# The analysis of a semiconductor single asymmetric and symmetric step-index laser for even and odd fields by Alpha Method 

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#### Abstract

In this work, semiconductor step-index single waveguide has been analyzed by Alpha Method. A requested quantity of wave guide can be obtained in terms of normalized propagation constant, which is represented by alpha belonging to active region. Based on this method, structural properties of the material containing any requested quantity of the waveguide is theoretically calculated, when the width of the active region, the refractive indices of the regions and the wavelength are given.

In the TE mode some important parameters, such as the propagation constants for regions of the semiconductor step-index single waveguide, the wave numbers, the effective index of refraction of the active region, the dielectric constant, the phase constant, the absorption coefficients, and the confinement factors of the regions, stray ratios of the field probabilities to the cladding regions from the active region, arised power per unit length and effective mass of electron in the active region, $\eta, \zeta$ coordinate variables for energy eigenvalues for charged carriers in the orthogonal coordinate system $\zeta-\eta$, have been estimated, calculated theoretically and validities of found formulas have been tested, numerically. Since the effective refractive index belonging to the active region is constant, the quantities such as the phase constant, the phase velocity, the dielectric constant, the parameters $\eta, \zeta$, the field amplitudes, the power raised per unit length, and effective mass of the electron being constant have been observed.

In this novel study, some design parameters such as the normalized frequency and a specially normalized propagation constant have been obtained, depending on some parameters which are functions of energy eigenvalues of the carriers such as electrons and holes confined in a semiconductor single asymmetric and symmetric step-index wave guide (SCSAaSSIWG) for even and odd fields. Some optical expressions about the optical power and probability quantities for the active region and cladding layers of the SCSAaSSIWG have been investigated in terms of these parameters.


Key Words: Normalized frequency, Normalized propagation constant, Wave number, Confinement factor, Field probability ratio

## 1. Introduction

Semiconductor single step-index laser (SCSIL) which has 3 regions is shown in Figure 1. Refractive indices (RIs) of the regions are $n_{I}, n_{I I}, n_{I I I}$ and the width of active region (AR) is $2 a$. Regions I and III are called cladding layers (CLs). Generally we take as $\left.\left.n_{I I}\right\rangle n_{I}\right\rangle n_{I I I}$ which shows a semiconductor single asymmetric step-index laser (SCSASIL). If it is taken as $\left.n_{I I}\right\rangle n_{I}=n_{I I I}=n_{I, I I I}$ the laser is called semiconductor single symmetric step-index laser (SCSSSIL) [1,2,3].


Figure 1. Active region (AR) and cladding layers (CLs).
In the AR the optical fields are represented by the even field $E_{y I I}=A \cos \alpha_{I I} x=A \cos \frac{n \pi x}{2 a}, \mathrm{n}=1,3,5, \ldots$, which is a cosine function or odd field $e_{y I I}=B \sin \alpha_{I I} x=B \sin \frac{n \pi x}{2 a}, \mathrm{n}=2,4, .6 \ldots$, which is a sinus function. The fields of the CLs are respectively given by $E_{y I}=A_{I} \exp \left[\alpha_{I}(x+a)\right], E_{y I I I}=A_{I I I} \exp \left[-\alpha_{I I I}(x-a)\right]$ for even field and $e_{y I}=B_{I} \exp \left[\alpha_{I}(x+a)\right]$, $e_{y I I I}=B_{I I I} \exp \left[-\alpha_{I I I}(x-a)\right]$ for odd field [1,2,3]. Amplitudes $A_{I}$ $\left(B_{I}\right)$ and $A_{I I I}\left(B_{I I I}\right)$ are given $[1,2,3]$ by $A_{I}=A_{I I I}=A \cos \zeta\left(B_{I}=B_{I I I}=B \sin \zeta\right)$. In this work quantities for odd fields are symbolized with a prime notation. Since the AR is a medium of the signal transmission to normalize the fields in AR, the integrals $I_{I I}=\int_{-a}^{a}\left|E_{y I I}(x)\right|^{2} d x=1, I_{I I}^{\prime}=\int_{-a}^{a}\left|e_{y I I}(x)\right|^{2} d x=1$ give the amplitude (AD) A and amplitude (AD) B as $A=\sqrt{\frac{2 \alpha_{I I}}{2 \zeta+\sin 2 \zeta}}$ and $B=\sqrt{\frac{2 \alpha_{I I}}{2 \zeta-\sin 2 \zeta}}[1,2,3]$. Propagation constants (PCs) $\alpha_{I}, \alpha_{I I}$ and $\alpha_{I I I}$ for even field of the regions are defined as follows:

