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Semi-active H$_\infty$ robust control of six degree of freedom structural system using MR damper

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Abstract

In this study, H_∞ robust control is designed for vibration attenuation of six DOF structure using MR damper. A semi-active seismically excited, nonlinear structural system is modelled. The controller is designed for a reduced order model and applied to the full order model. The controller performance is verified experimentally. It is shown that the H_∞ controller performance is satisfactory to suppress the vibration of the structure despite the presence of uncertainties.

Key Words: *MR damper, H_∞ robust control, mixed sensitivity problem, structural control, shaking table test*

1. Introduction

Significant progress has been made over the last three decades on structural protection against earthquake and strong wind loading. Many control algorithms and devices such as MR dampers have been investigated to vibration attenuation of high rise buildings [1-11]. MR dampers are nonlinear semi-active control devices that have significant potential to mitigate vibration and shocks. Because of their mechanical simplicity, high range, low power requirements, low cost, large force capacity, and robustness, these devices are suitable for various applications such as buildings [4-11], bridges[12] and suspension systems [13-15]. Although they can only remove the energy from system, recent studies have shown that the MR dampers can achieve the majority of the performance of fully active systems [7,9,16].

Nonlinearities and hysteresis dynamics in structures incorporating semi-active devices have considerable effects on the control performance. Various control algorithms used with the semi-active devices can be found in the literature such as Lyapunov method [5], clipped optimal control [4], optimal control [9,16], stochastic

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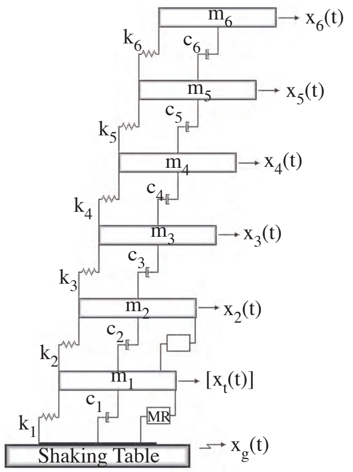

Figure 1. Six DOF structural system.

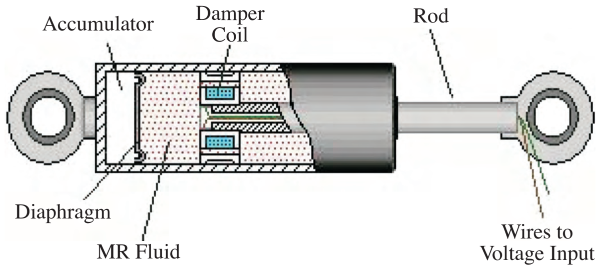

Figure 2. Schematic of MR damper.

optimal control [17], bilinear H_∞ control [18], backstepping control [19], adaptive control [11], sliding mode control [20], fuzzy logic [21], genetic algorithms [21].

In this paper an H_∞ robust controller is designed and applied experimentally for vibration attenuation of six DOF structural system using a semi-active MR damper. A structural system excited by an earthquake and controlled by using MR damper is modeled as a semi-active controlled, seismically excited and a nonlinear system with six DOF. The rest of the paper is organized as follows. In Section 2, mathematical model of a six DOF semi-active structural system model is represented. Dynamics of the MR damper is given in Section 3. Section 4 shows the general structure of the control scheme. Section 5 discusses the experimental results. Finally, Section 6 provides conclusions.

2. Mathematical model of semi-active structural system

Consider a six degree of freedom semi-active structural model depicted in Figure 1. The MR damper installed between the ground and the first floor via a rigid brace. The dynamical differential equation of motion of this system can be given as,

$$M_s\ddot{x} + C_s\dot{x} + K_s x = \Lambda_s f - M_s l\ddot{x}_g \quad (1)$$

where f is control force which is produced by the MR damper. M_s, C_s and K_s are mass, damping and stiffness matrices, respectively. $\ddot{x}_g$ is the earthquake acceleration excitation; $\ddot{x}$, $\dot{x}$ and x are the horizontal acceleration, velocity and displacement vectors, respectively. In this paper, only horizontal motion is considered, torsional and vertical motions are not considered. $\Lambda_s = [\,-1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\,]^T$ is location vector of the controller and the displacement vector is defined as $x = [\,x_1\ \ x_2\ \ x_3\ \ x_4\ \ x_5\ \ x_6\,]^T$.

