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# Performance analysis of bid calculation methods in multirobot market-based task allocation

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**Abstract:** In this study, the empirical results of a market-based task allocation method for heterogeneous and homogeneous robot teams and different types of tasks in 2 different environments are presented. The proposed method allocates robots to tasks through a parallel multiitem auction-based process. The main contribution of the proposed method is energy-based bid calculations, which take into account both the heterogeneity of the robot team and features of the tasks. The multirobot task allocation problem is considered as the optimal assignment problem and the Hungarian algorithm is used to clear the auctions. Simulations are carried out using energy-based, distance-based, and time-based bid calculation methods. The methods are implemented using a 3-type task set: cleaning a space, carrying an object, and monitoring. The tasks may have different sensitivities and/or priority levels. Simulations show that robot-task allocations of all of the methods result in similar utility values when single-type and/or same-featured tasks are used. However, for different-type and/or different-featured tasks, the proposed energy-based bid calculation method assigns a greater number of high-sensitivity tasks compared to the other 2 methods while consuming almost the same amount of energy in both environments. Additionally, the energy-based method has a filtering behavior for high-priority tasks. These properties of the proposed method increase the efficiency of the robot team.

**Key words:** Multirobot, task allocation, market-based, optimal assignment, Hungarian algorithm, bid calculation, energy efficient, heterogeneous robot team

## 1. Introduction

Many complex and diverse applications require the use of more than one robot to obtain faster completion of tasks, increased robustness, and high-quality solutions, or because some tasks cannot be performed by a single robot. In order to exploit the advantages of multirobot systems, many researchers attempt to achieve coordination among multiple robots. In recent years, the multirobot task allocation (MRTA) problem has become one of the main topics in the coordination of multirobot systems. The task allocation problem can be defined as the allocation of a set of resources to a set of tasks. For MRTA, robots and their abilities are resources and the efficiency of solutions is closely related to the structure of the robot team and features of the tasks. For example, assigning a high-skilled robot to a task that requires lower skills decreases the efficiency of the team.

All of the tasks can be assigned to robots at once when their sequential order is known. In the literature, this type of MRTA is classified as a time-extended assignment (TA) [1] and studies on this problem were presented in [2–4]. However, the announcement time and the order of the tasks could not be known in advance

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for search and rescue applications after disasters (earthquake, tsunami, fire, and flood) or service applications in an environment like a hospital or an office. In these applications, tasks must be assigned as they appear. In the literature, this type of MRTA is classified as instantaneous assignment (IA) [1]. In recent years, studies on IAs have increased substantially [5–8].

Market-based methods are widely used to allocate robots to tasks. In [9], Dias et al. defined market-based multirobot coordination approaches and their important components, such as auction mechanism, costs, utilities, and valuations. Next, they surveyed and analyzed the related studies in terms of classification of planning, solution quality, scalability, heterogeneous teams, and dynamic events and environments. Moreover, they stated that market-based approaches are being used for both TA and IA problems. In market-based methods, a set of tasks is announced by an auctioneer (producer) and the robots (consumers) make an offer for these tasks. The producer may offer a price for each task and the consumers calculate their bids. In the literature, generally, the traveled distance by the robot or the time required to perform a task are used to calculate the price, cost, and bid for the task. On the other hand, robots consume energy to perform tasks. Thus, the price, cost, and bid for a task can also be calculated in terms of energy consumption. For a single-type task and time-extended allocation problems such as exploration, routing, or patrolling [3,10,11], distance-based, time-based, and energy-based bid calculation methods may produce similar solutions. However, in many applications, there could be different tasks (cleaning, carrying an object, etc.) and the same types of tasks may have different features, such as priority and sensitivity. If the market-based method provides an energy-efficient solution, the robot team may perform a greater number of high-featured (i.e. high-sensitivity and/or high-priority) tasks with a given limited amount of energy. However, because of their nature, distance-based and time-based methods do not check the consumed energy when allocating robots to tasks. In this case, a robot would not be allocated to a high-featured task that requires almost the same amount of energy as a low-featured task, or if it is a little farther or takes a little more time to complete than the low-featured task. Additionally, a high-skilled robot could be allocated to a low-featured task even though a nearby low-skilled robot is available. This may cause inefficient use of the robot team's resources.

