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A pseudo spot price of electricity algorithm applied to environmental economic active power dispatch problem

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Abstract

In this study, an environmental economic active power dispatch problem is transformed into an optimization problem with a single-objective function by applying the weighted sum method (WSM) and the conic scalarization method (CSM). A pseudo spot price of electricity algorithm (PSPA) is used to solve the transformed problem. This is demonstrated on an example problem, which is a lossy electrical power system having only thermal units

The WSM and CSM methods are applied to the example problem, and the weight factor values of these methods is increased from 0 to 1 by a step size of 0.1. For each possible value of the coefficient, a different total generation cost rate that is to be minimized is obtained, and an optimal solution is also calculated for it (Pareto optimal solutions). Solutions obtained from both methods are not only compared with each other, but also compared with other solutions obtained using different methods such as the first order gradient method, the lambda iteration method, nonlinear programming, linear programming, and fuzzy linear programming.

In this study, the CSM is used for the first time in the transformation of an environmental economic active power dispatch problem into a single-objective optimization problem. Thus, it is shown that the PSPA can also be used in the solution of the environmental economic power dispatch problem.

Key Words: *Environmental economic active power dispatch, pseudo spot price of electricity algorithm, Pareto optimal solutions, weighted sum scalarization, conic scalarization*

1. Introduction

In a traditional economic power dispatch problem, an attempt is made to calculate the amount of the units' active generation power in a considered electric power system so that the total thermal cost rate is kept to a minimum. The active generation power amount should also satisfy the electric constraints in the considered electric power system [1,2].

Presently, environmental pollution created by some types of thermal units has become an important issue. Thermal units, which burn fossil fuel, emit carbon dioxide, sulfur dioxide, nitrogen oxide, and ash. An increase of these emissions in large amounts can result in some deadly environmental effects such as global warming [3,4].

The solution obtained from a traditional economic dispatch cannot be taken as the best one since the environmental costs are not taken into consideration. In order to have a cleaner environment, the amount of emissions produced by the thermal units must be decreased. This can be achieved by using fuels with a lower sulfur content and by adding units to the generation plants that decrease carbon dioxide, sulfur dioxide, nitrogen oxide and ash emissions, together with the use of new dispatch techniques like the one considered above [2,3].

In some optimization problems, there may be more than one objective function to be optimized. None of these objectives are comparable with the others. Generally, in these types of optimization problems, there is no unique solution, but rather a set of solutions. If all of the objectives are taken into consideration, none of the solutions in the solution set can be taken as the best one. These types of solutions are called Pareto optimal solutions [5]. The problem can be considered as a multiobjective optimization problem when both the cost rate function and the emission rate function are to be minimized. In the literature, 2 different approaches are used to solve multiobjective optimization problems. One of these approaches is able to solve multiobjective optimization problems directly, while the other approach transforms the multiobjective optimization problem into a single-objective optimization problem and then applies an appropriate method to solve the single-objective optimization problem. Transforming a multiobjective optimization problem into a single-objective optimization problem using an appropriate conversion is called *scalarization*. Some of the scalarization methods are the weighted sum method (WSM), the ε-constraining method, the elastic constraining method, the Benson scalarization method, the compromise programming method, the conic scalarization method (CSM), and the goal programming method [6-9].

In the literature, genetic (or modified genetic) algorithms and linear programming, which are used directly to solve multiobjective optimization problems, are shown in [1,10,11] and [12], respectively. The hierarchical system approach, fast Newton Raphson algorithm, fuzzy linear programming, and first order gradient method, which are applied after transforming the multiobjective optimization problem into a single-objective optimization problem using the WSM, are also shown in [2], [13], [14], and [15], respectively. The fuzzified multiobjective particle swarm optimization algorithm and multiobjective evolutionary algorithm, which can be applied directly to both a single-objective optimization problem obtained via WSM scalarization and a multiobjective optimization problem, are explained in [3] and [16], respectively. The solution of the problem using scalarization with a combination of the WSM and ε-constraining method can be found in [17]. In [4], a survey of environmental economic dispatch algorithms is given.

In this paper, a solution to a lossy environmental economic power dispatch problem with 2 objective functions is given. First, the optimization problem with the objectives is transformed into an optimization problem with a single-objective using the WSM and the CSM. After that, the pseudo spot price of electricity algorithm (PSPA) solution technique is used to solve the optimization problem.

The WSM has been widely used in the literature for the scalarization of various multiobjective optimization problems [2,3,13-17], whereas the CSM is a new approach compared to the WSM and, to the best of our knowledge, it will be applied to the scalarization of an environmental economic dispatch problem for the first time.

In the literature, the WSM as well as the ε-constraining method are used in the scalarization of the environmental economic power dispatch problem. In this study, it is shown that the CSM can also be applied in addition to the mentioned methods. While the WSM can be used for scalarization of convex functions only, the CSM can be used for both convex and nonconvex functions [6-9]. Furthermore, the requests of system operators can be modeled mathematically using parameters supplied by the CSM, which appears to be an advantage of the CSM. Those parameters supplied by the CSM do not exist in the WSM.

The PSPA solution technique has been utilized for the solutions of various problems. It was used in the solution of an active and reactive power dispatch problem in a lossy electric power system comprising only normal thermal units [18], in the solution of an active generation dispatch problem in a lossy electric power system containing a pumped storage hydraulic unit [19], in the solution of a lossy short-term hydrothermal coordination problem [20], in the solution of an active generation dispatch problem in a lossy electric power system that included normal and limited energy supply thermal units [21], and in the solution of a lossy hydrothermal coordination problem with limited energy supply thermal units [22].

2. Problem statement

The solution to an environmental economic power dispatch problem gives active power generations for all of the generation units, which will minimize the total thermal cost rate and total NO_x emission rate functions together. The solution also satisfies all of the possible electric constraints.

The thermal cost rate (cost per hour) functions of the thermal units in the considered electric power system are taken as follows [12,14,15,17,23]:

$$F_n(P_{G,n}) = a_n + b_n P_{G,n} + c_n P_{G,n}^2, n \in N_G, (R/h). \tag{1}$$

The NO_x emission rate functions of the thermal units in the considered electric power system are also taken as [12,14,15,17]:

$$E_n(P_{G,n}) = d_n + e_n.P_{G,n} + f_n P_{G,n}^2, n \in N_G, (ton/h). \tag{2}$$

The value of $P_{G,n}$ in Eqs. (1) and (2) represents the active power generation (as MW) of the thermal unit that is connected to bus n in the considered electric power system. N_G in Eqs. (1) and (2) also denotes the set containing all of the thermal units in the considered electric power system.

