

An improved distributed power-control scheme for cellular mobile systems

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Abstract

In this paper, a new distributed power-control (DPC) scheme is suggested to improve convergence speed and system robustness against carrier-to-interference-ratio (CIR) estimation errors. To expedite the CIR balancing in our DPC scheme, an instability detection rule was used. As compared with Foschini's DPC (FDPC) method, numerical results indicated that the proposed algorithm achieves performance improvements in terms of outage probability as well as in the algorithm's convergence speed. More specifically, by appropriate selection of some parameters, the algorithm speed reduces from about 90 iterations in FDPC to 9 iterations in the proposed algorithm. The system robustness against CIR estimation errors was also explored.

Key Words: Cellular systems, CIR balancing, distributed algorithms, instability detection, power control

1. Introduction and related work

Frequency reuse in wireless cellular networks results in increased system capacity. It increases, however, the cochannel interference that imposes limitations on the minimum reuse distance. An efficient technique in reducing the cochannel interference is to control the transmitted power in order to provide each receiver with a satisfactory reception. A commonly used measure of the quality of communications is the carrier-to-interference ratio (CIR) at the receiver.

The principle idea of power control was first introduced by Aein [1] for CIR balancing in satellite systems. After that, Nettleton and Alavi [2,3] applied the idea to spread-spectrum cellular satellite systems. Evolution in power-control schemes was then constructed by Zander [4], who reformulated the problem of power control and CIR balancing in terms of eigenvalues and eigenvectors of the transmit gain matrix in the absence of background noise. Zander developed an optimum power-control scheme to minimize system outage probability, which is the probability of a randomly chosen mobile having a CIR less than the system protection ratio. Although Zander's power-control scheme was not implemented in practice, since it was a central power-control (CPC) method and required all path-gain information and a central controller to process a huge amount of data, CPC analysis

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can be used for providing theoretical limits and performance evaluation of distributed power-control (DPC) schemes. The DPC schemes [5-13] have been developed to cope with computational complexities arising from CPC schemes. A framework on the convergence of generalized uplink power control was provided by Yates [14] and extended by Huang and Yates [15]. The results in [14] and [15] were a breakthrough, providing guidelines for designing and analyzing new algorithms.

In the present paper, we present a new DPC algorithm that achieves CIR balancing with unit probability. Numerical analysis through computer simulations indicated that our algorithm, in conjunction with the use of the instability detection rule, has excellent performance compared to Foschini's DPC algorithm (FDPC) in terms of outage probability and convergence speed.

2. System model

The cellular mobile system assumptions and notations in this paper are the same as those made in [7]. Let us consider a set containing K cells sharing a particular (reused) channel. It is assumed that the mobiles have been evenly deployed throughout the system. Suppose that P_i and G_{ij} denote the power transmitted by the i th mobile and the link gain from the j th mobile to the i th base station (BS), respectively. The CIR at the i th BS, Γ_i , can be expressed as:

$$\Gamma_i = \frac{G_{ii}P_i}{\sum_{\substack{j=1 \\ j \neq i}}^K G_{ij}P_j + \nu_i}, \quad i = 1, 2, \dots, K. \quad (1)$$

Assuming a snapshot model assumption in this paper, the gain matrix, $\mathbf{G} = [G_{ij}]$, is a $K \times K$ matrix of random elements, and Γ_i is a random variable. For successful base-to-mobile transmissions, the following criterion should be satisfied:

$$\Gamma_i \geq \gamma_0, \quad i = 1, 2, \dots, K, \quad (2)$$

in which γ_0 is called the system protection ratio or the user's minimum required (target) CIR. If Eq. (2) is met, a corresponding effective power vector for a simultaneous satisfactory transmission quality can be defined as:

$$\mathbf{P}^* = [P_1^*, P_2^*, \dots, P_K^*] > 0. \quad (3)$$

In situations in which Eq. (2) is not met, one or more users must be required to be disconnected or handed over to other channels in order to maintain transmission quality. This is achieved through the cell-removal process.