In this study, a market-based method is proposed for a single-task, single-robot, and instantaneous-assignment (ST-SR-IA) MRTA problem. In the proposed method, the cost, price, and bid of a task are determined based on energy consumption. To the best of the authors' knowledge, energy-based cost, price, and bid calculations were not used in the market-based (or auction-based) methods for ST-SR-IA [1] problems. This is the main contribution of the proposed method. Additionally, the MRTA problem is considered as the optimal assignment problem (OAP) and the Hungarian algorithm is used to clear the auctions [12]. In order to show the effectiveness of the proposed method, distance-based, time-based, and energy-based bid calculation methods in market-based task allocation are analyzed for heterogeneous and homogeneous robot teams and tasks in 2 different environments. The rest of the paper is organized as follows: Section 2 reviews the related work in the literature; 2 examples are given in Section 3 to explain the motivation of the study; Section 4 relates the MRTA to the OAP and the Hungarian algorithm is explained; the proposed method is given in Section 5, followed by simulations in Section 6; and in the last section, conclusions and planned future studies are given.

## 2. Related work

In the literature, there are several economy-based task allocation studies through negotiations between the members of the robot team. Economy-based approaches were first proposed by Smith [13]. He developed the contract net protocol (CNP) to control multiagent systems. Later, CNP-based approaches were applied in multirobot applications [14–16].

In recent years, market-based methods have been widely used to allocate robots to tasks. Auctions are the most common mechanisms used in market-based approaches. In robotic applications, the items for sale are typically tasks, roles, or resources. The bid reflects the robots' costs or utilities that are associated with completing a task, satisfying a role, or utilizing a resource. Several auction methods are being used in the MRTA problem. The simplest kind of auction is a single-item auction [17,18], in which only one item is offered. In such auctions, each participant submits a bid, and the auctioneer awards the item to the best bidder. Some other auction methods are the sequential single-item auction [19] and repeated parallel single-item auction [20]. In [21], an auction method was used that combines the sequential single-item and repeated parallel single-item auctions. Combinatorial auctions are more complex: multiple items are offered and each participant can bid on any combination of bundles (i.e. subsets) of these items. In general, there are an exponential number of bundles to consider, which makes bid valuation, communication, and auction clearing intractable if all of the bundles are considered [22]. Studies in [23–25] used the combinatorial auction approach for task assignments.

In market-based approaches, several auction types are being used to increase the efficiency of the robot team. However, for efficient task allocation, robots must determine the costs as precisely as possible and must transform this cost into a meaningful bid. Dias et al. [4] claimed that including the features of the environment and robots in the price and cost calculations may increase the efficiency of the robot team. Their study was the first to give the relation between the cost estimation and task allocation efficiency in market-based approaches. They used utility maximization instead of cost minimization. Hence, the flexibility of the system increases and task priorities can be included in the bid calculations.

In [3], how to produce the bids and which bid structure could be used for a given objective function for a multirobot time-extended exploration task assignment was discussed. Mosteo and Montano [10] followed a similar approach for minimum total time and minimum resource usage optimization objective functions for the IA object-searching task. In these studies, single-type tasks with the same features were used and robot teams were homogeneous. In the following years, some studies included task features into cost and utility calculations for TA task assignment problems. In the study by Melvin et al. [26], high-priority targets were required to be visited in a specified time interval. In another study, using a time-decreasing reward approach, tasks were completed with the minimum possible cost [11]. In these 2 studies, a homogeneous robot team was also used for TAs. As seen from the above discussion, studies in the literature generally dealt with single-type tasks in TAs for homogeneous robot teams. However, in our study, the IA of various-type tasks with different features for heterogeneous robot teams is considered. Additionally, in the previous studies, assignments were done using distance-based and/or time-based cost calculations. In our study, an energy-based bid calculation method is used for task allocations. The tasks are assumed to be independent of each other. All of the tasks auctioned at an instant may be announced by a single robot or they could be announced by different robots. This auction type can be considered as a parallel multiitem auction method.

