

# Low-complexity parameters estimator for multiple 2D domain incoherently distributed sources

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## Abstract

*A new low-complexity parameters estimator for multiple two-dimensional (2D) domain incoherently distributed (ID) sources is presented. One 2D domain ID source is parameterized with four parameters, the central azimuth direction-of-arrival (DOA), azimuth angular spread, central elevation DOA and elevation angular spread. Based on the eigenstructure between the steering matrix and signal subspace, an average total least-squares via rotational invariance technique (TLS-ESPRIT) is used to estimate the central elevation DOA, and then a generalized multiple signal classification (GMUSIC) algorithm is derived to estimate the central azimuth DOA. Utilizing preliminary estimates obtained at a pre-processing stage, the angular spread parameters can be obtained by matrix transform. To estimate four-dimensional parameters, our algorithm only needs one-dimensional search. Compared with earlier algorithms, our method has lower computational cost. In addition, it can be applied to scenarios with multiple sources that may have different angular power densities. Finally, the Cramér-Rao Lower bound (CRLB) for parameters estimation of 2D domain ID sources is also derived. Numerical simulations are carried out to study the performance of the suggested estimator.*

**Key Words:** *Distributed sources, parameter estimation, angular spread, direction-of-arrival, source localization*

## 1. Introduction

In mobile communications, due to local scattering in the vicinity of the mobile, the sources are no longer viewed as point sources by the array in the base station, in that they represent spatially distributed sources with some central angles and angular spreads. Angular spreads can increase up to  $10^\circ$  depending on the distance between the mobile and the base station as well as the base station's height in practice [1]. Hence, in such case, the distributed source model is more appropriate than the point source one. The direct application of many classical direction-of-arrival (DOA) estimators [2, 3, 4, 5, 6], which are designed for point source, will cause deteriorated performance.

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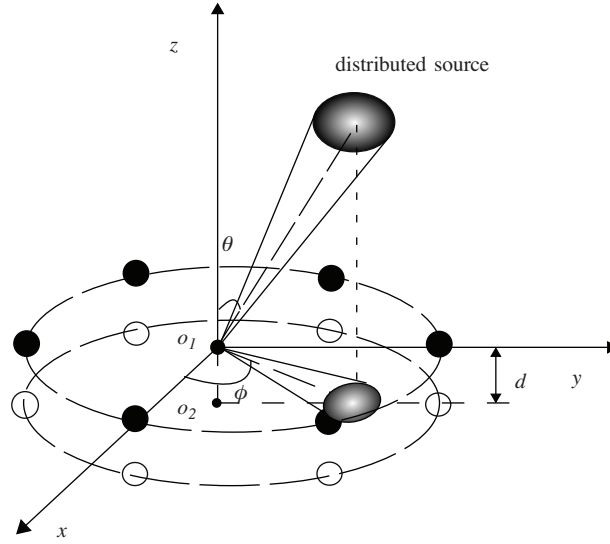
In [7], the distributed source models are classified into coherently distributed (CD) source and incoherently distributed (ID) source, depending on the relationship between the channel coherency time and the observation period. For a CD source, all signal components arriving from different directions are time delay replicas of the same signal, whereas for the ID source case, these components are considered to be uncorrelated.

Several techniques have been proposed for distributed source parameters estimation. A class of extensions to the MUSIC algorithm, such as distributed source parameter estimator (DSPE) in [7] and the dispersed signal parameter estimator (DISPARE) in [8], have been proposed. Both algorithms involve a 2D search and, therefore, are computationally intensive. A maximum likelihood (ML) [9] and weighted subspace fitting (WSF) [10] have been proposed to estimate distributed source parameters, the computational costs of these algorithms remain prohibitively high since a multidimensional search is required. Recently, the covariance matching estimation technique (COMET) [11], which is based on the extended invariance principle (EXIP), has been used to compute the central DOA and angular spread of single distributed source using two successive one-dimensional searches. However, this technique has an ambiguity [12] and cannot be extended to multiple sources case. To overcome the drawbacks, a modified COMET-EXIP method [12] has been used to solve the ambiguity problem. Based on an approximation of the array covariance matrix, [13] proposed a new multiple distributed sources parameters estimation algorithm with iterative search technique. The algorithm needs preliminary DOA estimates of the sources and subjects to local minimum problem. A parameters estimation algorithm using ESPRIT for the distributed sources model was proposed in [14], in which the central DOAs of the sources are estimated by using TLS-ESPRIT with two closely spaced uniform linear arrays (ULAs) and the angular spreads are estimated by one-dimensional DSPE spectrum.

However, all the algorithms mentioned above are designed for 1D domain distributed sources. That is, it commonly assumes that the sources and the base arrays are in the same plane and the parameters to be estimated are azimuths-only and their angular spreads. However, in most cases, sources and receiving arrays are not in the same plane, which correspond to a 2D domain distributed sources model. In this study, we consider parameters estimation problem for multiple 2D domain ID sources. The 1D domain ID sources are only special cases of our model [15].

The computational complexity of the parameters estimator for 2D domain ID sources is normally high since four-dimensional optimization is needed. In [16] and [17], two kinds of 2D domain CD source parameters estimation techniques using uniform circular arrays (UCAs) and L-shape ULA, respectively, have been proposed. These algorithms can give more exact parameters estimation results, however, they need two two-dimensional searches and the computation burden is still large. In [18], the authors consider the central azimuth DOAs and central elevation DOAs estimation problem using a pair of UCAs under 2D domain CD sources model. The central azimuth DOAs and central elevation DOAs are estimated by sequential one-dimensional searching (SOS) algorithm. Two-dimensional search is simplified as one-dimensional one. However, it can not estimate the azimuth angular spreads and elevation angular spreads parameters. In addition, the algorithms listed above are designed only for 2D domain CD sources. Additionally, for 2D domain CD sources, an improved source localization method based quadric rotational invariance property (QRIP) method has been developed in our recent work [19]. In this work, only 2D domain ID sources model is considered.

In this study, we develop a new low-complexity parameters estimation algorithm for 2D domain ID sources using a pair of uniform circular arrays (UCAs). Based on the eigenstructure between the steering matrix and signal subspace, an average TLS-ESPRIT algorithm is adopted to estimate the central elevation DOAs, and a new GMUSIC algorithm is derived to estimate the central azimuth DOAs. Utilizing the preliminary estimation



**Figure 1.** A pair of uniform circular array geometry.

results, the angular spreads parameters can be obtained by matrix transform. To estimate the four-dimensional parameters, our approach only needs one-dimensional search. Compared with the existing method, our method has lower computational cost and does not exist parameters pairing problem. In addition, it can be applied to scenarios with multiple distributed sources that may have different angular power densities.

## 2. 2D domain incoherently distributed sources signal model

Assume that the signals impinge on UCAs of  $2L$  sensors from  $q$  far-field narrowband incoherently distributed sources. The geometry structure of receiving arrays using a pair of UCAs is shown in Figure 2. The two UCAs  $X$  and  $Y$  are displaced from each other by a known distance  $d$  vertically and the origin of the coordinate system is located at the center of the array  $X$ . Each of the UCAs has  $L$  sensors. The  $L \times 1$  array receiving signal vectors of the two arrays can be modeled as

$$\mathbf{x}(t) = \sum_{i=1}^q \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \mathbf{a}(\vartheta, \varphi) s_i(\vartheta, \varphi; \mu_i, t) d\vartheta d\varphi + \mathbf{n}_x(t), \quad (1)$$

$$\mathbf{y}(t) = \sum_{i=1}^q \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \mathbf{a}(\vartheta, \varphi) e^{-j\kappa \cos \varphi} s_i(\vartheta, \varphi; \mu_i, t) d\vartheta d\varphi + \mathbf{n}_y(t), \quad (2)$$

