Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2013) 21: 120 – 130

doi:10.3906/elk-1107-29

Modified iterated extended Kalman particle filter for single satellite passive tracking

Panlong WU, Jianshou KONG*, **Yuming BO**
School of Automation, Nanjing University of Science and Technology,
Nanjing 210094 P.R. China

Received: 16.07.2011 • **Accepted:** 03.10.2011 • **Published Online:** 27.12.2012 • **Printed:** 28.01.2013

Abstract: Single satellite-to-satellite passive tracking techniques have great significance in space surveillance systems. A new passive modified iterated extended Kalman particle filter (MIEKPF) using bearings-only measurements in the Earth-Centered Inertial Coordinate System is proposed. The modified iterated extended Kalman filter (MIEKF), with a new maximum likelihood iteration termination criterion, is used to generate the proposal distribution of the MIEKPF. Moreover, a new measurement update equation of the MIEKF is derived by modifying the objective function of the Gauss–Newton iteration. The approximated second-order linearized state propagation equation, Jacobian matrix of state transfer, and measurement equations are derived in satellite 2-body movement. The tracking performances of the MIEKPF, iterated extended Kalman particle filter (IEKPF), extended Kalman particle filter (EKPF), and extended Kalman filter (EKF) are compared via Monte Carlo simulations through simulated data from STK8.1. The simulation results indicate that the proposed MIEKF is capable of passively tracking a low earth circular orbit satellite with a high earth orbit satellite using bearings-only measurements and has higher tracking precision than the traditional algorithms.

Key words: Bearings, Gauss–Newton iteration, modified iterated extended Kalman filter, particle filter, passive tracking

1. Introduction

Satellite-to-satellite tracking is used for various measurements and orbit restrictions to give the target satellite's motion state estimation. The ordinary measurements are the range and its changing rate, the angles and their changing rates, and a combination of these measurements [1]. Compared with the cooperative tracking mode, the satellite-to-satellite passive tracking system can obtain angles and frequencies by means of optical or radioed measurements. The angles are the most easily acquired target information. Research on passive tracking using bearings-only measurements has great significance in space surveillance systems [2,3]. Low earth orbit (LEO) satellites usually need signals relayed to transmit signals to the ground station. If a high earth orbit (HEO) satellite is used to passively receive these signals, i.e. navigation and communication, the satellite tracking system can be realized through analysis and can estimate the parameters of these signals using filtering algorithms.

The measurements of satellite-to-satellite passive tracking are nonlinear. For nonlinear problems, several variants of the filters such as the extended Kalman filter (EKF), the divided difference filter (DDF), the unscented Kalman filter (UKF), and the particle filter (PF) are introduced. A new satellite-to-satellite EKF tracking method with the bearings-only measurements in the J2000.0 Earth-Centered Inertial (ECI) frame was

*Correspondence: plwu@163.com

proposed in [4]. Two different observation models were established based on bearings-only detection data in [5], and the EKF method was used to track LEO satellites with a single space-based optic platform. A simplified DDF algorithm was proposed for satellite-to-satellite passive orbit determination and tracking in [6]. A UKF tracking method was proposed in [7], and it can use passive tracking to locate the orbit of a satellite by bearings. In recent years, the PF has attracted significant attention in the target-tracking field.

A new passive modified iterated extended Kalman particle filter (MIEKPF) tracking algorithm using bearings-only measurements is proposed in this paper. The rest of this paper is organized as follows. In Section 2, the tracking model for the satellite passive tracking is formulated. The state transfer matrix and measurement Jacobian matrix are also derived in a satellite 2-body problem. In Section 3, the MIEKPF tracking algorithm is proposed, and the modified iterated extended Kalman filter (MIEKF) with new measurement update and iteration termination criteria is used to generate the proposal distribution of the MIEKPF, which can reasonably approximate the posterior distribution by integrating the latest measurements into the system state transition density. The simulation results of 100 Monte Carlo experiments based on simulated data from the Satellite Tool Kit (STK8.1) are given in Section 4. The conclusions are presented in Section 5.

2. Tracking model

2.1. State motion equation

The relative movement of Earth and a satellite is a 2-body movement without perturbation according to the law of universal gravitation. LEO and HEO satellites can be regarded as 2 simplified particles in a geocentric celestial sphere coordinate. The trajectories of the LEO and HEO satellites are shown in Figure 1.

