip (which has similarity with our annular strips in that it also has flat structure), one can speculate that $|\lambda_{max}$(M)$|$ has to be proportional to $(kL)^2$ where L is the width of the widest annular strip in our FZPA system. Given estimate for ν_N(M) should be considered as the lowest boundary for the rate of the condition numbers growing. Unlucky choice of the basis functions may transform the rate of ν_N(M) even to be exponential.

Examples of such bad behavior of the condition numbers are presented in [4] and [6]. Recommendations for proper choice of basis are given in [4] (see also discussion about Lagrangian bases in [18]), where it is admitted that numerical implementation of what they recommend is not always easily implementable in practice.

Thus we have clear reason to transform equations in (8) into functional equations of the second kind in space l_2; we do this next according to the scheme mentioned in introduction.

Using equation (12), one can rewrite equation (8) as

$$\left\{\begin{array}{l} \int\limits_{-1}^{1}\left\{{}^{l}\hat{Z}_m^{\rho}(v)\left(-\frac{1}{\pi}\ln|u-v|+{}^{l,l}K_m(u,v)\right)+{}^{l}\hat{Z}_m^{\varphi}(v)\,{}^{l,l}\hat{G}_m^{s}(u,v)\right\}dv \\ +\sum\limits_{\substack{j=1\\ j\neq l}}^{N}\int\limits_{-1}^{1}\left\{{}^{j}\hat{Z}_m^{\rho}(v)\,{}^{j,l}\hat{G}_m^{c}(u,v)+{}^{j}\hat{Z}_m^{\varphi}(v)\,{}^{j,l}\hat{G}_m^{s}(u,v)\right\}dv \end{array}\right\}={}^{l}\hat{g}_m^{\rho}(u),$$

$$\left\{\begin{array}{l} \int\limits_{-1}^{1}\left\{-{}^{l}\hat{Z}_m^{\rho}(v)\,{}^{l,l}\hat{G}_m^{s}(u,v)+{}^{l}\hat{Z}_m^{\varphi}(v)\left(-\frac{1}{\pi}\ln|u-v|+{}^{l,l}K_m(u,v)\right)\right\}dv \\ +\sum\limits_{\substack{j=1\\ j\neq l}}^{N}\int\limits_{-1}^{1}\left\{-{}^{j}\hat{Z}_m^{\rho}(v)\,{}^{j,l}\hat{G}_m^{s}(u,v)+{}^{j}\hat{Z}_m^{\varphi}(v)\,{}^{j,l}\hat{G}_m^{c}(u,v)\right\}dv \end{array}\right\}={}^{l}\hat{g}_m^{\varphi}(u), \tag{13}$$

$$u\in[-1,1]\ ,\ l=1,2,3,...,N,\quad m=0,\pm1,\pm2,...$$

with new unknown functions pair ${}^{l}Z_m^{\rho,\varphi}$ and right-hand side ${}^{l}\hat{g}_m(u)$:

$$ {}^{l}\hat{Z}_m^{\rho,\varphi}(v)=\left[\eta^{l}(v)\right]^{1/2}\ \eta^{l'}(v)\ {}^{l}\tilde{Z}_m^{\rho,\varphi}(v),\quad {}^{l}\hat{g}_m^{\rho,\varphi}(u)=2\left[\eta^{l}(u)\right]^{1/2}\,{}^{l}\tilde{g}_m^{\rho,\varphi}(u). \tag{14}$$

with regard to conditions (7), the solution pair ${}^{l}Z_m^{\rho,\varphi}$ should have the representation below (with r defined in (7)):

$$ {}^{l}\tilde{Z}_m^{\rho,\varphi}(v)=\left(1-v^2\right)^{r}\,{}^{l}h_m^{\rho,\varphi}(v),\, v\in(-1,1), \tag{15}$$

where ${}^{l}h_m^{\rho,\varphi}(v)\in C^{0,\alpha}[-1,1]$ are functions of Hölder class. Thus it can be proved that integral equation (2) is equivalent to the infinite set of coupled integral equation of the first kind of type (13). System of kind (13) can be referred as the canonic coupled integral equations system for OPM each kernel of which has $\pi^{-1}\ln|u-v|$ as the strongest singularity.

