

# A Robust Method for State Estimation of Power System with UPFC

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## Abstract

*This paper presents a Least Absolute Value (LAV) state estimator for power system using a new method called Recursive Least Squares (RLS). Also, a suitable model is used for state estimation of power system that includes a Unified Power Flow Controller (UPFC). The IEEE 14-bus test system is used to show the validity of the proposed algorithm in estimation of power system states as well as the states of controllable parameters of UPFC.*

**Key Words:** *RLS method, State estimation, UPFC, Power system states estimator*

## 1. Introduction

Due to the enlargement of interconnected electric power system and increasing the complexity of power system structure, Energy Management System (EMS) is becoming more critical for modern power system [1–3]. State estimation plays an important role in EMS and provides a reliable and consistent system data by processing real time redundant telemetric and pseudo measurements. These measurements typically consist of bus voltage magnitudes, active and reactive line flows and power injections. The Weighted Least Square (WLS) state estimation has been widely used in the past for power system state estimation [4]. One criterion called the Weighted Least Absolute Value (WLAV) is used to improve the robustness of state estimation [5]. The WLAV estimator is able to reject the bad measurements as long as these are not leverage point method [6, 7]. Recently the application of the Interior Point (IP) method for WLAV state estimation of the conventional power system has been presented [8, 9]. Generally, the aim is good estimation of state variables of power system with little error.

On the other hand, the Flexible AC Transmission System (FACTS) controllers are used increasingly in many power systems, since they can control the utilization of the power transfer capability, as well as improving the security and stability of power systems [10, 11]. However, very limited efforts have been made to study the impact of FACTS controllers on power system state estimation. In order to estimate the state of a power system containing FACTS controllers, an improved sequential method has been proposed in [12]. However,

the constraints of these controllers are not considered. The state estimation problem is then solved by using a solution method based on IP method by considering the constraints of power system and FACTS controller [13]. That will be responded by flat initial guess. Otherwise the inequality function would not be satisfied. Also, if the control variables associated with the UPFC, initialized by flat guesses, it will lead to a singular coefficient matrix. Hence, these variables are assigned very small but nonzero values.

In this paper, we propose a new method for solving the state estimation problem of power system containing UPFC by formulating the problem as a LAV optimization. The RLS method is then used to solve this problem. The estimator proposed in this paper is based on solving a sequence of  $L_1$ -regression problems (i.e. linearised LAV problems). The RLS method is based on iteratively solving the set of equations that yields the critical points of the  $L_1$ -regression problem.

This paper is organized as follows. The steady-state and power injection model of UPFC including its operating constraints are given in section 2. In section 3 the LAV state estimator is introduced and in section 4, the proposed method for state estimation is presented. Also, variations in measurement matrix for power system state estimation with UPFC are shown in section 5. The performance of the proposed state estimation method is demonstrated by using the IEEE 14-bus test system, which has been modified by means of UPFC. Simulation results for IEEE test system in section 6 are provided to illustrate the performance of the proposed implementation. The results show that the estimated states using the proposed algorithm are very close to the true values, i.e. the state variables of system and control variables of UPFC are estimated properly in this work.

## 2. Modeling of UPFC in state estimation

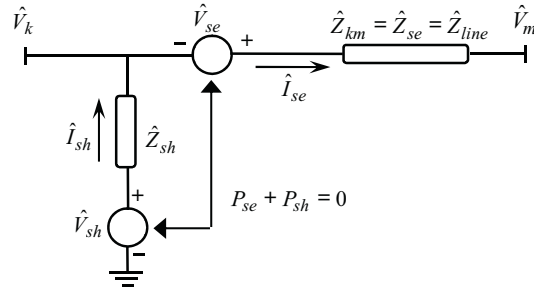
### 2.1. Steady state model of the UPFC

One of the many purposes of using a FACTS device is to reroute power flows by way of controlling the effective impedance of a line, voltage magnitude of a chosen bus or the phase shift between two buses in the system. Among the many different types of FACTS devices, this paper concentrates on the UPFC which is capable of simultaneously controlling the voltage magnitude as well as the active and reactive power flows. The UPFC converters are assumed lossless in this model. This implies that there is no absorption or generation of active power by the two converters and the active power demanded by the series converter at its output is supplied from the AC power system by the shunt converter via the common DC link. The DC link capacitors voltage,  $V_{dc}$  remains constant. Hence, the active power supplied to the shunt converter  $P_{sh}$  must be equal to the active power demanded by the series converter  $P_{se}$  at the DC link. Then  $P_{sh}$  and  $P_{se}$  must be constrained as

$$P_{se} + P_{sh} = 0 \quad (1)$$

This UPFC constraint should be included in the estimation equations.