where  $s_i(\vartheta, \varphi; \mu_i, t)$  is the complex random time-varying angular distribution of the  $i^{\text{th}}$  source; and  $\kappa = 2\pi d/\lambda$ , with  $\lambda$  being the wavelength.  $\mathbf{a}(\vartheta, \varphi)$  is the response of array to unit energy source emitting from the direction  $(\vartheta, \varphi)$ .  $\mathbf{n}_x(t)$  and  $\mathbf{n}_y(t)$  are the additive zero-mean spatially white noise in arrays  $X$  and  $Y$ . The parameter vector  $\mu_i = [\theta_i, \phi_i, \sigma_{\theta_i}, \sigma_{\phi_i}]^T$ , which characterizes the angular distribution of the  $i^{\text{th}}$  2D domain ID source, is to be estimated. The symbols  $\theta_i$ ,  $\phi_i$ ,  $\sigma_{\theta_i}$  and  $\sigma_{\phi_i}$  are the central elevation DOA, central azimuth DOA, elevation angular spread and azimuth angular spread of the  $i^{\text{th}}$  2D domain ID source, respectively. The steering vector

and matrix of UCA can be expressed as

$$\mathbf{a}(\theta_i, \phi_i) = [e^{j\eta \sin \theta_i \cos(\phi_i - \gamma_1)}, e^{j\eta \sin \theta_i \cos(\phi_i - \gamma_2)}, \dots, e^{j\eta \sin \theta_i \cos(\phi_i - \gamma_L)}]^T$$

and

$$\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1), \mathbf{a}(\theta_2, \phi_2), \dots, \mathbf{a}(\theta_q, \phi_q)],$$

respectively. Among them,  $\eta = 2\pi r/\lambda$  and  $\gamma_k = 2\pi(k-1)/L$  ( $k = 1, 2, \dots, L$ ), with  $r$  being the radius of UCA and  $T$  being a transposition operator.  $\mathbf{n}_x$  and  $\mathbf{n}_y$  are  $L \times 1$  additive Gaussian vectors with zero-mean and covariance matrix  $E[\mathbf{n}_x \mathbf{n}_x^H] = E[\mathbf{n}_y \mathbf{n}_y^H] = \sigma_n^2 \mathbf{I}_L$ , where  $\sigma_n^2$  is noise variance and  $E\{\}$  is expectation operator. The integer limits for  $\varphi$  and  $\vartheta$  are  $0 \leq \varphi \leq 2\pi$  and  $-\pi/2 \leq \vartheta \leq \pi/2$ , respectively.

For 2D domain ID source, the angular cross-correlation kernel function can be defined as [14]

$$p_{ij}(\vartheta, \varphi, \vartheta', \varphi'; \mu_i, \mu_j) = E\{s_i(\vartheta, \varphi; \mu_i, t) s_j^*(\vartheta', \varphi'; \mu_j, t)\}, \tag{3}$$

since the components arriving from different directions in the same 2D domain ID source are uncorrelated, and we also assume that all distributed sources are mutual uncorrelated yields

$$p_{ij}(\vartheta, \varphi, \vartheta', \varphi'; \mu_i, \mu_j) = \sigma_{s_i}^2 \rho_i(\vartheta, \varphi; \mu_i) \delta(\theta - \theta') \delta(\varphi - \varphi') \delta_{ij}, \tag{4}$$

in which  $\sigma_{s_i}^2$  is the power of the  $i^{\text{th}}$  source,  $\rho_i(\vartheta, \varphi; \mu_i)$  is deterministic angular power density and  $\delta(\cdot)$  is Dirac function. It is reasonable to assume that it is a unimodal symmetrical function with respect to  $(\theta_i, \phi_i)$ . The index  $i$  shows that different sources may have different deterministic angular power density functions. It satisfies the relationship as

$$\int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \rho_i(\vartheta, \varphi; \mu_i) d\vartheta d\varphi = 1, \quad i = 1, 2, \dots, q. \tag{5}$$

Assuming that  $\vartheta$  and  $\varphi$  are independent each other, i.e., it follows  $\rho_i(\vartheta, \varphi; \mu_i) = f_\vartheta(\vartheta; \mu_i) f_\varphi(\varphi; \mu_i)$ , in which  $f_\vartheta(\vartheta; \mu_i)$  and  $f_\varphi(\varphi; \mu_i)$  are the probability density function of  $\vartheta$  and  $\varphi$ , respectively. Using the first-order and second-order central moment information of the angular power density, the central DOA  $\theta_i, \phi_i$  and angular spreads  $\sigma_{\theta_i}$  and  $\sigma_{\phi_i}$  in vector  $\mu_i$  can be defined as [13]

$$\theta_i = M_{\theta_i}^1 \triangleq \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \vartheta \rho_i(\vartheta, \varphi; \mu_i) d\vartheta d\varphi, \tag{6}$$

$$\phi_i = M_{\phi_i}^1 \triangleq \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \varphi \rho_i(\vartheta, \varphi; \mu_i) d\vartheta d\varphi, \tag{7}$$

$$\sigma_{\theta_i}^2 = M_{\theta_i}^2 \triangleq \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} (\vartheta - \theta_i)^2 \rho_i(\vartheta, \varphi; \mu_i) d\vartheta d\varphi, \tag{8}$$

$$\sigma_{\phi_i}^2 = M_{\phi_i}^2 \triangleq \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} (\varphi - \phi_i)^2 \rho_i(\vartheta, \varphi; \mu_i) d\vartheta d\varphi. \tag{9}$$

Assuming that the sources and the noises are uncorrelated, so the covariance matrix of array receiving signal vectors  $\mathbf{x}(t)$  and  $\mathbf{y}(t)$  can be given by

$$\mathbf{R}_x = \mathbf{R}_y = \sum_{i=1}^q \sigma_{s_i}^2 \mathbf{R}_i + \sigma_n^2 \mathbf{I}_L, \quad (10)$$

where

$$\mathbf{R}_i = \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \rho_i(\vartheta, \varphi; \mu_i) \mathbf{a}(\vartheta, \varphi) \mathbf{a}^H(\vartheta, \varphi) d\vartheta d\varphi \quad (11)$$

is normalized covariance matrix for the  $i^{\text{th}}$  source. The expression of  $\mathbf{R}_i$  for different deterministic angular power density functions can be seen from Appendix 6.

### 3. A low-complexity parameters estimator for multiple 2D domain ID sources

#### 3.1. TLS-ESPRIT algorithm for central elevation DOAs estimation

In what follows, an approximate invariance property between the receiving signal vectors of  $\mathbf{x}(t)$  and  $\mathbf{y}(t)$ , which is based on first-order Taylor series expansion, is derived to estimate the central elevation DOAs

Let the centroid of  $\rho_i(\vartheta, \varphi; \mu_i)$  be  $(\theta_i, \phi_i)$ , the first-order Taylor series expansion of the steering vector  $\mathbf{a}(\vartheta, \varphi)$  around  $(\theta_i, \phi_i)$  can be expressed as

$$\mathbf{a}(\vartheta, \varphi) = \mathbf{a}(\theta_i, \phi_i) + \mathbf{a}'_{\vartheta}(\theta_i, \phi_i)(\vartheta - \theta_i) + \mathbf{a}'_{\varphi}(\theta_i, \phi_i)(\varphi - \phi_i), \quad (12)$$

where  $\mathbf{a}'_{\vartheta}(\theta_i, \phi_i)$  and  $\mathbf{a}'_{\varphi}(\theta_i, \phi_i)$  are the first-order derivative of  $\mathbf{a}(\vartheta, \varphi)$  with respect to  $\vartheta$  and  $\varphi$  around  $(\theta_i, \phi_i)$ , respectively.

