# A novel three-phase transformer hysteresis model in MATLAB ${ }^{\text {TM }}$ Simulink 

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#### Abstract

This paper presents a new transformer model displaying hysteresis using the MATLAB ${ }^{T M}$ Simulink environment. The proposed model displays a complete scheme for the simulation of 3-phase transformers. The new model is mainly based on the transmission line model and the Jiles-Atherton model of a power system using lumped parameters. Jiles-Atherton model parameters were determined by curve fitting and numerical optimization iteration. The developed model is particularly suitable for fault analysis and protective relaying studies under harmonic conditions where the transformer is driven into the nonlinear regime. The performance of the method was evaluated by both theorical and experimental data.


Key Words: Transformer modeling, hysteresis, Jiles-Atherton

## 1. Introduction

The power transformer is one of the most important elements of power systems, since the continuity of transformer operation has vital importance in maintaining the reliability of the power supply. Therefore, various modeling schemes have been developed over the decades to provide a proper protective method for the transformers, and there have been many attempts to precisely estimate the hysteresis behavior of ferromagnetic materials, especially in the transformer's core [1].

The development and validation of protective algorithms for the transformers requires the preliminary determination of the transformer model. The model used is required to simulate the power system's normal

[^0]and faulted conditions and to estimate the behavior of the protection algorithm under abnormal conditions. In particular, it must allow for the simulation of internal and external fault conditions. Most of the present electromagnetic transient programs are able to accurately simulate some of the phenomena occurring in the transformer, like magnetizing inrush current, excitation current, and transformer saturation [2]. However, the implementations of the present monitoring methods [3,4] tend to cost too much to be applied to distribution transformers. Such analysis also usually requires commercial software such as FEMLAB, MAGNET, or ANSYS Maxwell. However, most of the present simulation programs are not able to properly simulate the hysteresis characteristic of the core material [5].

This paper presents a 3-phase transformer model displaying hysteresis based on Jiles-Atherton (J-A) magnetization with a modified Langevin function and determination of model parameters; it is an extended version of [1]. The nonlinear behavior of the transformer's core was modeled using the MATLAB ${ }^{\text {TM }}$ Simulink environment. The proposed model is based on the transmission line model (TLM) method for simulating transformers, which can consider nonlinear hysteresis. To validate the proposed technique, a 3 -phase, 2 -winding laboratory-type transformer was modeled using the TLM method including the J-A technique with parameter estimation. The TLM method converts a second-order circuit into a first-order one, which simplifies the solution of the circuit at any discrete time. In addition to this, it facilitates the incorporation of time-dependent models such as the J-A model, which is described in Section 2. The proposed modeling technique is suitable for modeling transformer-related phenomena such as magnetizing inrush, internal faults, and loading conditions.

## 2. Three-phase, two-winding transformer model

The TLM method was first developed in the early 1970s for modeling 2-dimensional field problems. Since then, it has been extended to cover 3 -dimensional problems and circuit simulations. For circuit simulation, the TLM method can be used to develop a discrete circuit model directly from the system without setting up any integro-differential equations. The TLM algorithm is discrete in nature and ideally suited for implementation on computer-based systems [6].

The hysteresis specs of the transformer are based on the J-A technique in TLM simulations. The JA technique describes the relationship between magnetic moment M and magnetic field intensity H using the current physical theories of magnetic domains in ferromagnetic materials [7], and it is able to present a possible anisotropic steel behavior in transformers. The J-A model requires the following input parameters: magnetization saturation, thermal energy parameter, domain flexing constant, domain anisotropy constant, and interdomain coupling parameter. These are not parameters that transformer manufacturers or manufacturers of transformer magnetic material can provide for users. In fact, they cannot even be determined directly through measurements. The various core hysteresis parameters required in this model are theorical and can be calculated from experimental measurements of coercivity, remanence, saturation flux density, initial anhysteretic susceptibility, initial normal susceptibility, and the maximum differential susceptibility [8]. Moreover, the J-A model requires little memory storage, as its status is totally described by only 5 parameters. On the other hand, convergence problems may be encountered in the identification of these parameters by using iterative procedures. More details about the J-A technique can also be found in [9-11].