Steady-state model of the UPFC including the transmission line is shown in Figure 1. This model consists of one series voltage source  $\hat{V}_{se}$  in addition to one shunt voltage source  $\hat{V}_{sh}$  and their source impedances  $\hat{Z}_{se}$  and  $\hat{Z}_{sh}$ , respectively.



**Figure 1.** Steady-state model for the UPFC.

The quantities in Figure 1 are defined as follows:

$\hat{V}_k$  and  $\hat{V}_m$  are the bus voltage phasors at bus  $k$  and  $m$ , respectively ( $V_k \angle \theta_k$ ,  $V_m \angle \theta_m$ );  
 $\hat{V}_{sh}$  and  $\hat{V}_{se}$  are the voltage phasors of the shunt and the series voltage sources, respectively;  
 $\hat{I}_{sh}$  and  $\hat{I}_{se}$  are the current phasors of the shunt and the series voltage sources, respectively;  
 $\hat{Z}_{sh}$  and  $\hat{Z}_{se}$  are the impedance of the shunt and the series voltage sources, respectively;  
 $\hat{Z}_{line}$  is the impedance of transmission line;  
 $P_{sh}$  and  $P_{se}$  are real the power of the shunt and the series voltage sources, respectively.  
 The current phasors of the shunt and the series voltage sources are:

$$\hat{I}_{sh} = (\hat{V}_{sh} - \hat{V}_k) \cdot \hat{Y}_{sh} \quad (2)$$

$$\hat{I}_{se} = (\hat{V}_k + \hat{V}_{se} - \hat{V}_m) \cdot \hat{Y}_{km}, \quad (3)$$

where  $\hat{Y}_{sh} = 1/\hat{Z}_{sh}$  and  $\hat{Y}_{km} = 1/\hat{Z}_{km}$ . The apparent power flow through branch  $k - m$  and  $m - k$  can be expressed as

$$\hat{S}_{km} = \hat{V}_k \hat{I}_{km}^* = P_{km} + jQ_{km} \quad (4)$$

$$\hat{S}_{mk} = \hat{V}_m \hat{I}_{mk}^* = P_{mk} + jQ_{mk}, \quad (5)$$

where  $\hat{I}_{km} = \hat{I}_{se} - \hat{I}_{sh}$  and  $\hat{I}_{mk} = -\hat{I}_{se}$ . The apparent powers of the shunt and the series voltage sources are given by

$$\hat{S}_{sh} = \hat{V}_{sh} \hat{I}_{sh}^* = P_{sh} + jQ_{sh} \quad (6)$$

$$\hat{S}_{se} = \hat{V}_{se} \hat{I}_{se}^* = P_{se} + jQ_{se}. \quad (7)$$

## 2.2. Power injection model of UPFC

The models of UPFC in power system can be classified into the categories: power injection model, voltage source model and impedance model. Equations of the power injection model for the UPFC are taken from [14]. Power injection model is a suitable model in state estimation because it is very simple in terms of implementation in power system state estimation and can retain the structure of the original system [15]. Therefore, it is convenient for implementation in existing power system state estimation program. Power injection model of

UPFC consists of one controllable series injected voltage source and one controllable shunt injected current source. The equivalent circuit of UPFC placed in the line between bus  $k$  and bus  $m$  is shown in Figure 2.

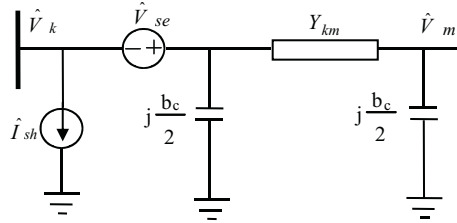


Figure 2. UPFC equivalent circuit.