Equation (13), as well as equation (8), is of the first kind. The degree of its coordinate-free ill-conditioning can be estimated as follows. Formula (B6) means that integral of the kind (B1), but with $K(u,v)\equiv 0$ has eigenvalues $\gamma_n^2=|n|$ for big n with $\hat{T}_n(u)$ as corresponding eigenvectors. If $K(u,v)$ is smooth enough in comparison with $\ln|u-v|$ (which is the case for the kernels in (13)), integral operator in (B1) has eigenvalues asymptotically close to $1/|n|$ for big indices $|n|$. Consequently, the operator of an algebraic system of size N obtained from (B1) by any direct method will have the smallest eigenvalue close to $1/N$ for big enough N. That is why the inverse operator will have the biggest eigenvalue close to N. Thus, condition number of the system will be proportional to $|\lambda_{\max}|N$, where $|\lambda_{\max}|$ is the eigenvalue with the biggest modulus of integral operator in (B1). System (13) is similar to (B1), but with a doubled structure. Consequently, coordinate-free ill-conditioning of equation (13) means qualitatively that the condition numbers of truncated systems derived from (13) will grow with at least with linear dependence on the system size. As well, for the equation (8), unlucky choices of the basis in Galerkin methods may lead to much faster (even exponential) increase of the condition numbers.

That is why we apply the procedure in Appendix B. The system right hand sides include "free constants" $A^l_{m,}B^l_m$ ((3)–(6)), which are yet unknown. Solution pair of this equation has to have the representation of the form (15), which can be reformulated and taken as two additional equations for A^l_m and B^l_m. System (13) is the subject of the procedure for solution suggested in Appendix C (where it is reduced to a functional equation of the second kind). Thus, initial electromagnetic diffraction problem for N annular strips is equivalently reduced to the set of algebraic systems of the second kind depending on $m = 0, \pm1, \pm2, \pm3, \ldots$.

4. Numerical results and discussion

In this section, we discuss numerical results obtained for two configurations of annular strips. The first of them is a Fresnel zone plate configuration consisting of six annular strips, with optical focus F=4λ, where λ is wavelength of incident field. As well known, the circular boundaries of Fresnel zones have radii r_n of the kind (see [2]) and that

$$kr_n = n\pi\sqrt{1 + 2kF/(n\pi)},\ \ k = 2\pi/\lambda,\ \ n = 1, 2, 3, \ldots,\ \ kF = 8\pi. \tag{16}$$

We have taken the following a_j, b_j, and z_j to define surface S of annular strips:

$$kz_j = 0; ka_j{=}\ kr_{2j-1}; kb_j{=}\ kr_{2j}; j = 1, 2, 3, 4, 5, 6. \tag{17}$$

The same configuration of annular strips is considered in [2], where it is referred as the first configuration. Figure 2 includes plots of principal feature of an algebraic system of the second kind: it is clear seen that truncated matrices of algebraic system that (13) has been reduced to for m=1 are uniformly bounded for configuration (17). When this configuration is illuminated by plane wave of unit amplitude and the wave propagates in negative direction of z-coordinate (see Figure 1), according to Physical Optics (PO) approach, the current density on perfectly conductive flat ring is equal to doubled amplitude of incident field, and the current's phase is constant. With this in mind, one can clearly see on Figure 3 the deviation of exact current from its PO approach. Figure 4 shows the same data for the same excitation, but for another structure, which is single annular strip with a_1=10λ and b_1=12λ. This strip is much wider of any strip of (17), and because of this PO approach is much better for this obstacle.

Figure 3 and Figure 4 presents scattered far-field patterns for both of the same obstacles. And here, as well, we see much better agreement of exact solution with PO approach for single strip, and the reason for this is, roughly speaking, the same. Nevertheless, it is possible to think that far field pattern demonstrates acceptable accuracy from engineering point of view for PO approach even for the worst case shown.

The most typical task of Fresnel zone plate is its usage as an antenna. In particular, it means that total field excited by a point source should have high directivity in the main lobe, and much smaller fields in the others lobes. PO gives solution of this task according to formula (17) and under supposition that the point source is placed in focus point: in axis OZ (see Figure 1) on distance F from the plate.

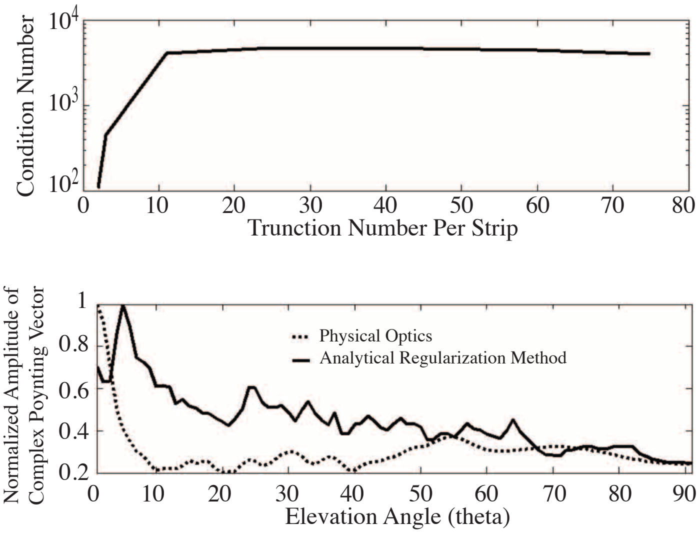

Figure 2. Condition number of final algebraic system and far scattered field patterns (by physical optics and analytical regularization method - point source excitation) concerning the Fresnel Zone Plate Antenna model of 6 annular strips.