3. Cell-removal and instability detection rule

Let us assume that the DPC algorithm meets Eq. (2) after L iterations. Thus, \mathbf{P}^* would be the effective power vector, and the system is called stable at the current state. Otherwise, if $\Gamma_i^{(L)} < \gamma_0$, the cell-removal process is invoked, so that all users remaining maintain a minimum CIR of γ_0 . In [4] and [7], the cell-removal process was used to improve the system performance in terms of the outage probability in conjunction with CIR balancing and a DPC algorithm. In the following section, we proceed with an augmented DPC algorithm using the cell-removal process.

Table. Outage probability mean number of iterations.

25 dB	23 dB	20 dB	Target CIR \rightarrow	DPC algorithm
113.08	75.616	26.814	Mean no. of iterations	AFM
0.3194	0.2083	0.0656	Outage probability	
0.3203	0.2104	0.0667	Outage probability	AFMD
112.57	75.104	26.312	Mean no. of iterations	
100.52	69.740	26.312	Mean no. of iterations	APM
0.2774	0.1774	0.0569	Outage probability	
0.2795	0.1797	0.0584	Outage probability	APMD
77.097	53.616	20.906	Mean no. of iterations	

4. Augmented DPC algorithm

The augmented DPC (ADPC) algorithm [8] can be summarized in the following steps.

- Step 1 (initialization): For all users, choose an initial power vector $\mathbf{P}^0 = [p_i^0]_{K \times 1}$, and measure and store the initial CIR vector $\mathbf{\Gamma}^0 = [\Gamma_i^0]_{K \times 1}$. If:

$$\Gamma_i^0 > \gamma_0, \text{ for all } i, \quad (4)$$

then stop. Otherwise, go to step 2.

- Step 2 (CIR balancing): Perform a DPC algorithm for a maximum of L iterations. If, at iteration ν ($\nu \leq L$), and for any i , $\Gamma_i^\nu \geq \gamma_0$, then stop. Otherwise, go to step 3.
- Step 3 (cell removal): Remove the i th cell with smallest CIR, Γ_i^0 , and return to step 1.

To expedite the CIR balancing, we benefit from the instability detection rule [8] and use it as Step 2 in the ADPC algorithm. Let us assume that ε_1 and ε_2 are 2 small positive numbers. The cell-removal process will then be evoked if the following constraints are met:

$$\left| \Gamma_i^{(\nu+1)} / \Gamma_i^{(\nu)} - 1 \right| < \varepsilon_1 \text{ and } \Gamma_i^{(\nu+1)} < (1 - \varepsilon_2) \gamma_0, \quad (5)$$

in which the first constraint checks the convergence of the power levels and the second inspects whether or not the target CIR, γ_0 , is being obtained.

5. The proposed DPC algorithm

The DPC algorithms in [5-12] provide CIR balancing with a probability of 1. They were developed in order to adjust the transmitted power level in accordance with the received interference power level. Although the FDPC algorithm in Foschini and Miljanic's method [6] exhibited faster convergence than the distributive balancing (DB) algorithm of Zander's method [7], their convergence trajectories fluctuate so much that they make these methods quite sensitive to CIR estimation errors. To improve the system performance, we introduce, in the sequel, a new DPC algorithm that achieves improved system performance, compared to FDPC, in terms of higher convergence speed and less sensitivity to CIR estimation errors.

Our principle idea behind the proposed DPC algorithm is based on the assumption that there would always be a fixed CIR improvement at the present iteration, with respect to a fraction (α) of the CIR obtained at the previous one:

$$\Gamma_i^{(\nu+1)} - \alpha\Gamma_i^{(\nu)} = \beta\gamma_0, \tag{6}$$

in which $0 < \alpha \leq 1$ and $\beta > 0$. Since the power at iteration $(\nu + 1)$ can be calculated using its value at iteration (ν) from $K - 1$ other users, we then have:

$$\frac{G_{ii}P_i^{(\nu+1)}}{\sum_{\substack{j=1 \\ j \neq i}}^K G_{ij}P_j^{(\nu)} + \nu_i} - \alpha\Gamma_i^{(\nu)} = \beta\gamma_0. \tag{7}$$

