Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2013) 21: 90 – 106
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doi:10.3906/elk-1104-36

Composite power system adequacy assessment based on postoptimal analysis

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Received: 30.04.2011 • **Accepted:** 18.09.2011 • **Published Online:** 27.12.2012 • **Printed:** 28.01.2013

Abstract: The modeling and evaluation of enormous numbers of contingencies are the most challenging impediments associated with composite power system adequacy assessment, particularly for large-scale power systems. Optimal power flow (OPF) solution, as a widely common approach, is normally employed to model and analyze each individual contingency as an independent problem. However, mathematical representations associated with diverse states are slightly different in one or a few generating units, line outages, or trivial load variations. This inherent attribute brings a promising idea to speed up the contingency evaluation procedure. In this paper, postoptimal analysis (POA), as a well-recognized technique to attack a set of similar problems with minimal effort, is adopted to solve the contingency OPFs. Instead of solving all similar problems independently, POA exploits the similarity among them and accelerates the solving procedure. The proposed method here derives a base case model and obtains its respective solution. Thereafter, the solution associated with a contingency is determined by imposing differences due to unit or line outages or load variation with respect to the base case solution. The proposed approach is applicable in both sorts of analytical and simulation-based evaluation methods. The Roy Billinton test system, the IEEE reliability test system, and the IEEE 118-bus test system are used to demonstrate the performance and efficiency of the proposed approach. A significant reduction in the computational efforts is experienced.

Key words: Composite power system, linear programming, postoptimal analysis, reliability assessment, simplex method

1. Introduction

The reliability assessment of power systems based on probabilistic notions and measures is of particular interest for both the power system operating and planning phases. The evaluation incorporating both the functional zones of generation and transmission systems is referred to as the composite system reliability assessment or hierarchical level II, and it is the most complicated study among all levels of reliability evaluation [1]. This attribute is basically due to the complexity in the composite system state modeling and analysis [2]. Optimal power flow (OPF), in both AC and DC forms, could be utilized to analyze the states accommodated. However, the application of DC models is very frequent since the computation of the AC model is nonlinear and extensive [3–5]. The approximation introduced by the DC model is negligible [6].

Composite system reliability assessment consists of 3 major steps: state selection, state analysis, and reliability indices calculation. The most time consuming part is the state analysis, which usually involves an optimization procedure. This process turns restrictive when the number of states to be analyzed is large [7]. Two fundamental categories of reliability evaluation techniques are state enumeration (SE) techniques and

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Monte Carlo simulations (MCSs) [3,4,7]. The former group considers all of the system states with an associated probability greater than a specific level or those including outages up to a given order. They exploit several techniques, such as state merging, truncation, implicit enumeration, and sequential model building, to deal with the computational burden [7]. In the MCS methods, states are sampled randomly to the point that the reliability indices become approximately stable or a given number of contingencies are simulated. Various attempts were made to enhance the efficiency of the state selection of MCSs [1–5,7,8]. Among them, Billinton and Li [1] studied the well-known importance sampling approach, and variance reduction-based methods were used in [7] and [8].

In order to improve composite system reliability evaluation, either class 1, reducing the number of states (e.g., the scenario reduction tricks), or class 2, enhancing the state analysis procedure, has been investigated by researchers. Both classes have generated remarkable efforts and a brief literature review on each is provided below to clarify the contribution of the work presented in this paper.

Regarding class 1, diverse methods have been developed based on the intelligent search of the state space. Metaheuristic simulations have brought about a great deal of interest in this respect. In [3], an innovative state sampling using the genetic algorithm (GA) was presented to truncate the huge state space by tracing failure states, i.e. states resulting in load curtailments. Other versions of the GA were also successfully examined in [9] and an enhanced GA-based approach, which is capable of considering multistate components, was addressed in [10]. The efficiency of the self-organizing map in the state selection of the MCS was revealed in [7]. Singh and Wang [11] explored the conceptual basis of the overall reliability evaluation process and the role of artificial intelligence methods, including the ant colony system, artificial immune system, etc. That paper provided detailed comparisons between those methods, as well. In [7] and [8], the variance reduction method was used to lessen the sampled states. In [12–16], it was proposed to decompose the whole state space into the acceptable, unclassified, and loss of load categories, and the reliability indices were calculated in terms of the loss of load category. A performance index ranking algorithm to enhance the procedure of the reliability assessment by eliminating noninfluencing system states was presented in [17].

Class 2 attempts to analyze the system states in a faster manner to speed up the whole composite system reliability assessment. An efficient method for detecting transmission line overloads was presented in [18], in which distribution factors were employed to compute transmission line flows. In [19], postcontingency transmission line flows were obtained with low computational effort using linear sensitivity coefficients along with precontingency line flows. Other solutions to alleviate transmission line overloads were proposed in [20] and [21]. A direct method was devised in [20] to release line overloads by means of generation rescheduling and load shedding based on the sensitivity between the line flows and power injections. The local load shedding optimization method was discussed in [21], which improved the efficiency by lessening the optimization problem dimension. The objective function of the method is to minimize the sum of transmission line overloads. In [22], a heuristic approach tailored to the local load shedding was developed based on the power flow tracing in the vicinity of the overloaded section. The key drawback associated with such local optimizations is that the solution might be suboptimal or even that the problem is likely infeasible. This phenomenon is due to the restricted control variables and/or load shedding possibilities in the vicinity of the overloaded section. In [23] and [24], the artificial neural network and expert system-based methods were utilized to evaluate system states. In [25], radial basis function networks were used to analyze contingencies. In [26], an improved algorithm based on the Tellegen theorem was proposed to determine voltage magnitudes and transmission line flows. Parallel processing approaches to balance the computation among high performance computers were proposed in [27–29] to analyze system states.