In Figure 2,  $Y_{km} = G_{km} + jB_{km}$  is the line admittance. The controllable series injected voltage source and controllable shunt injected current source are defined as  $\hat{V}_{se} = V_{se}\angle\theta_{se}$  and  $\hat{I}_{sh} = \frac{\hat{V}_k - \hat{V}_{sh}}{jX_{sh}} = I_{sh}\angle\theta_{sh}$ , respectively. Let  $\Delta\hat{I}_k$  and  $\Delta\hat{I}_m$ , denote the additional injection currents to the buses  $k$  and  $m$ , respectively by using UPFC. Then

$$\begin{aligned} \Delta\hat{I}_k &= -(Y_{km} + j\frac{b_c}{2}) \cdot \hat{V}_{se} + \hat{I}_{sh} = [-(Y_{km} + j\frac{b_c}{2}) \cdot (V_{se}\angle\theta_{se}) + I_{sh}\angle\theta_{sh}] \\ \Delta\hat{I}_m &= Y_{km} \cdot \hat{V}_{se} = [Y_{km} \cdot (V_{se}\angle\theta_{se})] \end{aligned} \tag{8}$$

Also, let  $\Delta\hat{S}_k = \Delta P_k + j\Delta Q_k$  and  $\Delta\hat{S}_m = \Delta P_m + j\Delta Q_m$  denote the additional injection power with UPFC to buses  $k$  and  $m$ , respectively. Then

$$\begin{aligned} \Delta\hat{S}_k &= \hat{V}_k \cdot \Delta\hat{I}_k^* = (V_k\angle\theta_k) \cdot [-(Y_{km} + j\frac{b_c}{2}) \cdot (V_{se}\angle\theta_{se}) + I_{sh}\angle\theta_{sh}]^* \\ \Delta\hat{S}_m &= \hat{V}_m \cdot \Delta\hat{I}_m^* = (V_m\angle\theta_m) \cdot [Y_{km} \cdot (V_{se}\angle\theta_{se})]^* \end{aligned} \tag{9}$$

Based on the above mentioned model, the power flow can be derived to the relations

$$\begin{aligned} \Delta P_k &= [V_k I_{sh} \cos(\theta_k - \theta_{sh})] - V_k V_{se} [G_{km} \cos(\theta_k - \theta_{se}) + (B_{km} + \frac{b_c}{2}) \sin(\theta_k - \theta_{se})] \\ \Delta Q_k &= [V_k I_{sh} \sin(\theta_k - \theta_{sh})] - V_k V_{se} [G_{km} \sin(\theta_k - \theta_{se}) - (B_{km} + \frac{b_c}{2}) \cos(\theta_k - \theta_{se})] \\ \Delta P_m &= V_m V_{se} [G_{km} \cos(\theta_m - \theta_{se}) + B_{km} \sin(\theta_m - \theta_{se})] \\ \Delta Q_m &= V_m V_{se} [G_{km} \sin(\theta_m - \theta_{se}) - B_{km} \cos(\theta_m - \theta_{se})] \end{aligned} \tag{10}$$

Here,  $\theta_k$  and  $\theta_m$  stand for phase angle of phasors  $\hat{V}_k$  and  $\hat{V}_m$ , respectively. The UPFC power injection model is shown in Figure 3.

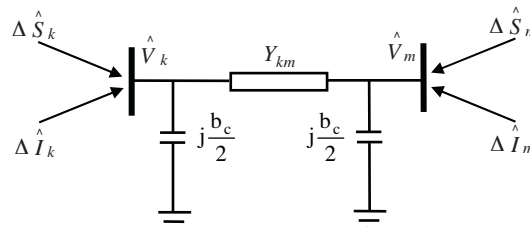


Figure 3. UPFC power injection model.

### 3. LAV state estimator

The measurement set and the measurement errors are related by

$$Z_i = h_i(X) + e_i, \quad i = 1, \dots, m, \quad (11)$$

where,  $Z_i$  is the  $i^{th}$  component of the measurement vector  $Z (m \times 1)$ ;  $h_i(\cdot)$  is the  $i^{th}$  component of the vector  $h (m \times 1)$  of known non-linear functions;  $X$  is the State vector ( $n \times 1$ );  $e_i$  is the  $i^{th}$  component of the error vector  $e (m \times 1)$ ;  $m$  is the number of measurements;  $n$  is the number of state variables.