Eq. (7) can be rewritten as:

$$P_i^{(\nu+1)} = \zeta P_i^{(\nu)} \left(\alpha + \frac{\beta\gamma_0}{\Gamma_i^{(\nu)}} \right), \tag{8}$$

where ζ is used to rescale the power vector. It is worth noting that with $\beta = (1 - \tau\Gamma_i^{(\nu)})$, $-\infty < \tau < -1$, and $\alpha = 1$ in Eq. (7), the proposed algorithm would be equivalent to the group of algorithms in [5]. Also, with $\alpha = 1$ in Eq. (7), the constraint improvement power-control (CIPC) algorithm in [8] would be a special case of the proposed algorithm.

Theorem 1: If γ_0 is achievable, then by choosing $\zeta = 1/(1 + \beta)$ and $\beta > 0$, the proposed algorithm will definitely achieve the effective power vector irrespective of the value of the initial power vector, \mathbf{P}^0 , or:

$$\lim_{\nu \rightarrow \infty} \mathbf{P}^{(\nu)} = \mathbf{P}^*, \forall \mathbf{P}^0. \tag{9}$$

See proof in Appendix.

6. Numerical results

The simulations regarding the proposed DPC algorithm in this paper were performed for a 2D hexagonal cellular system with 19 cochannel cells and a cluster size of 7. In order to calculate the outage probability, we used the Monte Carlo simulation for 1000 independent runs for different sets of parameters. It was also assumed that the mobiles were evenly deployed on a cell area, and the path gains regarding each link in the environment of propagation were calculated from:

$$G_{ij} = \frac{A_{ij}}{d_j^a}, \tag{10}$$

in which d_{ij} is the distance between the mobile at the i th cell and the BS at the j th cell and $a = 4$, which obeys the transmitter-decay law with an exponent of 4. A_{ij} is the attenuation factor and indicates the changes in power caused by shadowing. It is modeled as a random log-normal variable with a zero mean ($\mu = 0$) and a standard deviation of 6 dB ($\sigma = 6$ dB). The background noise is neglected. It can be seen from Figure 1 that our algorithm has a higher convergence speed compared to FDPC and DB algorithms. Figure 2 shows the mean number of iterations versus α , required for the algorithm to reach a CIR within 99% of the optimal one.

The appropriate choice of α expedites convergence. For example, by choosing $\alpha = 0.35$, the mean number of iterations reduces from 89.5 in FDPC to 8.68 in the proposed algorithm. It is worth noting that a value of 0.5 has been considered for α throughout the simulations. This has led to less sensitivity against estimation errors while the number of iterations has remained almost unchanged. In Figures 3 and 4, we demonstrate the ADPC scheme combined with our proposed algorithm and FDPC. A variety of DPC schemes have been compared, including Zander's limited-information stepwise removal algorithm (LI-SRA), the augmented FDPC method (AFM), AFM with instability detection rule (AFMD), the augmented proposed method (APM), and APM with instability detection rule (APMD). The maximum number of iterations, L , is 20, and $\varepsilon_1 = \varepsilon_2 = 0.001$. Figure 3 depicts the convergence speed for different DPC schemes. It indicates that the proposed algorithm is able to find the effective power vector, \mathbf{P}^* , more quickly with or without usage of the detection rule. Graphs of the outage probability as a function of target CIR are depicted in Figure 4 for various DPC schemes. It is worth noting that, referring to the Table and Figures 3 and 4, APMD has a faster convergence, but APM is better in terms of outage probability. In order to evaluate the impact of CIR estimation errors on the performance of

