

Weak penetration and radiation through apertures in conducting bodies of revolution

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Abstract

The simplest way to solve for penetration through small apertures in conducting surfaces is to treat the body as a scatterer and determine the interior field as the sum of the incident field and the scattered field contributed by the current induced on the surface of the body. However, it is well known that, if the aperture is very small or if the penetrated field is very weak this method yields very inaccurate results, which, in turn, prohibits proper design of electronic systems, especially for electromagnetic compatibility and interference. Previously, alternative integral equations formulations were proposed to remedy this problem and applied to two-dimensional conducting cylinders with slots [1–2]. Application of these alternative techniques to three-dimensional conducting bodies of revolution (BOR) is studied in this work. In addition, the reciprocity principle is used to recast a weak penetration problem into a weak radiation problem, and it is shown that the alternative formulations useful for weak penetration are also useful for weak radiation. The important features and relative accuracies of each formulation together with numerical results are detailed for mock missile-shape structures.

1. Introduction

The outer metallic surface of almost all electronic equipment contains apertures that may allow electromagnetic energy to radiate from, or penetrate into, the system. The apertures may be intentional (such as for ventilation ports, display screens, or a receiving/transmitting probe) or unintentional (such as nonconductive weather gaskets, bonded seams). At low frequencies, the electrical size of an aperture may be so small that radiation from or penetration into the system is minimal. However, as the operation frequency increases, the apertures become electrically larger, and the device may eventually be affected by electromagnetic interference or may create an interference problem.

Penetration through very small apertures in conducting surfaces has been of interest to researchers for many years. Of particular interest in this investigation is determination of the field that penetrates holes in conducting bodies of revolution (BOR's) and the coupling of this field to an interior probe. The straightforward

solution of this problem is to sum the incident field and the scattered field contributed by the current induced on the body surface, and this approach is referred to as the *scatterer method*. This approach can yield inaccurate results if the aperture is very small or is located on the body in such a way that the penetration is very weak. The inaccuracy inherent to the *scatterer method* is mainly attributed to *subtractive cancellation* which results due to the summation of two almost equal large numbers with opposite signs. Alternative integral equation methods, known as the *short-circuit current method* and the *equivalent current method*, have been proposed to remedy such inaccuracies for conducting two-dimensional cylinders [1–2]. The main advantage of the alternative formulations is that they provide more accurate results than the scatterer method. However, the two alternative methods require greater computational resources than does the scatterer method.

Although penetration through apertures on two-dimensional structures has been investigated, the accurate determination of field penetration through apertures on three-dimensional conducting structures has received little attention. The application of Schelkunoff's equivalence principle [3], also known as the *short-circuit current method*, to field penetration into BOR structures was first studied by Schuman and Warren [4] in order to improve the accuracy of field computations for cases when the aperture is small and/or when the field at points deep inside a cavity is sought. Glisson and Wilton [5] later recast their formulation into a more computationally efficient form and also applied it to determine the on-axis penetrated field for scatterers illuminated by arbitrarily incident plane waves. In the present study, we follow a different approach to determine the penetration for this problem, which will yield accurate results when the penetrated field is weak. Using reciprocity, we first recast the original penetration problem into a radiation problem in which an antenna resides axially inside the BOR structure. One only needs to solve the *zero-order* BOR equation in the radiation problem, as in the penetration problem, thus saving effort compared to the use of the m^{th} -order BOR equations that might otherwise be used. Using the radiation paradigm, along with a formula derived from reciprocity allows one to determine the signal coupled to an axially located loaded antenna due to an arbitrary angle-of-illumination plane wave. The alternative integral equation formulations are applied to geometries of interest, and it is shown that these methods yield accurate results in the determination of weak radiation, as well as for weak penetration.

In the next section, the solution methods are briefly described. Then, the moment method solution procedures to solve the integral equations are outlined. Three methods are applied to a mock missile structure and numerical results are presented to illustrate their relative accuracies. Observations and concluding remarks follow.