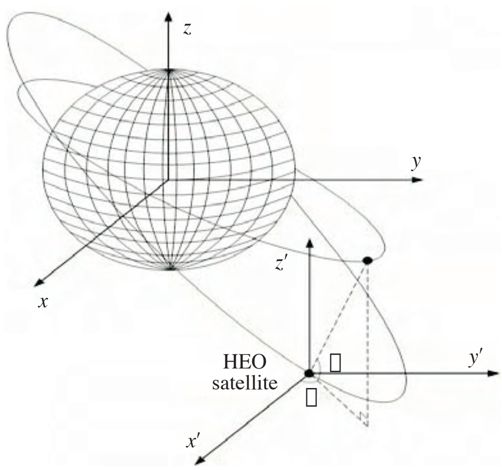

Figure 1. Trajectory of the satellite tracking system.

Select the position and velocity vectors in the ECI coordinate system as state vector $X = (x, y, z, \dot{x}, \dot{y}, \dot{z})^T$. According to the law of universal gravitation, the motion equation can be expressed in a 2-body problem as:

$$\begin{cases} \ddot{x} = -\mu \dfrac{x}{(x^2+y^2+z^2)^{3/2}} \\ \ddot{y} = -\mu \dfrac{y}{(x^2+y^2+z^2)^{3/2}} \\ \ddot{z} = -\mu \dfrac{z}{(x^2+y^2+z^2)^{3/2}} \end{cases}, \tag{1}$$

where μ is the Keplerian constant.

The motion state differential equation is:

$$\dot{X}(k) = (\dot{x}(k)\ \dot{y}(k)\ \dot{z}(k)\ \ddot{x}(k)\ \ddot{y}(k)\ \ddot{z}(k))^T = f(X(k), k). \tag{2}$$

The second-order discretization of Eq. (2) is:

$$X(k+1) = X(k) + f(X(k))T + F(X(k))f(X(k))\frac{T^2}{2} + q(k) \ = \Phi(X(k)) + q(k), \tag{3}$$

where $F(\mathbf{X}(k)) = \partial f(\mathbf{X}(k))/\partial \mathbf{X}(k)$, and $q(k)$is zero-mean Gaussian white noise with variance $Q(k)$.

At time k, suppose that the state vectors of the HEO satellite are $X_a = [x_a(k)y_a(k)z_a(k)\dot{x}_a(k)\dot{y}_a(k)\dot{z}_a(k)]^T$ and the state vectors of the observed LEO satellite are $X_b = [x_b(k)y_b(k)z_b(k)\dot{x}_b(k)\dot{y}_b(k)\dot{z}_b(k)]^T$.

The Jacobian of the state equation is:

$$F(X_b) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ F_{41} & F_{42} & F_{43} & 0 & 0 & 0 \\ F_{51} & F_{52} & F_{53} & 0 & 0 & 0 \\ F_{61} & F_{62} & F_{63} & 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

where $\rho(k) = x_b^2(k) + y_b^2(k) + z_b^2(k)$, $F_{41} = -\mu(\rho^{3/2}(k) - 3x_b^2(k)\rho^{1/2}(k))/\rho^3(k)$, $F_{42} = -3\mu x_b(k)y_b(k)\rho^{-5/2}(k)$, $F_{43} = -3\mu x_b(k)z_b(k)\rho^{-5/2}(k)$, $F_{51} = -3\mu x_b(k)y_b(k)\rho^{-5/2}(k)$, $F_{52} = -\mu(\rho^{3/2}(k) - 3y_b^2(k)\rho^{1/2}(k))/\rho^3(k)$, $F_{53} = -3\mu y_b(k)z_b(k)\rho^{-5/2}(k)$, $F_{61} = -3\mu x_b(k)z_b(k)\rho^{-5/2}(k)$, $F_{62} = -3\mu y_b(k)z_b(k)\rho^{-5/2}(k)$, $F_{63} = -\mu(\rho^{3/2}(k) - 3z_b^2(k)\rho^{1/2}(k))/\rho^3(k)$.