Conversion between the B-H loop and an M-H loop is straightforward and is calculated using Eq. (1).

$$
\begin{equation*}
B=\mu_{0}(H+M) \tag{1}
\end{equation*}
$$

The J-A hysteresis model decomposes the whole magnetization M into the reversible component $M_{\text {rev }}$ and the irreversible component $M_{i r r}$ [12].

$$
\begin{equation*}
M=M_{r e v}+M_{i r r} \tag{2a}
\end{equation*}
$$

Its current-related equation is given in Eq. (2b).

$$
\begin{equation*}
I_{m}=I_{r e v}+I_{i r r} \tag{2b}
\end{equation*}
$$

The reversible component is defined as in Eq. (3).

$$
\begin{equation*}
M_{r e v}=c\left(M_{a n}-M_{i r r}\right) \tag{3a}
\end{equation*}
$$

Its current-related equation is given in Eq. (3b).

$$
\begin{equation*}
I_{r e v}=c\left(I_{a n}-I_{i r r}\right) \tag{3b}
\end{equation*}
$$

$M_{a n}$ is the anhysteretic magnetization provided by the Langevin equation. The magnetization relationship between B and H is replaced by the anhysteretic magnetization curve between H and M as follows:

$$
\begin{equation*}
M_{a n}=M_{s} . f\left(H_{e}\right) \tag{4}
\end{equation*}
$$

where $H_{e}=H+\alpha M$ and is called the Weiss effective field. This is linked with magnetic field intensity $H$ by the modified Langevin function in Eq. (5) and the differential equation in Eq. (6).

$$
\begin{equation*}
M_{a n}=M_{s}\left(\operatorname{coth} \frac{H+\alpha M}{a}-\frac{a}{H+\alpha M}\right) \tag{5a}
\end{equation*}
$$

Its current-related equation is given in Eq. (5b).

$$
\begin{gather*}
I_{a n}=I_{s}\left(\operatorname{coth}\left(\frac{I_{L}+\alpha I_{m}}{a}-\frac{a}{I_{L}+\alpha I_{m}}\right)\right.  \tag{5b}\\
\frac{d M_{i r r}}{d H}=\frac{\delta_{m}\left(M_{a n}-M_{i r r}\right)}{k \delta-\alpha\left(M_{a n}-M_{i r r}\right)} \tag{6a}
\end{gather*}
$$

Its current-related equation is given in Eq. (6b).

$$
\begin{equation*}
\frac{d I_{i r r}}{d I_{L}}=\frac{\delta_{m}\left(I_{a n}-I_{i r r}\right)}{k \delta-\alpha\left(I_{a n}-I_{i r r}\right)}, \tag{6b}
\end{equation*}
$$

where $\delta_{m}$ and $\delta$ are given by:

$$
\delta_{m}=\left\{\begin{array}{c}
1: I F \frac{d H}{d t}>0 \text { and } M_{a n}>M_{i r r}  \tag{7a}\\
1: I F \frac{d H}{d t}<0 \text { and } M_{a n}<M_{i r r} \\
0: \text { otherwise }
\end{array}\right.
$$

Its current related equation is given in Eq. (7b).

$$
\begin{gather*}
\delta_{m}=\left\{\begin{array}{c}
1: I F \frac{d I_{L}}{d t}>0 \text { and } I_{a n}>I_{\text {irr }} \\
1: I F \frac{d I_{L}}{d t}<0 \text { and } I_{\text {an }}<I_{\text {irr }} \\
0: \text { otherwise }
\end{array}\right.  \tag{7b}\\
\delta=\left\{\begin{array}{l}
1: I F \frac{d H}{d t}>0 \\
1: I F \frac{d H}{d t}<0
\end{array}\right. \tag{8a}
\end{gather*}
$$

Its current-related equation is given in Eq. (8b).

$$
\delta=\left\{\begin{array}{l}
1: I F \frac{d I_{L}}{d t}>0  \tag{8b}\\
1: I F \frac{d I_{L}}{d t}<0
\end{array}\right.
$$

$a, \alpha, c, k$, and $M_{s}$ are the parameters of the model, where $a$ is a form factor, $\alpha$ is the interaction between the domains, $c$ is the coefficient of reversibility of the movement of the walls, $k$ represents the hysteresis losses, and $M_{s}$ is the saturation magnetization. Numerical iterative techniques such as least-square curve fitting quasi-Newton approaches are used to determine these parameters.