Introducing the following random variables

$$\alpha_{0i} = \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} s_i(\vartheta, \varphi; \mu_i, t) d\vartheta d\varphi, \quad (13)$$

$$\alpha_{\theta i} = \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} (\vartheta - \theta_i) s_i(\vartheta, \varphi; \mu_i, t) d\vartheta d\varphi, \quad (14)$$

$$\alpha_{\phi i} = \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} (\varphi - \phi_i) s_i(\vartheta, \varphi; \mu_i, t) d\vartheta d\varphi. \quad (15)$$

Therefore, for  $q$  2D domain ID sources, equation (1) can be approximated as

$$\mathbf{x} \approx \bar{\mathbf{A}}\mathbf{s} + \mathbf{n}_x, \quad (16)$$

in which

$$\bar{\mathbf{A}} = [\bar{\mathbf{a}}(\theta_1, \phi_1), \bar{\mathbf{a}}(\theta_2, \phi_2), \dots, \bar{\mathbf{a}}(\theta_q, \phi_q)], \quad (17)$$

$$\mathbf{s} = [s_1, s_2, \dots, s_q]^T, \quad (18)$$

where

$$\bar{\mathbf{a}}(\theta_i, \phi_i) = [\mathbf{a}(\theta_i, \phi_i), \mathbf{a}'_{\vartheta}(\theta_i, \phi_i), \mathbf{a}'_{\varphi}(\theta_i, \phi_i)], \tag{19}$$

$$s_i = [\alpha_{0i}, \alpha_{\theta_i}, \alpha_{\phi_i}]^T, \tag{20}$$

If different sources are mutually uncorrelated and  $\vartheta$  and  $\varphi$  are independent each other simultaneously, one obtains

$$E \{ \alpha_{\nu i} \alpha_{\kappa j}^* \} = \begin{cases} \begin{cases} \sigma_{s_i}^2, \nu = \kappa = 0 \\ M_{\theta_i}^2, \nu = \kappa = \theta \\ M_{\phi_i}^2, \nu = \kappa = \phi \end{cases}, i = j \\ 0, \nu \neq \kappa, \quad i \neq j \end{cases} \tag{21}$$

Hence, the covariance matrix of array receiving signal vector  $\mathbf{x}(t)$  can be approximated as

$$\mathbf{R}_{ax} \approx \bar{\mathbf{A}} \mathbf{\Lambda}_s \bar{\mathbf{A}}^H + \sigma_n^2 \mathbf{I}_L, \tag{22}$$

where

$$\begin{cases} \mathbf{\Lambda}_s = \text{diag}(\mathbf{\Lambda}_{s1}, \mathbf{\Lambda}_{s2}, \dots, \mathbf{\Lambda}_{sq}) \\ \mathbf{\Lambda}_{si} = \sigma_{s_i}^2 \text{diag}(1, M_{\theta_i}^2, M_{\phi_i}^2). \end{cases} \tag{23}$$

Now, we define

$$\mathbf{b}(\vartheta, \varphi) = \mathbf{a}(\vartheta, \varphi) e^{-j\kappa \cos \vartheta}, \tag{24}$$

similar to equation (12), the first-order Taylor series expansion of  $\mathbf{b}(\vartheta, \varphi)$  around  $(\theta_i, \phi_i)$  can be given by

$$\mathbf{b}(\vartheta, \varphi) = \mathbf{b}(\theta_i, \phi_i) + \mathbf{b}'_{\vartheta}(\theta_i, \phi_i)(\vartheta - \theta_i) + \mathbf{b}'_{\varphi}(\theta_i, \phi_i)(\varphi - \phi_i). \tag{25}$$

Then the derivative of  $\mathbf{b}(\vartheta, \varphi)$  with respect to the variables  $\vartheta$  and  $\varphi$  can be written as

$$\mathbf{b}'_{\vartheta}(\theta_i, \phi_i) = \mathbf{a}'_{\vartheta}(\theta_i, \phi_i) e^{-j\kappa \cos \theta_i} + j\kappa \sin \theta_i \mathbf{a}(\theta_i, \phi_i) e^{-j\kappa \cos \theta_i}, \tag{26}$$

$$\mathbf{b}'_{\varphi}(\theta_i, \phi_i) = \mathbf{a}'_{\varphi}(\theta_i, \phi_i) e^{-j\kappa \cos \theta_i}, \tag{27}$$

Under the assumption  $d/\lambda \ll 1$ , equation(26) can be approximated as

$$\mathbf{b}'_{\vartheta}(\theta_i, \phi_i) \approx \mathbf{a}'_{\vartheta}(\theta_i, \phi_i) e^{-j\kappa \cos \theta_i}. \tag{28}$$

Let the steering matrix of array  $Y$  be  $\bar{\mathbf{B}}$  and expressed as  $\bar{\mathbf{B}} = [\bar{\mathbf{b}}_1, \bar{\mathbf{b}}_2, \dots, \bar{\mathbf{b}}_q]$ , in which

$$\bar{\mathbf{b}}_i = [\mathbf{b}(\theta_i, \phi_i), \mathbf{b}'_{\vartheta}(\theta_i, \phi_i), \mathbf{b}'_{\varphi}(\theta_i, \phi_i)]. \tag{29}$$

Hence, the receiving signal vector  $\mathbf{y}(t)$  can be approximated as

$$\mathbf{y} \approx \bar{\mathbf{B}}\mathbf{s} + \mathbf{n}_y = \bar{\mathbf{A}}\mathbf{\Phi}\mathbf{s} + \mathbf{n}_y, \tag{30}$$

where the diagonal matrix  $\mathbf{\Phi}$  has the form

$$\begin{aligned} \mathbf{\Phi} &= \text{diag}(\mathbf{\Phi}_1, \mathbf{\Phi}_2, \dots, \mathbf{\Phi}_q) \\ \mathbf{\Phi}_i &= e^{-j\kappa \cos \theta_i} \mathbf{I}_3, \end{aligned} \tag{31}$$

where  $\mathbf{I}_3$  denotes  $3 \times 3$  identity matrix. From above description, the central elevation DOAs can be given by ESPRIT-type algorithm. Now we give the central elevation DOAs estimation method using an average TLS-ESPRIT as follows.

The total receive vector of the arrays  $X$  and  $Y$  is  $\mathbf{z} = [\mathbf{x}^T, \mathbf{y}^T]^T$ . The covariance matrix of  $\mathbf{z}$  denotes by  $\mathbf{R}_z$ . Let  $2L \times 3q$  matrix  $\mathbf{E}_s$  be the signal subspace which corresponds to the  $3q$  large eigenvalues of  $\mathbf{R}_z$ . Consequently, the column space spanned by  $\mathbf{E}_s$  can equivalently given by

$$\mathbf{A} = \begin{bmatrix} \bar{\mathbf{A}} \\ \bar{\mathbf{A}}\Phi \end{bmatrix}. \quad (32)$$

So there is an  $3q \times 3q$  matrix  $\mathbf{U}$ , which satisfies  $\mathbf{A} = \mathbf{E}_s \mathbf{U}$ . Let  $\mathbf{E}_{s1}$  and  $\mathbf{E}_{s2}$  are the upper  $L \times 3q$  and lower  $L \times 3q$  matrices of  $\mathbf{E}_s$ , respectively. Therefore, we have

$$\bar{\mathbf{A}} = \mathbf{E}_{s1} \mathbf{U}, \quad (33)$$

$$\bar{\mathbf{A}}\Phi = \mathbf{E}_{s2} \mathbf{U}. \quad (34)$$

Combining equations (33) and (34), one obtains

$$\mathbf{E}_{s2} = \mathbf{E}_{s1} \mathbf{U} \Phi \mathbf{U}^{-1}. \quad (35)$$

Based on equations (31) and (35), the central elevation DOAs  $\theta_i$  can be given by an average TLS-ESPRIT algorithm as

$$\hat{\theta}_i = \frac{1}{3} \sum_{k=1}^3 \arccos \left[ -\frac{\arg(l_{(i-1)q+k})}{\kappa} \right], \quad (36)$$

in which  $l_j$  is the  $j$ th eigenvalue of the matrix  $\Xi = \mathbf{U} \Phi \mathbf{U}^{-1}$ , ( $j = 1, \dots, 3q$ ). Noted that equation (36) is not a conventional TLS-ESPRIT but an average of three TLS-ESPRIT solvers.

### 3.2. A new generalized MUSIC (GMUSIC) for central azimuth DOAs estimation

Next, based on the conventional MUSIC algorithm, we derive a new generalized MUSIC algorithm (GMUSIC) to estimate the central azimuth DOAs. For a single 2D ID source, the normalized covariance matrix is a full rank matrix even in noise free environment its noise subspace is generally degenerate [8]. However, for several cases of practical interest, most of the energy of the signal is concentrated in a few eigenvalues of the covariance matrix. Assuming that the number of those eigenvalues is  $q_e$ , which is referred to as the effective dimensional of the signal subspace and it corresponds to the effective signal subspace  $\mathbf{E}_{qs1}$ . From equation (22), we know that the space spanned by the column of  $\bar{\mathbf{A}}$  is equivalent with the one spanned by  $\mathbf{E}_{qs1}$ . So we can construct a new one-dimensional generalized MUSIC (GMUSIC) spectrum based on Frobenius-norm minimization to obtain the central azimuth DOAs estimation.