The active power balance constraint is given as follows:

$$\sum_{n \in N_G} P_{G,n} - P_{load} - P_{loss} = 0, \tag{3}$$

where P_{load} and P_{loss} stand for the total active load and loss in the system, respectively.

The active power generation limits of the thermal units are given as below:

$$P_{G,n}^{\min} \leq P_{G,n} \leq P_{G,n}^{\max}, n \in N_G. \tag{4}$$

In this study, the objective function to be minimized consists of the total thermal cost rate and total NO_x emission rate functions. In order to solve this multiobjective optimization problem with the PSPA technique, the WSM and the CSM are used as scalarization methods.

2.1. The weighted sum method (WSM)

This method is one of the oldest and widely used scalarization methods. In this method, scalarization is accomplished by multiplying objective functions with positive coefficients and then summing up the resultant weighted objective function values. In order to use the WSM for scalarization, the multiobjective function should comprise only convex functions [2,3,13-17].

The scalarized objective function of the considered electric power system (SOF_w) *that is to be minimized* is given as:

$$SOF_w = min\left(w\sum_{n\in N_G} F_n(P_{G,n}) + (1-w)\,\gamma \sum_{n\in N_G} E_n(P_{G,n})\right), \tag{5}$$

where γ and w represent a scaling factor and a weight factor $(0 \leq w \leq 1)$, respectively [3,16]. If the value of w is taken as equal to 1, only the total thermal cost rate is considered, but if the value of w is taken as equal to 0, only the total NO_x emission rate is considered in the solution process.

2.2. The conic scalarization method (CSM)

The conic scalarization method (CSM), developed by Gasimov [6-9], transforms objective functions into a single-objective function by combining them without any restrictions on the objective functions and the other constraints. This scalarization method uses support cones in calculating Pareto efficient values.

In the scalarization of the environmental economic power dispatch problem with the CSM, W is defined as below:

$$W = \left\{ (\beta, w) \in R \times R^2 \middle|\, 0 \leq \beta < \min(w_1, w_2), w_1 > 0, w_2 > 0 \right\}. \tag{6}$$

In that case, the objective function of the scalarized environmental economic power dispatch problem is given as shown in Eq. (7).

$$SOF_{w,\beta} = min\left\{\beta\sum_{i=1}^{2}|F_i(x) - B_i| + \sum_{i=1}^{2} w_i\left(F_i(x) - B_i\right)\right\} \tag{7}$$

Here, parameters B_1 and B_2 can be randomly chosen between the neighbor-supported Pareto optimal points, which correspond to the weights assigned by the decision maker and are relatively distant from each other. After calculating additional efficient points, the intervals used to choose nonzero values for B_1 and B_2 in Eq. (7) can be narrowed successively and, thus, the decision maker may find other nonsupported efficient solutions if any exist. In Eq. (7), the vertex of cone β is chosen in such a way that $0 \leq \beta < \min(w_1, w_2)$. If $\beta = 0$ in Eq. (7), the CSM is simplified to the WSM [6-9].

In this study, the objective function of the environmental economic power dispatch problem scalarized by the CSM is formulized by taking $w_1 = w$, $w_2 = 1 - w$, $F_1(x) = \sum_{n\in N_G} F_n(P_{G,n})$, $F_2(x) = \sum_{n\in N_G} E_n(P_{G,n})$, $B_1 = CTTCR$, and $B_2 = CTER$ in Eq. (7). In the equations, the chosen total thermal cost rate ($CTTCR$) (as R/h) and chosen total NO_x emission rate ($CTER$) (as t/h) are arbitrary fixed values, which are chosen among points on the Pareto optimal surface. The way to find these values is explained in Section 4. The objective function of the environmental economic power dispatch problem scalarized by the CSM is given as a scalarized objective function ($SOF_{w,\beta}$) in Eq. (8).

$$SOF_{w,\beta} = min\ \{\beta\,[|NTCR(P_{G,n})| +\ \gamma\ \ |NTER(P_{G,n})|] + wNTCR(P_{G,n}) + (1-w)\,\gamma\, NTER(P_{G,n})\} \tag{8}$$

The net total cost rate ($NTCR(P_{G,n})$) and net total NO_x emission rate ($NTER(P_{G,n})$) values in the above equation are defined below.

$$\begin{aligned} NTCR(P_{G,n}) &= \sum_{n\in N_G} F_n(P_{G,n}) - CTTCR \ , \quad (R/h) \\ NTER(P_{G,n}) &= \sum_{n\in N_G} E_n(P_{G,n}) - CTER \ , \quad (ton/h) \end{aligned} \tag{9}$$

In Eq. (8), the vertex of the cone, β, is chosen in such a way that $0 \le \beta < \min[w,(1-w)]$ inequality is satisfied.

3. The PSPA technique

An AC load flow is solved using active generations determined in the last iteration (at the beginning, using chosen initial active generations satisfying Eq. (4)). Active generation of the unit connected to the swing bus, active power flows, and losses on each line in the system are obtained from this AC load flow calculation [18-22]. The scalarized objective function values are then calculated using the WSM and the CSM, respectively, as follows:

$$SOF_w^{old} = w \sum_{n\in N_G} F_n(P_{G,n}^{old}) + (1-w)\,\gamma \sum_{n\in N_G} E_n(P_{G,n}^{old}), \tag{10}$$

$$SOF_{w,\beta}^{old} = \beta\left[\left|NTCR(P_{G,n}^{old})\right| + \ \gamma\ \left|NTER(P_{G,n}^{old})\right|\right] + wNTCR(P_{G,n}^{old}) + (1-w)\,\gamma\, NTER(P_{G,n}^{old}). \tag{11}$$