## 3. Motivation

In market-based MRTA problems, only the robots with adequate abilities can bid for a task. The bid can be for the cost or the utility of the task. A producer robot evaluates the bids and assigns the robot that offers the lowest cost or the highest utility to the task. In previous studies, generally, the bid of a task was calculated using either the traveled distance by the robot or the time required to perform the task. Although robots consume energy during task execution, distance-based or time-based bid calculations do not consider the energy consumption. Additionally, these methods generally ignore task characteristics such as priority, sensitivity, or total completion time. Thus, distance-based and time-based task allocation methods may lead to inefficient task-robot assignments. In the following, some possible inefficient task-robot assignment examples are given. It is shown that these inefficient assignments could be avoided if energy-based task allocation is used.

**Motivating example 1:** In this example, 2 robots bid for a task. Robot $R_1$ has ultrasonic and laser range finders, a gripper, and a camera. Robot $R_2$ has only an ultrasonic range finder and a gripper. The other properties of $R_1$ and $R_2$ are the same. We assume that a robot cannot bid for another task while performing a task and that both robots have the same velocity. The performance of bid calculation methods are compared for a high-featured task (Figure 1) and a low-featured task (Figure 2).

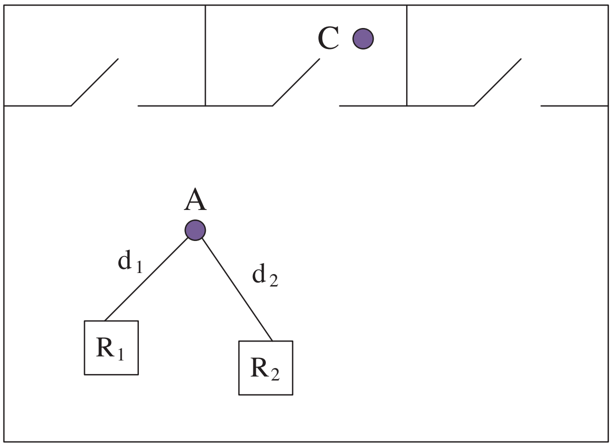


**Figure 1.** Two robots and 1 task (Case 1).

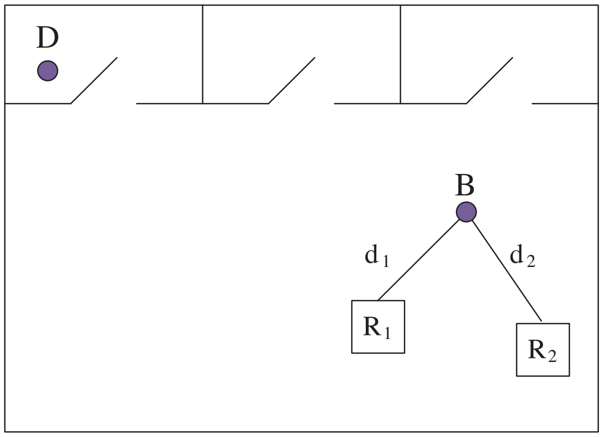


**Figure 2.** Two robots and 1 task (Case 2).

**Case 1:** Assume that a fragile object is to be carried from A to C. This task requires the carrying robot to have an ultrasonic range finder, a gripper, a laser range finder, and a camera. $R_1$ is $d_1 = 3$ m and $R_2$ is $d_2 = 3.2$ m away from point A. In this case, only $R_1$ can send a bid for the task and all of the bid calculation methods assign the task to $R_1$. This task assignment is considered efficient because a high-skilled robot is allocated to a high-featured task.

**Case 2:** Assume that a box is to be carried from B to D. For this task, the minimum requirement for the carrying robot is to have an ultrasonic range finder and a gripper. $R_1$ is $d_1 = 3$ m and $R_2$ is $d_2 = 3.2$ m away from point B. In this case, both $R_1$ and $R_2$ can perform the task. Therefore, bid calculation methods could make different assignments.

The distance-based bid calculation method uses only the distance to be traveled by the robot and assigns the tasks to the robots to minimize the total traveled distance by the team. In this case, the task is assigned to $R_1$.