Let  $\mathbf{E}_{qn1}$  be a  $(L - q_e)$  matrix whose columns are the eigenvectors of covariance matrix  $\mathbf{R}_{ax}$  corresponding to the smallest eigenvalues. After obtaining the central elevation DOAs estimation, we can make use of orthogonal principle between steering response matrix  $\bar{\mathbf{A}}$  and noise subspace  $\mathbf{E}_{qn1}$  to estimate the central azimuth DOAs such that

$$\hat{\phi}_i = \arg \min_{\hat{\theta}_i, \phi} \left\| \bar{\mathbf{a}}^H(\hat{\theta}_i, \phi) \hat{\mathbf{E}}_{qn1} \hat{\mathbf{E}}_{qn1}^H \bar{\mathbf{a}}(\hat{\theta}_i, \phi) \right\|_{\mathbf{F}}^2. \quad (37)$$

The corresponding generalized MUSIC (GMUSIC) spectrum can be given by

$$P(\hat{\phi}_i) = \frac{1}{\left\| \bar{\mathbf{a}}^H(\hat{\theta}_i, \phi) \hat{\mathbf{E}}_{qn1} \hat{\mathbf{E}}_{qn1}^H \bar{\mathbf{a}}(\hat{\theta}_i, \phi) \right\|_F^2}. \quad (38)$$

The spectrum peaks of  $P(\hat{\phi}_i)$  give the locations of the central azimuth DOAs estimation. It can transform two-dimensional search to one-dimensional one successfully. It is noticed that the new GMUSIC spectrum has remarkable differences with that of conventional MUSIC. The expression of  $\bar{\mathbf{a}}(\theta_i, \phi_i)$  in equation(38) is  $\bar{\mathbf{a}}(\theta_i, \phi_i) = [\mathbf{a}(\theta_i, \phi_i), \mathbf{a}'_{\vartheta}(\theta_i, \phi_i), \mathbf{a}'_{\varphi}(\theta_i, \phi_i)]$ , which is a  $L \times 3$  matrix. The conventional MUSIC makes use of the orthogonality between the first column vector of  $\bar{\mathbf{a}}(\theta_i, \phi_i)$  and noise subspace to construct spectrum. Hence, for 2D domain ID sources, GMUSIC can give more precise estimation results compared with MUSIC. This will be verified with simulation result.

### 3.3. The elevation angular spreads and azimuth angular spreads estimation using matrix transform

From equations (22) and (23), since the diagonal elements of matrix  $\mathbf{\Lambda}_s$  include the second central moment information  $M_{\theta_i}^2$  and  $M_{\phi_i}^2$  of  $\rho_i(\vartheta, \varphi; \mu_i)$ , they can be used to estimate the angular spread parameters  $\sigma_{\theta_i}$  and  $\sigma_{\phi_i}$  by simple matrix transform as

$$\hat{\mathbf{\Lambda}}_s = \bar{\mathbf{A}}^\dagger(\hat{\theta}_i, \hat{\phi}_i) (\mathbf{R}_{ax} - \hat{\sigma}_n^2 \mathbf{I}_q) (\bar{\mathbf{A}}^H(\hat{\theta}_i, \hat{\phi}_i))^\dagger, \quad (39)$$

where  $\hat{\sigma}_n^2$  is the estimated noise power, which can be approximated by the average of the  $L - q_e$  smallest eigenvalues of covariance matrix and the signal powers  $\sigma_s^2$  are often known. The relationships between  $\hat{\mathbf{\Lambda}}_s$  and  $\sigma_{\theta_i}, \sigma_{\phi_i}$  are

$$\begin{cases} \hat{\sigma}_{\theta_i}^2 = \hat{M}_{\theta_i}^2 = \hat{\mathbf{\Lambda}}_s(iq - 1, iq - 1) / \sigma_{s_i}^2 \\ \hat{\sigma}_{\phi_i}^2 = \hat{M}_{\phi_i}^2 = \hat{\mathbf{\Lambda}}_s(iq, iq) / \sigma_{s_i}^2. \end{cases} \quad (40)$$

From the above steps, we find that for each central elevation DOA estimation  $\hat{\theta}_i$ , the proposed GMUSIC spectrum can give the corresponding central azimuth DOA estimation  $\hat{\phi}_i$ . Furthermore, the angular spreads parameters can be given by equation (40). The proposed estimator avoids parameters pairing problem successfully.

## 4. Cramér-Rao Lower Bound for 2D domain ID sources

In this section, we give the Cramér-Rao Lower Bound (CRLB) for 2D domain ID sources based on UCA model. The covariance matrix equation (10) of the receive vector can be rewritten as [20]

$$\begin{aligned} \mathbf{r} = \text{vec}(\mathbf{R}_x) &= \sum_{i=1}^q \sigma_{s_i}^2 [(\mathbf{a}_i^* \otimes \mathbf{a}_i)] \odot [\text{vec}(\mathbf{B}_i)] + \text{vec}(\mathbf{R}_n), \\ &= [(\mathbf{A}^* \circ \mathbf{A}) \odot \mathbf{B}] \mathbf{p}_s + \text{vec}(\mathbf{R}_n), \end{aligned} \quad (41)$$

in which  $\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1), \mathbf{a}(\theta_2, \phi_2), \dots, \mathbf{a}(\theta_q, \phi_q)]$  and  $\mathbf{B} = [\text{vec}(\mathbf{B}_1), \dots, \text{vec}(\mathbf{B}_q)]$  are the steering matrix and vectorized angular spread matrix, respectively. Also,  $\text{vec}()$ ,  $\otimes$ ,  $\odot$  and  $\circ$  are vector operator, kronecker



product, Hadamard product and Khatri-Rao product, respectively. Here,  $\mathbf{p}_s = [\sigma_{s,1}^2, \dots, \sigma_{s,q}^2]^T$  is the signal power vector. In what follows, we define  $\psi = [\theta_1 \cdots \theta_q, \phi_1, \dots, \phi_q]^T$  and  $\Delta = [\sigma_{\theta_1} \cdots \sigma_{\theta_q}, \sigma_{\phi_1} \cdots \sigma_{\phi_q}]^T$  are the central DOAs vector and the angular spreads vector, respectively. The unknown noise power vector is defined as  $\mathbf{I}_n = [\xi_1 \cdots \xi_L]^T$ . The vector to be estimated is denoted as  $\mu = [\psi^T, \Delta^T]^T$ . And  $\beta = [\mathbf{p}_s^T, \mathbf{I}_n^T]^T$  is a nuisance vector, which is not required in the context of solving for the central DOAs and angular spreads. The overall vector can be written as  $\eta = [\mu^T, \beta^T]^T$ .

Now we derive the CRLB of the parameters estimation for multiple 2D domain ID sources. From [21] and [22], the CRLB matrix for the parameters estimation of 2D domain ID sources can be given by

$$\mathbf{M}_{\text{CRB}}(\eta) = \frac{1}{N} [\mathbf{D}^H (\mathbf{R}_x^{-T} \otimes \mathbf{R}_x) \mathbf{D}]^{-1}, \quad (42)$$

where  $N$  is the number of snapshot,  $\mathbf{D}$  is  $\partial \mathbf{r} / \partial \eta^T$ . According to block matrix inversion lemma, the CRLB matrix of the parameters vector to be estimated can be given by

$$\mathbf{M}_{\text{CRB}}(\mu) = \frac{1}{N} [\bar{\mathbf{C}}_0^H \mathbf{\Pi}_{\bar{\mathbf{C}}_1}^\perp \bar{\mathbf{C}}_0]^{-1}, \quad (43)$$