2. Numerical solution of alternate integral equations

Application of reciprocity principle and three methods for determining the penetrated field through apertures in conducting BOR's are outlined and the numerical procedures for applying the methods are presented. Detailed derivations of integral equation formulations of these alternative methods were described in [1–2].

Application of Reciprocity Principle: A representative body with conducting surface S and aperture A is illustrated for reception and radiation modes in Figures 1(a) and 1(b), respectively. The indirect approach adopted here makes use of the reciprocity theorem and allows one to compute the signal at the coaxial terminal load from knowledge of the field radiated by the mock missile, under the condition that the excitation results from a current generator impressed at the terminal end of the coax, as illustrated in Figure 2. First, a voltage of one volt across the coaxial aperture is assumed. With this excitation, the current on the antenna and the

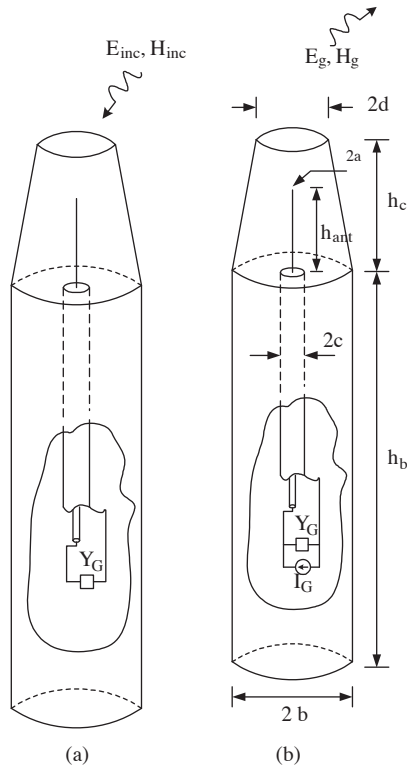


Figure 1. Mock missile with a bulkhead and an antenna on its axis: (a) penetration problem, (b) radiation problem. The dimensions are: $h_b = 118.7$ cm, $h_c = 14.6$ cm, $a = 0.0787$ cm, $b = 7.875$ cm, $c = 0.2286$ cm, $d = 4.25$ cm.

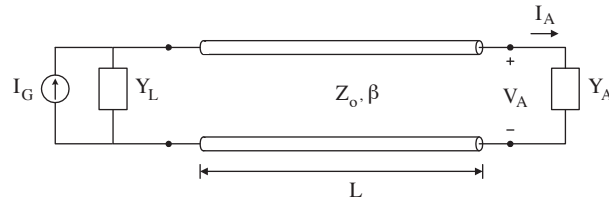


Figure 2. Transmission line model of the coax, the current generator, and the admittance Y_L .

structure is computed. From the knowledge of this current, driven by a one-volt generator, one can compute the input admittance Y_A at the base of the antenna. Since the antenna is driven by the coax, Y_A can be viewed as a terminating admittance at the end of the coax where its center conductor becomes the antenna. From transmission line theory, it can be easily shown that the current generator causes the voltage V_A , at the base of the antenna, to be

$$V_A = \frac{1 + \Gamma_A}{(Y_L + Y_A) + (1 + \Gamma_A e^{-2j\beta L})} e^{-j\beta L} I_g, \quad (1)$$

where β is the propagation factor of the transmission line and Γ_A is the reflection coefficient at the antenna

base, which is expressed as

$$\Gamma_A = \frac{Y_o - Y_A}{Y_o + Y_A}, \tag{2}$$

where Y_o is the characteristic admittance of the line. The actual voltage driving the antenna can be used to compute the actual currents on the structure and the radiated field caused by the current generator. Recall that the currents and the fields are computed with the assumption of 1 volt, so one can simply scale all the currents and field values by the ratio of $V_A:1$. The expression for V_g is