2.2. Measurement equation

Suppose the HEO has a receiver, which can obtain the azimuth θ and pitching angle ε between the observed LEO and HEO satellites. The angles are defined as:

$$\begin{cases} \theta(k) = \tan^{-1}\left(\dfrac{y_b(k) - y_a(k)}{x_b(k) - x_a(k)}\right) \\ \varepsilon(k) = \tan^{-1}\left(\dfrac{z_b(k) - z_a(k)}{\sqrt{(x_b(k) - x_a(k))^2 + (y_b(k) - y_a(k))^2}}\right) \end{cases}. \tag{5}$$

The measurement equation is described as:

$$Y(k) = \begin{bmatrix} \theta(k) \\ \varepsilon(k) \end{bmatrix} = h(X_a(k), X_b(k)) + V(k), \tag{6}$$

where $V(k)$is the measurement noise.

The Jacobian matrix of the measurement equation is:

$$H(k) = \frac{\partial h(k)}{\partial X_b(k)} = \begin{bmatrix} H_{11} & H_{12} & 0 & 0 & 0 & 0 \\ H_{21} & H_{22} & H_{23} & 0 & 0 & 0 \end{bmatrix}, \tag{7}$$

where $r_x(k) = x_b(k) - x_a(k), r_y(k) = y_b(k) - y_a(k)$, $r_z(k) = z_b(k) - z_a(k), r^2(k) = r_x^2(k) + r_y^2(k) + r_z^2(k), H_{11} = -r_y(k)/(r_x^2(k) + r_y^2(k)), H_{12} = r_x(k)/(r_x^2(k) + r_y^2(k)), H_{21} = -r_z(k)r_x(k)/((r_x^2 + r_y^2)^{1/2}r^2), H_{22} = -r_z(k)r_y(k)/((r_x^2(k) + r_y^2(k))^{1/2}r^2), H_{23} = (r_x^2(k) + r_y^2(k))^{1/2}/r^2$.

3. The modified iterated extended Kalman particle filter

The PF utilizes sequential Monte Carlo methods to approximate the posterior distribution using a set of weighted samples, and thus it can theoretically represent any distribution [8]. However, due to the huge calculated amount and the serious degeneracy problem, the sample impoverishment after the resampling step is a handicap in applying the PF to the state estimation [9]. To solve this problem, some researchers have adopted analytical methods. For example, the EKF or UKF approximation is used as the proposal distribution for a PF. Freitas et al. [10] combined the PF and EKF to form the extended Kalman particle filter (EKPF). The EKPF can get a better state equation of dynamic systems by EKF than PF; however, the EKF is a typical suboptimal estimate filter. When applying the EKF to nonlinear systems, the estimation error can be magnified and the estimation will be far from the optimal values [11]. A new iterated extended Kalman particle filter (IEKPF) was proposed in [12], and the iterated extended Kalman filter (IEKF) is used to generate the proposal distribution.

3.1. Iterated extended Kalman filter

The IEKF approximates the optimal state estimation using the measurement information effectively [13]. An approximate maximum a posteriori (MAP) estimate can be obtained through the iterative updating linear measurement equation. The IEKF is given as follows.

$$X(k|k-1) = \Phi(X(k-1)) \tag{8}$$

$$P(k|k-1) = F(k|k-1)P(k-1|k-1)F^T(k|k-1) + Q(k-1) \tag{9}$$

$$X^1(k|k) = X(k|k-1), \quad P^1(k|k) = P(k|k-1)$$

For $i = 1, 2 \cdots n$

$$H^i(k) = \partial h(X)/\partial X|_{X=X^i(k|k)} \tag{10}$$

$$K^i(k) = P^1(k|k)(H^i(k))^T \left(H^i(k)P^1(k|k)(H^i(k))^T + R(k)\right)^{-1} \tag{11}$$

$$X^{i+1}(k|k) = X^1(k|k) + K^i(k)\left(Y(k) - h(X^i(k|k)) - H^i(k)(X^1(k|k) - X^i(k|k))\right) \tag{12}$$

$$P^{i+1}(k|k) = (I - K^i(k)H^i(k))P^1(k|k)(I - K^i(k)H^i(k))^T + K^i(k)R(k)\left(K^i(k)\right)^T \tag{13}$$

End For

$$X(k|k) = X^{i+1}(k|k) \tag{14}$$

$$P(k|k) = P^{i+1}(k|k) \tag{15}$$

Here, n is the maximum iterative number. The update algorithm of the IEKF reduces to that of the EKF in the case of a single iteration. Inevitably, the iteration will increase the filter time and improve the tracking precision. Compromise always has to be made between the tracking precision and computation cost.

