# Antenna systems with beam forming and beam steering capabilities for HF skywave radars 

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#### Abstract

Radiation characteristics of linearly phased, periodic, planar dipole arrays, which can be used as transmitting/receiving antenna systems for HF skywave radars, are investigated. Rectangular, triangular and trapezoidal arrays are proposed in obtaining different beam shapes in the desired directions. Beam steering is achieved by adjusting the inter-element phase increments coherently. The effects of vertical array-tilt with a desired take-off angle (TOA) $\alpha$ are presented by a number of radiation pattern examples. The ground effect is investigated using the image theory by assuming the Earth's surface as perfectly electric conductor (PEC). Radiated fields are obtained using Floquet wave representations plus the synthetic aperture approach. Validation is done against element-by-element summation representations.


Key Words: HF radars, HF communication, Skywaves, ionospheric reflections, Earth- ionosphere waveguide, planar arrays, phased array antennas, beam forming, beam steering.

## 1. Introduction

Long range, over-the-horizon radar and communication systems use high frequencies (HF) and Earth-Ionosphere waveguides $[1-3]$. Skywaves refer to the propagation of electromagnetic (EM) waves reflected back and forth between the Earth's surface and the Ionosphere. Skywaves propagate between the Earth and Ionosphere, which occupies a region above the troposphere, about 50 to 250 miles above the Earth, where neutral air is ionized by solar photons and cosmic rays. Groundwave propagation, skywave propagation and the effects of the Ionosphere are illustrated in Figure 1.


Figure 1. Groundwave-skywave propagation and the effects of the ionosphere.

Skywaves can propagate many thousands of kilometers via multiple reflections called hoping. While HF groundwave propagation over the sea has been used for distances up to several hundred kilometers, radars operating in the HF band and using skywaves can detect targets at ranges of thousands of kilometers beyond-the-horizon well into diffraction region.

Skywave HF radars and/or communication systems necessitate antenna systems with broad transmit and narrow (angle sensitive) receive capabilities [4]. HF receive antenna systems also require high beam forming and beam steering capabilities. Resonant wire antennas of dipoles and/or monopoles are widely used in HF antenna systems. Since the wavelengths are very long in the HF band ( 100 m to 10 m ), so are the sizes of the HF antennas. Therefore HF radars do not physically rotate antennas to change the direction of the beam. Instead the beam steering is achieved by adjusting the amplitude and the phase of the signals from each element. Such arrays, where relative phases of the elements are changed in such a way that the main beam in the radiation pattern shows a desired direction are called phased arrays.

A large number of different array configurations are investigated in the literature [4-12]. Periodic, linear and planar dipole arrays have been extensively investigated by the late L. B. Felsen and his collaborators [8, 9]. This paper aims to investigate periodic, linearly phased, planar, dipole arrays confined in different shaped regions such as rectangular, triangular or trapezoidal areas.

The HF radars are mostly used for maritime surveillance where the skywave propagates over the water. Due to the high conductivity of the water, the Earth's surface can be supposed as PEC (Perfect Electric Conductor) where the conductivity is infinite ( $\sigma=\infty$ ) and the ground effect can be taken into account by using the image theory. Therefore, for the investigated arrays, the ground is replaced by a mirror array of the original array with respect to the Earth's surface and the total field is obtained via superposition of the fields radiated by the original array and the mirror array. So the effect of the ground is also investigated.

## 2. Statement of the Problem and Formulation

### 2.1. Element-by-Element Summation Representation

The geometry of the rectangular periodic array of $N_{x} \times N_{z}$ infinitesimal z-directed dipoles with unit current amplitude is shown in Figure 2(a). The interelement spacing is $d_{x}$ and $d_{z}$ in the x and z directions. The dipoles
are linearly phased with $k \eta_{x} d_{x}$ and $k \eta_{z} d_{z}$, which denote the interelement phase increments along the x and z coordinates, respectively.

(b)

Figure 2. Geometry of the (a) rectangular (b) triangular array of electric current dipoles.