$$V_g = \frac{-\hat{\theta} \cdot \mathbf{E}_g(\mathbf{r}_\theta) (1 + \Gamma_A) e^{-j\beta L}}{V_A [(Y_L + Y_A) + (1 + \Gamma_A e^{-j2\beta L})]} \tag{3}$$

where \mathbf{E}_g is the far-zone electric field due to current generator displayed in Figure 2. A detailed derivation of the above expression is presented in [6]. Hence, the induced voltage across the load of Figure 1(a) can be computed via (3) if one solves the radiation problem of Figure 1(b). Since the conducting body and on-axis coax excitation of the radiation problem are circularly symmetric, the currents and the fields everywhere are ϕ -independent. Therefore, the solution of the radiation problem requires only the *zero-order* mode of a Fourier expansion of the unknown currents and fields.

Scatterer Method: The scatterer method is the simplest technique to determine the field that penetrates an aperture in a conducting surface. Hence, we will only consider the radiation problem as depicted in Figure 1(b) and study the radiation into the exterior region; that is, the region exterior to the bounded surface which encloses the body. In the scatterer method, the body is treated as a scatterer and the radiated field can be determined as the sum of the incident and scattered field. The total electric field can be viewed as the sum of the incident field $\mathbf{E}^{inc}(\mathbf{r})$, due to radiation from a known source in the absence of the scatterer, and the scattered field $\mathbf{E}^s(\mathbf{J}; \mathbf{r}; S)$, which is due to radiation by the current on a surface S :

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}^s(\mathbf{J}; \mathbf{r}; S) + \mathbf{E}^{inc}(\mathbf{r}). \tag{4}$$

In (1), $\mathbf{E}(\mathbf{r})$ is the total electric field and the tangential components of the fields must satisfy the relation

$$\mathbf{E}_{tan}^s(\mathbf{J}; \mathbf{r}; S) + \mathbf{E}_{tan}^{inc}(\mathbf{r}) = 0; \quad \mathbf{r} \in S \tag{5}$$

on the conducting surface of the body.

Consider the body with an aperture shown in Figure 1(b) excited by a current generator inside the body. For such an excitation the electric current has two components: one in the direction of surface tangential vector $\hat{\mathbf{t}}$, and the other in the azimuthal direction $\hat{\phi}$. The integral equations resulting from each unit vector are coupled. However, using the azimuthal symmetry of the BOR, one can simplify the solution procedure by expanding the current and field components in terms of Fourier series. When this is carried out, one arrives at decoupled integral equations for each mode. If method of moments (MoM) [7] solution technique is applied for each Fourier mode m , one obtains

$$\begin{bmatrix} \beta_{11}^m & \beta_{12}^m \\ \beta_{21}^m & \beta_{22}^m \end{bmatrix} \begin{bmatrix} \rho J_m^t \\ J_m^\phi \end{bmatrix} = \begin{bmatrix} V_m^{t,inc} \\ V_m^{\phi,inc} \end{bmatrix} \tag{6}$$

where β_{ij} represents integral operators acting on the current components. A detailed derivation of (6) is given in [8–10]. When the excitation is also symmetric, an on-axis radiating antenna for instance, (6) is further simplified by solving $m = 0$ mode only:

$$[\beta_{11}^0] [\rho J_0^t] = [V_0^{t,inc}] \quad (7)$$

Short-Circuit Current Method: The theory developed by Schelkunoff allows one to replace the effect of an aperture on the surface of a body with equivalent “impressed” currents in the aperture. With a small modification to the numerical scheme used to implement the scatterer method, the same numerical procedure can be employed for the short-circuit current method. The zero-order ($m = 0$) BOR equation in matrix form is presented below. One solves the following matrix equation to determine the short-circuit current on the structure:

$$\begin{bmatrix} Z_{mn}^{aa,0} & Z_{mn}^{as,0} \\ Z_{mn}^{sa,0} & Z_{mn}^{ss,0} \end{bmatrix} \begin{bmatrix} \rho J_n^{a,0} \\ J_n^{s,0} \end{bmatrix} = \begin{bmatrix} V_m^{a,0} \\ V_m^{s,0} \end{bmatrix} \quad (8)$$