3.2. Modified iterated extended Kalman filter

The IEKF update is an application of the Gauss–Newton method for approximating a maximum likelihood estimation. The Gauss–Newton method can guarantee the global convergence but cannot guarantee the achievement of the likelihood surface. The objective function of the IEKF is given by [14]:

$$f(X(k)) = \frac{1}{2}\left\| S(Z(k) - g(X(k)))\right\|^2, \tag{16}$$

where $S^T S = \bar{Q}^{-1}$, $Z(k)$=$[Y(k), X^1(k)]^T$, $Z(k) \sim \mathbf{N}(g(X(k)), \bar{Q}(k))$, $g(X(k)) = [h(X(k)), X(k)]^T$, $\bar{Q}(k) = \begin{bmatrix} R(k) & 0 \\ 0 & P^1(k) \end{bmatrix}$.

Eq. (16) indicates that the disadvantage of the IEKF is caused by its objective function, which largely depends on the initial estimate. The objective function can be modified as:

$$\tilde{f}(X(k)) = \frac{1}{2}\left\| \tilde{S}(\tilde{Z}(k) - g(X(k)))\right\|^2, \tag{17}$$

where $\tilde{S}^T \tilde{S} = \tilde{Q}^{-1}$, $\tilde{Z}(k)$=$[Y(k),X^i(k)]^T$, $\tilde{Z}(k)\sim \mathbf{N}(g(X(k)), \tilde{Q}(k))$, $g(X(k)) = [h(X(k)), X(k)]^T$, $\tilde{Q}(k) = \begin{bmatrix} R(k) & 0 \\ 0 & P^i(k) \end{bmatrix}$.

In the above equation, the initial estimate $X^1(k)$ and its covariance $P^1(k)$ are replaced by the iterative estimate $X^i(k)$and its covariance $P^i(k)$. Thus, the influence of the initial estimate error for the whole iterative process can be decreased. According to the properties of the Gauss–Newton method, a new update equation is obtained as:

$$X^{i+1}(k) = X^i(k) + K^i(k)(Z(k) - h(X^i(k))), \tag{18}$$

where

$$K^i(k) = ((H^i(k))^T R^{-1}(k) H^i(k) + (P^i(k))^{-1})^{-1} (H^i(k))^T R(k)^{-1}. \tag{19}$$

Accordingly, its covariance matrix can be written as:

$$P^{i+1}(k) = P^i(k) - K^i(k) H^i(k) P^{-1}(k). \tag{20}$$

Although the iteration can be completed by Eqs. (18), (19), and (20) [15], its gain requires computing the inverse matrix of the estimate covariance matrix and the inverse matrix of the measurement covariance matrix. In practice, the inverse operation is difficult to achieve and too many inverse operations may cause instability. Therefore, we deduce a new gain equation. Since the iterative procedure is a process of approximating a maximum likelihood estimate, the likelihood function can be written as:

$$L(X(k)) = \frac{1}{\sqrt{(2\pi)^{m+n}\left|\tilde{Q}(k)\right|}} exp(-\frac{1}{2}(\tilde{Z}(k) - g(X(k))^T \tilde{Q}^{-1}(k)(\tilde{Z}(k) - g(X(k))), \tag{21}$$

where m and n denote the dimensions of the measurement vector and the state vector. According to the property of the maximum likelihood estimate, we have:

$$(g'(X(k)))^T \tilde{Q}^{-1}(k)(\tilde{Z}(k) - g(X(k))) = 0. \tag{22}$$

Replacing g by its first-order approximation at $X^{i+1}(k)$, we can obtain:

$$X^{i+1}(k) - X(k) = ((g'(X^{i+1}(k)))^T Q^{-1}(k) g'(X^{i+1}(k)))^{-1} (g'(X^{i+1}(k)))^T Q^{-1}(k)(\tilde{Z}(k) - g(X^{i+1}(k))). \quad (23)$$

Thus, the estimate covariance matrix can be computed as:

$$\begin{aligned} P^{i+1}(k) &= \mathbf{E}((X^{i+1}(k) - X(k))(X^{i+1}(k) - X(k))^T) \\ &= ((H^i(k))^T R^{-1}(k) H^i(k) + (P^i(k))^{-1})^{-1} \end{aligned} . \quad (24)$$

It follows that:

$$(P^{i+1}(k))^{-1} = (H^i(k))^T R^{-1}(k) H^i(k) + (P^i(k))^{-1}. \quad (25)$$

According to the matrix inversion lemma [16], we have another form of the estimate covariance matrix, shown below.