in which

$$\bar{\mathbf{C}}_0 = \mathbf{\Lambda}^{1/2} \mathbf{C}_0, \quad (44)$$

$$\bar{\mathbf{C}}_1 = \mathbf{\Lambda}^{1/2} \mathbf{C}_1, \quad (45)$$

$$\mathbf{\Pi}_{\bar{\mathbf{C}}_1}^\perp = \mathbf{I} - \bar{\mathbf{C}}_1 (\bar{\mathbf{C}}_1^H \bar{\mathbf{C}}_1)^{-1} \bar{\mathbf{C}}_1^H, \quad (46)$$

and in which  $\mathbf{C}_0$  and  $\mathbf{C}_1$  are defined as

$$\mathbf{C}_0 = [\partial \mathbf{r} / \partial \psi^T, \partial \mathbf{r} / \partial \Delta^T], \quad (47)$$

$$\mathbf{C}_1 = \partial \mathbf{r} / \partial \mathbf{p}_s^T. \quad (48)$$

The matrix  $\mathbf{\Lambda}$  can be expressed as

$$\mathbf{\Lambda} = (\mathbf{R}_x^{-T} \otimes \mathbf{R}_x^{-1}) [\mathbf{I} - \mathbf{\Pi}], \quad (49)$$

where

$$\mathbf{\Pi} = \dot{\mathbf{R}}_n \left( \dot{\mathbf{R}}_n^H (\mathbf{R}_x^{-T} \otimes \mathbf{R}_x^{-1}) \dot{\mathbf{R}}_n \right)^{-1} \dot{\mathbf{R}}_n^H (\mathbf{R}_x^{-T} \otimes \mathbf{R}_x^{-1}), \quad (50)$$

and

$$\dot{\mathbf{R}}_n = \partial \mathbf{r} / \partial \mathbf{I}_n^T = \partial \text{vec}(\mathbf{R}_n) / \partial \mathbf{I}_n^T. \quad (51)$$

The concrete expression of  $\mathbf{C}_0$ ,  $\mathbf{C}_1$  and  $\mathbf{\Lambda}$  can be seen from Appendix 6.

## 5. Simulation results

In this section, the performance of our algorithm is investigated through some simulation experiments. Assume that each of the uniform circular arrays (UCA) has  $L = 16$  sensors. The distance between the two UCAs is  $d = 0.1\lambda$  and the signal-to-noise-ratio (SNR) is defined as  $\text{SNR} = 10 \log_{10} (\sigma_s^2 / \sigma_n^2)$ , the snapshot number  $N$  is 500.

In the first experiment, we consider that two equipower uncorrelated narrowband ID sources with Gaussian shape deterministic angular power density functions both in elevation and azimuth and their parameters are

$$\mu_1 = (30^\circ, 2^\circ, 50^\circ, 3^\circ);$$

$$\mu_2 = (50^\circ, 4^\circ, 70^\circ, 5^\circ).$$

Next, we compare our GMUSIC algorithm with the conventional MUSIC for estimating the central azimuth DOA. Firstly, we use the average TLS-ESPRIT solver of equation (36) to estimate the central elevation DOA with SNRs are 5 dB and 15 dB, respectively. Based on the preliminary estimation of  $\hat{\theta}_i$ , GMUSIC and MUSIC algorithms are used to search the central azimuth DOA, respectively. Figure 2 shows the central azimuth DOA estimation results. GMUSIC algorithm can give more precise estimation whenever the SNR is 5 dB or 15 dB compared with MUSIC. And it has more acute spectrum than MUSIC. Since MUSIC algorithm does not take the distribution character of distributed source into consideration. As a result, the MUSIC spectrum gives the wrong central azimuth DOA estimation results whenever the SNR is low or high.

Figure 3 shows the central elevation DOAs and central azimuth DOAs estimation results with 50 runs. Similarly, the elevation angular spread and azimuth angular spread estimation results are depicted in Fig. 4. From the Figure 3 and 4, it is observed that our estimator can give more precise parameters estimation results and it does not need parameters pairing. As expected, the parameters estimation error becomes smaller with increasing SNR.

Towards this end, the performance of the proposed estimators is studied. The SNRs vary from 5 dB to 40 dB in each figure and 200 Monte Carlo simulations were run to calculate the root-mean-square error (RMSE) of the estimators. Figure 5 displays the RMSE and CRLB of the central azimuth DOA  $\phi$ . Figure 6 considers the RMSE and CRLB of the elevation angular spread  $\sigma_\theta$ . Similarly, the RMSE and CRLB of the azimuth angular spread  $\sigma_\phi$  are plotted in Figure 7. From the inspection of these figures, it can be observed that our proposed estimator is an efficient estimator for 2D domain ID sources, as predicted by the theory. For low SNR, the RMSE departs slightly from the CRLB, but for high SNR, the RMSE draws close to the CRLB. This can be explained intuitively as follows. Since our method is derived from the first-order Taylor series expansion of steering vector, the noise level will bring influence in this approximation. As a result, the estimation precision will degenerate with reduction, and improve with increasing SNR.

In the second experiment, we validate the robustness of our estimator. Here, we consider the influence of different deterministic angular power density functions to our method. Assume that three signals impinge on the arrays. The first one has Gaussian deterministic angular power density function, the second and the last one have Uniform and Laplace deterministic angular power density functions, respectively. The parameters are:

$$\mu_1 = (30^\circ, 2^\circ, 50^\circ, 3^\circ),$$

$$\mu_2 = (35^\circ, 4^\circ, 40^\circ, 3^\circ),$$

$$\mu_3 = (45^\circ, 3^\circ, 50^\circ, 2^\circ).$$

The central elevation DOAs and central azimuth DOAs estimation results via different SNRs are plotted in Figure 8 and the elevation angular spreads and azimuth angular spreads estimation results are shown in Figure 9. As expected, although for the case of different sources with different deterministic angular power density functions, the proposed estimator can give the more precise parameters estimation results, which can not be done by other previous algorithm (for example, DSPE, DISPARE, COMET, etc.).

We use the CPU time as a measure of complexity. Although it is not an exact measure, it gives a rough estimation of the complexity, for comparing the classical 2D searching algorithm (2D MUSIC [23]) and our algorithms. Our simulations are performed in MATLAB7 environment using an AMD Athlon sempron 2400+, 1.67 GHz processor with 512 MB of memory, and under Microsoft Windows XP operating system. The searching grid intervals in azimuth angle and elevation angle are set to 1° for compared methods. For 500 independent trials, the total run time of our method and 2D searching method are 101.55 s and 2914.1 s, respectively. It shows that our method is faster than classical 2D searching method. Additionally, we also shown the performance of method for SNR of 0 dB case in Table. 1.

## 6. Conclusion

Herein, a low-complexity parameters estimator for multiple 2D domain incoherently distributed (ID) sources is proposed. The proposed method can estimate four-dimensional parameters with only one-dimensional search.

Firstly, based on the approximate rotational invariant property between the steering vector and signal subspace, a average TLS-ESPRIT is used to estimate the central elevation DOAs. Then, the central azimuth DOAs can be found from the peaks of the reconstructed one-dimensional GMUSIC spectrum. Finally, the elevation angular spreads and azimuth angular spreads are estimated by matrix transform. The proposed estimator is a low-complexity parameters estimation technique for 2D domain ID sources. It avoids parameters pairing problem and it can be applied to the situation that different 2D domain ID sources have different deterministic angular power density functions.