The techniques reviewed above have particular pros and cons. Some have trivial impacts on the reduction of computation burden and some might lead to approximations or inaccuracies. However, one can conclude that there is no dominant solution and the gate for further research is still open.

A new contingency analysis method based on postoptimal analysis (POA) was presented in [30], in which the reliability evaluation was overlooked. This paper proposes a methodology based on POA, which is a well-recognized method in mathematics to jointly solve a set of similar problems. In this sense, the method derives a single OPF problem for the base case and determines the associated solution. The base case is assumed to be the state in which all of the components of the system are healthy and the load forecast is set to its mean value. Other system states, which have just slight differences in the component states and the load level from those of the base case, are then analyzed by applying the effects of the generating unit, transmission line outages, and load fluctuations to the base case solution. In other words, instead of solving independent optimization problems, the proposed method seeks the solution of contingencies around the base case result with lower calculations. The suggested approach does not require the reduction of the number of sampled states (for class 1) and is an innovative technique to enhance the state analysis procedure, and therefore it can decrease errors and computing time. The proposed approach is implemented on the Roy Billinton test system (RBTS), the IEEE reliability test system (RTS), and the IEEE 118-bus test system, and the results are obtained for cases of SE and MCS applications and with/without POA accommodation. The results are discussed thoroughly and compared with each other to demonstrate the performance of the method.

The paper is outlined as follows: Sections 2 and 3, respectively, review the composite system reliability assessment and POA; the proposed methodology is presented in Section 4; the simulation results and discussions are addressed in Section 5; and Section 6 outlines the concluding remarks.

2. Composite system reliability assessment

The aforementioned 3 main steps of the composite system reliability assessment are pictorially shown in Figure 1. The first step specifies the system load level, the operating status of the transmission lines and generating units, and the system operating policy. In the second step, the sampled states are to be judged as to whether they are a success or a failure. For failure states, the remedial actions available are taken and the amount of curtailed load is determined for the load points as well as the whole system. Accordingly, in the second step,

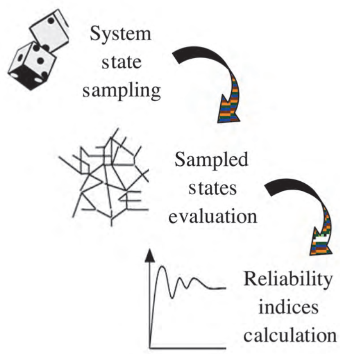

Figure 1. The 3 steps of the composite system reliability assessment.

an OPF is usually performed to reschedule generating units and alleviate constraint violations, and the load shedding is considered as the last resort. The last step calculates the reliability indices based on the state evaluation outcomes.

In the reliability evaluation, it is common practice to employ the network DC power flow model, as it leads to a linear programming (LP) optimization [31]. The studies described in this paper assume the DC model of the network. The objective function of the DC-OPF model is the total load curtailment or the total interruption cost minimization, subject to a set of operating constraints.

Load curtailment can be performed using different philosophies if required. Among them, the most important ones are based on the importance of load points or the value of lost load (VOLL) for each load point [10]. In this paper, we assume that the VOLLs associated with the load points are given and are not identical to consider the load point priorities in the load shedding procedure.

The LP minimization model for the problem associated with state s is formulated as follows:

$$\min Total\, Interruption\, Cost = \sum_{n\in B} VOLL_n \,.\, LC_n^s, \tag{1}$$

subject to:

$$PF_l = \frac{1}{x_l}\sum_{n\in B} A_{nl}\,.\,\delta_n; \quad \forall l \in L, \tag{2}$$

$$\sum_{i\in G_n} P_i - \sum_{l\in L} A_{nl}\,.\,PF_l = PD_n - LC_n^s; \quad \forall n \in B, \tag{3}$$

$$-\overline{PF}_l \le PF_l \le \overline{PF}_l; \quad \forall l \in L, \tag{4}$$

$$\underline{P_i} \le P_i \le \overline{P_i}\ \forall i \in G_n; \quad \forall n \in B, \tag{5}$$

$$0 \le LC_n^s \le PD_n; \quad \forall n \in B, \tag{6}$$

$$-\pi \le \delta_n \le \pi; \quad \forall n \in B. \tag{7}$$

In the above formulation, the objective function is to minimize the hourly interruption cost, as expressed in Eq. (1); the line flows are calculated based on the bus voltage angle differences, as in Eq. (2); the nodal power balance is satisfied in Eq. (3); the line flow capacity limits are enforced in Eq. (4); the generating unit limits are considered in Eq. (5); load shedding at the load points are capped with the associated total load in Eq. (6); and the bus voltage angles are bound in Eq. (7). It has to be noted that in the above formulation, the decision variables are the power output of the generating units and the load curtailments at the load point buses. The dependent variables are the bus voltage angles and line power flows. Annualized reliability indices, including loss of load probability ($LOLP$), expected demand not served ($EDNS$), expected energy not supplied ($EENS$), and system minutes (SM) are considered in this paper. These indices are calculated as follows [2]:

$$LOLP = \sum_{s\in S} Pr_s, \tag{8}$$

$$EDNS = \sum_{s\in S}\sum_{n\in B} LC_n^s\,.\,Pr_s, \tag{9}$$

$$EENS = EDNS\,.\,8760, \tag{10}$$

$$SM = \frac{EENS}{system\,peak\,load}\,.60. \tag{11}$$

Equations associated with the individual load point reliability indices are the same as those appearing above, but in Eqs. (8) and (9), S should cover states in which the relevant load bus experiences an interruption. Thus far, the majority of techniques to solve the composite system reliability assessment are different in the system state sampling step, while the steps "Sampled states evaluation" and "Reliability indices calculation" are the same. The obvious evidence is the difference in the state sampling of the SE and MCS approaches. Usually in the step of sampled states evaluation, the OPF model is separately constituted and solved for each state, and this process would be extremely time-consuming when thousands of OPF problems have to be examined, even for a medium-scale network.

POA recommends obtaining the solution of a problem through the result of another problem, but with slight differences. In the reliability assessment, the contingency states are usually due to one or few generating units and/or transmission line outages and small load variations. Accordingly, differences between the base case and the contingency states are not remarkable and the application of POA seems to be profitable.

As the POA notion might be novel in the power system context, a summary description of the simplex algorithm and POA is reviewed in the next section and the detailed presentation of the methodology is then addressed.

3. Simplex algorithm and postoptimal analysis

A major proportion of real-world problems, such as those in economics, management, production, and transportation, are expressed in the LP format, in which the objective function and equality and inequality constraints are all linear. The simplex method, developed by George Dantzig in 1947 [32], is the most efficient procedure to solve LP problems and its simplicity and elegance made it an essential numeric tool. In LP problems, each inequality constraint halves the solution space. The aggregation of all of the problem constraints shapes a convex polyhedron. Due to the linearity of the objective function, the optimal solution must lie at vertices of the polyhedron. The simplex method begins from the origin and walks along a path on the edges of the polyhedron to vertices with an enhancement in the objective function value, up to the point that the optimum solution is achieved. A pictorial representation of the optimization evolution associated with the simplex algorithm is shown in Figure 2. The simplex method works in a systematic manner, and at a given vertex it assesses all of the adjacent vertices and moves to the best one. Understanding the geometric description of the algorithm provides an overall insight on the performance of the simplex algorithm.

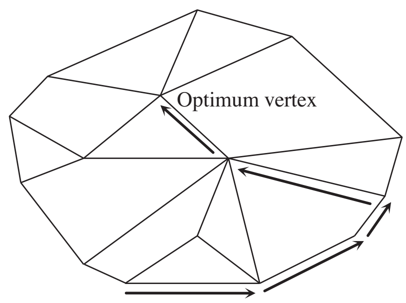

Figure 2. Optimization evolution of the simplex algorithm.

Below, we discuss finding the new optimum solution after making changes in the problem. In many practical contexts, it is the case that one wants to solve slightly different versions of an optimization problem repeatedly. Of course, the simplest way is to solve an entire optimization problem for each scenario, but this is clearly undesirable. Therefore, the raised question is: how can we solve the model of each scenario without starting from the beginning? The answer, it turns out, is that only minor revisions to the solved scenario are required to achieve similar problems' solutions. POA is developed in this respect. For the case where we have many similar problems, POA recommends to solve just one arbitrary problem via the simplex algorithm and determine the other problems' solutions by POA. The proof of the POA method, its comprehensive calculating steps, and several illustrative examples were discussed in [32] and [33]. Although the efficiency of the POA approach depends mainly on the degree of similarity between the LP models, it is assuredly applicable even in cases with a large number of differences.

Figure 3 depicts the performance of the POA method, where some constraints of the problem are changed and the optimum solution travels to a new location. The dashed lines illustrate the original problem surfaces and the new polyhedron is shown by solid lines. As shown in Figure 3, the POA procedure begins with the original model optimum solution versus the simplex method that uses the origin as the initial point. One therefore would expect that this action reduces both the computational time and effort. This experience is confirmed by much empirical evidence. It is worth emphasizing that there is no mathematical proof or guarantee that POA results in a shorter time. However, there are reported cases in which POA solves problems in as short as 1% of the time required to solve the problem independently and from the origin.

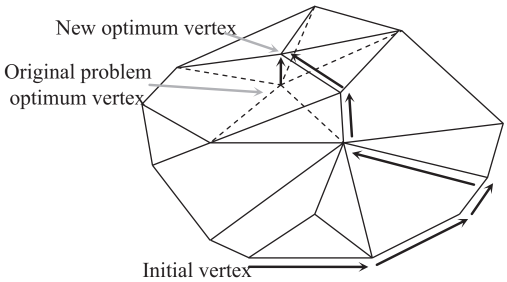

Figure 3. Optimization evolution of the POA after some changes in the model.

For small-size problems, POA might be trivially useful since the independent solution is acquired promptly. However, POA is extremely fruitful for larger problems and it becomes more valuable with the increase of problem dimension. Another aspect demanding further attention is that the computational time reduction raised by POA depends on the extent of changes made in the original problem. Generally speaking, for the problem more similar to the original version, POA leads to a faster solution.