where $[\rho J_n^{a,0}]$ and $[\rho J_n^{s,0}]$ are the unknown zero-order current coefficients over the shorted aperture and the conductor surfaces, respectively. The matrix elements $[Z_{mn}^{aa,0}]$, $[Z_{mn}^{as,0}]$, $[Z_{mn}^{sa,0}]$, and $[Z_{mn}^{ss,0}]$ represent the contributions from the shorted aperture to itself, the conductor to the shorted aperture, the shorted aperture to the conductor, and the conductor to itself, respectively. The tested incident electric field is $[V_m^{a,0}]$ on the aperture and $[V_m^{s,0}]$ on the conductor surface.

After solving for the short-circuit current from (8), this current is placed at the aperture with the original source removed from the problem. The induced current on the conducting surface is subsequently determined due to the impressed electric field created by the short-circuit current at the aperture. This is written in matrix form as

$$[Z_{mn}^{ss,0}] [\rho J_n^{ind,0}] = -[Z_{mn}^{sa,0}] [\rho J_n^{imp,0}] \quad (9)$$

where $[\rho J_n^{ind,0}]$ is the unknown induced current and $[\rho J_n^{imp,0}]$ is the known impressed current which is equal and opposite to the aperture short-circuit current ($J^{imp,0} = -J^{sc,0}$). The penetrated field is simply the sum of the fields due to the induced current on the conductor and that due to the impressed current at the aperture.

Equivalent Current Method: In the equivalent current method, an electromagnetic model valid in the interior region and another model valid in the exterior region are utilized. The equivalence principle is used to express the field components valid for the equivalent interior and exterior regions. The unknown equivalent magnetic current at the aperture is expanded by piecewise constant functions with unknown current coefficients $\{M_n\}$. Upon substituting the approximation for the equivalent magnetic current, the resulting equation is tested with piecewise constant functions. The three coupled equations for the unknown zero-order currents are written in matrix form as

$$\begin{bmatrix} [Z_{aa}^{ee,0}] & 0 & [Z_{ac}^{em,0}] \\ 0 & [Z_{bb}^{ii,0}] & [Z_{bc}^{im,0}] \\ [Z_{ca}^{me,0}] & [Z_{cb}^{mi,0}] & [Z_{cc}^{mm,0}] \end{bmatrix} \begin{bmatrix} [\rho J_a^{ext,0}] \\ [\rho J_b^{int,0}] \\ [\rho M_c^0] \end{bmatrix} = \begin{bmatrix} 0 \\ [V_b^{int,0}] \\ [V_c^{int,0}] \end{bmatrix} \quad (10)$$

where $[\rho J_a^{ext,0}]$, $[\rho J_b^{int,0}]$, and $[\rho M_c^0]$ are the unknown zero-order current coefficients over the exterior conductor, the interior conductor and shorted aperture surfaces, respectively. The matrix elements, $[Z_{aa}^{ee,0}]$,

$[Z_{ac}^{em,0}]$, $[Z_{bb}^{ii,0}]$, $[Z_{bc}^{im,0}]$, $[Z_{ca}^{me,0}]$, $[Z_{cb}^{mi,0}]$, $[Z_{cc}^{mm,0}]$ represent the contributions from exterior conductor to itself, exterior conductor to the shorted aperture, interior conductor to itself, interior conductor to the shorted aperture, the shorted aperture to exterior conductor, the shorted aperture to interior conductor, and the aperture to itself, respectively. The tested incident electric field and the tested incident magnetic field are displayed as $[V_b^{int,0}]$ and $[V_c^{int,0}]$, respectively. The interior and exterior fields are then determined from the knowledge of these equivalent currents.