## 3. Parameter estimation

The literature presents several step-by-step methods for the identification of the J-A model parameters from experimental B-H loops, which are based on the physical meaning of the parameters. Among them, random and deterministic searches of the J-A model parameters [13], particle swarm optimization [14], the simplex method and simulated annealing algorithm [15], estimations by using a global optimization technique known as "branch and bound" [16], and usage of the genetic algorithm for parameter estimation [17] are the recent studies in the literature. In this work, parameters are extracted using a least-square method, imposing the condition of the local minimum, and solving the equations using Newton's method.

### 3.1. Anhysteretic susceptibility

The anhysteretic susceptibility at the origin can be used to define a relationship among $M_{s}, a$, and $\alpha$ [18].

$$
\begin{gather*}
X_{a n}=\left(\frac{d M_{a n}}{d H}\right)_{M=0, H=0}  \tag{9a}\\
\alpha=\frac{M_{s}}{3}\left(\frac{1}{X_{a n}}+\alpha\right) \tag{9b}
\end{gather*}
$$

Its current-related equation is given in Eqs. (9b) and (10b).

$$
\begin{align*}
X_{a n} & =\left(\frac{d I_{a n}}{d I_{L}}\right)_{I_{m}=0, I_{L}=0}  \tag{10a}\\
\alpha & =\frac{I_{s}}{3}\left(\frac{1}{X_{a n}}+\alpha\right) \tag{10b}
\end{align*}
$$

### 3.2. Initial susceptibility

The reversible magnetization component is expressed via parameter $c$ in the hysteresis equation, Eq. (6a), defined by:

$$
\begin{equation*}
x_{i n i}=\left(\frac{d M}{d H}\right)_{M=0, H=0}=\frac{c M_{s}}{3 \alpha} . \tag{11a}
\end{equation*}
$$

Its current-related equation is given in Eq. (11b).

$$
\begin{equation*}
x_{i n i}=\left(\frac{d I_{m}}{d I_{L}}\right)_{I_{m}=0, I_{L}=0}=\frac{c I_{s}}{3 \alpha} \tag{11b}
\end{equation*}
$$

### 3.3. Coercivity

The hysteresis loss parameter k can be determined from coercivity $H_{c}$ and the differential susceptibility at coercive point $X_{a n}\left(H_{c}\right)$.

$$
\begin{equation*}
k=\frac{M_{a n}\left(H_{c}\right)}{1-c}\left[\alpha+\frac{1}{X\left(H_{c}\right)-\left(\frac{c}{1-c}\right) \frac{d M}{d H}}\right] \tag{12a}
\end{equation*}
$$

Its current-related equation is given in Eq. (12b).

$$
\begin{equation*}
k=\frac{I_{a n}\left(I_{c}\right)}{1-c}\left[\alpha+\frac{1}{X\left(I_{c}\right)-\left(\frac{c}{1-c}\right) \frac{d I_{m}}{d I_{L}}}\right] \tag{12b}
\end{equation*}
$$

### 3.4. Remanence

The coupling parameter $\alpha$ can be determined independently if it is known by using remanence magnetization $M_{r}$ and the differential susceptibility at remanence.

$$
\begin{equation*}
M_{r}=M_{a n}\left(M_{r}\right)+\frac{k}{\frac{a}{a-c}+\frac{k}{X\left(M_{r}\right)-c \frac{d M}{d H}}} \tag{13}
\end{equation*}
$$

## 4. Modeling procedure

Figure 1 shows the transformer core type under modeling consideration. In Figure 1, $N_{a}$ and $N_{b}$ represent the number of turns of primary and secondary windings, respectively. Lowercase subscripts represent the primary side parameters, while capitalized subscripts represent secondary side parameters. The linear transformer model is shown in Figure 2.