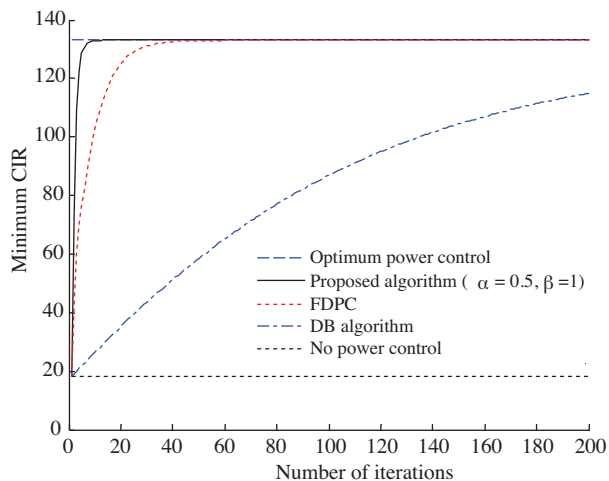


Figure 1. Convergence speed versus number of iterations.

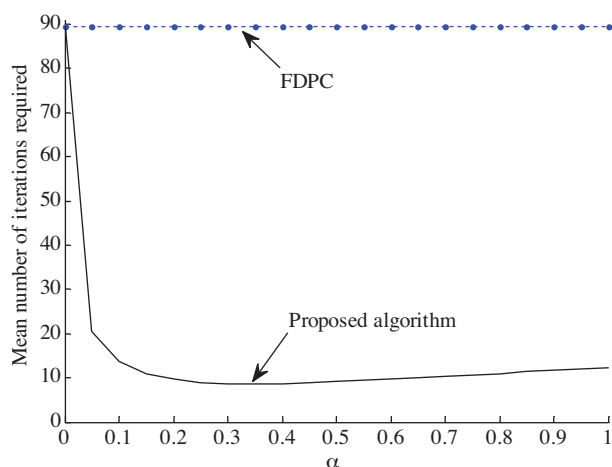


Figure 2. Convergence speed versus α .

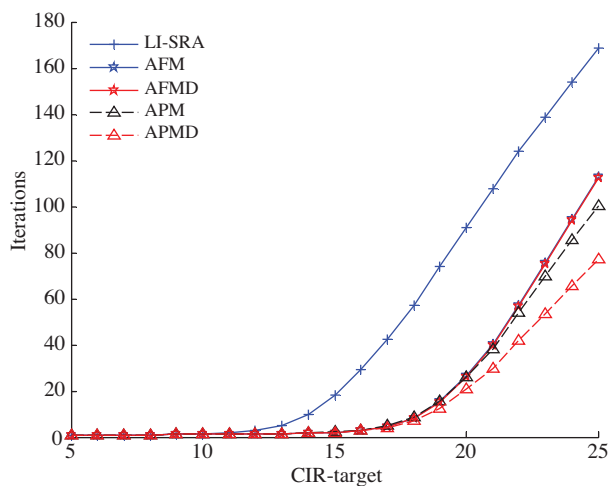


Figure 3. Convergence speed versus CIR target (γ_0).

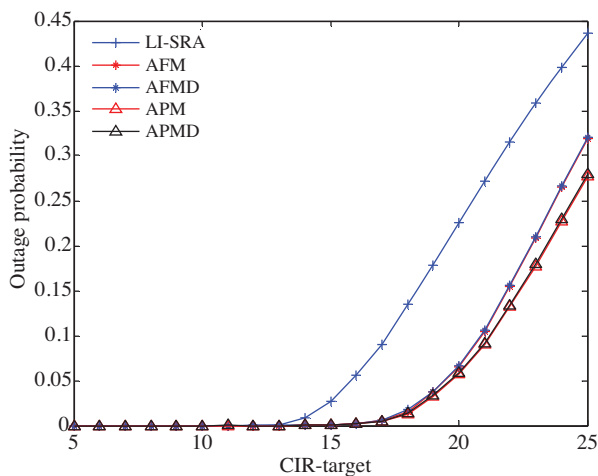


Figure 4. Outage probability versus CIR target (γ_0).

various DPC algorithms, the estimation error model used in [7] and [8] was exploited in this paper. It is stated as:

$$(\Gamma_i^{(v)})_e = \eta_i^{(v)} \Gamma_i^{(v)}, \quad (11)$$

in which $\eta_i^{(v)}$ is a zero-mean random log-normal variable with log variance σ_m .