The radiated field is determined by the z-component of the magnetic vector potential $A_{z}(\omega)$ and under the assumed $\exp (j \omega t)$ time-dependence $A_{z}(\omega)$ can be expressed as

$$
\begin{gather*}
A_{z}(\omega)=\sum_{m=0}^{N_{x}-1} \sum_{n=0}^{N_{z}-1} \frac{e^{-j k R_{n m}}}{4 \pi R_{n m}} e^{-j\left(k \eta_{x} m d_{x}+k \eta_{z} n d_{z}\right)}  \tag{1a}\\
R_{n m}=\sqrt{\left(x-m d_{x}\right)^{2}+y^{2}+\left(z-n d_{z}\right)^{2}} \tag{1b}
\end{gather*}
$$

Figure 2(b) shows an array consisting of $N_{x}$ rows along x direction each with $N_{z}-2 m$ elements ( $\mathrm{m}=0,1,2$, $\left.\ldots, N_{x}-1\right)$ along $z$ direction, so that the first $(\mathrm{m}=0)$ row has $N_{z}$ and the last ( $\mathrm{m}=N_{x}-1$ ) row has $\left(N_{z}-2 N_{x}+2\right)$ elements. If $N_{z}=2 N_{x}-1$, the last row has only one element and the array is triangular, otherwise the last row has more than one element and the array becomes trapezoidal. The interelement period is $d_{x}$ and $d_{z}$. along the x and z coordinates and the normalized interelement phasings in the x and z directions are $\eta_{x}$ and $\eta_{z}$, respectively. $A_{z}(\omega)$ radiated by the specified triangular / trapezoidal array can be expressed as

$$
\begin{equation*}
A_{z}(\omega)=\sum_{m=0}^{N_{x}-1} \sum_{n=m}^{\left(N_{z}-1-m\right)} \frac{e^{-j k R_{n m}}}{4 \pi R_{n m}} e^{-j\left(k \eta_{x} m d_{x}+k \eta_{z} n d_{z}\right)} \tag{2}
\end{equation*}
$$

A short MatLab code for the calculation of vertical radiation patterns of a triangular array by using the element-by-element summation is provided in Table.

### 2.2. Floquet wave (FW) representation

The calculation of element-by-element summation in (1a) and (2) is numerically inefficient for arrays consisting large number of dipoles. In order to improve numerical efficiency Floquet waves can be used. We will start our Floquet wave analysis with the field radiated by a semi-infinite line array given in [9] and reproduced below:

$$
\begin{equation*}
A_{z}(\omega)=\frac{e^{-j k R_{d}}}{4 \pi R_{d}}+\sum_{q} A_{q}^{F W} U\left(\beta_{q}-\theta\right) \tag{3a}
\end{equation*}
$$

where

Table. A MatLab script for plotting the vertical radiation pattern of a triangular array.

```
\(\mathrm{c}=3 \mathrm{e} 8\); freq=4e6; lamd=c/freq; k=2*pi/lamd;
\(\mathrm{dx}=\) lamd \(/ 4 ; \mathrm{dz}=\) lamd \(/ 4 ;\) etax \(=1 ;\) etaz \(=0\);
\(\mathrm{Nz}=15 ; \mathrm{dN}=1 ; \mathrm{Nx}=8 ; \mathrm{RR}=100^{*}\) lamd; \(\mathrm{z}=0 ; \mathrm{nn}=1\);
for theta=.01:.01:2*pi
    \(\mathrm{x}=\mathrm{RR}^{*} \boldsymbol{\operatorname { c o s }} \boldsymbol{\operatorname { s }}\) (theta) \(; \mathrm{y}=\mathrm{RR}^{*} \boldsymbol{\operatorname { s i n }}(\) (theta) \(; \mathrm{Azz}=0 ;\)
    for \(\mathrm{m}=0\) : \((\mathrm{Nx}-1)\)
        for \(\mathrm{n}=\mathrm{m}^{*} \mathrm{dN}:(\mathrm{Nz}-1)-\mathrm{m}\) * dN
            \(\operatorname{Rnm}=\operatorname{sqrt}\left(\left(\mathrm{x}-\mathrm{m}^{*} \mathrm{dx}\right)^{\wedge} 2+\left(\mathrm{y}-0.2^{*} \mathrm{lamd}\right)^{\wedge} 2+\left(\mathrm{z}-\mathrm{n}^{*} \mathrm{dz}\right)^{\wedge} 2\right) ;\)
            \(\mathrm{P} 1=\boldsymbol{\operatorname { e x p }}\left(-\mathrm{j}^{*} \mathrm{k}^{*} \mathrm{Rnm}\right) /\left(4^{*} \mathrm{pi}{ }^{*} \mathrm{Rnm}\right)\);
            \(\mathrm{P} 2=\boldsymbol{\operatorname { e x p }}\left(-\mathrm{j}^{*}\left(\mathrm{k}^{*} \operatorname{etax}^{*} \mathrm{~m}^{*} \mathrm{dx}+\mathrm{k}^{*} \operatorname{etaz}{ }^{*} \mathrm{n}^{*} \mathrm{dz}\right)\right) ;\)
            \(\mathrm{Azz}=\mathrm{Azz}+\mathrm{P} 1 * \mathrm{P} 2\);
        end
        end
        \(\operatorname{Thetar}(\mathrm{nn})=\) theta; \(\mathrm{Az}(\mathrm{nn})=\mathrm{Azz} ; \mathrm{nn}=\mathrm{nn}+1\);
end
\(\mathrm{A} 1=(\mathrm{abs}(\mathrm{Az})) ; \mathrm{A} 2=\mathrm{A} 1 / \max (\mathrm{A} 1) ; \boldsymbol{p o l a r}(\) Thetar, A 2\()\)
```