The main modeling equations, such as $V_{a-b-c}, V_{A-B-C}$ (primary and secondary voltages, respectively), and $M_{a-b-c}$ (magnetization intensities of the phases), are defined in Eq. (14), respectively. For simplicity, only phase $a$ and phase $A$ equations are explained here.

$$
\begin{align*}
& V_{a}=\frac{L_{a a} d I_{a}}{d t}+\frac{L_{a A} d I_{A}}{d t}+N_{a} \frac{L_{m} d I_{m a}}{d t} \\
& V_{A}=\frac{L_{A A} d I_{A a}}{d t}+\frac{L_{a A} d I_{a}}{d t}+N_{A} \frac{L_{m} d I_{m a}}{d t} \tag{14}
\end{align*}
$$



Figure 1. A 3-phase, 2-winding star/star-connected transformer.







Figure 2. Linear modeling of the 3-phase, 2-winding transformer.

Here, $L_{a a}=\frac{\mu_{0} N_{a}^{2} A}{l}, L_{A A}=\frac{\mu_{0} N_{A}^{2} A}{l}, L_{m}=\frac{\mu_{0} A}{l}, A$ is the cross-sectional area in square meters, and $l$ is the length of the magnetic path in meters. These are initially calculated as the simulation starts. The characteristic impedances of the primary and secondary sides are then calculated as follows:

$$
\begin{align*}
& Z_{a a}=\frac{2 L_{a a}}{d t} \\
& Z_{A A}=\frac{2 L_{A A}}{d t} \tag{15}
\end{align*}
$$

Similarly, the characteristic impedances of the magnetizing branches are calculated using the following equations.

$$
\begin{align*}
L_{a A} & =\left(\mu_{0} N_{a} N_{A} A\right) / l \\
Z_{a A} & =2 L_{a A} / d t \tag{16}
\end{align*}
$$

Leakage inductances and related characteristic impedances are calculated using Eq. (10).

$$
\begin{align*}
& Z_{L a S}=2 L_{a S} / d t \\
& Z_{L A S}=2 L_{A S} / d t \tag{17}
\end{align*}
$$

$Z_{\text {LaS }}$ and $Z_{L A S}$ are the leakage characteristic impedances of the primary and secondary sides, respectively. For a lossless linear transformer, the mutual TLM voltages are defined in Eq. (18) and Eq. (14) as follows:

$$
\begin{align*}
& V_{a A}=Z_{a A} I_{A}+2 V_{a A}^{i}  \tag{18}\\
& V_{A a}=Z_{A a} I_{a}+2 V_{A a}^{i} \tag{19}
\end{align*}
$$

where $Z_{a A}=\frac{2 L_{a A}}{d t}$ and $d t$ is the sampling interval.
After these calculation steps, the primary and secondary currents, $I_{a}$ and $I_{A}$, are calculated using Eqs. (20) and (21).

$$
\begin{gather*}
Z_{a a} I_{a}+Z_{a A} I_{A}=V_{a}-2\left(V_{a a}^{i}+V_{a A}^{i}\right)  \tag{20}\\
Z_{a A} I_{a}+Z_{A A} I_{A}=-2\left(V_{A a}^{i}+V_{A A}^{i}\right) \tag{21}
\end{gather*}
$$

Here, $Z_{a a}=\frac{2 L_{a a}}{d t}$.
In Eq. (14), $V_{a}$ is the source voltage of phase a. The calculations are repeated until the end of the simulation time. Figure 3 shows a 3 -phase, 2 -winding transformer model displaying hysteresis.

The required calculations for the hysteresis model of a 3-phase transformer are somewhat complicated in the linear model. To have a complete model, source and load impedances are added to the simulation. To simplify the whole procedure, the equations are given for only phase a here.

The whole calculation procedure, step by step, is explained below.
Step 1. $V_{a}^{k}$ is obtained, $k=1,2,3, \ldots N . N$ is the number of samples.
Step 2. Phase-neutral voltage is calculated as:

$$
V_{s a}^{k}=V_{a}^{k}+R_{a} I_{a}^{k},
$$

where $R_{a}$ is the source resistance.
Step 3. J-A calculations are begun from this step. Total ampere turns is calculated as:

$$
I_{h a}^{k}=N_{a} I_{a}^{k}+N_{A} I_{A}^{k} .
$$

Step 4. The exciting current is calculated as:

$$
I_{e a}^{k}=I_{h a}^{k}+\alpha I_{m a}^{k} .
$$

Step 5. Updating of the total ampere turns of the phases as:

$$
d I_{h a}^{k+1}=I_{h a}^{k}-\left(N_{a} I_{a}^{k-1}+N_{A} I_{A}^{k-1}\right) .
$$

Step 6. Directional flag is defined according to $d_{h a}^{k}$ as:
IF $d_{h a}^{k}<0$ THEN $\Delta_{a}^{k}=-1$ ELSE $\Delta_{a}^{k}=1$.