The LAV state estimator calculates the state vector  $X$ , by solving the following non-linear minimization problem:

$$\min_X f(X) = \min_X \sum_{i=1}^m |h_i(X) - Z_i| \quad (12)$$

Given an estimate  $X^{(k)}$  ( $k$ : iteration index), a better estimate of the system state is obtained by solving the problem

$$\min_{\Delta X^{(k)}} f(X^{(k)} + \Delta X^{(k)}) = \min_{\Delta X^{(k)}} \sum_{i=1}^m \left| h_i(X^{(k)}) + \sum_{j=1}^n H_{ij}^{(k)} \Delta X_j^{(k)} - Z_i \right|, \quad (13)$$

where

$$H_{ij}^{(k)} = \left. \frac{\partial h_i(X)}{\partial X_j} \right|^{(k)}. \quad (14)$$

On solution of equation (13) via the RLS method, the new iteration is computed as

$$X^{(k+1)} = X^{(k)} + \Delta X^{(k)}. \quad (15)$$

This process is repeated until the following convergence criterion is met:

$$\frac{\|\Delta X^{(k)}\|_1}{1 + \|X^{(k)}\|_1} \leq tol_{LAV}, \quad (16)$$

where  $\|\cdot\|_1$  stands for the  $L_1$  norm.

The starting point is initialized by setting all bus voltage magnitudes to 1 p.u. and all bus voltage phase angles to zero radians. The phase angle at the reference bus (slack bus) is fixed to zero radians during all iterations (bus number one is chosen as the reference bus in our test system).

Note that the LAV estimator can be extended to the WLAV estimator by simply assigning different weights to the measurement residuals in equation (12).

### 4. The proposed state estimation algorithm (RLS method)

The RLS iterative method is used to compute the solution of the equation (13). To simplify the notations, the iteration superscript  $k$  in (13) is ignored and this equation is rewritten as

$$\min_{\Delta X} f(\Delta X) = \min_{\Delta X} \sum_{i=1}^m \left| r_i - \sum_{j=1}^n H_{ij} \Delta X_j \right|, \quad (17)$$

where  $r_i$  is the residual of  $i^{th}$  measurement and is defined as

$$r_i = Z_i - h_i(X). \tag{18}$$

The RLS method finds the critical points of (17) by simply using elementary calculus. Hence, the critical points are thus found as

$$\frac{\partial f}{\partial \Delta x_k} = \sum_{i=1}^m \frac{r_i - \sum_{j=1}^n H_{ij} \Delta x_j}{\left| r_i - \sum_{j=1}^n H_{ij} \Delta x_j \right|} (-H_{ik}) = 0, \quad k = 1, \dots, n. \tag{19}$$

If the notation

$$e_i(\Delta X) = \left| r_i - \sum_{j=1}^n H_{ij} \Delta x_j \right| \tag{20}$$

is introduced for the deviations, we can rewrite equation (19) as

$$\sum_{i=1}^m \frac{H_{ik} r_i}{e_i(\Delta X)} = \sum_{i=1}^m \sum_{j=1}^n \frac{H_{ik} H_{ij}}{e_i(\Delta X)} \Delta X_j, \quad k = 1, \dots, n. \tag{21}$$

Now, if  $E_{\Delta X}$  ( $m \times m$ ) and  $\mathbf{r}$  ( $m \times 1$ ) denote the diagonal matrix containing the elements of  $e_i(\Delta X)$  on the diagonal and measurement residual vector containing the elements of  $r_i$ , respectively, we can write equation (21) in matrix form as

$$H^T E_{\Delta X}^{-1} \mathbf{r} = H^T E_{\Delta X}^{-1} H \Delta X. \tag{22}$$

Equation (22) can be rearranged by multiplying both sides by the inverse of  $H^T E_{\Delta X}^{-1} H$ . So, we have

$$\Delta X = (H^T E_{\Delta X}^{-1} H)^{-1} H^T E_{\Delta X}^{-1} \mathbf{r}. \tag{23}$$

The iterations terminate when the following criterion is satisfied:

$$\frac{\|\Delta X^{(k+1)} - \Delta X^{(k)}\|_1}{1 + \|\Delta X^{(k)}\|_1} \leq tol_{RLS}, \tag{24}$$

where  $tol_{RLS}$  is a small number that stands for the permissible error. The RLS algorithm is initialized by setting  $\Delta X^{(0)} = 0$ .