3. Dynamics of MR damper

The MR damper typically consist of a hydraulic cylinder which houses the piston, the magnetic circuit, an accumulator and MR fluid containing micron-sized magnetically polarizable ferrous particles. Figure 2 shows the schematic drawing of an MR damper. The damper piston contains damper coil and annular flow channels.A series of experiment is conducted to determine the MR damper dynamic behavior at Gebze Institute of Technology (GIT) Control Technologies and Robotics Laboratory. The force-velocity and force-displacement characteristics of the MR damper due to a 0.25 Hz sine wave excitation with amplitude of 0.03 m is depicted in Figure 3. As shown in Figure 3, each plot includes five curves that correspond to the damper behavior for various input voltages applied to the MR damper is maintained at a constant level of 0, 2, 4, 6 and 8 V. The flow properties of MR fluids depend on magnetic field that is generated by input voltage in a magnetic coil installed at rod head.

4. H$_\infty$ robust control design

4.1. System definition and model reduction

In this study a modal approach is chosen to design H$_\infty$ controller. H$_\infty$ controller is designed for the reduced order model and applied to the full order model. Controller can also be designed for full order model. Nevertheless effects of lower modes are greater than higher modes in vibrations of flexible systems such as elastic rotors, beams and high rise buildings. Besides, there are some difficulties in applying control to the system using high order controller. Therefore, controlling of lower modes reduce the amplitude of structural system effectively. In this study, structural system is modeled as a full order model and the model reduction is applied to obtain a reduced order model for the control design. System dynamics that is neglected in model reduction is defined as uncertainty. Full order system model can be defined as,

$$P_f = \begin{bmatrix} A_f & B_f \\ C_f & 0 \end{bmatrix} \tag{2}$$

where

$$A_f = \begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix}, B_f = \begin{bmatrix} 0 \\ M_s^{-1}\Lambda_s \end{bmatrix}, C_f = \begin{bmatrix} C_y & 0 \end{bmatrix}, C_y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