Figure 3. A 3-phase, 2-winding transformer model displaying hysteresis.
Step 7. To avoid numerical instability, the following condition is defined in TLM calculations for anhysteretic current.

IF $\left|I_{e a}^{k}\right|>0.001 I_{a c}$ THEN $I_{a n a}^{k}=I_{s c}\left(\operatorname{coth} \frac{I_{e a}^{k}}{I_{a c}}-\frac{I_{a c}}{I_{e a}^{k}}\right)$ and

$$
\left(\frac{d I_{a n a}}{d I_{h a}}\right)^{k}=\frac{I_{s c}}{I_{a c}}\left(\frac{-1}{\sin \left(\frac{I_{e a}^{k}}{I_{a c}}\right)^{2}}+\left(\frac{I_{a c}}{I_{e a}^{k}}\right)^{2}\right) \text { ELSE }
$$

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$$
I_{a n a}^{k}=\frac{I_{s c} I_{e a}^{k}}{3 I_{a c}} \text { and }\left(\frac{d I_{a n a}}{d I_{h a}}\right)^{k}=\frac{I_{s c}}{3 I_{a c}}
$$

Step 8. Irreversible current is defined as:

$$
d I_{i r r a}^{k}=d I_{h a}^{k} \frac{I_{a n a}^{k}-I_{i r r a}^{k}}{\Delta_{a}^{k} I_{c c}-\alpha\left(I_{a n a}^{k}-I_{i r r a}^{k}\right)} .
$$

Step 9. A migration flag is defined for each phase as:
IF $\Delta_{a}^{k}>0$ and $I_{a n a}^{k}>I_{i r r a}^{k}$ or $\Delta_{a}^{k}<0$ and $I_{a n a}^{k}<I_{i r r a}^{k}$ THEN

$$
m i g_{a}^{k}=1
$$

ELSE

$$
m i g_{a}^{k}=0
$$

Step 10. Updating of the irreversible current as:

$$
I_{i r r a}^{k}=m i g_{a}^{k} d I_{i r r a}^{k}
$$

Step 11. Migration flags are defined as:
IF $m i g_{a}^{k}=1$ and $d I_{h a}^{k} \cong 0$ THEN

$$
\left(\frac{d I_{m a}}{d I_{h a}}\right)^{k}=\beta_{c}\left(\frac{d I_{a n a}}{d I_{h a}}\right)^{k}+m i g_{a}^{k}\left(1-\beta_{c}\right)\left(\frac{d I_{i r r a}^{k}}{d I_{h a}^{k}}\right)
$$

ELSE

$$
\left(\frac{d I_{m a}}{d I_{h a}}\right)^{k}=\beta_{c}\left(\frac{d I_{a n a}}{d I_{h a}}\right)^{k}
$$

Step 12. Magnetization current is defined as:

$$
I_{m a}^{k+1}=\left(1-\beta_{c}\right) I_{i r r a}^{k+1}+\beta_{c} I_{a n a}^{k}
$$

Step 13. The following procedure is required for calculation of the phase current.

$$
\begin{gathered}
U_{a}^{k}=V_{s a}^{k}-\left(S_{a a}^{k}-S_{a A}^{k}+N_{a} S_{m a}^{k}\right) \\
U_{A}^{k}=\left(S_{A a}^{k}-S_{A A}^{k}+N_{a} S_{m a}^{k}\right)
\end{gathered}
$$

Voltage components are written as a column vector in the following equation.