**Table 1.** Average RMSE of parameters estimation under SNR of 0 dB

SNR(dB)	var ( $\theta - \hat{\theta}$ )	var ( $\phi - \hat{\phi}$ )	var ( $\sigma_\theta - \hat{\sigma}_\theta$ )	var ( $\sigma_\phi - \hat{\sigma}_\phi$ )
0	1.0651	1.2170	1.9203	2.0421

## APPENDIX A

### Approximated expression of the normalized covariance matrix for 2D domain ID source

From equation (12), we have

$$\mathbf{R}_i = \int_{-\pi/2}^{\pi/2} \int_0^{2\pi} \rho_i(\vartheta, \varphi; \mu_i) \mathbf{a}(\vartheta, \varphi) \mathbf{a}^H(\vartheta, \varphi) d\vartheta d\varphi. \tag{52}$$

For any DOA  $(\vartheta, \varphi)$ , we have  $\vartheta = \theta_i + \tilde{\vartheta}$  and  $\varphi = \phi_i + \tilde{\varphi}$ , where  $\tilde{\vartheta}$  and  $\tilde{\varphi}$  are random angular deviations. We introduce the following variables:

$$\begin{cases} m_{kl}^i = \cos(\phi_i - \gamma_k) - \cos(\phi_i - \gamma_l) \\ n_{kl}^i = \sin(\phi_i - \gamma_k) - \sin(\phi_i - \gamma_l) \end{cases} \tag{53}$$

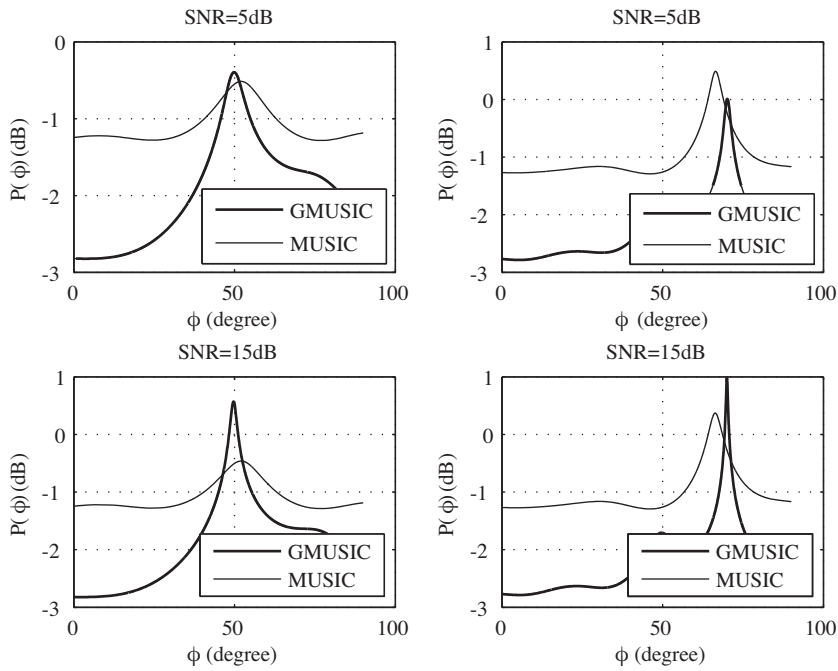


Figure 2. The central azimuth DOAs estimation with GMUSIC and MUSIC in 5 dB and 15 dB.

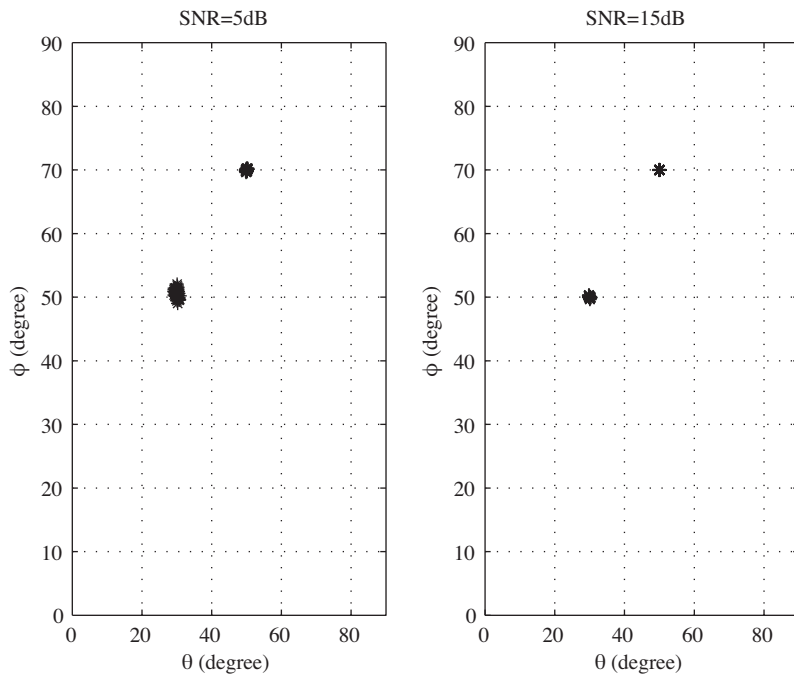
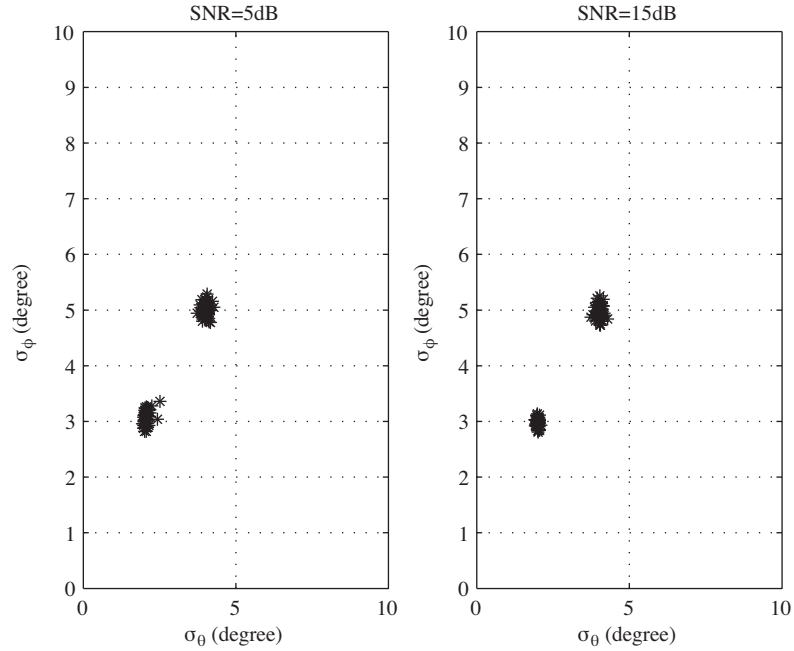


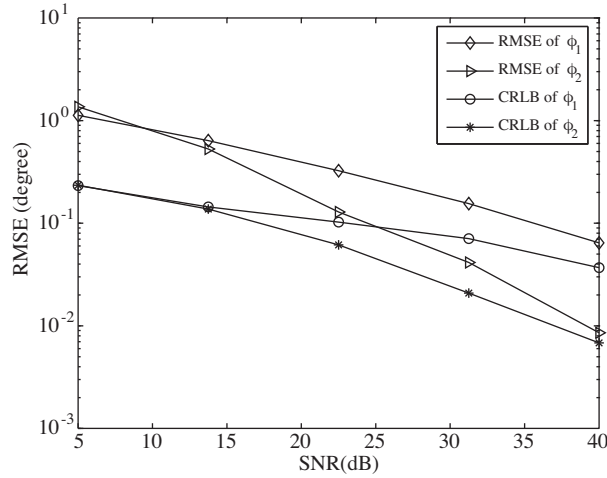
Figure 3. The central elevation DOAs and central azimuth DOAs estimation results with 5 dB and 15 dB.

Therefore, one obtains

$$\mathbf{a}(\vartheta, \varphi) \mathbf{a}^H(\vartheta, \varphi) \approx e^{j\eta \sin \theta_i m_{kl}^i} e^{j\eta (\cos \theta_i m_{kl}^i \tilde{\vartheta} - \sin \theta_i n_{kl}^i \tilde{\varphi})}, \quad (54)$$



**Figure 4.** The elevation angular spreads and azimuth angular spreads estimation results with 5 dB and 15 dB.



**Figure 5.** CRLB and RMSE of the central azimuth DOA estimation versus SNR,  $N=500$ .

where we use the facts that  $\cos \tilde{\theta} \approx 1$ ,  $\sin \tilde{\theta} \approx \tilde{\theta}$ ,  $\cos \tilde{\varphi} \approx 1$ ,  $\sin \tilde{\varphi} \approx \tilde{\varphi}$  and  $\tilde{\theta}\tilde{\varphi} \approx 0$  for small angular spreads. Now we consider three typical deterministic angular power density functions  $\rho_i(\vartheta, \varphi; \mu_i)$ . They are Gaussian, Uniform and Laplace shapes

$$\rho_i(\vartheta, \varphi; \mu_i) = \frac{1}{2\pi\sigma_{\theta_i}\sigma_{\phi_i}} e^{-1/2((\vartheta-\theta_i)^2/\sigma_{\theta_i}^2 + (\varphi-\phi_i)^2/\sigma_{\phi_i}^2)}, \quad (55)$$