Figures 5 and 6 represent the outage probability comparison for a variety of DPC algorithms in the presence of CIR estimation errors. The algorithms were tested for $\sigma_m = 3$ and 5 dB. As is illustrated in Figures 5 and 6, the proposed algorithm dominates the FDPC algorithm in terms of convergence speed and outage probability. Using the detection rule, it also shows less sensitivity against CIR estimation errors.

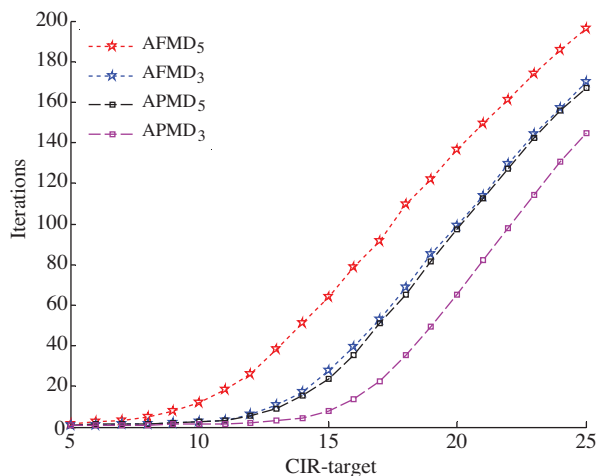


Figure 5. Convergence speed versus CIR target (γ_0).

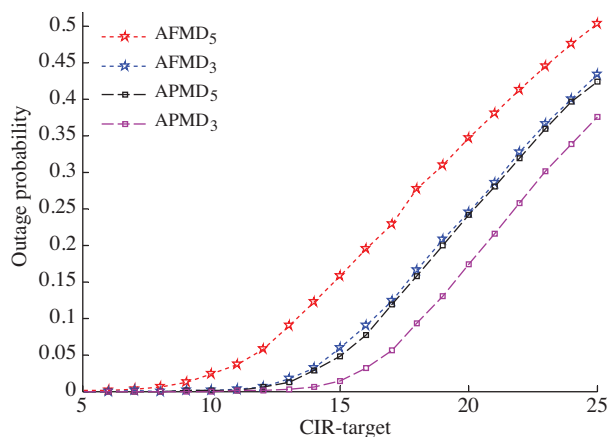


Figure 6. Outage probability versus γ_0 in the presence of CIR estimation error.

7. Conclusions

In this paper, we proposed a distributive power-control algorithm and compared it with the FDPC algorithm in terms of convergence speed and sensitivity to estimation errors. Results of simulations show that with the proper choice of $\alpha = 0.35$, the proposed DPC algorithm has a mean convergence speed of about 10 times that of the FDPC algorithm. We have also shown that by using the instability detection rule, our algorithm, compared to others, achieves a faster convergence while having the lowest sensitivity against CIR estimation errors.

Appendix

If γ_0 is achievable, then an effective power vector, \mathbf{P}^* , is calculated by solving the following equation:

$$(d\mathbf{I} + a\mathbf{C})\mathbf{P}^* = a\eta, \quad (A1)$$

in which a and d are constants, \mathbf{I} is the unit matrix of dimension $K \times K$,

$$\mathbf{C} = \begin{bmatrix} 1 & -\gamma_0 \frac{G_{12}}{G_{11}} & \cdots & -\gamma_0 \frac{G_{1k}}{G_{11}} \\ -\gamma_0 \frac{G_{21}}{G_{22}} & 1 & \cdots & -\gamma_0 \frac{G_{2k}}{G_{22}} \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_0 \frac{G_{k1}}{G_{kk}} & -\gamma_0 \frac{G_{k2}}{G_{kk}} & \cdots & 1 \end{bmatrix}_{K \times K}, \text{ and } \boldsymbol{\eta} = \begin{bmatrix} \gamma_0 \frac{v_1}{G_{11}} \\ \gamma_0 \frac{v_2}{G_{22}} \\ \vdots \\ \gamma_0 \frac{v_k}{G_{kk}} \end{bmatrix}_{K \times 1}. \quad (\text{A2})$$