$$
f=\left[\begin{array}{c}
I_{a}^{k}\left(R_{a}+Z_{a a}+Z_{L a S}\right)+I_{A}^{k} Z_{a A}+N_{a} I_{m a}^{k+1} Z_{m a}-U_{a}^{k} \\
\cdots \\
\cdots \\
I_{A}^{k}\left(R_{A}+Z_{A A}+Z_{L a S}+Z_{L A}\right)+I_{a}^{k} Z_{a A}+N_{a} I_{m a}^{k+1} Z_{m a}-U_{A}^{k} \\
\cdots \\
\cdots
\end{array}\right]
$$

The Jacobian matrix is defined as below.

$$
\begin{gathered}
J=\left[\begin{array}{cc}
J_{11} & J_{12} \\
J_{21} & J_{22}
\end{array}\right] \\
J_{11}=\left[\begin{array}{ccc}
R_{a}+Z_{L a S}+Z_{a a}\left(1+\left(\frac{d I_{m a}}{d I_{h a}}\right)^{k}\right) & \ldots & 0 \\
0 & \ldots & 0 \\
0 & 0 & \ldots
\end{array}\right] \\
J_{12}=\left[\begin{array}{ccc}
Z_{a A}\left(1+\left(\frac{d I_{m a}}{d I_{h a}}\right)^{k}\right) & 0 & 0 \\
0 & \ldots & 0 \\
0 & 0 & \ldots
\end{array}\right] \\
J_{21}=J_{12} J_{22}=\left[\begin{array}{llll}
R_{A}+Z_{L A S}+Z_{L A}+Z_{A A}\left(1+\left(\frac{d I_{m a}}{d I_{h a}}\right)^{k}\right) & 0 & 0 \\
0 & \ldots & 0 \\
0 & 0 & \ldots
\end{array}\right]
\end{gathered}
$$

The Newton-Raphson iteration method is applied to calculate the next sample of phase currents.

$$
\left[\begin{array}{c}
I_{a}^{k+1} \\
\ldots \\
\ldots \\
I_{A}^{k+1} \\
\ldots \\
\ldots
\end{array}\right]=\left[\begin{array}{c}
I_{a}^{k} \\
\ldots \\
\ldots \\
I_{A}^{k} \\
\ldots \\
\ldots
\end{array}\right]-J^{-1} f
$$

Step 14. Updating of the magnetizing current is defined as:

$$
\begin{gathered}
f_{m a}=I_{a}^{k+1}\left(R_{a}+Z_{a a}\right)+I_{A}^{k+1} Z_{a A}+N_{a} I_{m a}^{k+1} Z_{m a}-U_{a}^{k} \\
I_{m a}^{k+1}=I_{m a}^{k}-\frac{f_{m a}}{N_{a} Z_{m a}}
\end{gathered}
$$

Step 15. This step includes the updating procedures for accumulator values.

$$
\begin{gathered}
S_{a a}^{k+1}=-\left(S_{a a}^{k}+2 I_{a}^{k+1} Z_{a a}\right) \\
S_{A a}^{k+1}=-\left(S_{A a}^{k}+2 I_{a}^{k+1} Z_{a A}\right) \\
S_{A A}^{k+1}=-\left(S_{A A}^{k}+2 I_{A}^{k+1} Z_{A A}\right) \\
S_{m a}^{k+1}=-\left(S_{m a}^{k}+2 I_{m a}^{k+1} Z_{m a}\right)
\end{gathered}
$$

Step 16. This step includes the calculation of simulation outcomes.

$$
\psi_{a}=d t \frac{V_{a}^{k}-V_{a}^{k-1}}{2}
$$

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$$
\begin{gathered}
B_{a}^{k}=\frac{\mu_{0}}{l}\left(N_{a} I_{a}^{k+1}+N_{A} I_{A}^{k+1}+I_{m a}^{k+1}\right) \\
H_{a}^{k}=\frac{\left(N_{a} I_{a}^{k+1}+N_{A} I_{A}^{k+1}\right)}{l}
\end{gathered}
$$

These equations are executed in the MATLAB ${ }^{\text {TM }}$ Simulink environment. The simulation parameters of the modeled transformer can easily be changed. The user can easily change the required parameters by double clicking each block inside the model. The main TLM transformer block is suitable for implementing different types of 3-phase connections on both sides. The "powergui" option in the proposed model is used for waveform analysis. Basically, the TLM model consists of current-voltage measurement units, J-A implementation units, and a Jacobean matrix calculation module (subsystem4) for each phase.