$$
\begin{equation*}
A_{q}^{F W} \approx \frac{e^{-j\left(k_{\rho q} \rho+k_{z q} z+\pi / 4\right)}}{2 d_{z} \sqrt{2 \pi k_{\rho q} \rho}}, \quad \beta_{q}=\cos ^{-1}\left(k_{z q} / k\right) \quad \rho^{2}=x^{2}+y^{2} \tag{3b}
\end{equation*}
$$

The $z$-domain wavenumber $k_{z q}$ and the radial wavenumber $k_{\rho q}$ are determined by $k_{z q}=k \eta_{z}+2 \pi q / d_{z}$ and $k_{\rho q}=\sqrt{k^{2}-k_{z q}^{2}} . A_{q}^{d}$ in (3a) corresponding to the tip diffracted waves is expressed as

$$
\begin{equation*}
A_{q}^{d}=\frac{ \pm e^{-j k R_{d}} \sqrt{\pi} \delta_{q} e^{j \delta_{q}^{2}} e^{j \pi / 4} \operatorname{erfc}\left( \pm e^{j \pi / 4} \delta_{q}\right)}{4 \pi R_{d} j k d_{z}\left[\cos \left(\beta_{q}\right)-\cos (\theta)\right]}, \quad \delta_{q}=\sqrt{2 k R_{d}} \sin \left(\frac{\beta_{q}-\theta}{2}\right) \tag{3c}
\end{equation*}
$$

$\mathrm{U}($.$) is the Heaviside unit function and \operatorname{erfc}($.$) stands for the complementary error function.$
The field radiated by a finite line array consisting of $N_{z}$ dipoles can be found as a superposition of two semi-infinite line arrays where one of them is shifted distance $N_{z} d_{z}$ along the z coordinate and oppositely phased with respect to the other. Using a synthetic aperture approach [11], the field at an observation point radiated by the shifted array can be imagined as the field at a virtual oppositely shifted observation point radiated by the original array and weighted with an appropriate phase shift. The field radiated by a rectangular /triangular array can be expressed as a superposition of the fields radiated by finite line arrays. The synthetic aperture approach can be extended to two dimensions and the fields radiated by the finite line arrays shifted by distance $d_{x}$ along x with respect to each other can be imagined as the field radiated by the original finite line array at virtual oppositely shifted observation points.

For the following proposed array configurations horizontal and vertical radiation patterns are calculated using Floquet waves and also element-by-element summation representations.