POA can handle the following cases:

- Changes in the constraint constants,
- Changes in the coefficient of variables,
- Introduction of a new variable,
- Introduction of a new constraint.

The POA procedure, in contrast with the above cases, is discussed in brief below, and interested readers are referred to [32] and [33] for a detailed explanation. Readers are also recommended to first study the Appendix to become familiar with the simplex method principles and symbols.

3.1. Changes in the constraint constants

The feasibility of the current optimum solution might be affected by the constraint constant changes. This circumstance requires recomputing the value of the basic variables using $x_{Bnew} = B^{-1}b_{new}$. Next, it is necessary to continue the simplex method procedure to find the optimum solution. It should be noted that matrix B corresponds to the last iteration of the simplex algorithm.

3.2. Changes in the coefficient of the variables

If the objective function coefficients alter, the optimality of the solution might not hold anymore. To this end, it has to be determined whether or not the original solution is optimal; if so, the solution is given, and otherwise, the iteration of the simplex method should go forward. For the case in which the constraint coefficients alter, it is necessary to modify matrix A and continue the simplex method procedure to reach to the optimum solution.

3.3. Introduction of a new variable

Similar to "changes in the coefficient of the variables", introducing a new variable just affects the optimality of the solution. This change might provide a chance to improve the optimal value of the objective function. Thus, the optimality condition should first be checked and if the existing solution is no longer optimal, the simplex method recovers the optimal solution in some of the iterations. Note that adding a new variable does not make the problem infeasible, because by assuming that the variable is equal to zero, the problem would be the same as the original one.

3.4. Introduction of a new constraint

Similar to "changes in the constraint constants", the introduction of a new constraint to the existing model could lead to the infeasibility of the current optimum solution. In this case, it is necessary to add a new slack variable to the model; reform matrices b, x, A, and s; and continue the simplex method procedure to find the optimum solution. It should be remembered that the addition of a new constraint can never improve the final optimum value of the objective function.

4. Proposed methodology

As indicated in the introduction, the existing approaches to enhance the reliability evaluation process are based on either reducing the number of states to be analyzed or accelerating the state analysis procedure. The POA-based method proposed in this paper is recognized as the latter case. It is worth noting that in contrast to some available approaches with similar goals, the proposed method has no approximation and error. Following the introduction of the composite system reliability evaluation and POA bases, the proposed methodology is addressed in this section and an illustrative 3-bus system is employed to clarify the issue.

As implied before, POA exploits the similarity among different OPF models and avoids solving a full optimization problem for each system state. It solves an optimization problem for the base case and analyzes the other states through the base case solution. In the composite reliability assessment, the base case is assumed to be the state in which all of the system components are in the up state and the load forecast is set to its mean value. Subsequent to the base case OPF solution, the result is saved; thereafter, we should determine differences among the base case and selected state operating conditions, such as the generating unit and transmission line status, as well as the demand at each load point. POA derives the solution of the selected state by applying the differences to the base case solution.

It is necessary to find the changes of the above-mentioned differences in the OPF model associated with the base case. The single-line diagram of the illustrative 3-bus network is shown in Figure 4, where 2 generating units are connected at buses 1 and 2.

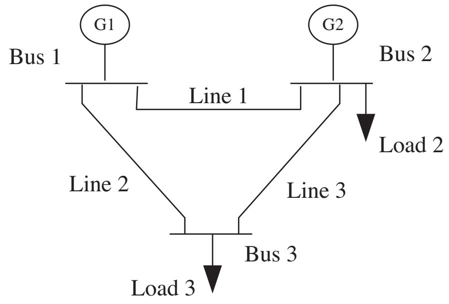

Figure 4. Single-line diagram of the 3-bus sample network.

The OPF model of the network associated with state s is as follows:

$$\min Total\,Interruption\,Cost \;=\; \sum_{n\in\{2,3\}} VOLL_n.LC_n^s, \tag{12}$$

subject to

$$P_1 - PF_1 - PF_2 = 0, \tag{13}$$

$$P_2 + PF_1 - PF_3 = PD_2 - LC_2^s, \tag{14}$$

$$PF_2 + PF_3 = PD_3 - LC_3^s, \tag{15}$$

$$-\overline{PF}_l \le PF_l \le \overline{PF}_l; \quad \forall l \in \{1,2,3\}, \tag{16}$$

$$\underline{P_i} \le P_i \le \overline{P_i}; \quad \forall i \in \{1,2\}, \tag{17}$$

$$0 \le LC_n^s \le PD_n; \quad \forall n \in \{2,3\}. \tag{18}$$

Note that other sets of equations for the power flow calculation of Eq. (2) and the bus voltage angle bounds of Eq. (7) are included in the programming, while not expressed above.

The necessary changes in the base case OPF model due to the generating unit and transmission line outages and load variations are described below. For a better comparison among the OPF model of system states, they are summarized in Table 1. The objective function of all of the states is identical to Eq. (12).

4.1. Generating unit outage

In the formulation of the OPF model, the real power of each unit is considered as a decision variable in the power balance of the relevant bus. The outage of a generating unit is equivalent to the drop of the corresponding variable from the power balance constraint. This change corresponds to the condition expressed in Section 3.2. In the event of generating unit G1 outage, the power balance constraint of bus 1 would be:

$$-PF_1 - PF_2 = 0. \tag{19}$$

Similarly, the outage of generating unit G2 leads to a change in the power balance constraint of bus 2, as follows:

$$PF_1 - PF_3 = PD_2 - LC_2^s. \tag{20}$$

Table 1. General formulation of the illustrative example for the base case and 3 scenarios.