3. Results and discussion

Rounding errors, due to the finite-precision arithmetic used in the numerical computations, are unavoidable. When the effects of rounding errors are amplified in an unstable algorithm, the results may be completely unreliable [11]. The best way to minimize these errors is to reformulate the original problem in a completely different way so that the numerical algorithm may become stable when it is applied to this new formulation. In the present context, the scatterer method is an unstable algorithm for a class of radiation problems where the radiation is weak. Therefore it is suggested that alternative formulations be employed that will yield reliable results.

The solution methods described in previous sections of this paper have been implemented on a mock missile. The θ -component of the far-zone electric field is computed and compared for every solution technique. It is well known that the Lorentz reciprocity theorem can be used to compute the signal coupled to a load at a receiving probe in the axi-symmetric structure from the knowledge of the far-zone electric field radiated by a probe. The normalized θ -component of the far-zone electric field is given by

$$|E_\theta|^{norm} = \left| \frac{E_\theta}{-jkn/4\pi} \right|, \quad (11)$$

where k is the wave number and $\eta = \sqrt{\mu/\varepsilon}$. When this value of normalized electric field is multiplied by an appropriate constant, the voltage induced at the load of the probe may be determined for various plane wave angles of illumination as given by (3).

The structure considered is a mock missile with an antenna protruding axially from the center of a bulkhead inside the nose cone. The structure and its relevant dimensions are illustrated in Figures 1(b). The equivalent models employed for this structure are shown in Figures 3(a-d). The operation frequency is selected as 300 MHz for all cases investigated. The far-zone electric field due to a monopole inside the missile nose cone is computed for monopoles of different heights.

For a monopole height of 5 cm, disagreements between the scatterer method and the short-circuit current and aperture methods are observed, as shown in Figure 4. Also, one observes that E_θ is not zero (computed by the scatterer method) if the structure is closed, which is theoretically impossible. This error in the numerical solution of the EFIE is identified as the noise floor of the solution. When the antenna height is raised to 8 cm, better agreement in the radiated field is observed, as illustrated in Figure 5. If the antenna height is further increased to 14 cm, the discrepancy between the scatterer method and the improved methods reduce further as does the noise floor, as depicted in Figure 6. One may improve the agreement between the numerical solutions of these methods by employing a finer discretization at the aperture region.

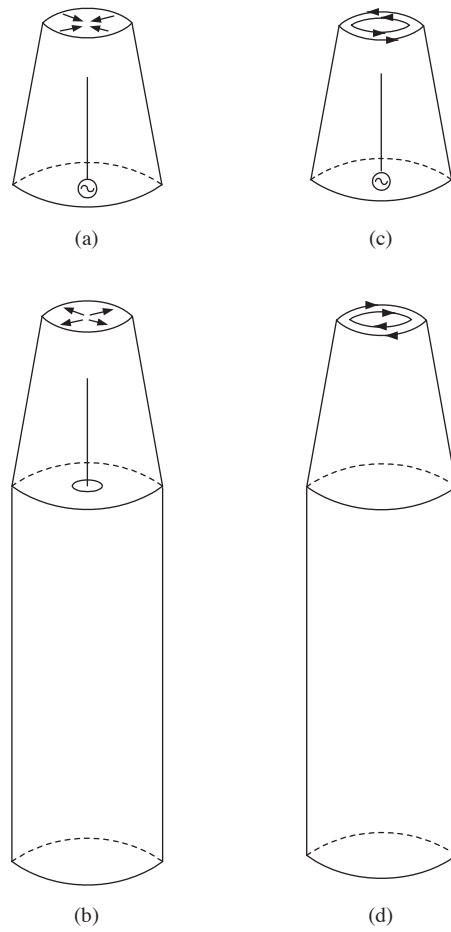


Figure 3. Structures employed in the computation of: (a) short-circuit current J^{sc} on the aperture, (b) $-J^{sc}$ exciting the original structure, (c) interior equivalence for aperture method, (d) exterior equivalence for aperture method.