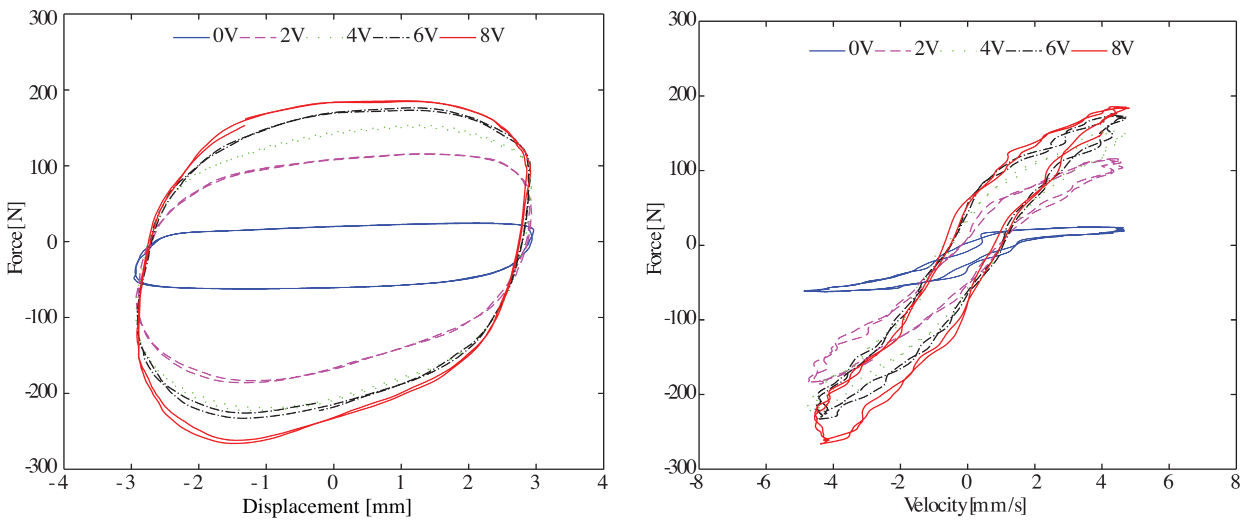

Figure 3. Characteristics of MR damper.

The transformation from physical space to modal space can be performed using

$$\eta = \begin{bmatrix} \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 & \eta_6 \end{bmatrix}^T \tag{4}$$

The equation in modal coordinates can be defined as

$$\ddot{\eta} + C_f \dot{\eta} + K_f \eta = \Lambda_f f \tag{5}$$

where $x = \phi\eta$, $\phi^T M_s \phi = I$ and $\ddot{x}_g = 0$. The matrices C_f, K_f and Λ_f are defined as

$$\begin{gathered} C_f = \phi^T C_s \phi = diag[c_{11}, c_{22}, c_{33}, c_{44}, c_{55}, c_{66}], \\ K_f = \phi^T K_s \phi = diag[\omega_{11}^2, \omega_{22}^2, \omega_{33}^2, \omega_{44}^2, \omega_{55}^2, \omega_{66}^2] \\ (\omega_{11} < \omega_{22} < \omega_{33} < \omega_{44} < \omega_{55} < \omega_{66}), \\ \Lambda_f = \phi^T \Lambda_s = [\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5, \Lambda_6]^T \end{gathered} \tag{6}$$

First two modes of the full order system are formed the reduced order model. Modal coordinate transformation vector and reduced order model is derived in the form of

$$\ddot{\eta}_r + C_r \dot{\eta}_r + K_r \eta_r = \Lambda_r f \tag{7}$$

where, $\eta_r = [\eta_1 \eta_2]^T$, $C_r = diag[c_{11}, c_{22}]$, $K_r = diag[\omega_{11}^2, \omega_{22}^2]$ and $\Lambda_f = \phi^T \Lambda_s = [\Lambda_1, \Lambda_2]^T$. Finally, reduced order model can be written in physical coordinates as follows

$$P_r = \begin{bmatrix} A_r & B_r \\ C_r & 0 \end{bmatrix} \tag{8}$$

where

$$A_r = \begin{bmatrix} 0 & I \\ -K_r & -C_r \end{bmatrix}, B_r = \begin{bmatrix} 0 \\ \Lambda_r \end{bmatrix}, C_r = \begin{bmatrix} C_r & 0 \end{bmatrix}, C_r = \begin{bmatrix} \phi_y & 0 \end{bmatrix}, \phi_y = C_y \phi_{12} \tag{9}$$

The frequency responses of the full and reduced order systems are shown in Figure 4.

4.2. Control design

Regulator problem in H_∞ control theory is defined as designing the controller that guarantees the stability of closed loop system and minimizes the H_∞ norm of closed loop system transfer function [22]. The control design structure is shown in Figure 5. First aim in design of H_∞ controller is providing robust stabilization of the feedback control system against the multiplicative uncertainty. There are two main transfer functions in this controller system. These are

$$S(s) = \frac{P_r(s)}{I - P_r(s)K(s)}, \quad T(s) = \frac{P_r(s)K(s)}{I - P_r(s)K(s)} \tag{10}$$

where $S(s)$ is the sensitivity function and $T(s)$ is the complementary sensitivity function. Assuming that $T(s)$ and $\Delta_m(s)$ are stable and provided that the upper limit of $\Delta_m(s)$ satisfies

$$|\Delta_m(j\omega)| \le |W_T(j\omega)|, \quad \Delta_m(s) = \frac{P_f(s) - P_r(s)}{P_r(s)} \tag{11}$$