Base case	G2 on outage	Line 1 on outage	Load variation at Bus 2
$P_1 - PF_1 - PF_2 = 0$	$P_1 - PF_1 - PF_2 = 0$	$P_1 - PF_2 = 0$	$P_1 - PF_1 - PF_2 = 0$
$P_2 + PF_1 - PF_3 = PD_2 - LC_2^s$	$PF_1 - PF_3 = PD_2 - LC_2^s$	$P_2 - PF_3 = PD_2 - LC_2^s$	$P_2 + PF_1 - PF_3 = PD_2^{new} - LC_2^s$
$PF_2 + PF_3 = PD_3 - LC_3^s$	$PF_2 + PF_3 = PD_3 - LC_3^s$	$PF_2 + PF_3 = PD_3 - LC_3^s$	$PF_2 + PF_3 = PD_3 - LC_3^s$
$-\overline{PF}_1 \leq PF_1 \leq \overline{PF}_1$	$-\overline{PF}_1 \leq PF_1 \leq \overline{PF}_1$	$-M \leq PF_1 \leq M$	$-\overline{PF}_1 \leq PF_1 \leq \overline{PF}_1$
$-\overline{PF}_2 \leq PF_2 \leq \overline{PF}_2$	$-\overline{PF}_2 \leq PF_2 \leq \overline{PF}_2$	$-\overline{PF}_2 \leq PF_2 \leq \overline{PF}_2$	$-\overline{PF}_2 \leq PF_2 \leq \overline{PF}_2$
$-\overline{PF}_3 \leq PF_3 \leq \overline{PF}_3$	$-\overline{PF}_3 \leq PF_3 \leq \overline{PF}_3$	$-\overline{PF}_3 \leq PF_3 \leq \overline{PF}_3$	$-\overline{PF}_3 \leq PF_3 \leq \overline{PF}_3$
$\underline{P_1} \leq P_1 \leq \overline{P_1}$	$\underline{P_1} \leq P_1 \leq \overline{P_1}$	$\underline{P_1} \leq P_1 \leq \overline{P_1}$	$\underline{P_1} \leq P_1 \leq \overline{P_1}$
$\underline{P_2} \leq P_2 \leq \overline{P_2}$	$\underline{P_2} \leq P_2 \leq \overline{P_2}$	$\underline{P_2} \leq P_2 \leq \overline{P_2}$	$\underline{P_2} \leq P_2 \leq \overline{P_2}$
$0 \leq LC_2^s \leq PD_2$	$0 \leq LC_2^s \leq PD_2$	$0 \leq LC_2^s \leq PD_2$	$0 \leq LC_2^s \leq PD_2^{new}$
$0 \leq LC_3^s \leq PD_3$	$0 \leq LC_3^s \leq PD_3$	$0 \leq LC_3^s \leq PD_3$	$0 \leq LC_3^s \leq PD_3$

4.2. Transmission line outage

The transmission line outage is equivalent to the drop of the corresponding capacity constraint and other modifications in the power balance constraints of the associated buses. Relaxing the capacity constraint of each transmission line is equivalent to enlarging the corresponding capacity to a large enough value. This change corresponds to the condition stated in Section 3.1. Moreover, the transmission line outage is equivalent to a drop in the corresponding variable from the power balance constraints, which corresponds to the condition expressed in Section 3.2. As an example, the power balance constraint in buses 1 and 2 would be as follows in the event of a Line 1 outage:

$$P_1 - PF_2 = 0, \tag{21}$$

$$P_2 - PF_3 = PD_2 - LC_2^s. \tag{22}$$

4.3. Load variation

The maximum amount of load curtailment, as well as the power balance constraints, is dependent on the load at the corresponding bus. Therefore, variation of the load at each bus leads to the change in a parameter in the relevant constraints. These changes correspond to the condition indicated in Section 3.1. For example, changes in the maximum amount of load curtailment and power balance constraints due to the load variation at bus 2 are as follows:

$$P_2 + PF_1 - PF_3 = PD_2^{new} - LC_2^s, \tag{23}$$

$$0 \leq LC_2^s \leq PD_2^{new}. \tag{24}$$

The procedure to calculate reliability indices for the composite power systems using the proposed POA approach consists of the following 3 steps. The corresponding algorithm is shown in Figure 5.

1. At first, the base case OPF, in which all of the generating and transmission facilities are in the operating state and the demand of the load points are equal to their mean value, is modeled and solved. The outcome is then saved as a basis to determine the solution of the contingency states.

2. Adopt system credible states and analyze them. First, the selected state is compared with the base case model and the differences are determined. Next, the differences are applied to the base case solution and the corresponding result is obtained.