The incremental scalarized objective function values of each thermal unit are also calculated using the WSM and the CSM, respectively, as below:

$$\lambda_n = w\frac{dF_n(P_{G,n})}{dP_{G,n}} + (1-w)\,\gamma\ \frac{dE_n(P_{G,n})}{dP_{G,n}}, \tag{12}$$

$$\lambda_n = (\beta + w)\frac{dF_n(P_{G,n})}{dP_{G,n}} + (\beta + 1 - w)\,\gamma\ \frac{dE_n(P_{G,n})}{dP_{G,n}}. \tag{13}$$

The amount of active power that is sold or bought by a bus is taken as the amount of active power flows entering or leaving the considered bus. They are fictitious values and used only to find better active generations giving smaller total scalarized objective function values as the iterative calculation procedure proceeds. The active power transmitted between any 2 buses is represented as TP_{ik}. Two indices indicate that active power flow is from bus i to bus k. The second index also shows that TP_{ik} is the active power flow at the border of bus k. According to this notation, $-TP_{ki}$ represents the active power flow going from bus i to bus k at the border of bus i. The pseudo spot active power price of bus i at the border of bus k is designated as SP_{ik}. It is calculated as:

$$SP_{ik} = \lambda_i\left(1 + \frac{-TP_{ki} - TP_{ik}}{TP_{ik}}\right)\ . \tag{14}$$

In the calculation of SP_{ik} seen in Eq. (14), the incremental scalarized objective function of bus i (λ_i) is increased according to the active loss *percentage* that occurs during the transmission of active power from bus $i$to bus k. Thus, the inclusion of active transmission losses into the solution procedure has been made possible.

In the proposed dispatch algorithm, it is assumed that there is a mechanism at every bus of the system that decides how much active power is to be bought from the other buses. In the determination of the new active power amount that is to be bought, the incremental scalarized objective function of the bus that buys the active power (λ_k) and the pseudo spot price of active bought power (SP_{ik}) are used according to:

$$TP_{ik}^{new} = \left[1 + \alpha\left(1 + \frac{\lambda_k - SP_{ik}}{\lambda_k}\right)\right] TP_{ik}^{old}. \tag{15}$$

If the incremental scalarized objective function of bus k is higher or lower than the spot price of active power bought from bus i, the new active power bought from bus i becomes higher or lower than its old value. To reduce the number of iterations, for the active generations that are far from their optimal values, α is taken as equal to 1. If an increase, instead of a decrease, in the total scalarized objective function rate occurs, α is decreased by a specific amount and used in the next iteration.

If there is a bus to which no generating unit is connected, the incremental scalarized objective function of such a bus should be calculated before calculating the new active power amount bought by it. The incremental scalarized objective function of such a bus is calculated as the weighted average of spot prices of bought active power with respect to bought active power.

$$\lambda_k = \frac{\displaystyle\sum_{i \in \left\{\begin{array}{c}\text{All buses from which}\\ \text{bus } k \text{ buys active power}\end{array}\right\}} TP_{ik} SP_{ik}}{\displaystyle\sum_{i \in \left\{\begin{array}{c}\text{All buses from which}\\ \text{bus } k \text{ buys active power}\end{array}\right\}} TP_{ik}} \tag{16}$$

If bus k, where there is no active generation, is connected to another bus g, where there is also no active generation ($i = g$ in Eq. (16)), the incremental scalarized objective function of bus g should have already been calculated using Eq. (16) before calculating λ_k. Therefore, the incremental scalarized objective function values of the buses at which there is no active generation have to be calculated *in a specific order.*

After finding all of the bought active powers for all of the buses of the system, the active powers sent from the other ends of the transmission lines (sold active powers) can *approximately* be calculated by adding the *old* active transmission losses ($P_{loss\ ik}^{old}$, obtained from the last load flow calculation) to the respective active powers bought.

$$-TP_{ki}^{new} = TP_{ik}^{new} + P_{loss\ ik}^{old} \tag{17}$$

After the calculation of bought and sold powers at each bus in the system, for buses whose active power balances are distorted, new active generations are calculated to reestablish their active power balance according to:

$$P_{G\,k}^{new} = P_{load\ k} - \sum_{a \in \{\text{All buses connected to bus } k\}} TP_{ak}^{new}, \tag{18}$$

where $P_{load\ k}$ is the active load value connected to bus k. The bought and sold active powers by bus k are taken as *positive* and *negative* quantities in the above summation, respectively. If there is no active generation at the considered bus, the active power imbalance of this bus ($P_{error\ k}$) is made 0 by correcting its bought active power according to the spot price of power at its border. The following equation gives the power imbalance of such a bus:

$$P_{error\ k} = P_{load\ k} - \sum_{b \in \{\text{All buses connected to bus } k\}} TP_{bk}^{new}. \tag{19}$$

If $P_{error\,k} > 0$, bus k needs to buy *extra* active power by the amount of $P_{error\ k}$. Therefore, the active power bought by bus k is *increased inversely proportional* to the spot price of the corresponding bought active power.

$$TP_{dk}^{new,\,corrected} = TP_{dk}^{new} + \frac{(1/SP_{dk})P_{error\ k}}{\sum\limits_{d \in \left\{ \substack{\text{All buses from which} \\ \text{bus } k \text{ buys active power}} \right\}} 1/SP_{dk}},\ P_{error\ k} > 0 \tag{20}$$

If $P_{error\ k} < 0$, the active power bought by bus k needs to be *decreased* by the amount of bus k's power imbalance. Therefore, the active power bought by bus k is decreased *proportional* to the spot price of the corresponding bought active power.

$$TP_{dk}^{new,\,corrected} = TP_{dk}^{new} + \frac{(SP_{dk})P_{error\ k}}{\sum\limits_{d \in \left\{ \substack{\text{All buses from which} \\ \text{bus } k \text{ buys active power}} \right\}} SP_{dk}},\ P_{error\ k} < 0 \tag{21}$$

After calculation of the corrected generations at each bus, the active generation of each unit is checked against its generation limits. If a bus where a generating unit violating one of its active generation limits is connected buys active power from the other buses, that bus is considered as if it were a bus where there is no active generation. The active generation at this bus is also taken as equal to the violated limit value. If a bus where a generating unit that violates one of its active generation limits is connected *never* buys any active power from the other buses, the adjustment of the incremental scalarized objective function value of this unit is done according to:

$$\lambda_k^{new} = \left[\theta_k \left(\frac{P_{load\ k} - \sum\limits_{d \in \left\{ \substack{\text{All buses to which} \\ \text{bus } k \text{ sells active power}} \right\}} TP_{dk} - P_{G,k}^{limit}}{P_{G,k}^{limit}} \right) + 1 \right] \lambda_k(P_{G,k}^{limit}), \quad 0 < \theta_k \leq 1. \tag{22}$$