The complete LAV algorithm which makes use of the RLS method to solve the state estimation problem is presented as a flowchart in Figure 4. The flowchart shows two loops. In the outer loop, the jacobian matrix  $\mathbf{H}$  and the measurement residual vector  $\mathbf{r}$  are computed and the RLS method is executed. The inner loop updates the correction of the state vector  $\Delta X$  according to inequality (24).

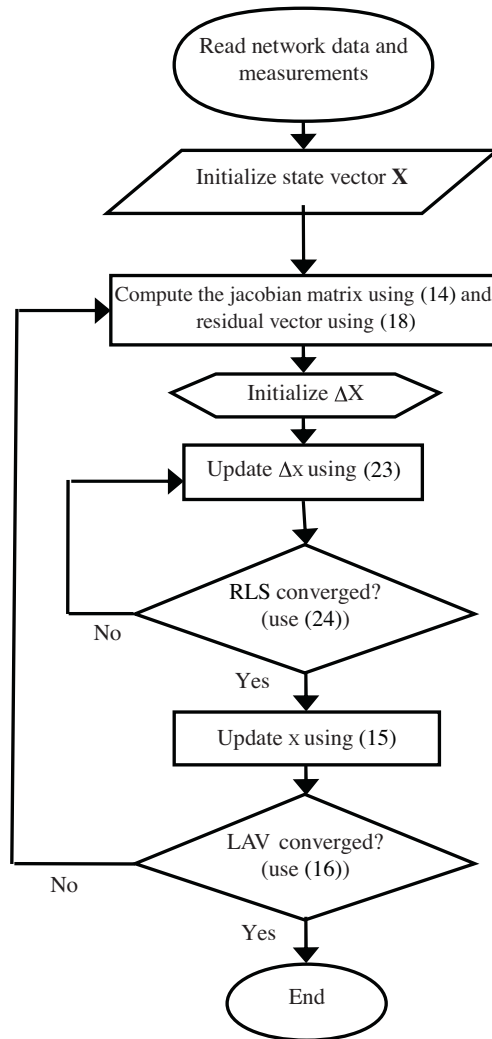


Figure 4. Complete LAV algorithm employing the RLS method.

## 5. Measurement matrix for power system state estimation with UPFC

In order to state estimation of a power system containing UPFC, the measurement equations for the nodal injected powers of the UPFC buses should be modified as

$$\begin{aligned}
 P_k^U &= P_k^\circ + \Delta P_k \\
 Q_k^U &= Q_k^\circ + \Delta Q_k \\
 P_m^U &= P_m^\circ + \Delta P_m \\
 Q_m^U &= Q_m^\circ + \Delta Q_m
 \end{aligned} \tag{25}$$

Where,  $\Delta P_k$ ,  $\Delta Q_k$ ,  $\Delta P_m$  and  $\Delta Q_m$  are defined in equation (10); the superscript "U" denotes the injected power with UPFC; and the superscript "o" denotes the injected power without UPFC. Other measurement equations remain unchanged.

The linearized system model based on Newton-Raphson algorithm, written in matrix form is

$$\begin{bmatrix} \Delta P^U \\ \Delta Q^U \end{bmatrix} = \begin{bmatrix} M^U & F^U \\ G^U & N^U \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}. \quad (26)$$

$\Delta \theta$  and  $\Delta V$  are vectors of incremental changes in nodal voltages.  $\mathbf{M}$ ,  $\mathbf{F}$ ,  $\mathbf{G}$  and  $\mathbf{N}$  denote the basic elements in the Jacobian matrix and correspond to partial derivatives of the real and the reactive powers with respect to the phase angles and the magnitudes of the nodal voltages

$$M = \frac{\partial P}{\partial \theta}, \quad F = \frac{\partial P}{\partial V}, \quad G = \frac{\partial Q}{\partial \theta}, \quad N = \frac{\partial Q}{\partial V}. \quad (27)$$