3. Reliability indices are updated and the termination criterion is checked. The criterion is used to determine whether or not the reliability assessment procedure was completed. The criterion considered is

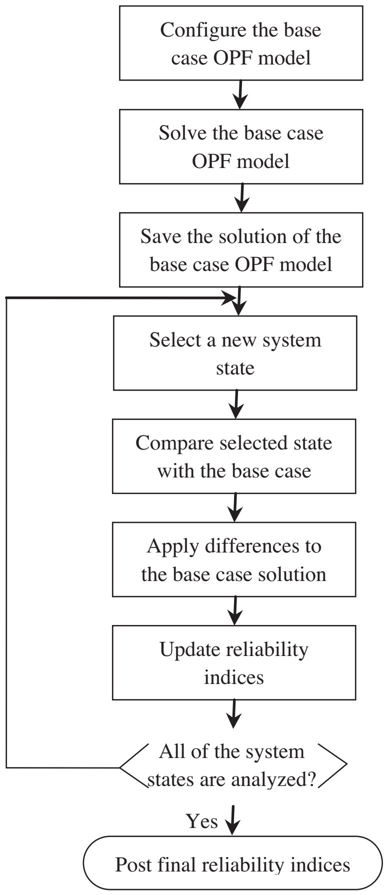

Figure 5. The proposed POA-based composite system reliability assessment.

dependent on the reliability evaluation approach. In the SE method, counting a given list of contingencies is usually accommodated and in the MCS technique, the procedure should proceed to the point that either the number of states analyzed reaches a given number or the variation of reliability indices is less than a prespecified threshold.

5. Study results

In this section, the RBTS [34], the IEEE RTS [35], and the IEEE 118-bus test system are all examined through the proposed approach to demonstrate its performance and efficiency. The proposed method is applied to both the SE and MCS techniques and the network OPF is based on the DC power flow. In the simulation studies, the simplex linear optimization algorithm is used to solve OPFs. The objective function is to minimize the total curtailment cost based on the load point VOLLs.

In the following sections, different studies are conducted and the respective results are compared from both the accuracy and execution time aspects. These studies include conventional SE, MCS, SE&POA, and MCS&POA. SE and MCS assess each contingency as an independent optimization problem. SE&POA and MCS&POA analyze contingency states using POA and are based on the base case solution. Note that in the MCS and MCS&POA methods, the number of simulations is adopted such that the variation of system indices is settled. The study results consist of EDNS, EENS, SM, LOLP, and execution time. All of the simulations are implemented on an Intel®Core 2 Duo 2.20 GHz processor with 2 GB RAM. The execution time is expressed per unit for the sake of simple comparison.

5.1. The RBTS

This educational test system has 6 buses, including 5 load buses and 2 generation buses. The system has 9 lines and 11 conventional generating units. The total installed capacity is 240 MW and the system annual peak load is 185 MW. Figure 6 shows the single-line diagram of the RBTS. Reliability indices are calculated for the system's hourly peak load and they are therefore designated as annual indices.

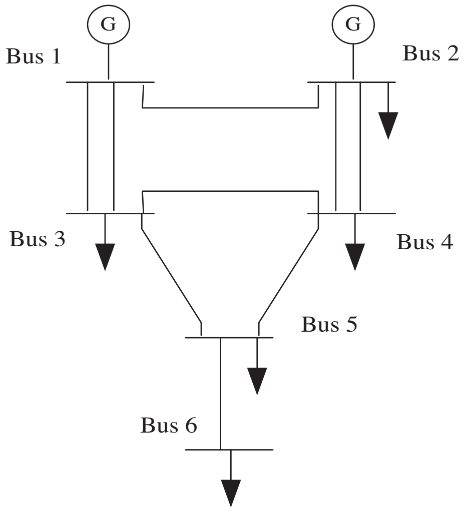

Figure 6. Single-line diagram of the RBTS.

Using SE, MCS, SE&POA, and MCS&POA, adequacy indices for the system are obtained. For both the SE and SE&POA methods, the contingency states considered are single to double line outages and single to triple generating unit outages. In MCS and MCS&POA, 20,000 system states are sampled and analyzed.

Table 2 presents various system indices and computation times. Tables 3 and 4, respectively, show the EDNS and EENS at different load points.

Table 2. System indices for the RBTS.

Method	EDNS [MW/year]	EENS [MWh/year]	SM [min/year]	LOLP	Time [p.u.]
SE	0.0147	128.7	41.7	0.0012	1 (=22,932 s)
MCS	0.0160	132.3	42.9	0.0013	11.1941
SE&POA	0.0147	128.7	41.7	0.0012	0.4122
MCS&POA	0.0160	132.3	42.9	0.0013	4.8371

Table 3. EDNS index for the load points of the RBTS.

Load point	SE	MCS	SE&POA	MCS&POA
2	0.0002	0.0001	0.0002	0.0001
3	0	0	0	0
4	0	0	0	0
5	0.0004	0.0004	0.0004	0.0004
6	0.0141	0.0146	0.0141	0.0146

Table 4. EENS index for the load points of the RBTS.

Load point	SE	MCS	SE&POA	MCS&POA
2	1.8	2.6	1.8	2.6
3	0	0	0	0
4	0	0	0	0
5	3.8	4.7	3.8	4.7
6	123.1	125	123.1	125

From Table 2, we might deduce that SE and SE&POA outperform MCS and MCS&POA. This conclusion, although it holds here because of the system's small size, is not a general observation since the superiority of SE and MCS techniques in the reliability evaluation strongly depends on the system dimension as well as on the equipment forced outage rates. On the contrary, comparison of the conventional and POA version of either SE or MCS is meaningful. Owing to the results shown in Table 2, a significant improving influence of POA on the execution time is observed for all of the methods. The computation time for the reliability assessment has been reduced by about 2.5 times by POA compared to that of the conventional methods. Moreover, comparing both the load point and system reliability indices obtained by the conventional method with those obtained by its POA-based version reveals that the same values are produced.