$$
\begin{gather*}
\alpha_{I}=\sqrt{\beta_{z}^{2}-\left(\frac{\omega n_{I}}{c}\right)^{2}}=\sqrt{\beta_{z}^{2}-k_{I}^{2}}, \alpha_{I I}=\sqrt{\left(\frac{\omega n_{I I}}{c}\right)^{2}-\beta_{z}^{2}}=\sqrt{\beta_{z}^{2}-k_{I I}^{2}}, \alpha_{I I I}=\sqrt{\beta_{z}^{2}-\left(\frac{\omega n_{I I I}}{c}\right)^{2}}=\sqrt{\beta_{z}^{2}-k_{I I I}^{2}}  \tag{1}\\
k_{I}=\frac{\omega n_{I}}{c}=k_{o} n_{I}=\frac{2 \pi}{\lambda} n_{I}, \quad k_{I I}=\frac{\omega n_{I I}}{c}=k_{o} n_{I I}=\frac{2 \pi}{\lambda} n_{I I}, \quad k_{I I}=\frac{\omega n_{I I I}}{c}=k_{o} n_{I I I}=\frac{2 \pi}{\lambda} n_{I I I} \tag{2}
\end{gather*}
$$

where $k_{i}, \mathrm{i}=\mathrm{I}, \mathrm{II}$, III, c, $\omega, \lambda, k_{o}$ ve $\beta_{z}$ are respectively wave number (WN), the speed of light, angular frequency, free space wavelength and wave number, and phase constant (PhC). The normalized propagation constant (NPC) for even (odd) field is represented by $\alpha\left(\alpha^{\prime}\right)$ and given by $\alpha=\sin ^{2} \zeta\left(L=\alpha^{\prime}=1-\alpha=\cos ^{2} \zeta\right)[1,2,3]$. The variables $\zeta, \eta$ for even field and $\zeta^{\prime}, \eta^{\prime}$ for odd field are respectively given by $\zeta=\alpha_{I I} a=\mathrm{V} \cos \zeta, \eta=\eta_{I}=\mathrm{V} \sin \zeta$

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and $\zeta^{\prime}=\alpha_{I I}^{\prime} a=\mathrm{V} \sin \zeta, \eta^{\prime}=\eta_{I}^{\prime}=\mathrm{V} \cos \zeta$. Here $\eta, \zeta$ are coordinate variables for energy eigenvalues for charged carriers in the orthogonal coordinate system $\zeta-\eta$. V is called normalized frequency (NF) [4], which is a function of the coordinate variables $(\zeta, \eta)$ and $\left(\zeta^{\prime}, \eta^{\prime}\right)$, and is given by $V_{a}=\frac{1}{\sqrt{1-\alpha}}\left[\frac{m}{2} \pi+\arctan \sqrt{\frac{\alpha}{1-\alpha}}\right]$, $\mathrm{m}=0,1,2, \ldots$ in a SCSASIL [4] and $V=\sqrt{\zeta^{2}+\eta^{2}}=\sqrt{\zeta^{\prime 2}+\eta^{\prime 2}}=a k_{o}$ NA in a SCSSSIL. Here NA is the numerical aperture given by $N A_{a}=\sqrt{n_{I I}^{2}-n_{I}^{2}}$ in a SCSASIL and $N A=\sqrt{n_{I I}^{2}-n_{I, I I I}^{2}}$ in a SCSSSIL $[1,2,3]$. The coordinate variables $\zeta, \eta\left(\zeta^{\prime}, \eta^{\prime}\right)$ create a circle with radius V in a SCSSSIL as shown in Figure 2. The intersection points of the circle and eigenvalue equation $\tan \zeta=\eta / \zeta\left(\cot \zeta=\eta^{\prime} / \zeta^{\prime}\right)$ of the SCSIL for even (odd) field in TE mode [3] give the solution points as shown in Figure 2. The PCs $\alpha_{I}$ and $\alpha_{I I I}$ and numeric apertures $N A_{a}$ in a SCSASIL and NA in a SCSSSIL are respectively equal to each other since $\left.n_{I I}\right\rangle n_{I}=n_{I I I}=n_{I, I I I}$ as $\alpha_{I I I}=\alpha_{I}=\alpha_{I, I I I}=\sqrt{\beta_{z}^{2}-\left(\frac{\omega n_{I I I I}}{c}\right)^{2}}$ and $N A_{a}=\mathrm{NA}$ and therefore the ordinate variable $\eta_{I}=\eta_{I I I}=\eta_{I, I I I}=\eta=\alpha_{I, I I I} a$. The propagation constant ( PC ) $\alpha_{I I}$ is given by $\alpha_{I I}=k_{o} N A \cos \zeta=k_{o} N A \sqrt{1-\alpha}$ in a SCSIL $[1,2,3]$.