The jacobian matrix for the nodal injected power from a UPFC located between bus  $k$  and bus  $m$  should be modified. Based on equation (10), the following additional elements of the jacobian matrix owing to the injections of the UPFC at bus  $k$  and bus  $m$ , between where the UPFC is installed, can be derived as follows for bus  $k$  when :

$$\begin{aligned} M_{kk}^U &= M_{kk}^\circ + \Delta M_{kk} = \frac{\partial P_k}{\partial \theta_k} + \frac{\partial \Delta P_k}{\partial \theta_k} \\ F_{kk}^U &= F_{kk}^\circ + \Delta F_{kk} = \frac{\partial P_k}{\partial V_k} + \frac{\partial \Delta P_k}{\partial V_k} \\ G_{kk}^U &= G_{kk}^\circ + \Delta G_{kk} = \frac{\partial Q_k}{\partial \theta_k} + \frac{\partial \Delta Q_k}{\partial \theta_k} \\ N_{kk}^U &= N_{kk}^\circ + \Delta N_{kk} = \frac{\partial Q_k}{\partial V_k} + \frac{\partial \Delta Q_k}{\partial V_k} \end{aligned} \quad (28)$$

which can be rewritten as

$$\begin{aligned} M_{kk}^U &= M_{kk}^\circ - V_k I_{sh} \sin(\theta_k - \theta_{sh}) + V_k V_{se} [G_{km} \sin(\theta_k - \theta_{se}) - (B_{km} + \frac{b_c}{2}) \cos(\theta_k - \theta_{se})] \\ F_{kk}^U &= F_{kk}^\circ + I_{sh} \cos(\theta_k - \theta_{sh}) - V_{se} [G_{km} \cos(\theta_k - \theta_{se}) + (B_{km} + \frac{b_c}{2}) \sin(\theta_k - \theta_{se})] \\ G_{kk}^U &= G_{kk}^\circ + V_k I_{sh} \cos(\theta_k - \theta_{sh}) - V_k V_{se} [G_{km} \cos(\theta_k - \theta_{se}) + (B_{km} + \frac{b_c}{2}) \sin(\theta_k - \theta_{se})] \\ N_{kk}^U &= N_{kk}^\circ + I_{sh} \sin(\theta_k - \theta_{sh}) - V_{se} [G_{km} \sin(\theta_k - \theta_{se}) - (B_{km} + \frac{b_c}{2}) \cos(\theta_k - \theta_{se})] \end{aligned} \quad (29)$$

When  $k \neq m$ ,

$$\begin{aligned} M_{km}^U &= M_{km}^\circ + \Delta M_{km} = \frac{\partial P_k}{\partial \theta_m} + \frac{\partial \Delta P_k}{\partial \theta_m} = M_{km}^\circ \\ F_{km}^U &= F_{km}^\circ + \Delta F_{km} = \frac{\partial P_k}{\partial V_m} + \frac{\partial \Delta P_k}{\partial V_m} = F_{km}^\circ \\ G_{km}^U &= G_{km}^\circ + \Delta G_{km} = \frac{\partial Q_k}{\partial \theta_m} + \frac{\partial \Delta Q_k}{\partial \theta_m} = G_{km}^\circ \\ N_{km}^U &= N_{km}^\circ + \Delta N_{km} = \frac{\partial Q_k}{\partial V_m} + \frac{\partial \Delta Q_k}{\partial V_m} = N_{km}^\circ \end{aligned} \quad (30)$$

The elements in the jacobian matrix remain unchanged.



Also, for bus  $m = k$ ,

$$\begin{aligned}
 M_{mm}^U &= M_{mm}^\circ + \Delta M_{mm} = \frac{\partial P_m}{\partial \theta_m} + \frac{\partial \Delta P_m}{\partial \theta_m} \\
 F_{mm}^U &= F_{mm}^\circ + \Delta F_{mm} = \frac{\partial P_m}{\partial V_m} + \frac{\partial \Delta P_m}{\partial V_m} \\
 G_{mm}^U &= G_{mm}^\circ + \Delta G_{mm} = \frac{\partial Q_m}{\partial \theta_m} + \frac{\partial \Delta Q_m}{\partial \theta_m} \\
 N_{mm}^U &= N_{mm}^\circ + \Delta N_{mm} = \frac{\partial Q_m}{\partial V_m} + \frac{\partial \Delta Q_m}{\partial V_m}
 \end{aligned} \tag{31}$$