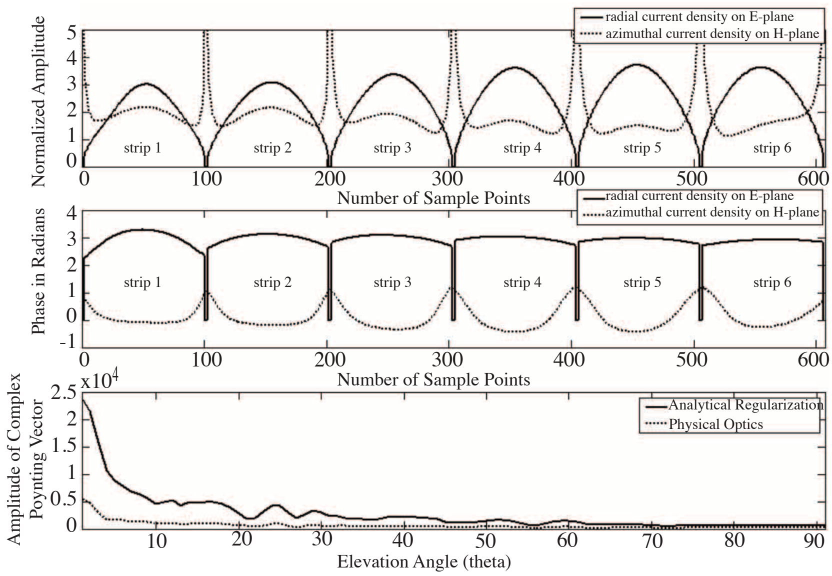

Figure 3. Normal incident unit amplitude plane wave on a FZPA model of 6 annular strips. Radial and azimuth currents in amplitude and phase, and far scattered fields of ARM and PO are given.

Figure 2 includes far field patterns given by PO approach and by exact solution of ARM. The first (and correct) impression from the Figure is that PO does not work in the case considered. Of course, the question arises: Why? The answer we believe in is the following. It is known, even in scope of PO, that effect of high directivity of Fresnel zone plate is very sensitive to the plate parameters. This means that such effect has resonant nature. Physical effects, which are negligible in other situations, may play important role in forming of resonance frequencies, etc. Many of such effects are out of PO consideration. The first of them is energy

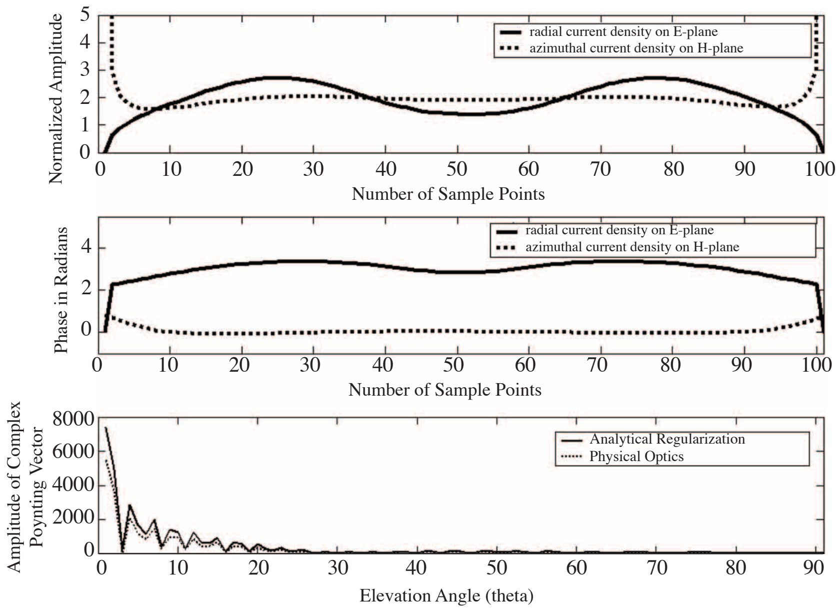

Figure 4. Normal incident unit amplitude plane wave on a single annular strip of width 2λ. Radial and azimuth currents in amplitude and phase, and far scattered fields of ARM and PO are given.