In the time-based method, the allocation criterion is to minimize the total task completion time of all of the tasks. The task completion time is composed of 2 components: the travel time from the current position to the task location and the task performance time. Assuming that both robots can perform the task in the same duration, the task assignment is determined by the relative velocities of the robots. Since both robots have the same velocity, the same task-robot assignment from the distance-based method is obtained.

In the energy-based method, the bid is composed of 2 components: the energy consumed to travel from the current position to the task location and the energy consumed to perform the task. Both components are functions of the mass and velocity of the robot. Since $R_1$ carries extra devices (a laser range finder and a camera), it is heavier than $R_2$. Thus, $R_1$ consumes more energy than $R_2$ to perform the task. In this case, the bid of $R_2$ for the task is greater than the bid of $R_1$ and the task is assigned to $R_2$.

For this case, assume that after a while, a high-featured task is announced. For the distance-based and time-based bid calculation methods, the only available robot is $R_2$, but it does not have the proper equipment and cannot bid for the task. Therefore, this task should wait for $R_1$ to finish the first task, or it is not performed at all. This causes a delay in the completion of the tasks and inefficient use of the robots. However, in the energy-based bid calculation method, $R_1$ is available and bids for the task. Thus, the energy-based bid calculation method assigns the tasks to the robots more effectively than the other methods.

**Motivating example 2:** In this example, there are 2 tasks and 1 robot. It is assumed that both tasks are announced at the same instant and that the robot is closer to point B than point A (Figures 3 and 4). The behaviors of the bid calculation methods are compared using a low-skilled robot (Figure 3) and a high-skilled robot (Figure 4).

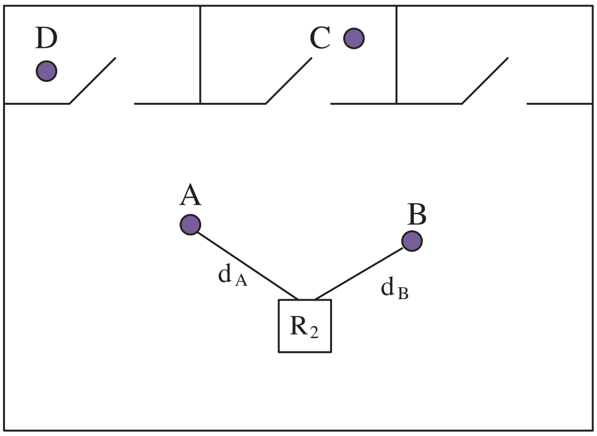


**Figure 3.** Two tasks and 1 robot (Case 1).

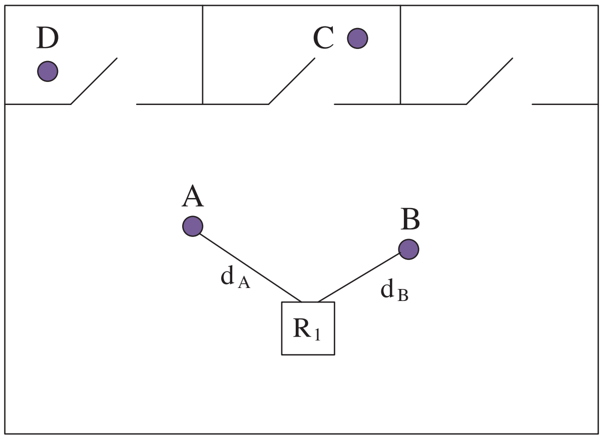


**Figure 4.** Two tasks and 1 robot (Case 2).

**Case 1:** Assume that the tasks and the low-skilled robot $R_2$ have the same properties as were given in the first example. In this case, all 3 bid calculation methods allocate $R_2$ to carry the box from B to D because $R_2$ cannot perform high-featured tasks.

**Case 2:** Now, instead of $R_2$, the high-skilled robot $R_1$ is going to be used. In this case, $R_1$ can bid for both the low-featured and the high-featured task.