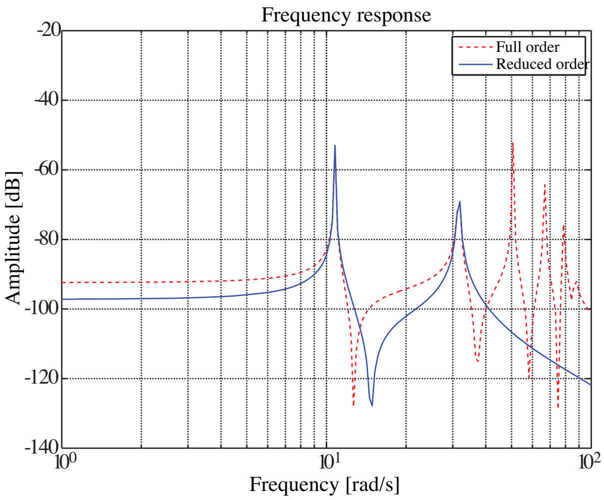

Figure 4. Frequency response of full and reduced order systems.

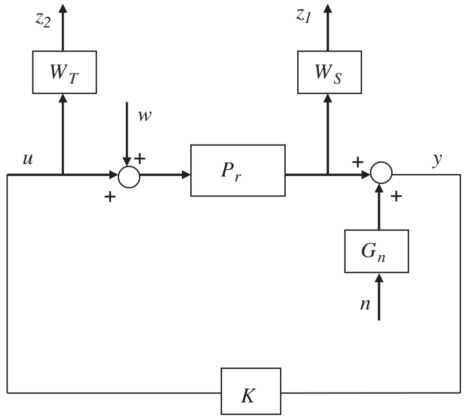

Figure 5. Block diagram of H$_\infty$ control

and the sufficient condition for the feedback system to be robustly stable against all perturbations can be written as

$$\|W_T T(s)\|_\infty < 1 \tag{12}$$

The second aim of the H$_\infty$ controller is to improve performance of the feedback control system. The problem in improving the response performance is how to attenuate the influence of the disturbance w on the output y of the plant. This issue is related to minimization of

$$\|S(s)\|_\infty = sup\overline{\sigma}[S(s)], \quad \|W_S S(s)\|_\infty < 1 \tag{13}$$

subjected to the condition of stability of the closed loop system. Here W_S is the filter for the system output. Control gain is chosen small on high frequency region for robust stability. Consequently, control system satisfies both robust stability and response performance. This type of H$_\infty$ controller is called as mixed sensitivity problem and defined as

$$\left\| \begin{bmatrix} W_S S \\ W_T T \end{bmatrix} \right\|_\infty < \gamma \tag{14}$$

where γ is a positive integer.

4.3. Frequency shape filters and controller performance

An important step in H$_\infty$ control is to decide the frequency shape filters. Multiplicative uncertainty is used for selecting W_T. Generally, filter should cover the uncertainty.

Frequency shape filters are

$$W_T = k_t\left(\frac{s^2 + 2\xi_{wn}\omega_{wn}s + \omega_{wn}^2}{s^2 + 2\xi_{wd}\omega_{wd}s + \omega_{wd}^2}\right)^2, \quad W_S = k_s \tag{15}$$

where $k_t = 0.6$, $\xi_{wn} = 0.1$, $\xi_{wd} = 0.2$, $\omega_{wn} = 32$, $\omega_{wd} = 50$ and $k_s = 850$. ω_{wn} is chosen as the frequency of the last controlled mode and ω_{wd} is chosen as the frequency of the first uncontrolled mode. The controller should