TP_{dk} in Eq. (22) represents the active power sold by bus k, and they are taken as negative quantities in the summation. $P_{G,k}^{\text{limit}}$ in Eq. (22) also denotes the violated active generation limit of the thermal unit connected to bus k. To change λ_k^{new} by a small amount, variable θ_k is used in Eq. (22). If the active generation of the thermal unit connected to bus $k$exceeds its upper generation limit, the expression inside the inner bracket in Eq. (22) becomes positive, and therefore λ_k^{new} becomes greater than $\lambda_k(P_{G,k}^{\text{limit}})$. Because of this, the active power sold by bus k decreases and, consequently, the active generation of the thermal unit connected to bus k approaches its upper generation limit. If the thermal unit's generation becomes smaller than its lower limit, that expression inside the inner bracket in Eq. (22) becomes negative and, therefore, λ_k^{new} becomes lower than $\lambda_k(P_{G,k}^{\text{limit}})$. Therefore, the active power sold by bus k increases and the active generation of the thermal unit connected to bus k approaches its lower limit.

The effect of corrections on bought active power at buses where there is no active power generation is reflected to the other buses, and the new corrected active generations are determined. An AC load flow calculation is made with the new corrected active generations, and new total scalarized objective function values, SOF_w^{new} or $SOF_{w,\beta}^{new}$, are calculated by using the WSM and the CSM, respectively, as follows:

$$SOF_w^{new} = w \sum_{n \in N_G} F_n(P_{G,n}^{new}) + (1-w)\,\gamma \sum_{n \in N_G} E_n(P_{G,n}^{new}), \tag{23}$$

$$SOF_{w,\beta}^{new} = \beta\left[\left|NTCR(P_{G,n}^{new})\right| +\ \gamma\ \left|NTER(P_{G,n}^{new})\right|\right] + wNTCR(P_{G,n}^{new}) + (1-w)\ \gamma NTER(P_{G,n}^{new}). \qquad (24)$$

After that, the stopping criteria are given as:

$$\begin{array}{ll} \Delta SOF \leq TOL_{\Delta SOF} & \text{if} \quad \Delta SOF \geq 0 \\ \alpha < \alpha_{\min} & \text{if} \quad \Delta SOF < 0, \end{array} \qquad (25)$$

where:

$$\Delta SOF = SOF_{w}^{old} - SOF_{w}^{new} \quad \text{or} \quad \Delta SOF = SOF_{w,\beta}^{old} - SOF_{w,\beta}^{new} \qquad (26)$$

is tested. $TOL_{\Delta SOF} > 0$ in Eq. (25) is a chosen tolerance value for ΔSOF. Testing of the stopping criteria is done as follows. If $\Delta SOF \geq 0$, then the first inequality in Eq. (25) is checked. If it is not satisfied, a new iteration is initiated with the newly determined active generations. If $\Delta SOF < 0$, then the second inequality is checked. If it is not satisfied, the value of α is decreased by a specific amount, and a new iteration is initiated with the last active generations that have given $TOL_{\Delta SOF} > 0$. If the second inequality in Eq. (25) is satisfied, the last active generations, which have given $\Delta SOF > 0$, are taken as solution values [18-22].

The optimal total thermal cost rate ($TTCR$) and total NO_x emission rate (TER) values are calculated by using those solution values. The values of $TTCR$ and TER are calculated via the following equations.

$$TTCR = \sum_{n \in N_G} F_n(P_{G,n}), (R/h) \qquad (27)$$

$$TER = \sum_{n \in N_G} E_n(P_{G,n}), (ton/h) \qquad (28)$$

4. Illustrative example and results

An example power system (10-bus, 5-generator model system [14]) of the environmental economic dispatch problem is shown in Figure 1. First, the environmental economic dispatch problem is transformed into a single-objective optimization problem via 2 different scalarization methods. The 2 resulting single-objective optimization problems are then solved by the PSPA. Transmission lines' equivalent π circuit pu parameters, S_{base} and U_{base} values, and unit types are also shown in Figure 1. In Table 1, the chosen pu load schedule for the example power system is given. Table 2 shows the cost rate coefficients, NO_x emission rate curve coefficients, and active generation limits of the thermal units. In Table 3, the selected initial pu active generations and reactive generation limits are given.

The scaling factor value in Eqs. (5) and (8) is taken as $\gamma = 1000$. Tolerance values in Eqs. (25) and (26) are chosen as $\alpha^{\min} = 0.25$ and $TOL_{\Delta SOF} = 1.0 \times 10^{-4}$. For some selected w values in Eqs. (5) and (8), active generations, total thermal cost rates, and total NO_x emission rates at the solution points when the WSM is used are given in Table 4.