$$\begin{aligned} P^{i+1}(k) &= P^i(k) - (P^i(k)(H^i(k))^T (H^i(k) P^i(k) (H^i(k))^T + R(k))^{-1}) H^i(k) P^i(k) \\ &= P^i(k) - K^i(k) H^i(k) P^i(k) \end{aligned} \quad (26)$$

Thus, a new gain equation is obtained:

$$K^i(k) = P^i(k)(H^i(k))^T (H^i(k) P^i(k) (H^i(k))^T + R(k))^{-1}. \quad (27)$$

Since the inverse matrix of the estimate covariance matrix and the inverse matrix of the measurement covariance matrix need not be computed in Eq. (27) and the number of the inverse operations decreases to 1 from 4, better feasibility and stability can be obtained.

A new maximum likelihood iteration termination criterion is also proposed in this paper. The maximum likelihood estimate for X is given by:

$$X^*(k) = \arg\min \left[q\left(X(k) \right) \right], \quad (28)$$

where

$$q\left(X(k) \right) = -\frac{1}{2} (\tilde{Z}(k) - g(X(k))^T \tilde{Q}^{-1}(k) (\tilde{Z}(k) - g(X(k)). \quad (29)$$

Suppose that $q\left(X^{i+1}(k)\right) < q\left(X^i(k)\right)$, the $q\left(X^{i+1}(k)\right)$ will be closer to the maximum likelihood surface than $q\left(X^i(k)\right)$, and $X^{i+1}(k)$ will be closer to the optimal solution than $X^i(k)$. From Eq. (29), we can get:

$$(\tilde{X}^i(k))^T (P^{i-1}(k))^{-1} \tilde{X}^i(k) + ((\tilde{Y}^i(k))^T R^{-1}(k) \tilde{Y}^i(k) < ((\tilde{Y}^{i-1}(k))^T R^{-1}(k) \tilde{Y}^{i-1}(k)), \quad (30)$$

where $\tilde{X}^i(k) = X^i(k) - X^{i-1}(k), \tilde{Y}^i(k) = \tilde{Y}(k) - h(X^i(k))$. Eq. (30) is the iteration termination condition.

In conclusion, the procedure of the MIEKF can be given as follows.

$$X(k|k-1) = \Phi(X(k-1)) \quad (31)$$

$$P(k|k-1) = F(k|k-1) P(k-1|k-1) F^T(k|k-1) + Q(k-1) \quad (32)$$

Set $X^1(k|k) = X(k|k-1)$, $P^1(k|k) = P(k|k-1)$

For $i = 1, 2 \cdots n$

$$H^i(k) = \frac{\partial h(X)}{\partial X}|_{X = X^i(k|k)} \quad (33)$$

$$K^i(k) = P^i(k)(H^i(k))^T(H^i(k)P^i(k)(H^i(k))^T + R(k))^{-1} \tag{34}$$

$$X^{i+1}(k|k) = X^i(k|k) + K^i(k)\left(Y(k) - h(X^i(k|k))\right) \tag{35}$$

$$P^{i+1}(k|k) = (I - K^i(k)H^i(k))P^i(k|k) \tag{36}$$

If Eq. (30) is satisfied, the iteration is terminated; otherwise, the iteration continues.

End

$$X(k|k) = X^{i+1}(k|k) \tag{37}$$

$$P(k|k) = P^{i+1}(k|k) \tag{38}$$

3.3. Modified iterated extended Kalman particle filter

As shown in the section above, the MIEKF has a bigger support overlap with the true posterior distribution than the overlap achieved by the IEKF and EKF estimates. In this paper, a novel filter named the MIEKPF is proposed. The MIEKF is used to generate the proposal distribution within the particle filter architecture. The procedure of MIEKPF is given as follows:

1. Initialization

Initializing a random particle set of N particles $\{(\hat{X}_b^i(0), w^i(0))|i = 1,2,\ldots,N\}$ from the prior probability distribution $p(\hat{X}_b(0))$, where $w^i(0) = 1/N$.

2. Recursion

For k = 1,2...