In the distance-based method, a low-featured task is assigned to a robot because it is closer to that robot. It is likely that performing the high-featured task is more profitable than performing the low-featured task, but it is ignored by the nature of the distance-based method. In the time-based method, the task with a smaller completion time is assigned to the robot. In this example, the high-featured task is more sensitive than the low-featured task and, most probably, the completion time of the high-featured task is longer than that of the low-featured task. Therefore, the low-featured task is assigned to the robot.

Gerkey and Matarić [20] mentioned that prices could be purposefully biased: "if there exists a relative priority among the tasks, initial prices could be skewed so that more money is offered for higher-priority tasks". Similarly, features of a task could skew the price of the task and a higher price could be offered for high-featured tasks. The distance-based method does not consider the properties of the tasks. In the time-based method, only some of the task features can be included in the price. Thus, in these 2 methods, a biased price that considers all of the features of a task cannot be included to calculate the price of a high-featured task. However, in the energy-based method, all of the features of a task could be included in the price calculation. Therefore, the energy-based method may assign a high-featured task to a robot because a higher price is going to be offered for this task than the price of the low-featured task.

As a result, when a high-skilled robot bids for low-featured and high-featured tasks, an efficient assignment is obtained if the high-featured task is assigned to the robot. The proposed energy-based bid calculation method has the ability to allocate high-skilled robots to high-featured tasks rather than low-featured tasks.

## 4. The optimal assignment problem

### 4.1. Relation between MRTA and OAP

The MRTA problem can be defined as follows:

Assume that $m$ robots with different skills and $n$ tasks that require some skills are given. Each task should be assigned to a robot only if the robot has the required skills. The goal is to allocate robots to tasks to maximize the overall utility of the robot team.

Gerkey and Matarić [1] defined the OAP as follows.

"Given are m workers, each looking for one job; and n possibly weighted jobs, each requiring one worker. Also given for each worker is a nonnegative skill rating estimating his/her performance for each job (if a worker is incapable of undertaking a job, then the worker is assigned a rating of zero for that job). The goal is to assign workers to jobs so as to maximize overall expected performance, taking into account the priorities of the jobs and the skill ratings of the workers".

As seen from the definitions, the MRTA and the OAP are very similar. Therefore, solution methods for the OAP may be used to solve the MRTA. The OAP, also known as the weighted bipartite matching problem, is a widely studied problem in combinatorial optimization literature. Given a weighted bipartite graph $G(V, U, E)$ with $|V| = |U|$ and arc profits $p_{ij}$, the weighted bipartite matching finds a matching that maximizes the total profit. An optimal assignment is that which makes the total assignment profit maximum or a one-to-one matching of robots to tasks [27]. A mathematical model of the maximum weighted bipartite matching problem is given below [28].

<u>Indices</u>

$i, j$: Task and resource index

<u>Sets</u>

$v_i$ : Set of tasks

$u_j$ : Set of resources

<u>Parameters</u>

$n$ :Number of tasks

$m$ : Number of robots

$p_{ij}$ : Profit of the assignment of $v_i$ to $u_j$

<u>Decision variables</u>

$$x_{ij} = \begin{cases} 1 \\ \text{If } v_i \text{ is assigned to } u_j \\ 0 \\ otherwise \end{cases}$$

Model

$$\max z = \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=1}^{m} p_{ij} \cdot x_{ij} \tag{1}$$

s.t.

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1, \ldots, m \tag{2}$$

$$\sum\nolimits_{j=1}^{m} x_{ij}= 1, i = 1, \ldots,n \tag{3}$$

$$x_{ij} = 0 \,\mathrm{or}\, 1, i = 1, \ldots, n \text{ and } j = 1, \ldots, m \tag{4}$$

Eq. (1) is the objective function of the model, which maximizes the sum of the assignment profits. Eq. (2) ensures that each robot has to be assigned exactly one task. Similarly, Eq. (3) ensures that each task has to be allocated to exactly one robot. Eq. (4) represents integrality constraints.