Figure 2. Intersection points of the normalized frequency $V=\sqrt{\zeta^{2}+\eta^{2}}\left(V=\sqrt{\zeta^{\prime 2}+\eta^{\prime 2}}\right)$ for even (for odd) field and eigenvalue equation $[1,2,3] \tan \zeta=\eta / \zeta\left(\cot \zeta=\eta^{\prime} / \zeta^{\prime}\right)$ of the SCSIL, which creates a circle part in the first dial in a SCSIL in TE mode.

## 2. Effective refractive index and phase velocity in a SCSASIL

The PhC $\beta_{z}$ in a SCSASIL is found by $\beta_{z}=k_{I I} \sqrt{1-\left(\frac{\alpha_{I I}}{k_{I I}}\right)^{2}}=k_{o} \sqrt{n_{I I}^{2}-(1-\alpha) N A^{2}}$ in which $\mathrm{NA}_{a}$ is transformed into $N A_{I, I I I}=N A=\sqrt{n_{I I}^{2}-n_{I, I I I}^{2}}$ since $n_{I I I}=n_{I}=n_{I, I I I}$ in a SCSSSIL. Therefore, we have $[4,5] \beta_{z} \rightarrow \beta_{z I . I I I}=k_{o} \sqrt{n_{I I}^{2}-(1-\alpha) N A_{I, I I I}^{2}}$ or $\beta_{z I . I I I}=k_{o} \sqrt{n_{I, I I I}^{2}-\alpha N A_{I, I I I}^{2}}$ and $V=a \sqrt{k_{I I}^{2}-k_{I, I I I}^{2}}=$
$a k_{o} N A_{I, I I I}[4]$. The effective refractive indices (ERI) of AR in a SCSASIL and SCSSSIL is given [5,6] by

$$
\begin{equation*}
n_{e f}=\frac{\beta_{z}}{k_{o}}=\sqrt{n_{I}^{2}+\alpha\left(n_{I I}^{2}-n_{I}^{2}\right)}=n_{I I} \sqrt{1-\left(\frac{\alpha_{I I}}{k_{I I}}\right)^{2}}=\sqrt{n_{I I}^{2}-(1-\alpha) N A_{a}^{2}} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
n_{e f I, I I I}=\frac{\beta_{z I, I I I}}{k_{o}}=\sqrt{n_{I I}^{2}-(1-\alpha) N A_{I, I I I}^{2}}=\sqrt{n_{I, I I I}^{2}-\alpha N A_{I, I I I}^{2}} . \tag{4}
\end{equation*}
$$

respectively and so phase velocity (PV) and value of dielectric constant (DEC) in AR are respectively $\mathrm{v}=\mathrm{c} / n_{\text {ef }}$ and $\varepsilon=\varepsilon_{o} n_{e f}^{2}$ where $\varepsilon_{o}$ is dielectric constant of the vacuum as $8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}$ in MKSA unit system.

## 3. Novel formulae in TE mode in SCSASIL and SCSSSIL

Using asymmetric factor $\mathrm{a}_{p}$ [4], the PCs $\alpha_{I}, \alpha_{I I}, \alpha_{I I I}$ and $\alpha_{I}^{\prime}, \alpha_{I I}^{\prime}, \alpha_{I I I}^{\prime}$ and abscissa $\zeta$, ordinates $\eta, \eta_{I I I}$ and $\eta^{\prime}, \eta_{I I I}^{\prime}$ in the regions I, II and III of the SCSIL are formulated by

$$
\begin{gather*}
\alpha_{I}=k_{o} N A_{a} \sqrt{\alpha}, \quad \alpha_{I I}=k_{o} N A_{a} \sqrt{(1-\alpha)}, \quad \alpha_{I I I}=k_{o} N A_{a} \sqrt{\left(1+a_{p}\right)-(1-\alpha)}  \tag{5}\\
\zeta=a k_{o} N A_{a} \sqrt{(1-\alpha)}, \quad \eta_{I}=\eta=a k_{o} N A_{a} \sqrt{\alpha}, \eta_{I I I}=a k_{o} N A_{a} \sqrt{\left(1+a_{p}\right)-(1-\alpha)} \tag{6}
\end{gather*}
$$