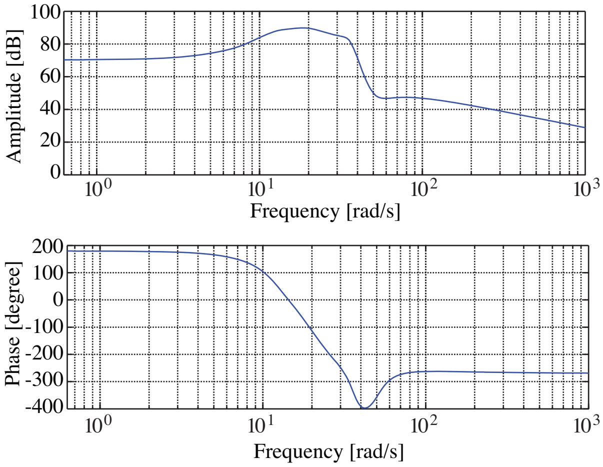

Figure 6. Frequency response of H_∞ control.

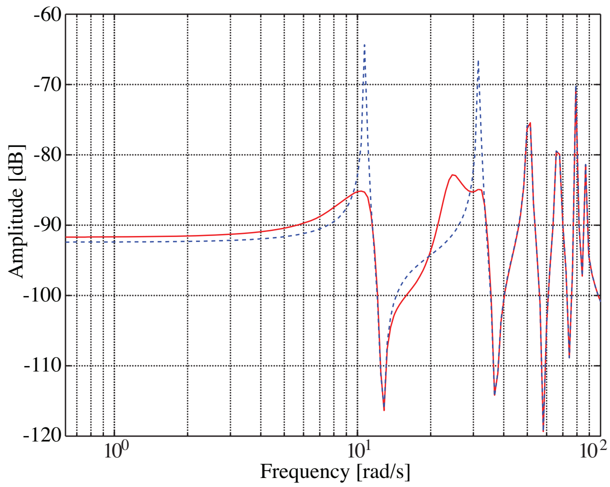

Figure 7. Open and closed loop responses of model.

suppress the first two modes without exciting the truncated modes. H_∞ control is designed using MATLAB Robust Control Toolbox [23]. Frequency response of H_∞ control is depicted in Figure 6. Open and closed loop responses of the full order model are shown in Figure 7.

Desired force can be produced by changing damper voltage. MR damper voltage is obtained as

$$\nu = V_{max} H\{(f_c - f)f\} \tag{16}$$

where V_{max} is the maximum voltage of MR damper[4], $H(.)$ is the Heaviside step function, f_c is the desired force and f is produced force by the MR damper.

5. Experimental setup and results

To demonstrate the performance of the designed H_∞ controller, shaking table tests were conducted in Yidiz Technical University Machine Theory System Dynamics and Control Laboratory. Figure 8 shows a photograph of the test structure. Structural model is 1.2 m height and 126 kg weight. Shaking table is one dimensional and is drived electromechanically. dSpace ACE Kit 1103 is used for data acquisition and implementation of the control. The MR damper installed between the ground and the floor via a rigid brace. Lord RD 1005-3 type MR damper is used as an actuator. First floor displacement is measured using Waycon SM-50 LVDT and sixth floor acceleration is measured by Bruel Kjaer 4507 B 002 piezoelectric accelerometer. Endevco Model 133 signal conditioner is placed between the accelerometer and dSpace Kit. Force produced by the MR damper is measured by Dytran 1051V3 force transducer. A schematic of experimental setup is depicted in Figure 9. The 1940 El Centro North-South component is inputted to shaking table. Feedback signal is the displacement of the first story for the control. Measurements used for comparing the control performance are first floor displacement and sixth floor acceleration.

Experimental results are compared in terms of uncontrolled and H_∞ control. Figure 10 shows the uncontrolled and controlled cases of the first floor displacement. It can be seen from the figure that the H_∞ controller is suppressed the first floor displacement. Sixth floor acceleration responses are depicted in Figure 11. H_∞ controller is also reduced the sixth floor acceleration.