For i = 1:N

Computing the Jacobian matrix $F^i(k)$ of the state equation with Eq. (4):

a. Updating the particles with the MIEKF algorithm.

b. Getting the mean value $\bar{X}^i(k)$ and covariance $P^i(k)$ from the particle set $\{\hat{X}^i(k)\}_{i=1}^N$. An approximate sample $X^i(k)$ can be drawn from the importance density function:

$$q(X_b^i(k)|X_b^i(k-1), Y(1:k)) = N(\hat{X}_b^i(k); \bar{X}_b^i(k), P^i(k)). \tag{39}$$

c. Updating the weights. Calculating the weights of the particle according to the measurements values of the new state:

$$w^i(k) = w^i(k-1)\frac{p(Y(k)|X_b^i(k))p(X_b^i(k)|X_b^i(k-1))}{q(X_b^i(k)|X_b^i(k-1), Y(1:k))}. \tag{40}$$

End For

For i = 1:N

Standardizing the weights:

$$\bar{w}^i(k) = w^i(k)/\sum_{i=1}^{N} w^i(k). \tag{41}$$

End For

3. Resampling

Getting the new particle set according to the weight standardization and setting the threshold points N_{th} for the sample (N_{th} equals the particle numbers N under normal circumstances).

Computing the valid particle numbers:

$$\hat{N}_{eff} = 1/\sum_{i=1}^{N}(w^i(k))^2. \tag{42}$$

Computing the posterior probability estimate of the target state:

$$X_b(k) = \sum_{i=1}^{N} X_b^i(k)\bar{w}^i(k). \tag{43}$$

4. Simulation and results

The circular orbit satellite is chosen as the LEO satellite, and the HEO satellite obtains the directions of the LEO satellite at intervals of T = 1 s. The 2 satellites' orbit elements are shown in Table 1, where a is the semimajor axis, e is the eccentricity, i is the inclination, Ω is the right ascension of the ascending node, ω is the argument of perigee, and τ is the time past perigee.

Table 1. Orbit elements of the HEO and LEO satellites.

Orbit elements	HEO	LEO
a (km)	42,000	7171
e	0.1	0
i (degrees)	120	30
Ω (degrees)	30	75
ω (degrees)	45	60
τ (s)	0	500

In order to guarantee the universality of the simulated data, STK8.1 is used to generate the tracking scenario, and afterwards, the HEO and LEO satellites are imported into the tracking scenario using the orbit wizard function in STK8.1. The 2 satellites' ephemeris data are generated by importing the orbital elements in Table 1 into STK8.1. Using the report tool in STK8.1, we can obtain the 2 satellites' orbital data, consisting of position, velocity, and start and stop time. Using the access analysis function in STK8.1, we can obtain the entire access time table from the HEO to the LEO. The former 13,500 data points in the longest access duration are chosen as the simulation data, and Eq. (5) is used to generate measurement data.

Monte Carlo simulation results are presented here in order to demonstrate the tracking performance of the MIEKPF, and 200 runs were performed. The scenario of LEO satellite tracking is defined as follows: the initial state vector of LEO is $\hat{X}_b(0) = [-4558.2185km\ -5248.8001\,km\ 1759.5401km\ 4.4925\,km/s\ -4.9885\,km/s\ -3.2428km/s]^T$, the particle numbers are $N = 50$, and the measurement noise variance is $\sigma_\theta = 0.1mrad$, $\sigma_\varepsilon = 0.1mrad$, $P(0) = diag[1000\ 1000\ 1000\ 10\ 10\ 10]$. The maximum iterative number is $n = 5$. The trajectories of the satellites, shown in Figures 2–5, show the position estimation error of the 4 algorithms in the directions x, y, and z, respectively. The average root mean square errors (RMSEs) of the target's position and velocity for the 4 algorithms are shown in Tables 2 and 3, separately. The simulation results show that the

PF can significantly reduce the tracking error compared to the EKF. Moreover, the combination of the IEKF with the PF can improve the tracking precision. Specifically, the proposed MIEKPF has the highest tracking compared to the other 3 algorithms by integrating the latest measurement into the system state transition density. The position tracking precision of the proposed MIEKPF decreased by 18.37% and 42.71% compared with the IEKPF and EKPF, respectively. The velocity tracking precision of the proposed MIEKPF decreased by 42.33% and 50.27% compared with the IEKPF and EKPF, respectively.