for even field and

$$
\begin{align*}
& \alpha_{I}^{\prime}=k_{o} N A_{a} \sqrt{1-\alpha}, \quad \alpha_{I I}^{\prime}=k_{o} N A_{a} \sqrt{\alpha}, \quad \alpha_{I I I}^{\prime}=k_{o} N A_{a} \sqrt{\left(1+a_{p}\right)-\alpha}  \tag{7}\\
& \zeta^{\prime}=a k_{o} N A_{a} \sqrt{\alpha}, \quad \eta_{I}^{\prime}=a k_{o} N A_{a} \sqrt{1-\alpha}, \quad \eta_{I I I}^{\prime}=a k_{o} N A_{a} \sqrt{\left(1+a_{p}\right)-\alpha} \tag{8}
\end{align*}
$$

for odd field in a SCSASIL and

$$
\begin{equation*}
\alpha_{I}=k_{o} N A_{I, I I I} \sqrt{\alpha}, \alpha_{I I}=k_{o} N A_{I, I I I} \sqrt{1-\alpha}, \quad \alpha_{I I I}=k_{o} N A_{I, I I I} \sqrt{\alpha} \tag{9}
\end{equation*}
$$

$$
\begin{equation*}
\zeta=a k_{o} N A_{I, I I I} \sqrt{(1-\alpha)}=V_{I, I I I} \sqrt{(1-\alpha)}, \eta_{I}=a k_{o} N A_{I, I I I} \sqrt{\alpha}=V_{I, I I I} \sqrt{\alpha}, \eta_{I I I}=a k_{o} N A_{I, I I I} \sqrt{\alpha}=V_{I, I I I} \sqrt{\alpha} \tag{10}
\end{equation*}
$$

for even field and

$$
\begin{gather*}
\alpha_{I}^{\prime}=k_{o} N A_{I, I I I} \sqrt{1-\alpha}, \alpha_{I I}^{\prime}=k_{o} N A_{I, I I I} \sqrt{\alpha}, \alpha_{I I I}^{\prime}=k_{o} N A_{I, I I I} \sqrt{(1-\alpha}, \eta_{I}^{\prime}=a k_{o} N A_{I, I I I} \sqrt{1-\alpha}=V_{I, I I I} \sqrt{1-\alpha}  \tag{11}\\
\zeta^{\prime}=a k_{o} N A_{I, I I I} \sqrt{\alpha}=V_{I, I I I} \sqrt{\alpha}, \quad \eta_{I}^{\prime}=a k_{o} N A_{I, I I I} \sqrt{1-\alpha}=V_{I, I I I} \sqrt{1-\alpha} \\
\eta_{I I I}^{\prime}=a k_{o} N A_{I, I I I} \sqrt{\left(1+a_{p}\right)-\alpha}=V_{I, I I I} \sqrt{\left(1+a_{p}\right)-\alpha} \tag{12}
\end{gather*}
$$

for odd field in a SCSSSIL. Here $V_{I, I I I}$ is given by $V_{I, I I I}=a k_{o} N A_{I, I I I}$.
The power per unit length (PPUL), impedance value (IV), effective mass of an electron (EME) in even or odd field in direction y in AR of a SCSIL are given by $P=\frac{\beta_{z}}{\mu_{o} \omega}, Z=\omega \mu_{o} / \beta_{z}$
$m *=0.067 \times 9.1095 \times 10^{-31} / \sqrt{1-1 / n_{e f}^{2}}{ }^{1}$, where $\omega, \mu_{o}$ are angular frequency and magnetic permeability. The expressions $E_{n}=n^{2} E_{1}$ and $E_{1}=\frac{\hbar^{2} \pi^{2}}{1.602 \times 10^{-19} \times 8 m^{*} a^{2}}$ (eV-electron volt), $\mathrm{n}=1,2,3, \ldots$, denote energy eigenvalue (EEV) in AR. Energy levels in AR is $e_{\nu}=V_{o}-n^{2} E_{1}, \nu=0,1,2, \ldots$, where $V_{o}=\frac{E_{n}}{\alpha}=\frac{E_{n} V^{2}}{\eta^{2}}$ is barrier potential energy (BPE). It must be always $V_{o}>E_{n}$ [3].