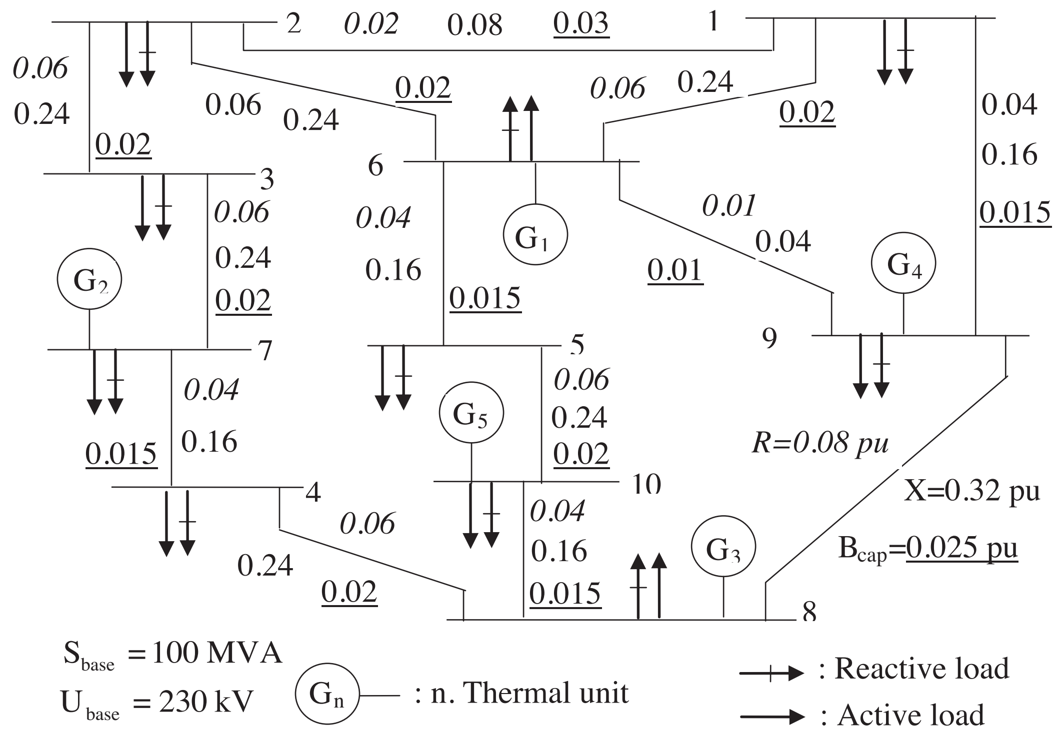

Figure 1. One-line diagram for the example power system [14].

Table 1. The chosen *pu* load schedule for the example power system [14].

Bus number	1	2	3	4	5	6	7	8	9	10
$P_{load,n}$	0.20	0.30	0.20	0.30	0.20	0.30	0.15	0.20	0.20	0.20
$Q_{load,n}$	0.097	0.145	0.097	0.145	0.097	0.145	0.0726	0.097	0.097	0.097
Bus type	1					2				3
Bus voltage	Not specified					1.0				1.05

Note: 1: PQ bus, 2: PV bus, and 3: swing bus.

Table 2. Cost rates, NO_x emission rate curve coefficients, and active generation limits of the thermal units [14].

Coefficients		Thermal unit number (n)				
		1	2	3	4	5
Cost	a_n	27.0	35.0	29.0	31.0	28.0
	b_n	0.06	0.10	0.05	0.08	0.06
	c_n	1.0E-4	1.0E-4	5.0E-5	1.0E-4	5.0E-5
Emission	d_n	4.258E-2	4.091E-2	5.326E-2	2.543E-2	5.326E-2
	e_n	–5.094E-4	–5.554E-4	–3.55E-4	–6.047E-4	–3.550E-4
	f_n	4.586E-6	6.490E-6	3.380E-6	5.5638E-6	3.380E-6
Generation limits	$P_{G,n}^{min}$ (MW)	5.0	5.0	5.0	5.0	5.0
	$P_{G,n}^{max}$ (MW)	150.0	150.0	150.0	150.0	150.0

Table 3. Selected initial *pu* active generations and reactive generation limits.

Thermal unit number, (n) number, (n)	$P_{G,n}^{init}$	$Q_{G,n}^{\min}$	$Q_{G,n}^{\max}$
1,2,3,4	0.400	–0.400	+0.400

In the case where the WSM is used, when the value of w in Eq. (5) is taken as equal to 1 (the NO_x emission rates are ignored), the total thermal cost rate is found as 165.0748 R/h. The NO_x emission rate in this case becomes 189.9477 *kg/h*. Once the value of w is increased from 0.0 to 1.0 by 0.1, the total thermal cost rate decreases, whereas the total NO_x emission rate increases. When the value of w is taken as equal to 0 (the thermal cost rates are ignored), the total thermal cost rate and the total NO_x emission rate become 166.5448 R/h and 155.1652 *kg/h*, respectively. As the value of w changes from 0.0 to 1.0, the changes in the total thermal cost and in the total NO_x emission rate are obtained as 1.470 R/h (decrease) and 34.7825 *kg/h* (increase), respectively.

Table 4. The *pu* active generations, total thermal cost rate, and total NO_x emission rate values at the solution point using the WSM for some selected w values.

w	$P_{G,1}^{sol.}$	$P_{G,2}^{sol.}$	$P_{G,3}^{sol.}$	$P_{G,4}^{sol.}$	$P_{G,5}^{sol.}$	$Q_{G,5}^{sol.}$	$TTCR(R/h)$	$TER(kg/h)$
1.0	0.500049	0.328211	1.237978	0.050000	0.176734	0.618749	165.0748	189.9477
0.9	0.530387	0.218716	0.646649	0.384459	0.496944	0.523460	165.4696	159.1388
0.8	0.507075	0.304472	0.546327	0.436201	0.478726	0.526503	165.9847	156.1619
0.7	0.500539	0.330362	0.508934	0.458272	0.473524	0.527270	166.1702	155.5963
0.6	0.499053	0.348053	0.486895	0.471203	0.465697	0.528824	166.2923	155.3569
0.5	0.498019	0.360496	0.472964	0.478997	0.459971	0.530006	166.3747	155.2509
0.4	0.497626	0.368807	0.463786	0.483873	0.456069	0.530818	166.4295	155.2041
0.3	0.496726	0.374935	0.457015	0.487885	0.453405	0.531373	166.4709	155.1809
0.2	0.496425	0.379458	0.452018	0.490472	0.451179	0.531842	166.5014	155.1704
0.1	0.496394	0.383008	0.448103	0.492639	0.449568	0.532180	166.5247	155.1662
0.0	0.495581	0.386007	0.444742	0.494484	0.448817	0.532329	166.5448	155.1652