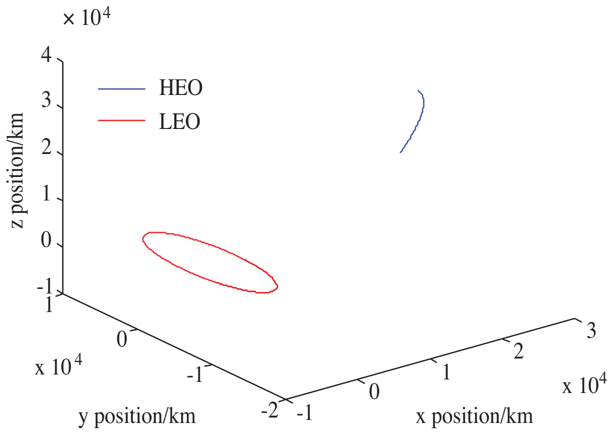

Figure 2. Trajectory of the LEO and HEO satellites.

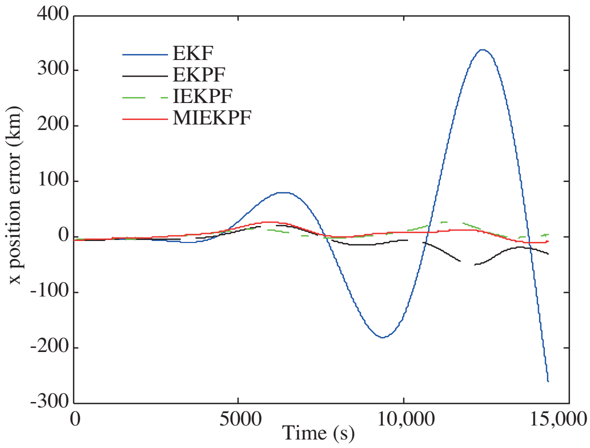

Figure 3. Position error of the x direction.

Table 2. The RMSE comparison of the position estimation.

Algorithms	x (km)	y (km)	z (km)
EKF	86.8234	90.8876	45.5191
EKPF	13.6351	8.2207	8.8515
IEKPF	8.0736	6.5008	7.4820
MIEKPF	7.6151	2.7976	6.5629

Table 3. The RMSE comparison of the velocity estimation.

Algorithms	x (km/s)	y (km/s)	z (km/s)
EKF	3.1580	3.8406	1.8640
EKPF	1.5280	1.7975	1.1247
IEKPF	1.2743	1.6108	0.9282
MIEKPF	0.8627	0.6382	0.7331

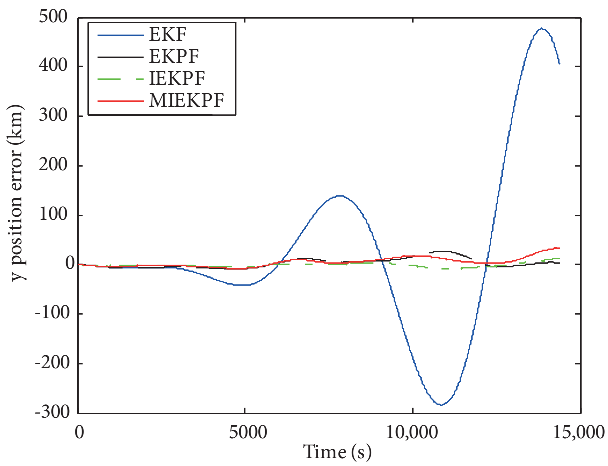

Figure 4. Position error of the y direction.

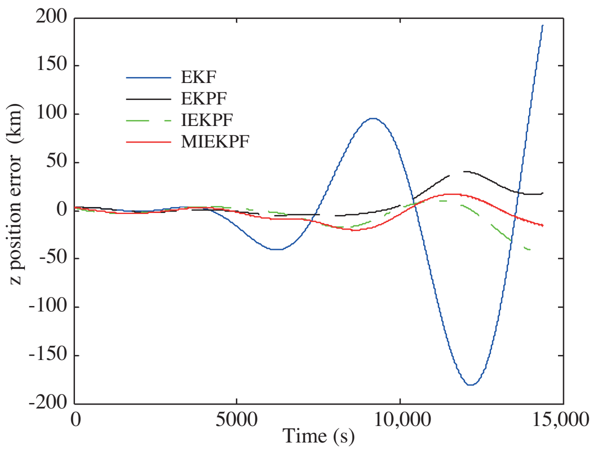

Figure 5. Position error of the z direction.