## 4. Field probability ratios in TE mode in a SCSASIL and SCSSSIL

Field Probability Ratio $\bar{R}_{a}$ (FPR) for even field [ $\bar{r}_{a}$ for odd field] in the regions I, II and III in a SCSASIL is given by $\bar{R}_{a}=\frac{I_{\ell}}{I_{I I}}\left[\bar{r}_{a}=\frac{I_{\ell}^{\prime}}{I_{I I}}\right]$. Here $I_{I I}$ and $I_{\ell}\left(I_{\ell}^{\prime}\right)$ are respectively field probability in AR and total evanescent field probability for even (odd) field in CLs $[1,2,3]$.

$$
\begin{equation*}
\bar{R}_{a}=\frac{I_{\ell}}{I_{I I}}=\frac{A_{I}^{2}}{2 \alpha_{I}}+\frac{A_{I I I}^{2}}{2 \alpha_{I I I}}, \quad \bar{r}_{a}=\frac{I_{\ell}^{\prime}}{\bar{I}_{I I}^{\prime}}=\frac{B_{I}^{2}}{2 \alpha_{I}}+\frac{B_{I I I}^{2}}{2 \alpha_{I I I}} \tag{13}
\end{equation*}
$$

which are transformated into the formulas

$$
\begin{equation*}
\bar{R}=\frac{1-\alpha}{\eta+\alpha}, \quad \bar{r}=\frac{1-\alpha}{\eta-\alpha} \tag{14}
\end{equation*}
$$

in a SCSSSIL $[1,2,3,7]$.
Denoting input probability with $\mathrm{I}_{i}=I_{I I}+I_{\ell}\left(I_{i}^{\prime}=I_{I I}^{\prime}+I_{\ell}^{\prime}\right)$, the ratios $\bar{K}_{a}$ and $\bar{q}_{a}$ of total evanescent field probabilities to the input probabilities, $\frac{I_{e}}{I_{i}}$ and $\frac{I_{e}^{\prime}}{I_{i}^{\prime}}$, for even and odd fields in a SCSASIL are respectively defined as

$$
\begin{equation*}
\frac{I_{\ell}}{I_{i}}=\bar{K}_{a}=\left[\frac{A_{I}^{2}}{2 \alpha_{I}}+\frac{A_{I I I}^{2}}{2 \alpha_{I I I}}\right] /\left[1+\frac{A_{I}^{2}}{2 \alpha_{I}}+\frac{A_{I I I}^{2}}{2 \alpha_{I I I}}\right], \frac{I_{\ell}^{\prime}}{I_{i}^{\prime}}=\bar{q}_{a}=\left[\frac{B_{I}^{2}}{2 \alpha_{I}^{\prime}}+\frac{B_{I I I}^{2}}{2 \alpha_{I I I}^{\prime}}\right] /\left[1+\frac{B_{I}^{2}}{2 \alpha_{I}^{\prime}}+\frac{B_{I I I}^{2}}{2 \alpha_{I I I}^{\prime}}\right], \tag{15}
\end{equation*}
$$

which are also transformed into the formulas $[1,2,3,7]$

$$
\begin{equation*}
\bar{K}=\frac{1-\alpha}{\eta+1}, \bar{q}=\frac{1-\alpha}{1+\eta-2 \alpha} . \tag{16}
\end{equation*}
$$

in a SCSSSIL $[1,2,3,7]$.
Note that $\bar{K}_{a}+F_{I I}=1$ and $\bar{q}_{a}+F_{I I}^{\prime}=1$. Absorption coefficients (ACs) or the confinement factors (CFs) of the regions I, II and III for even and odd fields in a SCSASIL are given [2] by