## 3. Characteristic examples

Characteristic examples and typical results are presented in this section. The first case, illustrated in Figure 3(a), belongs to a periodic line array consisting of $\mathrm{N}_{z}=15$ infinitesimal z-directed dipoles with unit current amplitude. The interelement spacing is chosen to be $\mathrm{d}_{z}=\lambda / 4$. For an HF radar operating at the frequency $\mathrm{f}=4 \mathrm{MHz}$, the
interelement spacing is $\mathrm{d}_{z}=18.75 \mathrm{~m}$. The dipoles are linearly phased with the normalized interelement phasing $\eta_{z}$. However, for this particular example $\eta_{z}=0$. Figure $3(\mathrm{~b})$ shows the amplitude of the magnetic scalar potential $\mathrm{A}_{z}(\omega)$ versus $\theta$ in the x-z plane at a radial distance $\mathrm{R}_{d}=100 \lambda$ radiated by this line array. This variation of $\left|\mathrm{A}_{z}(\omega)\right|$ in the far field can be considered as the horizontal radiation pattern of the array and it can be seen from the figure that the radiation is in the broadside direction. Figure 3(c) shows the variation of $\left|\mathrm{A}_{z}(\omega)\right|$ versus $\varphi$ in the $x-y$ plane at a radial distance $\mathrm{R}_{d}=100 \lambda$ which is the vertical radiation pattern of the line array. Since the ground effect is not taken into account in Figure 3(c), the vertical radiation pattern is isotropic. The effect of a PEC ground is added in Figure 3(d) by obtaining the total field as a superposition of the fields radiated by the original array and by a mirror array of the original array with respect to x-z plane. While the ground effect changes the vertical radiation pattern from being isotropic to a single beam in the $y$-direction, it does not affect the horizontal radiation pattern. In all radiation patterns in the following examples, throughout this paper, the blue solid curves correspond to element-by-element summation solutions and the red dashed curves correspond to Floquet wave representations.


Figure 3. (a) Periodic line array of axial electric current dipoles located along the z-axis (b) Horizontal radiation pattern (c) Vertical radiation pattern without the ground effect (d) Vertical radiation pattern with the PEC ground ( $\mathrm{N}_{z}=15, \mathrm{~d}_{z}=\lambda / 4, \eta_{z}=0$ ).

A rectangular array consisting of $\mathrm{N}_{z}=15$ and $\mathrm{N}_{x}=8$ dipoles in the z and x directions, respectively, is illustrated in Figure 4 (a) with its mirror array. The interelement spacings are $\mathrm{d}_{z}=\mathrm{d}_{x}=\lambda / 4$ and the interelement phasing along the x coordinate is $\eta_{x}=1$. The horizontal radiation patterns for different interelement phasings $\eta_{z}$ are plotted in Figures $4(\mathrm{c}-\mathrm{f})$. The interelement distances $\mathrm{d}_{z}, \mathrm{~d}_{x}$ and $\eta_{x}$ are so chosen that the array produces a single beam and the beam can be steered to the desired direction by appropriately changing $\eta_{z}$. For $\eta_{z}=0$ the beam is in the broadside direction $\left(\theta=90^{\circ}\right)$ as shown in Figure $4(\mathrm{c})$. For increasing positive $\eta_{z}$ values the beam is directed to $\theta$ values decreasing from $90^{\circ}$ as shown in Figures 4(d), 4(e) and for decreasing negative $\eta_{z}$ values the beam is directed to $\theta$ values increasing from $90^{\circ}$ as shown in Figure $4(\mathrm{f})$. The vertical radiation pattern is plotted in Figure $4(\mathrm{~b})$ which is independent from $\eta_{z}$.

Figure 5(a) shows a triangular array consisting of 8 rows ( $\mathrm{m}=0,1,2,3,4,5,6,7$ ) along x -direction each with $(15-2 \mathrm{~m})$ elements along z-direction, so that the first ( $\mathrm{m}=0$ ) row has 15 and the last ( $\mathrm{m}=7$ ) row has only one element. The interelement distances are $\mathrm{d}_{z}=\mathrm{d}_{x}=\lambda / 4$ and the normalized interelement phasings are $\eta_{z}=0, \eta_{x}=1$. The array is tilted by a take-off angle $\alpha$ in the $\varphi$-direction as shown in the figure. The horizontal radiation pattern, which is not effected by the angle $\alpha$, is plotted in Figure 5(b). The vertical radiation patterns for $\alpha=0^{\circ}, 15^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}, 75^{\circ}$ are presented in Figures $5(\mathrm{c}-\mathrm{h})$, respectively. It can be easily concluded from the figures that rotating the array by an angle of $\alpha$, rotates the main beam by $2 \alpha$. This attribute is analogous to the optical reflection rule, rotating a mirror by an angle $\alpha$ rotates the reflected ray by $2 \alpha$.