Here, we show that the proposed algorithm approaches an effective power vector of:

$$\mathbf{P}^* = (d\mathbf{I} + a\mathbf{C})^{-1} a\boldsymbol{\eta}, \quad (\text{A3})$$

in which $a = \beta/(1 + \beta)$ and $d = (1 - \alpha)/(1 + \beta)$.

In order to prove Eq. (A3), we benefit from [8] and [16] and the theorems therein.

Proof. Let the power rescaling factor be $\zeta = 1/(\beta + 1)$. Thus, Eq. (7) is rewritten as:

$$P_i^{(\nu+1)} = \frac{-\beta}{1 + \beta} \left(1 - \frac{\gamma_0}{\Gamma_i^{(\nu)}}\right) P_i^{(\nu)} + \left(\frac{\beta + \alpha}{1 + \beta}\right) P_i^{(\nu)}. \quad (\text{A4})$$

Now, substituting for $\Gamma_i^{(\nu)}$ from Eq. (1), we have:

$$P_i^{(\nu+1)} = \frac{-\beta}{1 + \beta} \left(\sum_{j=1}^K c_{ij} P_j^{(\nu)} - \eta_i \right) + \left(\frac{\beta + \alpha}{1 + \beta}\right) P_i^{(\nu)}, \quad (\text{A5})$$

where c_{ij} and η_i are elements of matrix \mathbf{C} and vector $\boldsymbol{\eta}$, respectively. Equivalently, using the matrix notation, we have:

$$\mathbf{P}^{(\nu+1)} = -a\mathbf{C}\mathbf{P}^{(\nu)} + a\boldsymbol{\eta} + b\mathbf{P}^{(\nu)}, \quad (\text{A6})$$

where $b = (\beta + \alpha)/(1 + \beta)$. Accordingly, solving the recursive Eq. (A5), we obtain:

$$\mathbf{P}^{(\nu)} = (\mathbf{I} + \mathbf{D} + \mathbf{D}^2 + \mathbf{L} + \mathbf{D}^{\nu+1}) a\boldsymbol{\eta} + \mathbf{D}^\nu \mathbf{P}^0, \quad (\text{A7})$$

where $\mathbf{D} = (b\mathbf{I} + a\mathbf{C})$. Since \mathbf{C} is diagonalizable, there is a matrix \mathbf{Q} such that:

$$\mathbf{C} = \mathbf{Q}\boldsymbol{\Lambda}\mathbf{Q}^{-1}, \quad (\text{A8})$$

where $\boldsymbol{\Lambda}$ is a diagonal matrix, all of whose entries have positive real parts. Thus:

$$\mathbf{D}^\nu = b^\nu \mathbf{Q} \left(\mathbf{I} - \frac{a}{b} \boldsymbol{\Lambda} \right)^\nu \mathbf{Q}^{-1}, \quad (\text{A9})$$

where the diagonal entries of $(\mathbf{I} - (a/b)\boldsymbol{\Lambda})$ are the eigenvalues of \mathbf{D} . Since $0 < a/b < 1$ for $\beta > 0$, $0 < \alpha \leq 1$, then the modulus of each eigenvalue of \mathbf{D} is strictly less than 1 (Lemma 3 in [8]). Using the geometric series formula:

$$\begin{aligned} \lim_{\nu \rightarrow \infty} \mathbf{P}^{(\nu)} &= (\mathbf{I} - \mathbf{D})^{-1} a\boldsymbol{\eta} \\ &= (d\mathbf{I} + a\mathbf{C})^{-1} a\boldsymbol{\eta} = \mathbf{P}^* \end{aligned} \quad (\text{A10})$$

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