### 4.2. Solution algorithm for the OAP (the Hungarian algorithm)

Several algorithms are available to solve the OAP. In this study, the Hungarian algorithm is used. The Hungarian or Kuhn–Munkres algorithm was originally proposed by Kuhn in 1955 [29] and refined by Munkres in 1957 [27]. The Hungarian algorithm solves the assignment problem in $O\left(n^3\right)$ time [30], where $n$ is the size of one partition of the bipartite graph. The Hungarian algorithm assumes the existence of a bipartite graph, $G = (V, U, E)$, where $V$ and $U$ are the sets of nodes in each partition of the graph and $E$ is the set of edges. As explained in the previous subsection, in the MRTA problem, the objective function is the maximization of the total profit. On the other hand, the Hungarian algorithm solves the assignment problem with cost minimization. A maximization problem can be transformed into a minimization problem as follows: first, find $t$ where $t = \max_{\forall(i,j)} p_{ij}$, and then obtain a cost matrix C as $c_{ij} = t - p_{ij} \forall(ij)$.

A detailed explanation of the Hungarian algorithm is given in [31]. A summary of the algorithm is given as follows:

**Input:** A bipartite graph, $G(V, U, E)G$, where $(|V|=|U|=n)$ and the $n \times n$ matrix of the edge costs $C$.

**Output:** A complete matching, $M$.

**Step 1:** For each row of the cost matrix, subtract the minimum element in the row from each element in the row.

**Step 2:** For each column of the resulting matrix, subtract the minimum element in the column from each element in the column. The resultant matrix is called the reduced matrix.

**Step 3:** Draw the minimum number of lines through the rows and columns to cover all of the zeros in the reduced matrix. If the minimum number of lines is equal to $n$, then an optimal solution is available; go to Step 5. Otherwise, go to Step 4.

**Step 4:** Select the minimum uncovered element of the reduced matrix. Subtract this element from each uncovered element and add it to each twice-covered element. Return to Step 3.

**Step 5:** Starting with the top row, make an assignment for each row. An assignment can be made when there is exactly 1 zero in a row. Once an assignment is made, delete that row and column from the reduced matrix.

The Hungarian algorithm uses a dual model of the problem while searching for the solution of the assignment problem. After the first 2 steps of the algorithm, the cost matrix $C$ of the dual model is obtained.

The optimal solution of the dual problem is obtained when there is exactly one $x_{ij}$ in each row equal to 1 and exactly one $x_{ij}$ in each column equal to 1. In this case, because of the optimality condition, $c_{ij}$ values corresponding to these $x_{ij}$ values must be zero. The third and fourth steps of the algorithm achieve these conditions. The last step assigns tasks to the robots.

## 5. The proposed method

A market-based method is proposed for the MRTA problem. Each robot assumes 2 roles in the market: producer and consumer. The producer announces the tasks to the market and the consumers give services to perform these announced tasks. Tasks are assigned to the robots through a parallel multiitem auction-based process. Since any robot in the team can start an auction, evaluate the bids, and determine the winners, the proposed method is a distributed market-based task allocation method. In the proposed method, the price, cost, and bid of a task are calculated based on the energy consumption of the robot. The Hungarian algorithm is used for auction clearance.

### 5.1. Producer, consumer, and auction clearing

A block diagram representation of a producer is shown in Figure 5. Each producer has a task list that holds the tasks to be announced. Starting from the first position, the prices of a randomly determined number of tasks are calculated. These tasks are announced along with their prices and are removed from the list after announcement. After the task announcement, the producer waits for a predefined amount time to receive the bids from the consumers. Next, the producer determines the winner(s) using the auction clearing algorithm. If the task is not assigned to any of the consumers, it is added to the end of the producer's task list. If the task is not assigned to any of the consumers after 3 announcements, it is deleted from the task list.