The triangular array in Figure $5(\mathrm{a})$ consisting of $\mathrm{N}_{z}=15-2 \mathrm{~m}(\mathrm{~m}=0,1,2,3,4,5,6,7)$ and $\mathrm{N}_{x}=8$ dipoles in the $z$ and $x$ directions, respectively, is illustrated in Figure 6(a) with its mirror array corresponding to the PEC ground. The interelement distances are again $\mathrm{d}_{z}=\mathrm{d}_{x}=\lambda / 4$ and $\eta_{x}=1$. The horizontal radiation patterns for $\eta_{z}=0,0.5,0.75,-0.75$ are shown in Figures 6 (b-e), respectively. Similar to the rectangular array in Figure 4(a), the beam steering in the horizontal pattern is achieved by changing $\eta_{z}$ coherently. The beam is in the broadside direction $\left(\theta=90^{\circ}\right)$ for $\eta_{z}=0$ as shown in Figure $6(\mathrm{~b})$. The beam is directed to $\theta$ values less than $90^{\circ}$ for positive $\eta_{z}$ values as shown in Figures $6(\mathrm{c}, \mathrm{d})$ and it is directed to $\theta$ values more than $90^{\circ}$ for negative $\eta_{z}$ values as shown in Figure 6(e). The vertical radiation patterns for $\eta_{z}=0,0.25,0.5,0.75,-0.5,-0.25$ are shown in Figures 7 (a-f), respectively. As mentioned before, the vertical radiation pattern of the rectangular array in Figure 4(a) was not effected by $\eta_{z}$. However unlike the rectangular array, the triangular array in Figure 6a has a vertical radiation pattern which varies with $\eta_{z}$. It can be easily concluded from the figures that the vertical radiation pattern depends on the absolute value of $\eta_{z}$ since $\eta_{z}=0.5$ and $\eta_{z}=-0.5$ produce the same pattern as in Figures 7(c) and (e). The same property can be observed from Figures 7(d) and (f) where the same vertical radiation patterns is obtained for $\eta_{z}=0.75$ and $\eta_{z}=-0.75$.

Figure 8(a) shows a triangular array consisting of 11 rows $(\mathrm{m}=0 \ldots 10)$ along the x -direction, each with (41-4m) elements along z-direction with its mirror array. The interelement spacings are $\mathrm{d}_{z}=\mathrm{d}_{x}=\lambda / 4$ and the normalized interelement phasings are $\eta_{z}=0.25, \eta_{x}=1$. While the vertical radiation pattern is shown in Figure 8(b), the horizontal radiation pattern is presented in Figure 8(c). As expected, using more dipoles causes the beam to be narrower.

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Figure 4. (a) Linearly phased, periodic, rectangular array of dipoles and its image array substituting the ground $\left(\mathrm{N}_{z}=15, \mathrm{~N}_{x}=8, \mathrm{~d}_{z}=\mathrm{d}_{x}=\lambda / 4, \eta_{x}=1\right)(\mathbf{b})$ Vertical radiation pattern, Horizontal radiation patterns for (c) $\eta_{z}=0$, (d) $\eta_{z}=0.5$, (e) $\eta_{z}=0.75$, (f) $\eta_{z}=-0.75$.