5. Conclusions

In this paper, a new particle filter that uses the MIEKF to generate the proposal distribution is proposed for single satellite-to-satellite passive tracking in the ECI Coordinate System. According to the essence of the MIEKF, where the Gauss–Newton method is used to approximate a maximum likelihood estimate, a new update method is obtained for the MIEKF. The MIEKF can generate an approximate MAP estimate of the system state; thus, the proposal distribution generated by the MIEKF is closer to the true posterior distribution than the IEKF or EKF. The linearized state propagation equation, the Jacobian matrix of the state transfer, and the measurement equation are derived in 2-body movement. Simulations have shown that the proposed MIEKPF has higher tracking precision than the EKF, EKPF, and IEKPF. The proposed MIEKPF is an effective algorithm for passive target satellite tracking systems.

Acknowledgments

This work was supported by the China Specialized Research Fund for the Doctoral Program of Higher Education (No.200802881017), NUST Research Funding (No.2010ZYTS051), and "Zijin Star" Research Funding (No.AB41381).

References

[1] S.X. Zhang, Trajectory Tracking and Satellite Orbit Measurement, Beijing, China, National Defense Industry Press, 1992.

[2] Q. Li, L.H. Shi, H.X. Wang, F.C. Guo, "Observability of satellite-to-satellite three dimensional passive tracking using bearing-only measurements", Journal of Astronautics, Vol. 30, pp. 1957–1966, 2009.

[3] G. Jauffret, D. Pillon, "Observability in passive target motion analysis", IEEE Transactions on Aerospace and Electronic Systems, Vol. 32, pp. 1290–1300, 1996.

[4] Q. Li, F.C. Guo, J. Li, Y.Y. Zhou, "Research of satellite-to-satellite passive tracking using bearings-only measurements in J2000 ECI frame", Proceedings of the IEEE International Conference on Radar, pp. 1–4, 2006.

[5] Z.G. Cao, Y. Gao, H.W. Cheng, "EKF tracking of low earth orbit satellites based on bearings-only data of a single space-based platform", Journal of Spacecraft TT&C Technology, Vol. 26, pp. 70–75, 2007.

[6] S.H. Wu, Q. Xin, J.W. Wan, "Simplified DDF algorithm and the application for satellite-to-satellite passive orbit determination and tracking", Journal of Astronautics, Vol. 30, pp. 1557–1563, 2009.

[7] F.C. Guo, Y. Fan, "A method of tracking satellite by single satellite using bearing-only measurement", Journal of Astronautics, Vol. 26, pp. 196–200, 2005.

[8] D. Crisan, A. Doucet, "A survey of convergence results on particle filtering methods for practitioners", IEEE Transactions on Signal Processing, Vol. 50, pp. 736–746, 2002.

[9] G.Y. Zhang, Y.M. Cheng, F. Yang, Q. Pan, "Particle filter based on PSO", Proceedings on Intelligent Computation Technology and Automation, Vol. 1, pp. 121–124, 2008.

[10] J.F.G. Freitas, M. Niranjan, A.H. Gee, A. Doucet, "Sequential Monte Carlo methods to train neural network models", Neural Computation, Vol. 12, pp. 955–993, 2000.

[11] S.Y. Yang, W.J. Huang, "A method of genetic algorithm optimized extended Kalman particle filter for nonlinear system state estimation", Proceedings of the 5th International Conference on Natural Computation, Vol. 5, pp. 313–316, 2009.

[12] L.Q. Li, H.B. Ji, J.H. Luo, "The iterated extended Kalman particle filter", IEEE International Symposium on Communications and Information Technology, Vol. 2, pp. 1213–1216, 2005.

[13] L.A. Johnston, V. Krishnamurthy, "Derivation of a sawtooth iterated extended Kalman smoother via the AECM algorithm", IEEE Transactions on Signal Processing, Vol. 49, pp. 1899–1909, 2001.

[14] M.B. Bell, W.C. Cathey, "The iterated Kalman filter update as a Gauss-Newton method", IEEE Transactions on Automatic Control, Vol. 38, pp. 294–297, 1993.

[15] Z.B. Yang, D.X. Zhong, F.C. Guo, Y.Y. Zhou, "Gauss-Newton iteration based algorithm for passive location by a single observer", Systems Engineering and Electronics, Vol. 29, pp. 2006–2009, 2007.

[16] R.A. Horn, C.R. Johnson, Matrix Analysis, Cambridge, Cambridge University Press, 1989.