For some selected w and β values, the corresponding solution points when the CSM is used are shown in Table 5. In order to determine the $CTTCR$ and $CTER$ values in Eq. (9), solutions are calculated by taking $w = 0.0$ and 1.0, since inequality $(0 \le \beta < \min[w,(1-w)])$ exists for β in Eq. (8). Therefore, $\beta = 0.0$ is taken when $w = 0.0$ or 1.0. On the other hand, for $\beta = 0.0$, the CSM is simplified to the WSM. In this context, when $CTTCR = CTER = 0.0$ is taken in Eq. (9), Eq. (8) becomes exactly equal to Eq. (5). When w is taken as 0 or 1, $CTTCR$ and $CTER$ should thus be taken as 0. However, in the solutions where w takes a value between 0.1 and 0.9, the $CTTCR$ and $CTER$ values are calculated via the following equations:

$$CTTCR = \sum_{n \in N_G} F_n(P_{G,n}),\ (R/h) \text{ for } \beta = 0.0,\ w = 1.0 in \min\ SOF_{w,\beta}; \tag{29}$$

$$CTER = \sum_{n \in N_G} E_n(P_{G,n}),(t/h) \text{ for } \beta = w = 0.0 in \min\ SOF_{w,\beta}. \tag{30}$$

In the solutions for different values of w, where w is increased from 0.1 to 0.9 by 0.1, the $CTTCR$ and $CTER$ values are taken as 165.07 R/h and 0.15516 t/h, respectively (see Table 5). When various values within the limits defined in Eq. (6) are assigned to β, different solution points can be obtained. This gives system operators the opportunity of selecting different operation points. In scalarization with the CSM, when w changes from 0.1 to 0.9, β changes as 0.01, 0.05, 0.10, 0.15, 0.20, ..., etc. As can be seen in Table 5, in contrast to the WSM, different running points related to β are obtained for the same w values in the CSM. For instance, when $w = 0.5$ in scalarization with the WSM in Table 4, it is obtained that $TTCR = 166.3747\ R/h$

and $TER = 155.2509\ kg/h$, whereas in scalarization with the CSM, 11 different solution points are found for changing values of β from 0.01 to 0.49 in Table 5. This is also the same for other values of w.

Table 5. The pu active generations, total thermal cost rate, and total NO_x emission rate values at the solution point by using the CSM for some selected w and β values.

w	β	$P_{G,1}^{sol.}$	$P_{G,2}^{sol.}$	$P_{G,3}^{sol.}$	$P_{G,4}^{sol.}$	$P_{G,5}^{sol.}$	$Q_{G,5}^{sol.}$	$TTCR(R/h)$	$TER(kg/h)$
1.0	0.0	0.500049	0.328211	1.237978	0.050000	0.176734	0.618749	165.0748	189.9477
0.9	0.01	0.530571	0.217072	0.649267	0.382771	0.497594	0.523354	165.4579	159.2336
	0.05	0.531206	0.205926	0.656829	0.378663	0.505313	0.521703	165.4032	159.6727
	0.09	0.532260	0.189574	0.670699	0.374786	0.511577	0.520445	165.3260	160.3537
0.8	0.01	0.506729	0.303693	0.547400	0.436215	0.478789	0.526501	165.9806	156.1770
	0.05	0.508175	0.300334	0.551843	0.434027	0.478597	0.526587	165.9590	156.2600
	0.10	0.509335	0.296630	0.557052	0.431223	0.478906	0.526573	165.9339	156.3612
	0.15	0.510770	0.292758	0.562349	0.428652	0.478788	0.526654	165.9089	156.4674
	0.19	0.512778	0.289363	0.566823	0.426279	0.478212	0.526824	165.8873	156.5640
0.7	0.01	0.500259	0.329813	0.509675	0.457835	0.474079	0.527154	166.1661	155.6061
	0.05	0.501013	0.327104	0.512983	0.456319	0.474344	0.527124	166.1489	155.6479
	0.10	0.501406	0.324111	0.516877	0.453848	0.475655	0.526869	166.1278	155.7028
	0.15	0.501989	0.321057	0.520793	0.451718	0.476470	0.526725	166.1073	155.7588
	0.20	0.503448	0.317875	0.524758	0.449287	0.476783	0.526689	166.0894	155.8200
	0.25	0.504313	0.314744	0.528731	0.447392	0.477097	0.526657	166.0658	155.8807
	0.29	0.504754	0.312544	0.531684	0.445565	0.477834	0.526523	166.0504	155.9296
0.6	0.01	0.499279	0.347374	0.487635	0.470334	0.466306	0.528693	166.2869	155.3656
	0.05	0.499523	0.344858	0.490454	0.469426	0.466753	0.528614	166.2719	155.3898
	0.10	0.499932	0.341831	0.493894	0.467112	0.468368	0.528280	166.2511	155.4263
	0.15	0.499970	0.338729	0.497438	0.465721	0.469394	0.528079	166.2319	155.4624
	0.20	0.500181	0.336062	0.500624	0.463822	0.470674	0.527821	166.2138	155.4989
	0.25	0.500950	0.333397	0.503822	0.461812	0.471486	0.527666	166.1956	155.5373
	0.30	0.501107	0.330789	0.506950	0.460425	0.472299	0.527512	166.1790	155.5746
	0.35	0.501476	0.328235	0.510048	0.458802	0.473114	0.527357	166.1622	155.6141
	0.39	0.502004	0.326258	0.512513	0.457112	0.473871	0.527210	166.1483	155.6484
0.5	0.01	0.498117	0.359929	0.473582	0.478733	0.460104	0.529980	166.3712	155.2545
	0.05	0.498613	0.357977	0.475704	0.477222	0.461018	0.529789	166.3578	155.2692
	0.10	0.498684	0.355450	0.478439	0.475959	0.462089	0.529567	166.3418	155.2880
	0.15	0.499319	0.352915	0.481201	0.474154	0.463120	0.529356	166.3247	155.3100
	0.20	0.499460	0.350310	0.484042	0.472862	0.464129	0.529150	166.3084	155.3327
	0.25	0.499527	0.347865	0.486717	0.471622	0.465159	0.528939	166.2929	155.3558
	0.30	0.499991	0.345399	0.489467	0.469982	0.466142	0.528739	166.2766	155.3818
	0.35	0.500059	0.342924	0.492185	0.468722	0.467185	0.528527	166.2610	155.4083
	0.40	0.500912	0.340386	0.495039	0.467135	0.467690	0.528435	166.2446	155.4377
	0.45	0.500511	0.337886	0.497839	0.465817	0.469215	0.528117	166.2286	155.4685
	0.49	0.500877	0.335960	0.500063	0.464475	0.469969	0.527967	166.2157	155.4944
0.4	0.01	0.497245	0.368447	0.464144	0.483698	0.456647	0.530691	166.4271	155.2059
	0.05	0.497587	0.366661	0.466094	0.482815	0.457078	0.530607	166.4158	155.2141
	0.10	0.498114	0.364675	0.468257	0.481189	0.458066	0.530399	166.4018	155.2254
	0.15	0.498549	0.362505	0.470641	0.480096	0.458578	0.530300	166.3881	155.2376
	0.20.	0.498557	0.360379	0.472941	0.478996	0.459569	0.530093	166.3745	155.2510
	0.25	0.498505	0.358277	0.475207	0.477994	0.460533	0.529893	166.3613	155.2652
	0.30	0.498991	0.356124	0.477525	0.476414	0.461535	0.529684	166.3466	155.2820
	0.35	0.498949	0.354044	0.479767	0.475381	0.462525	0.529478	166.3335	155.2984
	0.39	0.499314	0.352477	0.481503	0.474534	0.462889	0.529408	166.3235	155.3114