Figure 5. (a) Linearly phased, periodic, triangular array of dipoles tilted in the $\varphi$ direction at an $\alpha$ take off angle ( $\mathrm{N}_{z}=15-2 \mathrm{~m}, \mathrm{~m}=0,1,2,3,4,5,6,7, \mathrm{~N}_{x}=8, \mathrm{~d}_{z}=\mathrm{d}_{x}=\lambda / 4, \eta_{z}=0, \eta_{x}=1$ ) (b) Horizontal radiation pattern; Vertical radiation patterns for (c) $\alpha=0^{\circ}$, (d) $\alpha=15^{\circ}$, (e) $\alpha=30^{\circ}$, (f) $\alpha=45^{\circ}$, (g) $\alpha=60^{\circ}$, (h) $\alpha=75^{\circ}$.


Figure 6. (a) Linearly phased, periodic, triangular array of dipoles and its image array substituting the ground $\left(\mathrm{N}_{z}=15-2 \mathrm{~m}, \mathrm{~m}=0,1,2,3,4,5,6,7, \mathrm{~N}_{x}=8, \mathrm{~d}_{z}=\mathrm{d}_{x}=\lambda / 4, \eta_{x}=1, \alpha=0^{\circ}\right)$ Horizontal radiation patterns for (b) $\eta_{z}=0$, (c) $\eta_{z}=0.5$, (d) $\eta_{z}=0.75$, (e) $\eta_{z}=-0.75$.


Figure 7. Vertical radiation patterns of the array shown in Figure 6a for (a) $\eta_{z}=0$, (b) $\eta_{z}=0.25$, (c) $\eta_{z}=0.5$, (d) $\eta_{z}=0.75$, (e) $\eta_{z}=-0.5$, (f) $\eta_{z}=-0.75\left(\mathrm{~d}_{z}=\mathrm{d}_{x}=\lambda / 4, \eta_{x}=1, \alpha=0^{\circ}\right)$.

Figure $8(\mathrm{~d})$ shows a trapezoidal array consisting of 5 rows ( $\mathrm{m}=0,1,2,3,4$ ) along x -direction each with (518 m ) dipoles along z -direction. The interelement spacings are $\mathrm{d}_{z}=\mathrm{d}_{x}=\lambda / 4$ and the normalized interelement phasings are $\eta_{z}=0, \eta_{x}=1$. Figure $8(\mathrm{e})$ and $8(\mathrm{f})$ show the vertical and horizontal radiation patterns, respectively. The horizontal radiation pattern is in the broadside direction with a very narrow beam.

The radiation patterns at the preceding examples were obtained by plotting the amplitude of the magnetic scalar potential $\mathrm{A}_{z}(\omega)$. The electric field can be determined by the z-component of the magnetic vector potential $\vec{A}$ as $E_{z}=-j \omega \mu A_{z}+(1 / j \omega \varepsilon) \partial^{2} A_{z} / \partial z^{2}$. If the magnetic scalar potential $\mathrm{A}_{z}(\omega)$ is used as defined in (2), the electric field radiated by a triangular array shown Figure 2b can be expressed as

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Figure 8. (a) Linearly phased, periodic, triangular array of dipoles and its image array substituting the ground $\left(\mathrm{N}_{z}=41-4 \mathrm{~m}, \mathrm{~m}=0,1,2,3,4,5,6,7,8,9,10, \mathrm{~N}_{x}=11, \mathrm{~d}_{z}=\mathrm{d}_{x}=\lambda / 4, \eta_{z}=0.25 \eta_{x}=1, \alpha=0^{\circ}\right)$ (b) Vertical- (c) Horizontal radiation pattern of the triangular array (d) Linearly phased, periodic, trapezoidal array of dipoles and its image array substituting the ground $\left(\mathrm{N}_{z}=51-8 \mathrm{~m}, \mathrm{~m}=0,1,2,3,4, \mathrm{~N}_{x}=5, \mathrm{~d}_{z}=\mathrm{d}_{x}=\lambda / 4, \eta_{z}=0 \eta_{x}=1, \alpha=0^{\circ}\right.$ ) (e) Vertical- (f) Horizontal radiation pattern of the trapezoidal array.