As shown in Table 2, the results associated with the SE method are slightly different from those of the MCS method. Both techniques introduce some error. The error associated with SE is due to neglecting the higher order outages, while that of MCS is because of the termination criterion. It has to be emphasized that there is no serious concern about such trivial errors in the reliability evaluations. This is due to the fact that the reliability data themselves are inherently uncertain.

5.2. The IEEE RTS

This 24-bus test system has 32 generating units and 38 transmission lines. The system installed capacity is 3405 MW, and the annual peak load is 2850 MW. Simulations are fulfilled at the system peak load level; thus, the indices obtained are designated as annualized values. The single-line diagram of the network is presented in Figure 7. In this case, the contingencies accommodated in SE and SE&POA are single to triple line outages and single to quad generating unit outages. The number of samples accounted for in the MCS and MCS&POA are equal to 10,000.

Table 5 shows the reliability indices and the required computational time associated with different methods on the basis of the SE method. Similar to the RBTS, applying POA to both the SE and MCS methods leads to computational time improvement without affecting the accuracy. However, the order of acceleration is about 4.5 times, which is greater than that of the RBTS. Comparing Tables 2 and 5, it is evident that opposite to the RBTS, the simulation-based method is faster than SE in this case study.

Table 5. System indices for the IEEE RTS.

Method	EDNS	EENS	SM	LOLP	Time [p.u.]
SE	11.992	105,050	2212	0.0753	1 (=1211 s)
MCS	14.69	128,684	2709	0.0856	0.1029
SE&POA	11.992	105,050	2212	0.0753	0.2184
MCS&POA	14.69	128,684	2709	0.0856	0.0209

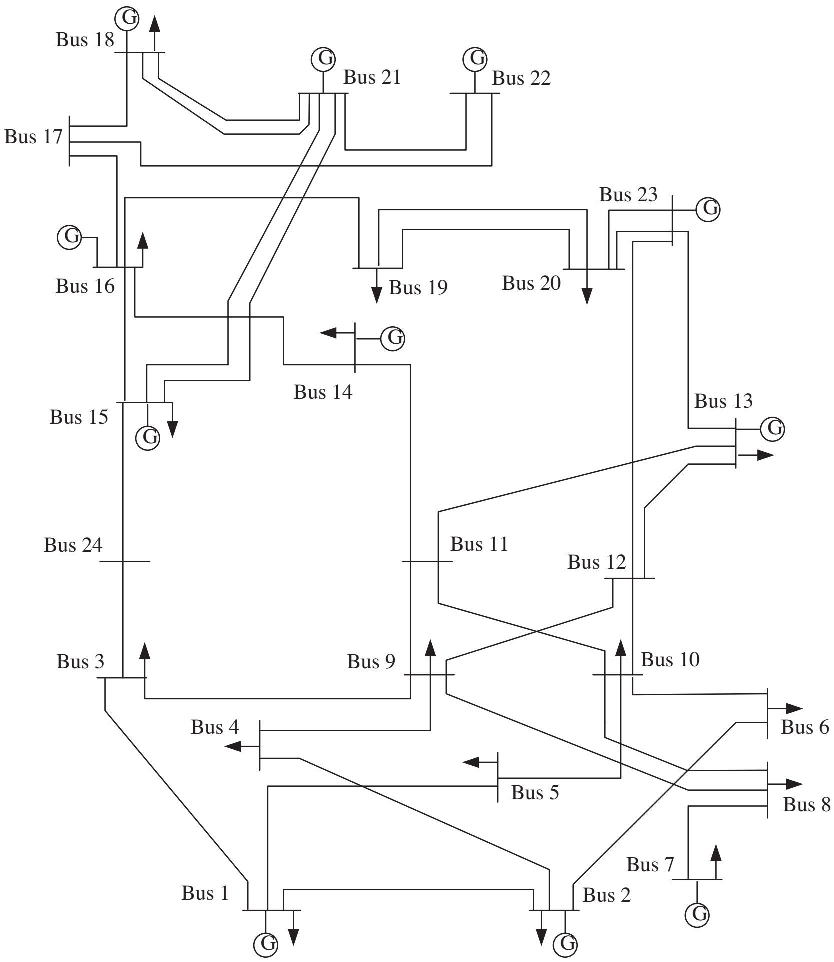

Figure 7. Single-line diagram of the IEEE RTS.

5.3. IEEE 118-bus system

This network has 118 buses, 186 transmission lines, and 54 generating units. The system annual peak load is 4242 MW. Similar to the IEEE RTS, the annualized indices are of interest here, too. The reliability data for this system are given at http://motor.ece.iit.edu/data/ltscuc. Up to the second order outages of the transmission lines and the third order outages of the generating units are considered in the SE and SE&POA methods. The total number of sampled states is equal to 20,000 in MCS and MCS&POA.

Table 6 shows the study results when implementing the noted 4 methods. The results of Table 6 validate the promising effect of POA on both the SE and MCS approaches and the enhancement proportion of acceleration is approximately equal to 8.

Table 6. System indices for the IEEE 118-bus network.