From which we can write following equations:

$$\begin{aligned}
 M_{mm}^U &= M_{mm}^\circ - V_m V_{se} [G_{km} \sin(\theta_m - \theta_{se}) - B_{km} \cos(\theta_m - \theta_{se})] \\
 F_{mm}^U &= F_{mm}^\circ + V_{se} [G_{km} \cos(\theta_m - \theta_{se}) + B_{km} \sin(\theta_m - \theta_{se})] \\
 G_{mm}^U &= G_{mm}^\circ + V_m V_{se} [G_{km} \cos(\theta_m - \theta_{se}) + B_{km} \cos(\theta_m - \theta_{se})] \\
 N_{mm}^U &= N_{mm}^\circ + V_{se} [G_{km} \sin(\theta_m - \theta_{se}) - B_{km} \cos(\theta_m - \theta_{se})]
 \end{aligned} \tag{32}$$

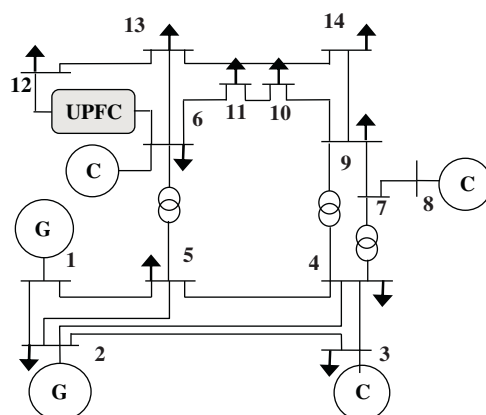
When  $m \neq k$ ,

$$\begin{aligned}
 M_{mk}^U &= M_{mk}^\circ + \Delta M_{mk} = \frac{\partial P_m}{\partial \theta_k} + \frac{\partial \Delta P_m}{\partial \theta_k} = M_{mk}^\circ \\
 F_{mk}^U &= F_{mk}^\circ + \Delta F_{mk} = \frac{\partial P_m}{\partial V_k} + \frac{\partial \Delta P_m}{\partial V_k} = F_{mk}^\circ \\
 G_{mk}^U &= G_{mk}^\circ + \Delta G_{mk} = \frac{\partial Q_m}{\partial \theta_k} + \frac{\partial \Delta Q_m}{\partial \theta_k} = G_{mk}^\circ \\
 N_{mk}^U &= N_{mk}^\circ + \Delta N_{mk} = \frac{\partial Q_m}{\partial V_k} + \frac{\partial \Delta Q_m}{\partial V_k} = N_{mk}^\circ
 \end{aligned} \tag{33}$$

## 6. Simulation results

In this section, a power system embedded with UPFC is used to test effectiveness of the LAV state estimation algorithm under application of the proposed RLS method. The solution accuracy and computational efficiency of the proposed method are verified by the test results and compared with those obtained from traditional state estimation (WLS) method. The data for testing the modified state estimation are obtained using the results from power flow analysis. For all simulations, the tolerance used to define convergence (i.e. LAV and RLS convergence) is  $10^{-4}$ .

The IEEE 14-bus system with a UPFC installed in the line 6-12 is shown in Figure 5.



**Figure 5.** IEEE 14-bus system with UPFC (G denotes generators and C for synchronous condensers).

Also, measurements which are assumed to be available for this power system are listed in Table 1. The measurement set consists of two voltages, 12 power injections and 32 flow measurements. Gaussian errors with zero mean value are added to the measurements in all the simulations. It should be noted that there are enough measurements to make the network and the UPFC parameters observable. The state estimation algorithm has been developed using MATLAB software.