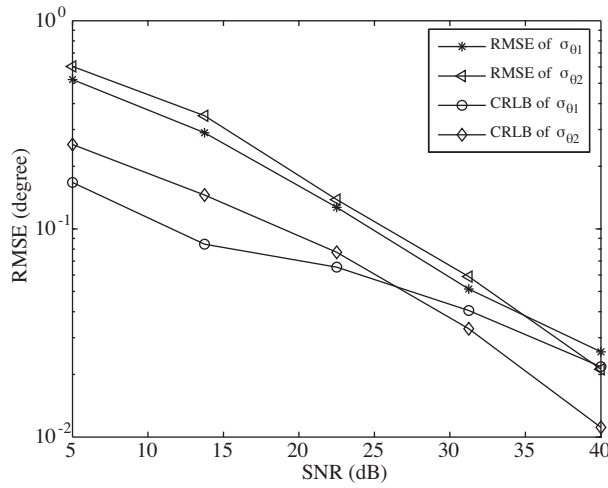


Figure 6. CRLB and RMSE of the elevation angular spread estimation versus SNR, N=500.

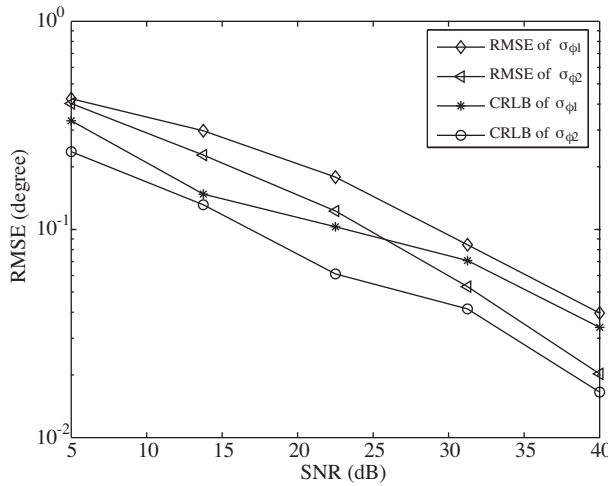


Figure 7. CRLB and RMSE of the central azimuth DOA estimation versus SNR, N=500.

$$\rho_i(\vartheta, \varphi; \mu_i) = \begin{cases} \frac{1}{2\sqrt{3}} |\vartheta - \theta_i| < \sqrt{3}\sigma_{\theta_i} \\ \frac{1}{2\sqrt{3}} |\varphi - \phi_i| < \sqrt{3}\sigma_{\phi_i} \\ \text{otherwise} \end{cases} \quad (56)$$

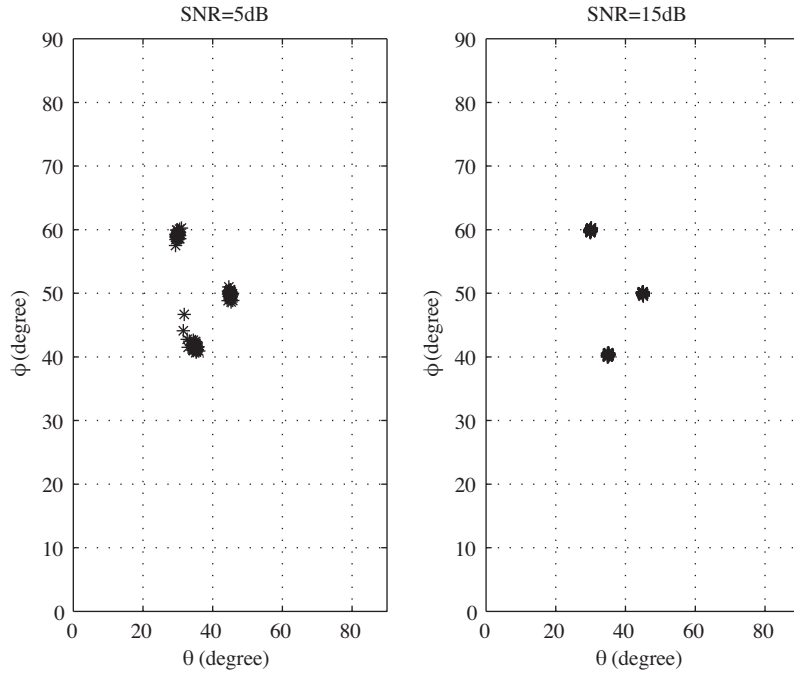
$$\rho_i(\vartheta, \varphi; \mu_i) = \frac{1}{2\sigma_{\theta_i}\sigma_{\phi_i}} e^{-(\sqrt{2}|\vartheta - \theta_i|/\sigma_{\theta_i} + \sqrt{2}|\varphi - \phi_i|/\sigma_{\phi_i})}, \quad (57)$$

respectively. Inserting equations (55), (56), (57) and (54) into equation (52), The different normalized covariance matrix expressions can be given by

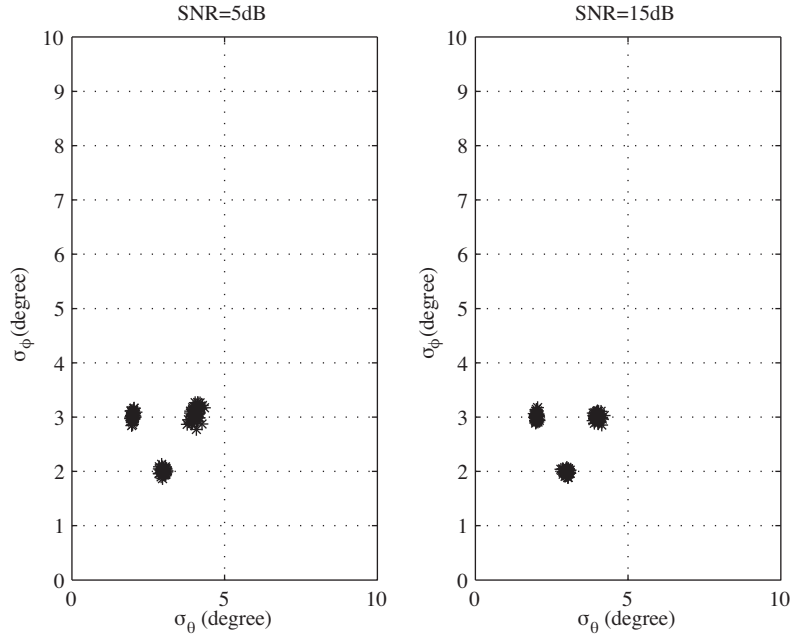
$$[\mathbf{R}_i^G]_{kl} \approx e^{j\eta \sin \theta_i m_{kl}^i} [\mathbf{B}_i^G]_{kl}, \quad (58)$$

$$[\mathbf{R}_i^U]_{kl} \approx e^{j\eta \sin \theta_i m_{kl}^i} [\mathbf{B}_i^U]_{kl}, \quad (59)$$

$$[\mathbf{R}_i^L]_{kl} \approx e^{j\eta \sin \theta_i m_{kl}^i} [\mathbf{B}_i^L]_{kl}. \quad (60)$$



**Figure 8.** The central elevation DOAs estimation and central azimuth DOAs results in different SNRs (50 runs).



**Figure 9.** The elevation angular spreads and azimuth angular spreads estimation results in different SNRs (50 runs).

For above three different cases,  $\mathbf{R}_i$  can be written as

$$\mathbf{R}_i = (\mathbf{a}_i \mathbf{a}_i^H) \odot \mathbf{B}_i, \quad (61)$$

where  $i$  denotes the  $i^{\text{th}}$  2D domain ID source and  $\mathbf{B}_i$  for the different cases are Gaussian:

$$[\mathbf{B}_i^G]_{kl} = e^{-\frac{\eta^2((\sigma_{\theta_i} \cos \theta_i m_{kl}^i)^2 + (\sigma_{\phi_i} \sin \theta_i n_{kl}^i)^2)}{2}}; \quad (62)$$