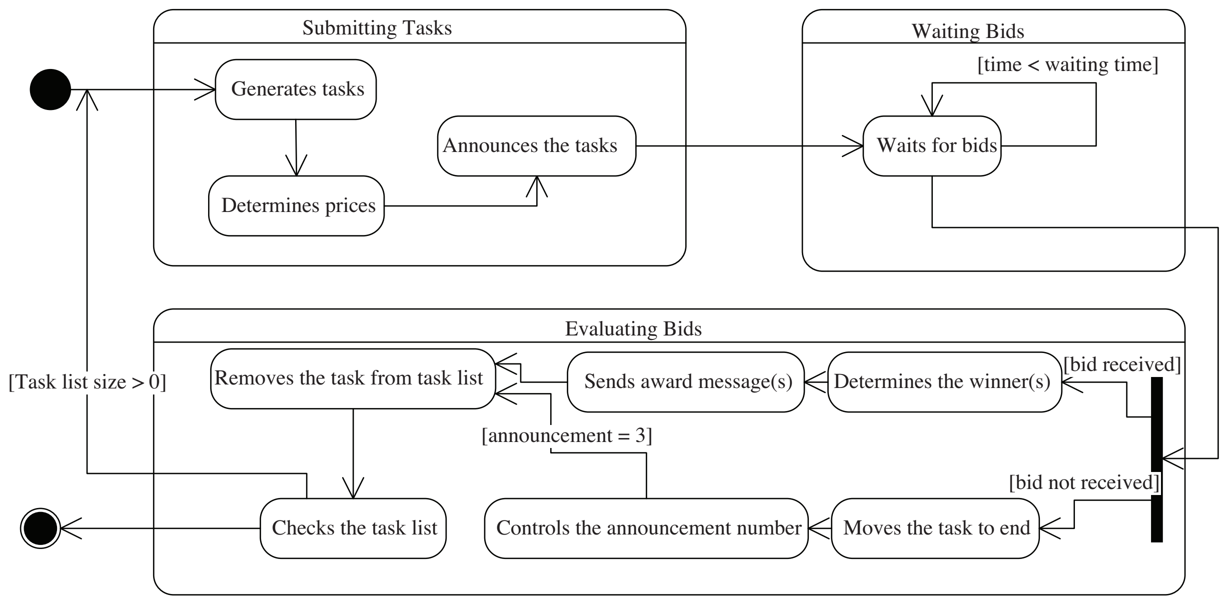


**Figure 5.** Block diagram representation of a producer.

Consumers supply goods or services to the producers. Tasks are assigned after some message exchanges between the producers and consumers. There are 3 types of messages: task, bid, and award messages. A task message contains the parameters, requirements, and prices of all of the tasks that are announced at an instant.

A consumer can bid for a task if it is not performing a task or is not at charging unit. After receiving a task message, all of the consumers that can bid for the tasks compare the requirements of each task with their resources. Each consumer calculates the cost of the task using its own parameters and its current state if it has adequate abilities and enough energy to perform this task, and repeats this procedure for all of the tasks within the task message. The bid of a task is determined by subtracting the cost from the price. Next, the consumer sends the bid message to the producer of the task. The producer evaluates the bids, determines the robot-task assignments, and sends award messages to the winning robots. The winner starts to perform the task after receiving the award message from the producer of the task. Each consumer controls its energy after completing the allocated task. If the energy of a consumer is lower than a specified threshold, it goes to a charging unit. Otherwise, it waits for new task announcements (Figure 6).