re-radiation by strips' edges. For example, correct currents in Figures 3 and 4 are very far from being constant (PO predicts two), which is a direct result of the existence of edges. Nevertheless, far field patterns in Figures 3 and 4 demonstrate acceptable agreement between PO and ARM, when the structure is excited by plane wave. Consequently, it should be something more which makes polar plots in Figure 2 such dramatically different. We suppose that this "something" is the presence of slow (surface) waves traveling along radial coordinate of the obstacle in both directions and re-radiating via the edges. Really, as follows from (16) and (17), widths of strips and distances between them tend to $\lambda/2$ as $n \to \infty$. That is why the system of strips considered can be roughly interpreted as periodic-like structure, especially for the most outer strips. As known, plane waves do not excite slow waves of a periodic structure. On the contrary, point source, as well as any strongly inhomogeneous field, is very efficient for such purpose. That is why we have relatively good agreement of PO and ARM for plane wave excitation, but complete disagreement for point source. Of course, all this said does not mean that Fresnel zone plate can not work well in reality. For example, presence of deep minimum of radiated energy in main lobe direction of exact solution clear indicates that z-coordinate Z_{ps} of the point source must be shifted from F. As well known, optical calculation of a mirror focus position may give error till $\lambda/2$. This value is negligible in optical range of frequencies (when, in our case, F, a_j, b_j, $b_j - a_j, a_{j+1} - b_j >> \lambda$), but is quite big for the structure considered. So, simple adjustment of Z_{ps} in real device may essentially improve the antenna characteristics. Nearly the same can be said about optimal choice of strips' radii a_j and b_j (which are not as easily changeable as Z_{ps}). Such optimization problems supposed to be the topic of our next publication.

5. Conclusion

A mathematically strong and numerically efficient method for electromagnetic waves diffraction by axially symmetrical system of annular strips (flat rings) is suggested. The method equivalently reduces the diffraction problem to the set of infinite algebraic systems of the second kind. Thus, being numerically stable, the truncation procedure allows obtaining the problem solution with, in principle, any accuracy required. Due to this, the method can be used as validation tool for the other (more general or less accurate) numerical methods and physical approaches. The method suggested has been applied to Fresnel zone plate antenna, and the results thus calculated were compared with ones of the PO approach. We found that the PO approach does not always give satisfactory solution, because the diffraction process involves a few complex physical phenomena which are not taken into account by the PO approach. We conclude that any physical model of the problem should take into account presence of the strips' edges (i.e. edge waves) and, possibly, exciting of slow waves traveling back and forth along radial direction, and re-radiated by the edges. The method suggested herein gives a simple but efficient alternative to complicated physical models of such a kind. It is based on conventional mathematical posing of the problem and does not require any additional physical supposition. Thus the design of a Fresnel zone plate antenna can not be based on the PO approach, only. The effect of high directivity is sensitive to values of the antenna parameters. That is why optimization of the parameters is necessary, which requires strong mathematical modeling of the antenna. Such optimization problem will be the subject of our next publications, as well as the method generalization for arbitrary shaped bodies and screens of revolution.

Appendix A: Abstract Scheme for ARM

Without going into mathematical details (see [12, 14, 18, 20, 21]), we explain here the algebraic scheme of ARM only. Let A be an operator given on a pair of functional (Banach or Hilbert) spaces, i.e. $A : B_1 \to B_2$. According to conventional terminology [21, 22]), A is said to be an operator of the second kind if, at first, $B_1 = B_2 = B$ for some space B, and

$$A = I + H, \tag{A1}$$

where I and H are the identical and some compact operator respectively. Otherwise A is an operator of the first kind. Let us suppose additionally that pair (L, R) of operators is given: $L : B_2 \to B$ and $R : B \to B_1$ is such that

$$LAR = I + H, \quad LAR : B \to B, \tag{A2}$$

where H is compact operator in B. In such a case we refer to the pair (L, R) as a two-sided regularizer if both $L \neq I$ and $R \neq I$. If $R = I$ or $L = I$, we refer to the pair (L, R) as a one-sided (left or right) regularizer.

When an equation of the first kind (i.e., with operator A of the first kind)

$$Ax = b \tag{A3}$$

is given and its regularizer (L, R) is known, the equation can be immediately reduced to one of the second kind ($LARy = (I + H)y$):

$$(I + H)y = c; \quad x = Ry, \quad c = Lb, \quad y, c \in B. \tag{A4}$$

The methods of construction of various regularizers (L, R) in closed explicit form, and applying the regularizer to equations of the first kind, is the key point of ARM in Electromagnetics and diffraction theory. The strong necessity of such or similar procedure is well known and outlined by many authors (see, for example, [3, 4]). From numerical point of view, the most convenient space B is Hilbert space l_2 of square-summable sequences $B = l_2$ for which I and H are matrix-operators.