$$
\begin{equation*}
\frac{I_{I}}{I_{i}}=F_{I}=\frac{A_{I}^{2}}{2 \alpha_{I}} /\left[1+\frac{A_{I}^{2}}{2 \alpha_{I}}+\frac{A_{I I I}^{2}}{2 \alpha_{I I I}}\right], \frac{I_{I I}}{I_{i}}=F_{I I}=1 /\left[1+\frac{A_{I}^{2}}{2 \alpha_{I}}+\frac{A_{I I I}^{2}}{2 \alpha_{I I I}}\right], \frac{I_{I I I}}{I_{i}}=F_{I I I}=\frac{A_{I I I}^{2}}{2 \alpha_{I I I}} /\left[1+\frac{A_{I}^{2}}{2 \alpha_{I}}+\frac{A_{I I I}^{2}}{2 \alpha_{I I I}}\right] \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{I_{I}^{\prime}}{I_{i}^{\prime}}=F_{I}^{\prime}=\frac{B_{I}^{2}}{2 \alpha_{I}^{\prime}} /\left[1+\frac{B_{I}^{2}}{2 \alpha_{I}^{\prime}}+\frac{B_{I I I}^{2}}{2 \alpha_{I I I}^{\prime}}\right], \frac{I_{I I}^{\prime}}{I_{i}^{\prime}}=F_{I I}^{\prime}=1 /\left[1+\frac{B_{I}^{2}}{2 \alpha_{I}^{\prime}}+\frac{B_{I I I}^{2}}{2 \alpha_{I I I}^{\prime}}\right], \frac{I_{I I I}^{\prime}}{I_{i}^{\prime}}=F_{I I I}^{\prime}=\frac{B_{I I I}^{2}}{2 \alpha_{I I I}^{\prime}} /\left[1+\frac{B_{I}^{2}}{2 \alpha_{I}^{\prime}}+\frac{B_{I I I}^{2}}{2 \alpha_{I I I}^{\prime}}\right] \tag{18}
\end{equation*}
$$

[^0]which give the results
\[

$$
\begin{equation*}
\Gamma_{I}=\Gamma_{I I I}=\Gamma_{I, I I I}=\frac{1}{2} \bar{K}, \quad \Lambda_{I}=\Lambda_{I I I}=\Lambda_{I, I I I}=\frac{1}{2} \bar{q}, \quad \Gamma_{I I}=\frac{\alpha+\eta}{1+\eta}, \quad \Lambda_{I I}=\frac{\eta-\alpha}{1+\eta-2 \alpha} \tag{19}
\end{equation*}
$$

\]

in a SCSSSIL $[1,2,3,7]$. Also note that $\bar{K}+\Gamma_{I I}=1, \bar{q}+\Lambda_{I I}=1$.

## 5. Some novel formulae in TM mode in a SCSASIL

The relations between the variables $\zeta, \zeta^{\prime}, \eta_{I}=\eta, \eta_{I}^{\prime}=\eta^{\prime}$ in TE mode and $\zeta_{T M}, \zeta_{T M}^{\prime}$ in TM mode for even and odd field in a SCSASIL, as shown in Figure 3, are respectively as follows:

$$
\begin{align*}
& \zeta_{T M}=V \sqrt{1+\alpha\left[\left(\frac{n_{I I}^{2}}{n_{I}^{2}}\right)^{2}-1\right]} \cos \zeta, \quad \eta_{T M}=V \sqrt{\alpha+\left(\frac{n_{I}^{2}}{n_{I I}^{2}}\right)^{2}(1-\alpha)} \sin \zeta  \tag{20}\\
& \zeta_{T M}^{\prime}=V \sqrt{1+(1-\alpha)\left(\frac{n_{I I}^{4}}{n_{I}^{4}}-1\right)} \sin \zeta, \quad \eta_{T M}^{\prime}=V \sqrt{1-\alpha+\frac{n_{I}^{4}}{n_{I I}^{4}} \alpha} \cos \zeta \tag{21}
\end{align*}
$$

which create the ellipses as

$$
\begin{gather*}
\frac{\zeta_{T M}^{2}}{r^{2}}+\frac{\eta_{T M}^{2}}{b^{2}}=1, \quad r=V \sqrt{1-\alpha+\alpha\left(\frac{n_{I I}^{2}}{n_{I}^{2}}\right)^{2}}, \quad b=V \sqrt{\alpha+\left(\frac{n_{I}^{2}}{n_{I I}^{2}}\right)^{2}(1-\alpha)}  \tag{22}\\
\frac{\zeta_{T M}^{\prime 2}}{r^{\prime 2}}+\frac{\eta_{T M}^{\prime 2}}{b^{\prime 2}}=1, \quad r^{\prime}=V \sqrt{1+(1-\alpha)\left(\frac{n_{I I}^{4}}{n_{I}^{4}}-1\right)}, \quad b^{\prime}=V \sqrt{1-\alpha+\left(\frac{n_{I}^{2}}{n_{I I}^{2}}\right)^{2} \alpha} . \tag{23}
\end{gather*}
$$