Figure 4 shows the overall flowchart for a single phase.


Figure 4. A complete flowchart for a single-phase modeling procedure.

## 5. Model verification

To demonstrate the validity of the modeling procedure, a 3-phase, 2-winding test transformer was used in the laboratory and modeled using the TLM method described in Section 2. The transformer was loaded with a star-connected R-L load to simulate normal and faulted conditions. As seen in Figure 4, the user defines the following parameters: source frequency, magnetic path length (m), core area $\left(\mathrm{m}^{2}\right)$, number of turns for primary and secondary windings, saturation magnetization $\left(\mathrm{m}^{-1}\right)$, anhysteretic form factor $\left(\mathrm{m}^{-1}\right)$, interdomain coupling coefficient, coercive field magnitude $\left(\mathrm{m}^{-1}\right)$, magnetization weight factor, permeability of free space, leakage inductances of primary and windings, source resistors, load impedances, and load factor.

The performance of the proposed technique was evaluated by root mean square error (RMSE). Figure 5 shows the primary phase currents obtained using the TLM and J-A models.

In this particular example, the 3-phase source voltage is defined as $V_{\text {peak }} \operatorname{Sin}(w t+\phi)\left(1-e^{-15 t}\right)$.

Figure 6 shows the B-H curve of phase a.


Figure 5. Simulated primary phase currents $\left(i_{a}, i_{b}, i_{c}\right)$.


Figure 6. Real and simulated (best-fit) B-H curve of the transformer for phase A. RMSE for H is 7.9803 , for B is $5.8065 \times 10^{-4}$.

To simulate the magnetizing inrush condition in the J-A model, a (zero crossing) source voltage of $\sqrt{2} * 220 * \sin (w * t)$ is applied. Figure 7 shows the typical inrush current of the modeled transformer and Figure 8 shows the real and simulated B-H curve of the transformer.


Figure 7. Real and simulated inrush currents of the modeled transformer for phase A. RMSE $=0.0029$.


Figure 8. Real and simulated B-H curve of the transformer during inrush phenomenon for phase A. RMSE for Ha is 34.94 , for B is 0.0144 .

## 6. Conclusion

This paper introduces a time domain model of a 3-phase, 2-winding transformer with nonlinear and hysteretic behavior. It is a complete 3-phase transformer model in the MATLAB ${ }^{\text {TM }}$ Simulink environment. The user can easily change the parameters by double clicking the "TLM Model of 3-Phase Transformer Model" block. The hysteretic model using the modified Langevin function is based on the J-A model of ferromagnetic hysteresis. The simulation results produce an acceptable transformer transient response and show that the proposed overall technique is ideal for simulating 3 -phase transformers.

The proposed modeling technique displaying hysteresis is suitable for most types of 3 -phase, 2 -windings transformers. The suggested modeling technique is also able to simulate the transformers during single or multiple internal faults.

## Appendix

Data for test transformer:
$\mathrm{U}_{1} / \mathrm{U}_{2}=380 / 220 \mathrm{~V}$, star/star-connected
$f=50 \mathrm{~Hz}$
$\mathrm{B}=1 \mathrm{~T}$
$\mathrm{S}=1 \mathrm{kVA}$
$l_{m}=24.10^{-3} \mathrm{~m}$, magnetic path length
$a=454.10^{-6} \mathrm{~m}^{2}$, core area
$N_{a}=565$, primary turns
$N_{A}=255$, secondary turns
$M_{s}=170000 \mathrm{~m}^{-1}$, saturation magnetization
$H_{a}=42 \mathrm{~m}^{-1}$, anhysteretic form factor
$\alpha=8.10^{-6}$, interdomain coupling coefficient
$H_{c}=78 \mathrm{~A} \mathrm{Tm}^{-1}$, coercive field magnitude
$\beta_{c}=0.55$, magnetization weighting factor
$R_{a}, R_{b}, R_{c}=10 \Omega$, source resistances
$\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} \mathrm{m}^{-1}$, permeability of the free space

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