The most essential advantage of equations of the second kind (in contrast to those of the first kind) is that solutions y^M of truncated systems $(I+H^M)y^M = c^M$ tend to the solution y^∞ of infinite system (A4), i.e. $||y^\infty - y^M|| \to 0$, if $||H^M - H|| \to 0$ and $||c^M - c|| \to 0$ for $M \to \infty$, and that this convergence is not just only theoretical, but numerical as well. Namely, the sequence $\nu^M = ||I+H^M|| \cdot ||(I+H^M)^{-1}||$ of condition numbers of truncated systems has the finite limit $\nu^\infty = ||I+H|| \cdot ||(I+H)^{-1}||$, and that is why it is uniformly bounded: $\nu^M \leq const$ (for sufficiently large M). That is why real numerical process of truncated system solving is stable relative to the round off errors for arbitrary big M.

An implementation of this idea requires, of course, analytical construction (in explicit form) of operators L and R. These operators are constructed herein for integral operator of every above-considered integral equation. Due to OPM, operators L and R have rather simple form, based on Fourier-Chebyshev transform and corresponding infinite diagonal matrices; see below. Thus each of the integral equations considered, as well as diffraction problems posed, have been equivalently reduced to the correspondent equation of the second kind of type (A4) with $B = l_2$ (or with B equal to finite direct sum of a few spaces l_2, which is the same from qualitative and numerical points of view).

Appendix B: Treating the Canonical Integral Equation with Logarithmic Singularity by OPM and ARM

Let us consider the canonical integral equation

$$\int_{-1}^{1} \left\{ -\frac{1}{\pi} \ln|u-v| + \mathrm{K}(u,v) \right\} z(v)\, dv = b(u), \quad u \in [-1,1], \tag{B1}$$

with unknown function $z(v)$. Suppose all other functions in (B1) are known and smooth enough for our purposes. In particular, $\mathrm{K}(u,v)$ is continuous with its first derivatives and all its mixed derivatives of the second order are square-integrable.

We are looking for a solutions $z(v)$ of the kind

$$z(v) = (1-v^2)^{-1/2} m(v), \quad v \in [-1,1], \tag{B2}$$

with function $m(v)$ belonging to Hölder class on [-1,1]. These properties of functions $K(u,v)$, $z(v)$ and $b(u)$ are quite natural for diffraction problem considered.

Any equation of (8) is of type (B1). We use below the orthonormal Chebyshev polynomials of the first kind $\hat{T}_n(x)$, which are connected with standard Chebyshev polynomials of the first kind $T_n(x)$ (i.e. $T_n(\cos\theta) = \cos n\theta$) by means of the formulae

$$\hat{T}_n(x) = d_n^{-1} T_n(x); \quad d_0 = \pi^{1/2}, \quad d_n = (\pi/2)^{1/2}, \quad n \neq 0; \tag{B3}$$

$$\int_{-1}^{1} \left(1-x^2\right)^{-1/2} \hat{T}_n(x)\, \hat{T}_s(x)\, dx = \delta_{s,n}, \quad s,n = 0,1,2,3..., \tag{B4}$$

with Kronecker delta δ_{sn} (i.e. $\delta_{nn} = 1$ and $\delta_{sn} = 0$ for $s \neq n$). Using formula 5.4.2.9 of [22],

$$\sum_{n=1}^{\infty} \frac{\cos nx}{n} = -\ln\left|2\sin\frac{x}{2}\right|, \quad x \in [-2\pi, 2\pi], \tag{B5}$$

one can after elementary transformations obtain

$$-\frac{1}{\pi}\ln|u-v| = \sum_{n=0}^{\infty} \frac{\hat{T}_n(u)\,\hat{T}_n(v)}{\gamma_n^2}, \quad u,v \in [-1,1]; \qquad \gamma_0 = (\ln 2)^{-1/2}, \quad \gamma_n = |n|^{1/2}, \quad n \neq 0. \tag{B6}$$