Method	EDNS	EENS	SM	LOLP	Time [p.u.]
SE	0.1576	1381	20	0.0028	1 (=4029 s)
MCS	0.4570	4174	59	0.0080	0.3417
SE&POA	0.1576	1381	20	0.0028	0.1274
MCS&POA	0.4570	4174	59	0.0080	0.0421

Table 7 summarizes the above analyses of the 3 test systems by presenting time improvements in percentages. The key conclusion raised here is that the effectiveness of POA appears more and more with an increase of the network dimension. This feature is reasonable and verified by the mathematic literature [32,33].

Table 7. Time improvement of POA in the test systems.

Network	Evaluation method	
	MCS	SE
RBTS	131%	143%
IEEE RTS	392%	358%
118-bus system	712%	685%

6. Conclusion

A computationally efficient technique for evaluating the reliability of composite power systems was presented in this paper. Based on the similarity of OPF models of system states, the proposed method uses POA to speed up the contingency evaluation procedure. The proposed methodology allows examiners to compute all types of reliability indices, including expected power/energy not supplied and LOLP and cost, at both the system and load point levels. Moreover, the developed approach is applicable in both sorts of analytical and simulation-based evaluation methods. Numerical case studies validated the effectiveness of the proposed POA-based method in the execution time saving while not sacrificing the accuracy. With an increasing network size, the proposed method would be even more efficient in reducing the run time. As a consequence, it is expected that the POA-based method would be more desirable and exploitable in evaluating the reliability of large-scale networks.

Appendix

In Section 3, a summary of the geometric concept of the simplex algorithm is described. Understanding the notion brings a general insight over the performance of the simplex algorithm. The goal of this appendix is to address the algebraic procedure of the simplex method, which is principally necessary to capture the key features of POA. The procedure is based on solving systems of equations. The first step, therefore, is converting the inequality constraints to equivalent equality constraints by introducing slack variables.

The standard form of a linear optimization problem is as follows:

$$\max Z = \mathbf{c}^T \mathbf{x} \tag{A.1}$$

subject to:

$$\mathbf{A}\mathbf{x} \leq \mathbf{b}, \tag{A.2}$$

$$\mathbf{x} \geq 0, \tag{A.3}$$

where Z denotes the objective function of the model, c and x are $n \times 1$ column vectors, b is a $m \times 1$ column vector, A is an $m \times n$ matrix, n is the number of variables, and m is the number of constraints. Using slack variables, the problem would be rewritten as follows:

$$\max \quad Z = \mathbf{c}^T \mathbf{x} \ , \tag{A.4}$$

subject to:

$$[\mathbf{A}\ \mathbf{I}]\begin{bmatrix}\mathbf{x}\\ \mathbf{s}\end{bmatrix} = \mathbf{b}, \tag{A.5}$$

$$\begin{bmatrix}\mathbf{x}\\ \mathbf{s}\end{bmatrix} \geq 0. \tag{A.6}$$

Here, I is an $n \times n$ identity matrix and s is an $n \times 1$ column vector. After applying slack variables, the number of variables is $n+m$, which is greater than the number of equations, i.e. m. The simplex method chooses n nonbasic variables to be set equal to 0 and the simultaneous solution of m equations for other m basic variables is a vertex. At each of the iterations, one of the nonbasic variables (entering variables) should be replaced by a basic variable (leaving variables) to improve the objective function.

x_B is the vector of basic variables and can be derived by elimination of nonbasic variables and relevant columns of matrix A in Eq. (A.5):

$$\mathbf{x}_B = \mathbf{B}^{-1}\mathbf{b}. \tag{A.7}$$

Here, B is the basic matrix and is obtained by eliminating the columns corresponding to the coefficients of the nonbasic variables from Eq. (A.1). The value of the objective function associated with each of the iterations is:

$$Z = \mathbf{c}_B\mathbf{x}_B. \tag{A.8}$$

Here, c_B is the vector whose elements are the objective function coefficients (including zeros for slack variables) for the corresponding elements of x_B.

The overall procedure of the simplex algorithm is summarized below in 3 main steps.

1. Initialization: in the first step, the model should be converted to the standard form, followed by converting all of the inequality constraints to the equivalent equality constraints by applying slack variables.

2. Iteration: the second step consists of 3 substeps, as follows:

a) Determine the entering basic variable,

b) Determine the leaving basic variable,

c) Derive the B^{-1} matrix and set $x_B = B^{-1}b$.

3. Optimality test: determine whether the solution is the optimum point or not and decide about iteration continuation.

The solution is optimal if change in the basic variable matrix could not improve the objective function.

Nomenclature

Indices and sets

n	Bus index
i	Generating unit index
l	Transmission line index
s	State index
L	Set of transmission lines
G_n	Set of generating units connected to bus n
B	Set of buses
S	Set of states resulting in load curtailment

Parameters

x_l	Reactance of line l
Pr_s	Probability of state s
$\overline{PF}_l$	Power flow capacity of line l
$\overline{P}_i$	Capacity of generating unit i
$\underline{P}_i$	Minimum power output of generating unit i
PD_n	Demand at bus n
A_{nl}	Element of the network incidence matrix, which is equal to 1 if bus n is the sending bus of line l, –1 if bus n is the receiving bus of line l, and 0 otherwise
$VOLL_n$	Value of lost load at bus n
M	A very large number

Variables

P_i	Power output of generating unit i
PF_l	Power flow of line l
δ_n	Voltage phase angle at bus n
LC_n^s	Load curtailment at bus n at state s

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