Table 5. Continued.

w	β	$P^{sol.}_{G,1}$	$P^{sol.}_{G,2}$	$P^{sol.}_{G,3}$	$P^{sol.}_{G,4}$	$P^{sol.}_{G,5}$	$Q^{sol.}_{G,5}$	$TTCR(R/h)$	$TER(kg/h)$
0.3	0.01	0.496789	0.374573	0.457415	0.487686	0.453515	0.531351	166.4685	155.1819
	0.05	0.497073	0.373187	0.458941	0.486912	0.453906	0.531273	166.4595	155.1862
	0.10	0.496954	0.371299	0.460924	0.485589	0.455318	0.530967	166.4465	155.1934
	0.15	0.497309	0.369352	0.463054	0.484627	0.455800	0.530873	166.4342	155.2009
	0.20	0.497673	0.367605	0.464984	0.483678	0.456256	0.530784	166.4229	155.2086
	0.25	0.498066	0.365727	0.467002	0.482254	0.457211	0.530582	166.4099	155.2186
	0.29	0.498367	0.364340	0.468544	0.481503	0.457548	0.530517	166.4010	155.2259
0.2	0.01	0.496484	0.379140	0.452370	0.490569	0.451269	0.531825	166.4994	155.1709
	0.05	0.496781	0.377907	0.453698	0.489425	0.452060	0.531654	166.4902	155.1736
	0.10	0.497070	0.376358	0.455406	0.488573	0.452509	0.531564	166.4802	155.1771
	0.15	0.496866	0.374648	0.457227	0.487834	0.453397	0.531376	166.4694	155.1816
	0.19	0.497101	0.373381	0.458623	0.487146	0.453758	0.531304	166.4612	155.1854
0.1	0.01	0.496443	0.382721	0.448418	0.492482	0.449656	0.532163	166.5229	155.1664
	0.05	0.496155	0.381605	0.449606	0.491991	0.450402	0.532002	166.5157	155.1675
	0.09	0.496356	0.380488	0.450840	0.491372	0.450734	0.531935	166.5084	155.1688
0.0	0.0	0.495581	0.386007	0.444742	0.494484	0.448817	0.532329	166.5448	**155.1652**

The environmental economic power dispatch problem, scalarized by both the WSM and the CSM, is solved by the PSPA. The resulting optimal solution values for different w (0.1∼0.9) values are shown in Figure 2.

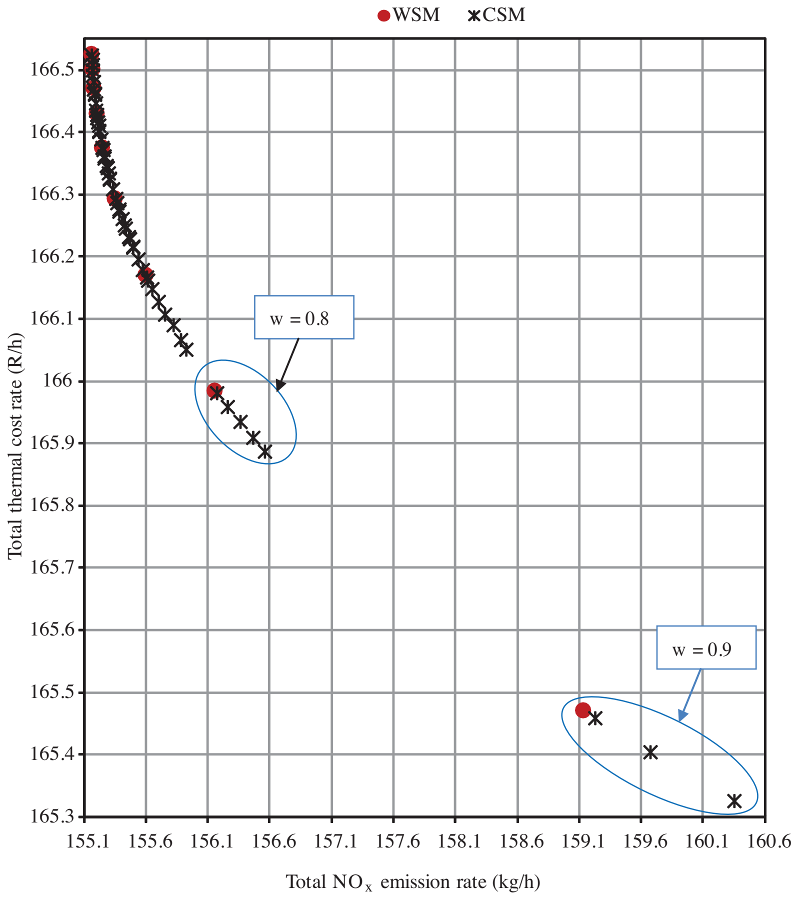

Figure 2. The effect of the w value on the total thermal cost rate and the total NO_x emission rate when the WSM and the CSM are used ($w = 0.1 \sim 0.9$).

The solution of the same example power system's dispatch problem is given (by not considering the transmission losses) in [14] via 4 different solution methods. This dispatch problem is also solved using the first order gradient method (FOGM) and WSM (by considering the transmission losses) for comparison. The solution methods used in [14] are the lambda iteration method (LMDIM), nonlinear programming (NLP), linear programming (LP), and fuzzy linear programming (FLP). The results obtained from those solution methods and the PSPA technique are given in Table 6, where the minimum total thermal cost rate (w = 1.0) and the minimum total emission rate (w = 0.0) are shown for each method.