Due to the properties of functions $z(v)$, $b(u)$ and $\mathrm{K}(u,v)$, they are equal to their Fourier-Chebyshev series:

$$z(v) = \left(1-v^2\right)^{-1/2} \sum_{m=0}^{\infty} z_m \hat{T}_m(v),; \quad b(u) = \sum_{n=0}^{\infty} b_n \hat{T}_n(u), \; u \in (-1,1) \tag{B7}$$

$$K(u,v) = \sum_{s=0}^{\infty}\sum_{n=0}^{\infty} k_{sn}\hat{T}_n(u)\,\hat{T}_s(v), \;\; u,v \in [-1,1], \tag{B8}$$

where $\{z_m\}_{m=0}^{\infty}$ are unknown coefficients and $\{b_n\}_{n=0}^{\infty}$ and $\{k_{sn}\}_{s,n=0}^{\infty}$ are Fourier-Chebyshev coefficients of the functions $b(u)$ and $\mathrm{K}(u,v)$ respectively:

$$b_n = \int_{-1}^{1} \left(1-u^2\right)^{-1/2} b(u)\,\hat{T}_n(u)\,du; \quad k_{sn} = \int_{-1}^{1}\int_{-1}^{1} \frac{K(u,v)\,\hat{T}_s(u)\,\hat{T}_n(v)}{\left(1-u^2\right)^{1/2}\left(1-v^2\right)^{1/2}}dudv \tag{B9}$$

Moreover, coefficients k_{sn} satisfy the following inequality (see [7, 8]):

$$\sum_{s=0}^{\infty}\sum_{n=0}^{\infty} \left(1+n^2\right)\left(1+s^2\right)|k_{sn}|^2 < \infty,. \tag{B10}$$

After substituting the right hand sides of series (B7), (B8) into equation (B1), changing the order of integration and summation, the orthogonal property (B4) gives us the equalities of the Fourier-Chebyshev coefficients of the left and right hand sides of equation (B1):

$$\gamma_n^{-2} z_n + \sum_{s=0}^{\infty} k_{ns} z_s = b_n, \; n = 0, 1, 2, \dots, \tag{B11}$$

which are equivalent to (B1) because of the completeness of functions system $\{\widehat{T}_n(u)\}_{n=0}^{\infty}$. Equalities (B11) can be considered an infinite algebraic system, which is possible, in principle, to solve by means of truncation procedure. It is found in paper [23] that condition number ν_M of the algebraic (finite-dimensional) system of Moment Method for diffraction by infinitely thin strip is growing proportionally to M, when M tends to infinity, where M is the size of the system. It can be easy seen that system (B11) has the same qualitative property: the condition number $\nu_M \approx const \cdot M$, where M is the size (truncation number) of the corresponding truncated system. Really, according to (B6) and (B10), $\gamma_n^{-2} \approx n^{-1}$, and k_{sn} are decaying much faster, when $s, n \to \infty$. That is why, eigenvalues of infinite matrix in (A11) are asymptotically equal to n^{-1} for large n. Consequently, eigenvalues of inverse matrix are proportional asymptotically to n, and evidently, spectral condition numbers ν_M of truncated matrices are proportional to M. In contrast, the system resulting from (B11) after ARM (see below) has uniformly bounded conditions numbers $\nu_M \le const, \quad M \to \infty$ of its truncated systems (and these ν_M are relatively small; see Figure 2, for example).

So, it is clear that (B11) is a system of the first kind. In order to apply corresponding ARM to (B11), let us define matrix-operators $L, R, K, \hat{K}_\alpha$ (see appendix A):

$$L = R = diag\{\gamma_n\}_{n=0}^{\infty}, \quad K = \{k_{sn}\}_{s,n=0}^{\infty};$$

$$\hat{K}_\alpha = L^{2-\alpha} K R^\alpha = \{\hat{k}_{sn}\}_{s,n=0}^{\infty}; \ \hat{k}_{sn} = \gamma_n^{2-\alpha}\gamma_s^\alpha k_{sn}, \tag{B12}$$

and vector-columns

$$z = \{z_n\}_{n=0}^{\infty}; \quad b = \{b_n\}_{n=0}^{\infty};$$

$$y = R^{-\alpha} z = \{y_n\}_{n=0}^{\infty}, \quad y_n = \gamma_n^{-\alpha} z_n; \quad \hat{b} = L^{2-\alpha} b = \{\hat{b}_n\}_{n=0}^{\infty}, \quad \hat{b}_n = \gamma_n^{2-\alpha} b_n, \tag{B13}$$

where diag denotes that L and R are diagonal matrix-operators, and $\alpha \in [0, 2]$ is some parameter to be chosen; see below. Using new unknowns y_n, instead of z_n, and multiplying each n^{th} equation of (B11) by $\gamma_n^{2-\alpha}$, one obtains infinite algebraic system with the following functional view:

$$(I + \hat{K}_\alpha) y = \hat{b}, \quad y, \hat{b} \in l_2. \tag{B14}$$

It can be proved that (B14) is system of the second kind, i.e. $\hat{K}_\alpha$ is compact in l_2 operator for any $\alpha \in [0, 2]$. It means that we constructed the family $(L^{2-\alpha}, R^\alpha)$, $\alpha \in [0, 2]$ of regulators, and each solves the initial problem: to obtain an algebraic system of the second kind. Thus, the optimization problem arises: to find the best in some sense α. From a numerical point of view, it is most naturally to think the "best" α as one providing the fastest convergence of truncation method for system (B14). Namely, let $\hat{K}_\alpha^M$ is finite matrix of size M, obtained as truncation of matrix $\hat{K}_\alpha$. It is necessary to find such $\alpha \in [0, 2]$, which provides the fastest asymptotic decaying of for $M \to \infty$. The investigation made shows that the solution of this problem (for Hilbert-Schmidt operator norm) is $\alpha = 1$.

In paper [24] (for a system similar to (B11)) the regularization is constructed in a way equivalent to the choice $\alpha = 0$ in our notation (with the exception operator R is absent in [24]). This approach is based on ideas of semi-inversion method explained in [12, 13], which is mistakenly mismatched in [25] with Analytical Regularization. We understand the terminology ARM in the sense [12–17] described above, and semi-inversion method as a very special case (when $R = I$) of Analytical Regularization in general. By introducing parameter α, one has the possibility to find not the best value of α only in sense mentioned, but, what is most important, to construct operator $\hat{K}_\alpha$ with very important qualitative properties. In particular, from formulae (A10) and (A12) it follows that

$$\sum_{n=0}^{\infty}\sum_{s=0}^{\infty} (1+n)(1+s)\left|\hat{k}_{ns}\right|^2 < \infty \text{ for } \alpha = 1, \tag{B15}$$

and evidently $\hat{K}_\alpha$ for $\alpha = 1$ is compact operator, but much more than compact operator only: its coefficient decaying much faster than necessary even for to be Hilbert-Schmidt operator. It can be proved (see [12, 14]) that equations (B1) and (B14) are equivalent in the sense of one to one correspondence between solutions of both equations (in relevant functional spaces). Truncation procedure applied to equation (B14) of the second kind enables us to obtain solutions of both equations with, in principle, any required accuracy.

We use $\alpha = 1$ in calculations and eliminate α from notation without any additional notice.

Appendix C: Treating the Canonical Integral Equation with Logarithmic Singularity Including "Free Constants" by OPM and ARM

Let us consider integral equation of the kind

$$\int_{-1}^{1} \left\{ -\frac{1}{\pi} \ln|u-v| + K(u,v) \right\} z(v)\, dv = b(u) + A\Phi^{+}(u) + B\Phi^{-}(u), \quad u \in [-1,1], \tag{C1}$$

where $\Phi^{\pm}(u)$ are two smooth-enough and linearly independent functions. Such an equation without free constants A and B (i.e. $A = B = 0$) is known as an OPM canonic integral equation given in Appendix B. This section includes below a brief explanation of the Analytical Regularization of equation (C1), when A and B are unknown. In general, for arbitrary A and B, the unknown function $z(v)$ is singular for $v = \pm 1$ and has the form

$$z(v) = z(A, B;\, v) = \left(1 - v^2\right)^{-1/2} \tilde{m}(A, B;\, v), \tag{C2}$$

where $\tilde{m}(A, B;\, v)$ is a smooth function that depends on parameters A and B. $\tilde{m}(A,B;v)$ can be represented by means of a Fourier-Chebyshev series $\tilde{m}(A,B;v) = \sum_{n=0}^{\infty} z_n \hat{T}_n(v)$, where $\hat{T}_n(v)$ are orthonormal Chebyshev polynomials of the second kind. Physical properties of the considered diffraction problems dictate the edge conditions of the type (7) for equations of the type (C1). In particular, when $z(v)$ is proportional to radial component of current density it should be of the type

$$z(v) = z(A, B;\, v) = \left(1 - v^2\right)^{1/2} m(A, B;\, v) \tag{C3}$$

with some smooth function $m(A,B;v)$. Thus it is necessary to find such constants A and B such that (C3) is valid. Evidently, the necessary (and sufficient) requirements are

$$\tilde{m}(A,B;\pm 1) = 0, \text{ i.e. } \sum_{n=0}^{\infty} z_n \hat{T}_n(\pm 1) = 0. \tag{C4}$$

Let the Fourier–Chebyshev expansions of functions $\Phi^{\pm}(u)$ be

$$\Phi^{\pm}(u) = \sum_{n=0}^{\infty} \phi_n^{\pm} \hat{T}_n(u), \ \ u \in [-1,1]; \quad \phi_n^{\pm} = \int_{-1}^{1} \left(1 - u^2\right)^{-1/2} \Phi^{\pm}(u)\, \hat{T}_n(u)\, du. \tag{C5}$$