$$
\begin{align*}
& E_{z}(\omega)=\sum_{m=0}^{N_{x}-1}\left(\sum_{z}-1-m\right)  \tag{4}\\
& \sum_{n=m}^{30 k^{2} e^{-j k R_{n m}} e^{-j\left(k \eta_{x} m d_{x}+k \eta_{z} n d_{z}\right)}}{ }^{j} \times \\
& {\left[\frac{1}{k R_{n m}}-\frac{j}{k^{2} R_{n m}^{2}}-\frac{1}{k^{3} R_{n m}^{3}}+(z-n d)^{2} k^{2}\left(\frac{-1}{k^{3} R_{n m}^{3}}+\frac{3 j}{k^{4} R_{n m}^{4}}+\frac{3}{k^{5} R_{n m}^{5}}\right)\right]}
\end{align*}
$$

Figure 9 (a) shows the variation of $\left|E_{z}(\omega)\right|$ versus $\varphi$ in the x-y plane radiated by the triangular array shown in Figure 6a. The array is located over an imperfect ground with the relative permittivity $\varepsilon_{r}=15$ and the conductivity $\sigma=0.01 \mathrm{~S} / \mathrm{m}$. The imperfect ground is modeled by multiplication of the field radiated by the image array with a reflection coefficient defined as


Figure 9. (a) Vertical (b) Horizontal radiation patterns of the array shown in Figure 6a over an imperfect ground with $\varepsilon_{r}=15$ and $\sigma=0.01\left(\mathrm{~N}_{z}=15-2 \mathrm{~m}, \mathrm{~m}=0,1,2,3,4,5,6,7, \mathrm{~N}_{x}=8, \mathrm{~d}_{z}=\mathrm{d}_{x}=\lambda / 4, \eta_{x}=1, \eta_{z}=0.25, \alpha=0^{\circ}\right)$ (c) Vertical (d) Horizontal radiation patterns of the array shown in Figure 5a over an imperfect ground with $\varepsilon_{r}=15$ and $\sigma=0.01$ $\left(\mathrm{N}_{z}=15-2 \mathrm{~m}, \mathrm{~m}=0,1,2,3,4,5,6,7, \mathrm{~N}_{x}=8, \mathrm{~d}_{z}=\mathrm{d}_{x}=\lambda / 4, \eta_{x}=1, \eta_{z}=0, \alpha=45^{\circ}\right)$.

$$
\begin{equation*}
\rho=\frac{\cos \beta-\sqrt{n^{2}-\sin ^{2}(\beta)}}{\cos \beta+\sqrt{n^{2}-\sin ^{2}(\beta)}} \tag{5}
\end{equation*}
$$

where $n^{2}=\varepsilon_{r}-j 60 \sigma \lambda$ and $\beta=-\varphi+90^{\circ}$. The blue solid curves correspond to element-by-element summation solutions and the red dashed curves correspond to outputs obtained by the software NEC which is based on Method of Moments. Figure 9b shows the horizontal radiation pattern of the same triangular array over imperfect ground with the same relative permittivity $\varepsilon_{r}=15$ and the same conductivity $\sigma=0.01$. Figure 9 c shows the vertical radiation pattern of the same triangular array but tilted by a take-off angle $\alpha=45^{\circ}$ in the $\varphi$-direction. The effects of the imperfect ground with $\varepsilon_{r}=15$ and $\sigma=0.01$ are taken into account in the same manner by multiplication of the field radiated by the image array with the reflection coefficient defined in (5). Finally, Figure 9(d) shows the horizontal radiation pattern of the tilted array. While tilting the triangular array by a take-off angle $\alpha$ causes a small change in the horizontal radiation pattern, the vertical radiation pattern totally changes and a new beam appears at $2 \alpha$. Figures $9(a-d)$ are plotted in dB scale and they are normalized to 30 dBs . It can be observed that the element by element summation solutions agree very well with the NEC solutions. Therefore we can conclude that the effects of imperfect ground are correctly modeled.

## 4. Conclusion

Skywave HF radar antenna systems are investigated. Different triangular, trapezoidal and rectangular, phased arrays of dipoles are proposed. Beam forming is achieved by using different number of dipoles in different geometrical configurations. The beams are steered by tilting the array by a take-off angle $\alpha$ and also by changing the interelement phasings. Ground effects are also taken into account. Fields are obtained using Floquet wave and element-by-element summation representations, and it's observed that both representations agree very well with each other within the accuracy of the plots.

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