Figure 8. Six DOF experimental model.

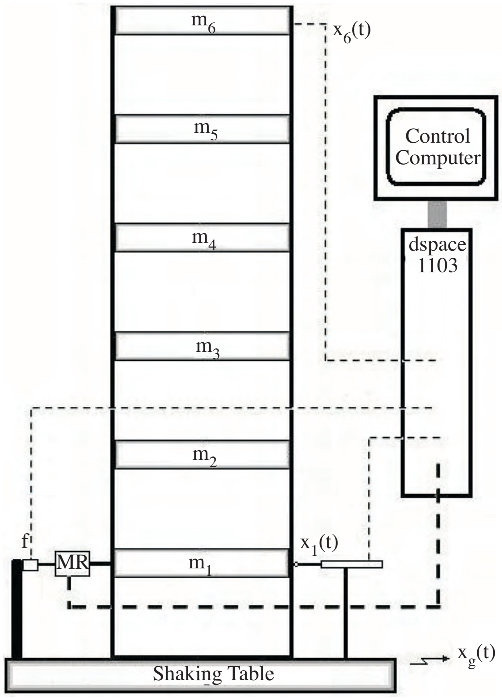

Figure 9. Schematic of experimental setup.

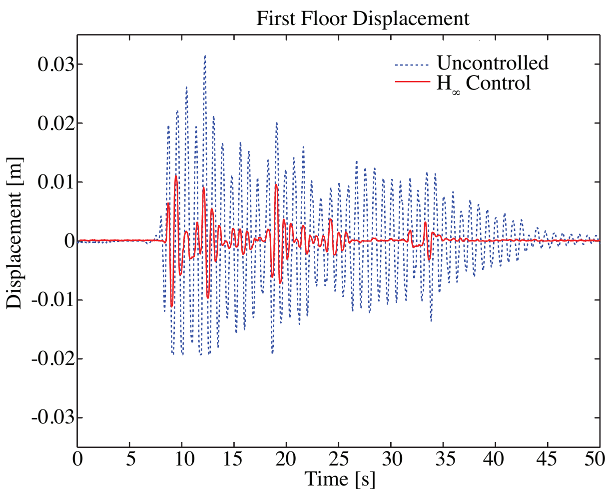

Figure 10. First floor displacement.

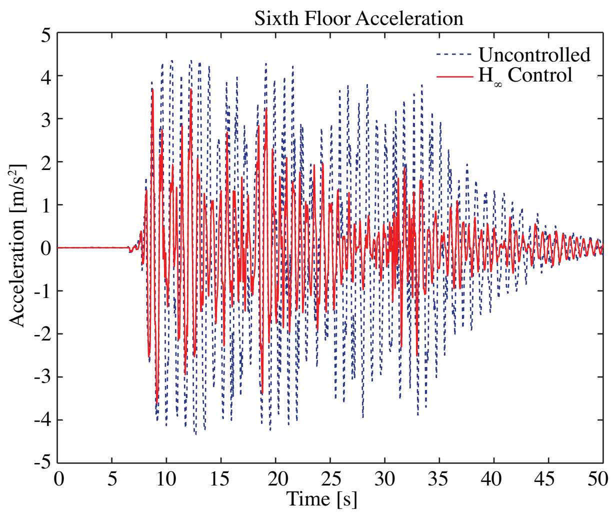

Figure 11. Sixth floor acceleration.

6. Conclusion

In this study, to reduce the vibrations of a six DOF structural model using an MR damper, H_∞ controller is designed and tested experimentally. H_∞ controller is designed for a reduced order model and applied to the full order model. The designed controller satisfies a strong robust stabilization and improves teh performance of the closed loop system. The controller performance is also verified experimentally. The performance of controller is compared with uncontrolled case. Experimental results demonstrate the effectiveness of the H_∞ controller.

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