Define $\hat{\phi}_n^{\pm} = \gamma_n \phi_n^{\pm}$, borrowing γ_n from (B6) and [15]: $\gamma_0 = (\ln 2)^{-1/2}, \gamma_n = (n)^{1/2},\ n \neq 0$. Then we can suppose for a moment that constants A and B and functions $\Phi^{\pm}(u)$ are known, and apply to (C1) the version of ARM constructed in [15] similar to the step followed from (B13) to (B14). Doing this, we arrive to the following algebraic system with new unknowns $y_n = z_n/\gamma_n$:

$$y_n + \sum_{s=0}^{\infty} \hat{k}_{ns} y_s = \hat{b}_n + A\hat{\phi}_n^+ + B\hat{\phi}_n^-, \quad n = 0, 1, 2, \ldots \ . \qquad (C6)$$

Condition (C4) can now be rewritten as

$$\sum_{n=0}^{\infty} \gamma_n y_n \hat{T}_n (\pm 1) = 0. \qquad (C7)$$

Equation (C7) must be considered together with (C6). The final equation of the second kind evidently dictates the further transform (regularization) of equation (C7). Let us substitute the expressions for y_n that follow from (C6) into (C7):

$$\sum_{n=0}^{\infty} \gamma_n \hat{T}_n (\pm 1) \left\{ -\sum_{s=0}^{\infty} \hat{k}_{ns} y_s + \hat{b}_n + A\phi_n^+ + B\phi_n^- \right\} = 0. \qquad (C8)$$

Then passage to the equations

$$\sum_{s=0}^{\infty} Q_s^{(\pm)} y_s - AP_+^{(\pm)} - BP_-^{(\pm)} = b^{(\pm)} \qquad (C9)$$

(with two variants: (+) and (-)) can be done by defining the following numbers s=0,1,2,...:

$$Q_s^{(\pm)} = \sum_{n=0}^{\infty} \gamma_n \hat{k}_{ns} \hat{T}_n (\pm 1) \ ; \ b^{(\pm)} = \sum_{n=0}^{\infty} \gamma_n \hat{b}_n \hat{T}_n (\pm 1) \ ; \ P_+^{(\pm)} = \sum_{n=0}^{\infty} \gamma_n \hat{\phi}_n^+ \hat{T}_n (\pm 1) \ ; \ P_-^{(\pm)} = \sum_{n=0}^{\infty} \gamma_n \hat{\phi}_n^- \hat{T}_n (\pm 1) \,. \qquad (C10)$$

Due to the property $\sum\limits_{n=0}^{\infty} \sum\limits_{s=0}^{\infty} (1+n)(1+s) \left| \hat{k}_{ns} \right|^2 < \infty$, concerning (C6) (see [15] and (B15)), numbers $Q_s^{(\pm)}$ exist and tend to zero sufficiently fast for the purpose, while index s increases. Functions $\Phi^{\pm}(u)$ are smooth and, consequently, their Fourier-Chebyshev coefficients $\phi_n^{\pm}$ are very fast decaying. This means that numbers $P_{\pm}^{(\pm)}$ exist. In the case of the diffraction problem considered, $\Phi^{\pm}(u)$ are eigenfunctions of the corresponding Sturm-Liouville problem, generated by Separation of Variables Method for Helmholtz equation. That is why $\Phi^{\pm}(u)$ are Bessel and Neumann functions (see (3), (4), and (13)), and consequently $\phi_n^{\pm}$ are proportional to the corresponding Bessel functions; and why $\phi_n^{\pm}$ tends to zero faster than n^{-l} with arbitrary l >0, as n$\to \infty$.

Thus equations (C6) and (C9) can be considered together as infinite algebraic system of the second kind in l_2 with unknowns A, B, $\{y_n\}_{n=0}^{\infty}$, and it is equivalent (in the sense of one-to-one correspondence between solutions) to integral equation (C1) with conditions (C3). This statement can be proved in strong mathematical sense; see [14].

Now, let us consider equations (13) for N=1. Such equation consists of coupled pair of canonic OPM equation and OPM equation with "free constants." It can be easy shown that the above described procedures of ARM for both equations give ARM for equation (13) with $N=1$. In the case $N>1$, the same procedures applied to block-diagonal of $N \times N$ matrix (13) give ARM for system (13) of coupled integral equation. The resulting algebraic system $(I+H)y=b$ is of the second kind—with compact operator H.

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