**Table 1.** Measurement data for the IEEE 14-bus system.

BUS VOLTAGE MEASUREMENTS					
BUS	VOLTAGE		BUS	VOLTAGE	
1	1.0683		2	1.0448	
3	1.0063		6	1.0725	
8	1.0900				
FLOW MEASUREMENTS					
BRANCH	P	Q	BRANCH	P	Q
1-2	1.5706	-0.1745	2-3	0.7399	0.0594
4-7	0.2756	-0.1400	6-11	0.0642	0.0825
6-12	0.1906	0.0226	7-8	0.0013	-0.2039
7-9	0.2800	0.1573	9-7	-0.2799	-0.1473
9-14	0.0808	0.0017	12-6	-0.1925	-0.0257
12-13	0.1206	0.0194	13-14	0.0694	0.0422
INJECTION MEASUREMENTS					
BUS	P	Q	BUS	P	Q
3	-0.9416	0.0101	5	-0.0963	-0.0508
6	-0.1188	-0.0404	8	-0.0017	0.2099
9	-0.2884	-0.1012	10	-0.0939	-0.0756
11	-0.0347	-0.0161	12	-0.0726	-0.0066
13	-0.1330	-0.0684			

Table 2 shows the state estimation results with two state estimator, i.e. WLS traditional state estimator and proposed LAV state estimator by applying the RLS method. From Table 2, it is noted that the proposed method estimates state variables with more accuracy and little error in comparison with WLS method.

**Table 2.** State estimation results.

Bus no.	True states		WLS method		Proposed method (RLS)	
	$V(p.u.)$	$\theta(\circ)$	$V(p.u.)$	$\theta(\circ)$	$V(p.u.)$	$\theta(\circ)$
1	1.063	0	1.0621	0	1.0629	0
2	1.048	-4.961	1.0432	-4.957	1.0477	-4.9608
3	1.013	-12.744	1.010	-12.7503	1.0124	-12.7439
4	1.024	-10.511	1.0210	-10.523	1.0236	-10.5089
5	1.032	-9.1010	1.0298	-9.0961	1.0315	-9.1007
6	1.070	-15.205	1.0663	-15.1907	1.0693	-15.2011
7	1.054	-13.568	1.0510	-13.5769	1.0531	-13.5687
8	1.088	-13.584	1.0831	-13.5903	1.0863	-13.5815
9	1.038	-15.180	1.0365	-15.1792	1.0376	-15.1792
10	1.035	-15.458	1.0332	-15.4671	1.0359	-15.4565
11	1.049	-15.453	1.0433	-15.4601	1.0493	-15.4493
12	1.081	-14.454	1.0792	-14.4492	1.0799	-14.4490
13	1.053	-15.452	1.050	-15.4479	1.0537	-15.4479
14	1.028	-16.342	1.0212	-16.3538	1.0279	-16.3379

For better comparison, it is possible to examine the deviation of the estimated states from the true state values. Normally, it is possible to use normalized error, NE, to assess the accuracy of the estimated state values as follows:

$$NE = \frac{\|x_{true} - x_{estimated}\|_2}{\|x_{true}\|_2} \tag{34}$$

The NE values which are obtained from the simulation results by applying the WLS and RLS methods are shown in Table 3.

**Table 3.** NE values for state variables from WLS and RLS methods.

	WLS method	Proposed method
$NE_{voltages}$	0.0035	0.0008
$NE_{phaseangles}$	0.0006	0.0002

As it is shown in Table 3, the proposed method gives less NE values in comparison with WLS method and also it shows that the estimated states using the proposed method are more close to the true state values. Furthermore, UPFC control variables in IEEE 14-bus system estimated by RLS method are shown in Table 4 and they are compared with UPFC control variables estimated by IP method [13]. This table shows that  $P_{se} + P_{sh} \cong 0$ , so there is no real power exchange between UPFC and the power system.

**Table 4.** Estimated values for UPFC control variables in IEEE 14-bus system.

UPFC control variables	RLS method		IP method [13]	
	SERIES SOURCE	SHUNT SOURCE	SERIES SOURCE	SHUNT SOURCE
$V$	0.0771	1.0905	0.077	1.090
$\theta(\circ)$	44.0199	-15.2127	44.08	-15.22
$P$	0.0056	-0.0055	0.0056	-0.0056
$Q$	0.0127	0.4388	0.0127	0.4378

## 7. Conclusion

This paper presents a new approach for power system state estimation containing UPFC using the LAV criterion. The proposed approach implements the LAV estimator using the RLS method. Based on the conventional power system state estimation model, this paper uses the model of state estimation containing UPFC, called power injection model. Simulations are carried out on IEEE 14-bus test system to illustrate the effectiveness of the proposed state estimator. The simulation results indicate that the proposed method yields good estimation of the power system states and UPFC control variables.

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