in which r and $\mathrm{b}\left(r^{\prime}\right.$ and $\left.b^{\prime}\right)$ are axises of ellipses (AEs) for even (odd) field. Consequently, we have seen that the geometrical position of the solution points on the eigenvalue equation of the SCSIL in TM mode is an ellipse part, since $\zeta>0, \eta>0[6]$. That is, the ellipse part is also contained on the first quadrant of the coordinate system $\zeta-\eta\left(\zeta^{\prime}-\eta^{\prime}\right)$ which is formed by the coordinate variables $\zeta, \eta\left(\zeta^{\prime}, \eta^{\prime}\right)$ for even (odd) field of eigenvalues (EEVs) of charged particles such as electrons and holes, as shown in Figure 3.


Figure 3 Solution points on the eigenvalue equation $[1,2,3] \tan \zeta=\left(\frac{n_{H}}{n_{I}}\right)^{2} \frac{\eta_{T M}}{\zeta_{T M}}\left[\cot \zeta=\left(\frac{n_{I}}{n_{I}}\right)^{2} \frac{\eta_{T M}^{\prime}}{\zeta_{T M}^{\prime}}\right]$ of the SCSIL for even (odd) field create ellipse part in TM mode.

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As a result, for numeric computational confirming, for example, for data $\lambda=1.55 \mu \mathrm{~m}$ $\left(f=1.772096565332690 \times 10^{13} \mathrm{~Hz}\right), a=35 \mathrm{~A}^{o}, n_{I}=10.922, n_{I I}=10.923$ ve $n_{I I I}=10.921$ for even and odd fields in a SCSASIL and SCSSSIL $\left(n_{I}=n_{I I I}=n_{I, I I I}=10.922\right)$, respectively, we have Table which present various computed quantities. Since $V<1.57$ in this example, there are no solutions for the odd field [8].

Table. For data $\lambda=1.55 \mu \mathrm{~m}\left(\mathrm{f}=1.772096565332690 \times 10^{13} \mathrm{~Hz}\right), a=35 \mathrm{~A}^{o}, n_{I}=10.922, n_{I I}=10.923$ ve $n_{I I I}=10.921$ for even field in a SCSASIL and SCSSSIL.