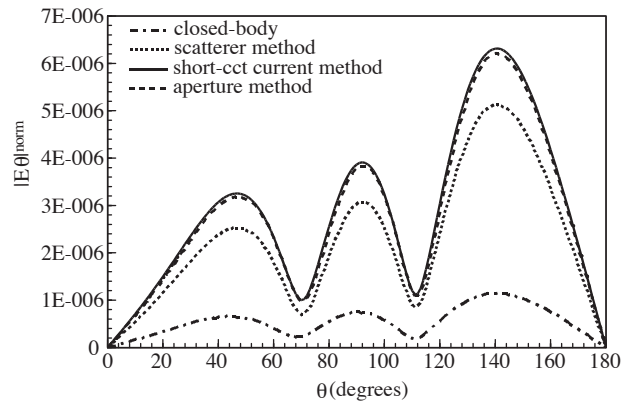


Figure 4. Far-zone electric field due to the probe shown in Figure (1b), $h_{ant} = 5$ cm.

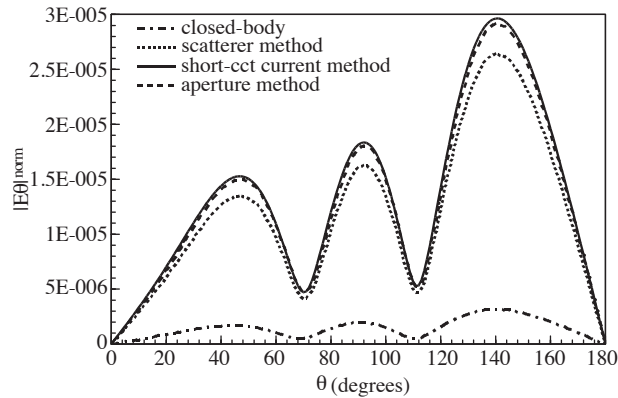


Figure 5. Far-zone electric field due to the probe shown in Figure (1b), $h_{ant} = 8$ cm.

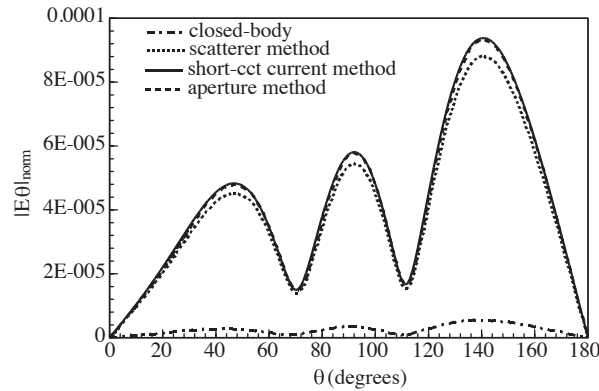


Figure 6. Far-zone electric field due to the probe shown in Figure (1b), $h_{ant} = 14$ cm.

4. Conclusions

When the field penetration through an aperture in the nose cone of a mock missile is very small, comparable to the noise floor of the numerical solution, the accuracy of the values obtained by the scatterer method is poor. This error is mainly attributed to *subtractive cancellation* or *catastrophic cancellation* which results due to the summation of two almost equal large numbers with opposite signs. This is the case when the scattered electric field produced by the induced current on the structure and the incident electric field are almost equal in magnitude but opposite in sign, in order to yield a sum near zero. To circumvent this problem in the case of BORs with axi-symmetric apertures, we apply two alternative formulations that have been found to be successful in 2D penetration problems. However, in an effort to demonstrate the greater overall facility of these formulations, we apply them in the context of weak radiation. The two alternative formulations are described, applied, and found to be applicable to weak radiation as well as weak penetration.

Although in theory, the application of alternate methods to an arbitrary aperture problem (not symmetric in azimuth) on a BOR seems feasible, the Fourier mode expansion of short-circuit and equivalent currents that reside very close to conducting surface exhibits slow numerical convergence and requires the numerical solution of alternate integral equations for many Fourier modes.

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