Uniform:

$$[\mathbf{B}_i^U]_{kl} = \frac{\sin(\sqrt{3}\eta \sin \theta_i n_{kl}^i \sigma_{\theta_i}) \sin(\sqrt{3}\eta \cos \theta_i m_{kl}^i \sigma_{\phi_i})}{\sqrt{3}\eta \sin \theta_i n_{kl}^i \sigma_{\theta_i} \sqrt{3}\eta \cos \theta_i m_{kl}^i \sigma_{\phi_i}}; \quad (63)$$

and Laplacian:

$$[\mathbf{B}_i^L]_{kl} = \left( \frac{1}{1 + (\eta \sin \theta_i n_{kl}^i \sigma_{\phi_i})^2} \right) \left( \frac{1}{1 + (\eta \cos \theta_i m_{kl}^i \sigma_{\theta_i})^2} \right). \quad (64)$$

## APPENDIX B

### Cramér-Rao Lower Bound for 2D domain ID sources

In spatial uniform white Gaussian environment, the noise covariance matrix can be expressed as

$$\mathbf{R}_n = \sigma_n^2 \mathbf{I}_L. \quad (65)$$

Accordingly, the derivative of the noise covariance matrix with respect to (w.r.t) the noise variance vector  $\mathbf{I}_n = [\xi_1, \dots, \xi_L]^T = \sigma_n^2 \mathbf{1}^T$  ( $\mathbf{1}^T = [1, \dots, 1]^T$  is a  $L \times 1$  column vector) can be written as

$$\dot{\mathbf{R}}_n = \text{vec}(\mathbf{I}_L). \quad (66)$$

Inserting equation (66) into equations (50) and (49), the matrix  $\mathbf{A}$  is

$$\mathbf{A} = \mathbf{R}_x^{-T} \otimes \mathbf{R}_x^{-1} - \text{tr}^{-1}(\mathbf{R}_x^{-2}) \text{vec}(\mathbf{R}_x^{-2}) \text{vec}^H(\mathbf{R}_x^{-2}), \quad (67)$$

where  $\text{tr}(\cdot)$  is the trace operator. From above description, we know that the vector  $\mathbf{r}$  can be rewritten as

$$\begin{aligned} \mathbf{r} = \text{vec}(\mathbf{R}_x) &= \sum_{i=1}^q \sigma_{s,i}^2 [(\mathbf{a}_i^* \otimes \mathbf{a}_i)] \odot [\text{vec}(\mathbf{B}_i)] + \text{vec}(\mathbf{R}_n), \\ &= [(\mathbf{A}^* \circ \mathbf{A}) \odot \mathbf{B}] \mathbf{p}_s + \text{vec}(\mathbf{R}_n) \end{aligned} \quad (68)$$

where  $\mathbf{A} = [\mathbf{a}(\theta_1, \phi_1), \mathbf{a}(\theta_2, \phi_2), \dots, \mathbf{a}(\theta_q, \phi_q)] = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_q]$ . So the derivatives of  $\mathbf{r}$  with respect to (w.r.t) the elevation central DOA vector  $\theta = [\theta_1, \dots, \theta_q]^T$ , the azimuth central DOA vector  $\phi = [\phi_1, \dots, \phi_q]^T$ , elevation angular spread vector  $\sigma_\theta = [\sigma_{\theta_1}, \dots, \sigma_{\theta_q}]^T$  and azimuth spread vector  $\sigma_\phi = [\sigma_{\phi_1}, \dots, \sigma_{\phi_q}]^T$  are, respectively,

$$\partial \mathbf{r} / \partial \theta^T = (\mathbf{A}_\theta^* \circ \mathbf{A} + \mathbf{A}^* \circ \mathbf{A}_\theta) \odot (\mathbf{B}\mathbf{P}) + (\mathbf{A}^* \circ \mathbf{A}) \odot (\mathbf{B}_\theta \mathbf{P}), \quad (69)$$

$$\partial \mathbf{r} / \partial \phi^T = (\mathbf{A}_\phi^* \circ \mathbf{A} + \mathbf{A}^* \circ \mathbf{A}_\phi) \odot (\mathbf{B}\mathbf{P}) + (\mathbf{A}^* \circ \mathbf{A}) \odot (\mathbf{B}_\phi \mathbf{P}), \quad (70)$$

$$\partial \mathbf{r} / \partial \sigma_\theta^T = (\mathbf{A}^* \circ \mathbf{A}) \odot (\mathbf{B}_{\sigma_\theta} \mathbf{P}), \quad (71)$$

$$\partial \mathbf{r} / \partial \sigma_\phi^T = (\mathbf{A}^* \circ \mathbf{A}) \odot (\mathbf{B}_{\sigma_\phi} \mathbf{P}), \quad (72)$$



where the matrices  $\mathbf{A}_\theta$ ,  $\mathbf{A}_\phi$ ,  $\mathbf{B}_\theta$ ,  $\mathbf{B}_\phi$ ,  $\mathbf{B}_{\sigma_\theta}$ ,  $\mathbf{B}_{\sigma_\phi}$  and  $\mathbf{P}$  are

$$\mathbf{A}_\theta = [\partial \mathbf{a}_1 / \partial \theta_1, \dots, \partial \mathbf{a}_q / \partial \theta_q], \quad (73)$$

$$\mathbf{A}_\phi = [\partial \mathbf{a}_1 / \partial \phi_1, \dots, \partial \mathbf{a}_q / \partial \phi_q], \quad (74)$$

$$\mathbf{B}_\theta = [\partial \text{vec}(\mathbf{B}_1) / \partial \theta_1, \dots, \partial \text{vec}(\mathbf{B}_q) / \partial \theta_q], \quad (75)$$

$$\mathbf{B}_\phi = [\partial \text{vec}(\mathbf{B}_1) / \partial \phi_1, \dots, \partial \text{vec}(\mathbf{B}_q) / \partial \phi_q], \quad (76)$$

$$\mathbf{B}_{\sigma_\theta} = [\partial \text{vec}(\mathbf{B}_1) / \partial \sigma_{\theta_1}, \dots, \partial \text{vec}(\mathbf{B}_q) / \partial \sigma_{\theta_q}], \quad (77)$$

$$\mathbf{B}_{\sigma_\phi} = [\partial \text{vec}(\mathbf{B}_1) / \partial \sigma_{\phi_1}, \dots, \partial \text{vec}(\mathbf{B}_q) / \partial \sigma_{\phi_q}], \quad (78)$$

$$\mathbf{P} = \text{diag}(\mathbf{p}_s). \quad (79)$$

Observing that  $\boldsymbol{\psi} = [\theta_1 \cdots \theta_q, \phi_1, \dots, \phi_q]^T$  and  $\boldsymbol{\Delta} = [\sigma_{\theta_1} \cdots \sigma_{\theta_q}, \sigma_{\phi_1} \cdots \sigma_{\phi_q}]^T$ , it follows that

$$\partial \mathbf{r} / \partial \boldsymbol{\psi}^T = [\partial \mathbf{r} / \partial \boldsymbol{\theta}^T, \partial \mathbf{r} / \partial \boldsymbol{\phi}^T], \quad (80)$$

$$\partial \mathbf{r} / \partial \boldsymbol{\Delta}^T = [\partial \mathbf{r} / \partial \boldsymbol{\sigma}_\theta^T, \partial \mathbf{r} / \partial \boldsymbol{\sigma}_\phi^T], \quad (81)$$

Inserting equations (80) and (81) into equation (49), we can get the expression of  $\mathbf{C}_0$

$$\mathbf{C}_0 = [\partial \mathbf{r} / \partial \boldsymbol{\psi}^T, \partial \mathbf{r} / \partial \boldsymbol{\Delta}^T]. \quad (82)$$

Obviously, the matrix  $\mathbf{C}_1$  can be directly given by

$$\mathbf{C}_1 = \partial \mathbf{r} / \partial \mathbf{p}_s^T = (\mathbf{A}^* \circ \mathbf{A}) \odot \mathbf{B}. \quad (83)$$

Inserting equations (67), (81) and (82) into equations (44), (45) and (46). The CRLB matrix can be obtained from equation (43).

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