|  | Asymmetric |  | Symmetric |  |
| :---: | :---: | :---: | :---: | :---: |
| Quantities | Symbol | Value of Quantity | Symbol | Value of Quantity |
| NF | $\mathrm{V}_{a}$ | 0.00209697010488701 | V | 0.00209697010488701 |
| NA | $\mathrm{NA}_{a}$ | 0.14780054127099 | NA | 0.14780054127099 |
| NPC | $\alpha$ | $4.39725773624949 \times 10^{-6}$ | $\alpha$ | $4.39725773624949 \times 10^{-6}$ |
| PC | $\alpha_{I}(1 / \mathrm{m})$ | $1.256363203854682 \times 10^{3}$ | $\alpha_{I}=\alpha_{I I I}=\alpha_{I, I I I}(1 / \mathrm{m})$ | $1.256363203854682 \times 10^{3}$ |
| PC | $\alpha_{I I}(1 / \mathrm{m})$ | $5.991329984064474 \times 10^{5}$ | $\alpha_{I I}(1 / \mathrm{m})$ | $5.991329984064474 \times 10^{5}$ |
| PC | $\alpha_{I I I}(1 / \mathrm{m})$ | $5.991082057765108 \times 10^{5}$ | $\alpha_{I}=\alpha_{I I I}=\alpha_{I, I I I}(1 / \mathrm{m})$ | $1.256363203854682 \times 10^{3}$ |
| AD | $\mathrm{A}_{\text {I }}$ | $1.195226857421494 \times 10^{4}$ | $\mathrm{A}_{I}=\mathrm{A}_{I I I}=\mathrm{A}_{I, I I I}$ | $1.195226857421494 \times 10^{4}$ |
| AD | $\mathrm{A}_{I I I}$ | $1.195226857421494 \times 10^{4}$ | $\mathrm{A}_{I}=\mathrm{A}_{I I I}=\mathrm{A}_{I, I I I}$ | $1.195226857421494 \times 10^{4}$ |
| WN | $k_{I}(1 / \mathrm{m})$ | $4.427416124194545 \times 10^{7}$ | $k_{I}=k_{I I I}=k_{I, I I I}(1 / \mathrm{m})$ | $4.427416124194545 \times 10^{7}$ |
| WN | $k_{\text {II }}(1 / \mathrm{m})$ | $4.427821490988556 \times 10^{7}$ | $k_{I I}$ | $4.427821490988556 \times 10^{7}$ |
| WN | $k_{\text {III }}$ | $4.427010757400533 \times 10^{7}$ | $k_{I}=k_{I I I}=k_{I, I I I}(1 / \mathrm{m})$ | $4.427416124194545 \times 10^{7}$ |
| ERI | $n_{e f}$ | 10.92200000439746 | $n_{e f}$ | 10.92200000439746 |
| DEC | $\varepsilon(\mathrm{F} / \mathrm{m})$ | $1.054755919207530 \times 10^{-8}$ | $\varepsilon(\mathrm{F} / \mathrm{m})$ | $1.054755919207530 \times 10^{-8}$ |
| PhC | $\beta_{z}(1 / \mathrm{m})$ | $4.427416125977129 \times 10^{7}$ | $\beta_{z}(1 / \mathrm{m})$ | $4.427416125977129 \times 10^{7}$ |
| Zeta | $\zeta$ | 0.00209696549442293 | $\zeta$ | 0.00209696549442293 |
| Eta | $\eta=\eta_{I}$ | $4.397270678500610 \times 10^{-6}$ | $\eta=\eta_{I}=\eta_{I I I}=\eta_{, I, I I I}$ | $4.397270678500610 \times 10^{-6}$ |
| $\mathrm{Eta}_{\text {III }}$ | $\eta_{I I I}$ | 0.00209687872022 | $\eta=\eta_{I}=\eta_{I I I}=\eta_{, I, I I I}$ | $4.397271213491386 \times 10^{-6}$ |
| CF | $F_{I}$ | 0.99788981757319 | $\Gamma_{I}=\Gamma_{I I I}=\Gamma_{I, I I I}$ | 0.49999560275457 |
| CF | $F_{I I}$ | $1.755202013010514 \times 10^{-5}$ | $\Gamma_{I I}$ | $8.794490864587430 \times 10^{-6}$ |
| CF | $F_{I I I}$ | 0.00209263040668 | $\Gamma_{I}=\Gamma_{I I I}=\Gamma_{I, I I I}$ | 0.49999560275457 |
| FPR | $\mathrm{R}_{a}$ | $5.697249892419534 \times 10^{4}$ | $R$ | $1.137065633974280 \times 10^{5}$ |
| FPR | $\mathrm{K}_{a}$ | 0.99998244797987 | $\bar{K}$ | 0.99999120551026 |
| AD | A (V/m) | $1.195229485290495 \times 10^{4}$ | A (V/m) | $1.195229485290495 \times 10^{4}$ |
| PPUL | $\mathrm{P}\left(\mathrm{nW} / \mathrm{nm}^{2}\right)$ | 0.31642677576226 | $\mathrm{P}\left(\mathrm{nW} / \mathrm{nm}^{2}\right)$ | 0.31642677576226 |
| IV | Z ( $\Omega$ ) | 3.16028881434272 | Z ( $\Omega$ ) | 3.16028881434272 |
| PV | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ | $2.746749678439964 \times 10^{7}$ | $\mathrm{v}(\mathrm{m} / \mathrm{s})$ | $2.746749678439964 \times 10^{7}$ |
| EME | $\mathrm{m}^{*}$ (kg) | $6.129109001012260 \times 10^{-32}$ | $\mathrm{m}^{*}$ | $6.129109001012260 \times 10^{-32}$ |
| EEV | $E_{1}(\mathrm{eV})$ | 114.0725396948354 | $E_{1}(\mathrm{eV})$ | 114.0725396948354 |
| BPE | $\mathrm{V}_{o}(\mathrm{eV})$ | 25941745.18233497 | $\mathrm{V}_{o}(\mathrm{eV})$ | 25941745.18233497 |
| AE | r | 0.00209697010658 | r | 0.00209697010658 |
| AE | b | 0.00209658616917 | b | 0.00209658616917 |
| $\mathbf{Z e t a}_{T M}$ | $\zeta_{T M}$ | 0.00209696549611 | $\zeta_{T M}$ | 0.00209696549611 |
| $\mathbf{E t a}_{T M}$ | $\eta_{T M}$ | $4.396465630771056 \times 10^{-6}$ | $\eta_{T M}$ | $4.396465630771056 \times 10^{-6}$ |

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[^0]:    ${ }^{1}$ This effective mass formula is given for the GaAs as the laser material.