Table 6. Minimization of each objective by different methods.

<table>
<tr><th rowspan="2">System loss</th><th rowspan="2" colspan="2">Method</th><th colspan="3">Minimum total cost rate ($w = 1.0$)</th><th colspan="3">Minimum total emission rate ($w = 0.0$)</th></tr>
<tr><th>$TTCR$ (R/h)</th><th>TER (t/h)</th><th>P_{loss} (MW)</th><th>$TTCR$ (R/h)</th><th>TER (t/h)</th><th>P_{loss} (MW)</th></tr>
<tr><td rowspan="4">Ignored</td><td colspan="2">LMDIM</td><td>163.5695</td><td>0.2117</td><td rowspan="4">0.000</td><td>166.4380</td><td>0.1554</td><td rowspan="4">0.000</td></tr>
<tr><td colspan="2">NLP</td><td>163.5695</td><td>0.2117</td><td>166.3943</td><td>0.1555</td></tr>
<tr><td colspan="2">LP</td><td>163.9523</td><td>0.1820</td><td>166.4388</td><td>0.1554</td></tr>
<tr><td colspan="2">FLP</td><td>163.8270</td><td>0.1815</td><td>166.3740</td><td>0.1554</td></tr>
<tr><td rowspan="3">Considered</td><td colspan="2">FOGM (WSM)</td><td>165.3487</td><td>0.1755</td><td>4.867</td><td>166.5492</td><td>0.1552</td><td>1.966</td></tr>
<tr><td rowspan="2">PSPA</td><td>WSM</td><td>165.0748</td><td>0.1899</td><td rowspan="2">4.477</td><td>166.5448</td><td>0.1552</td><td rowspan="2">1.963</td></tr>
<tr><td>CSM</td><td>165.0748</td><td>0.1899</td><td>166.5448</td><td>0.1552</td></tr>
</table>

As we see from Table 6, the PSPA method gives the smallest minimum total NO_x emission rate (w = 0.0) although it considers the system transmission loses. However, the minimum total cost rate (w = 1.0) appears high in the PSPA method when compared with the others methods where the system transmission losses are ignored. While the system transmission losses are being considered, relatively better results are obtained with the PSPA method compared with the FOGM.

As is seen from Table 6, the minimum total cost rate (w = 1.0) is obtained as 163.5695 R/h for both the LMDIM and NP methods in the lossless case. The minimum total emission rate (w = 0.0) is obtained as 0.1554 t/h for the LMDIM, LP, and FLP methods. In the lossy case, when the FOGM is used, the minimum total cost rate and the minimum total NO_x emission rate are obtained as 165.3486 R/h and 0.1552 t/h, respectively. When the WSM with the PSPA and the CSM with the PSPA are used for the lossy case, the same minimum total cost rate (165.0748 R/h) and the same minimum total NO_x emission rate (0.1552 t/h) are obtained for both methods.

In Figure 2, it is obvious that the solutions obtained using the PSPA and the CSM are the same as the solutions obtained via the PSPA and the WSM. Since the considered optimization problem is a convex one, the superiority of the CSM, which can scalarize both convex and nonconvex problems, cannot be exhibited in the considered example.

From Figure 2, it can be seen that different values are obtained for different values of β in the CSM, whereas only a unique value is obtained in the WSM for the same condition. For w = 0.0 and w = 1.0, the solution to the *SOF*, which is obtained using the WSM and the CSM via the PSPA method, gives the same solution points; therefore, those points for w = 0.0 and w = 1.0 are not shown in Figure 2. However, different values obtained for w = 0.1 and w = 0.9 are shown in the graph. Thus, this gives the opportunity of selection to system operators.

5. Conclusion

In this study, an environmental economic power dispatch problem was transformed into an optimization problem with a single-objective function via the WSM and the CSM. The PSPA technique was applied to minimize the single-objective function scalarized by both methods. The solution technique was tested on an example electric power system whose environmental economic dispatch problem was solved previously by some different methods in the literature.

In the solution process, the value of w was increased from 0.0 to 1.0 by 0.1. The total thermal cost rate and total NO_x emission rate values at solution points for each value of w were given for the cases where the WSM and the CSM scalarization methods were used. The solution values giving the minimum total thermal cost rate ($w = 1.0$) and the minimum total NO_x emission rate ($w = 0.0$) were compared with those calculated using some other solution methods stated in Table 6.

This study shows the application of the CSM to the scalarization of the environmental economic power dispatch problem as an alternative to the WSM. To the best of our knowledge, the CSM has not been used in the scalarization of the environmental economic power dispatch problem.

List of symbols

R	A fictitious monetary unit
$SOF_w, SOF_{w,\beta}$	Scalarized objective function of the WSM and the CSM, respectively
$TTCR$	Total thermal cost rate (R/h)
TER	Total NO_x emission rate (t/h)
$CTTCR$	Chosen total thermal cost rate (R/h)
$CTER$	Chosen total NO_x emission rate (t/h)
$NTCR(P_{G,n})$	Net total cost rate (R/h)
$NTER(P_{G,n})$	Net total NO_x emission rate (t/h)
$P_{G,n}$	Active generation of the nth thermal unit (MW)
$P_{G,n}^{sol.}$	Solution point active generation of the nth thermal unit (MW)
$Q_{G,1}^{sol.}$	Solution point reactive generation of the unit connected to the swing bus (MW)
$F_n(P_{G,n})$	Thermal cost rate of the nth thermal unit (R/h)
$E_n(P_{G,n}) = NO_x$	emission rate of the nth thermal unit (t/h)
γ, w	Scaling factor and weight factor, respectively
β	The vertex of a cone, $\{0 \leq \beta < \min[w, (1-w)]\}$
P_{load} , P_{loss}	Total system active load and loss, respectively (MW)
$P_{G,n}^{\min}$, $P_{G,n}^{\max}$	Lower and upper active generation limits of the nth thermal unit, respectively (MW)
N_G	Set of all of the thermal units in a given power system

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