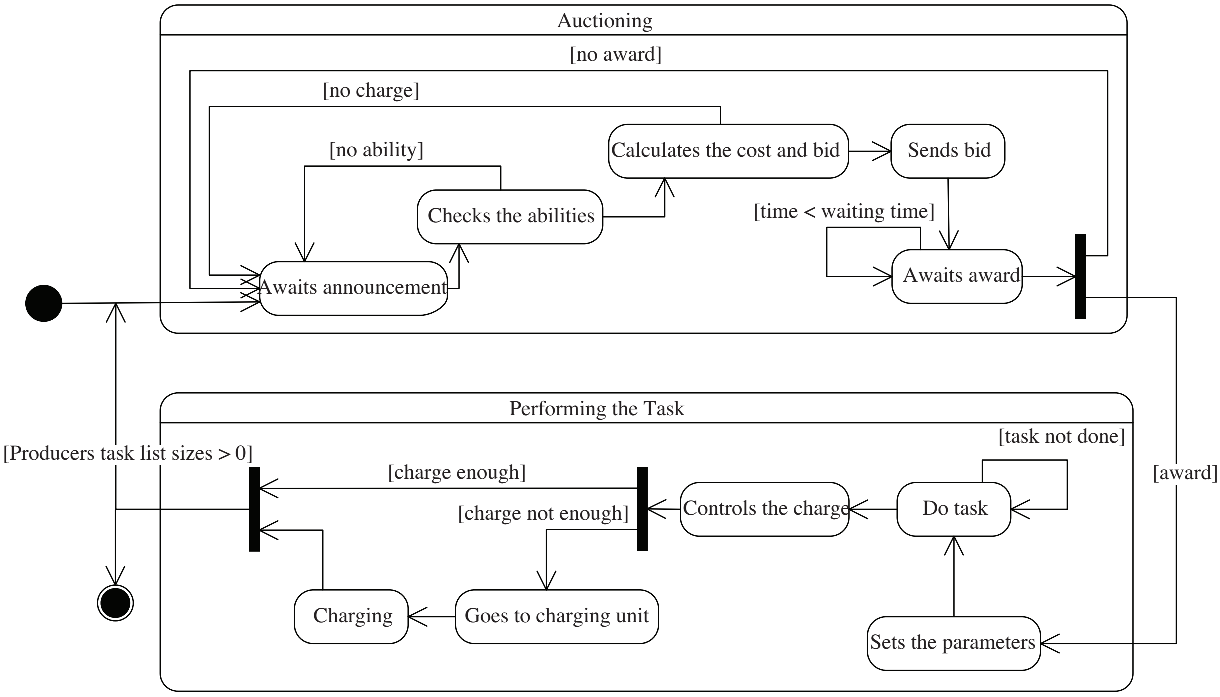


**Figure 6.** Block diagram representation of a consumer.

### 5.2. Price and cost calculations: power model of a robot platform

In market-based approaches, price and cost determine the position of the producers and consumers in the market. If the price and cost reflect the real positions of the producers and consumers, the efficiency of the market increases. Robots in the team may differ in terms of sensing abilities (i.e. sonar, camera, and laser range finder) and parameters (i.e. velocity, location, and current charge). Additionally, tasks may have different features such as priority, sensitivity, and completion time. For an efficient task allocation, robot abilities, parameters, and task features must be taken into account during bid calculations. Distance-based price and cost calculations include some robot parameters, such as location, but ignore task features and robot abilities. Time-based price calculations may consider task features and time-based cost calculations may consider some robot parameters (location and velocity). However, robot parameters and task features are not used together to calculate the price and cost of the task. Thus, the price and cost calculations of these 2 methods do not reflect the real positions of the producers and consumers.

In our paper, an energy-based method is proposed to determine the price and cost of a task using robot abilities, parameters, and task features all together. The proposed method requires the power model of the robot. The robot consumes energy for motion and sensing. In our paper, the power model developed by Mei et al. [32] is used.

1) Motion power:

$$P_m\left(m, v, a\right)=P_l+m\left(a+g\mu\right)v \tag{5}$$

Here, $P_m$ is the motion power, $P_l$ is the transformation loss, $m$ is the mass of the robot, $g$ is the gravity constant, $\mu$ is the ground friction constant, and $v$ and $a$ are the linear velocity and acceleration of the robot, respectively.

2) Sonar power:

$$P_s\left(f_s\right)=c_0+c_1 f_s \tag{6}$$

Here, $f_s$ is the sensing frequency, and $c_0$ and $c_1$ are 2 positive constants.

The energy consumption of other devices (microcontroller, computer, range sensors, etc.) on the robot is assumed to be constant. These constant values are included in the price and cost calculations.

### 5.3. Auction clearance: the Hungarian algorithm in task assignment

After receiving the bids, the producer constructs a bid matrix. Assume that, at an instant, the number of announced tasks by a producer is $n$ and the number of robots in the team is $m$. The bids are kept in a square matrix with a dimension of $p = \max(m, n)$. The rows and columns of the matrix represent the tasks and the robots, respectively. If robot $j$ has not bid for task $i$, a zero is placed in the $(i, j)$ entry of the matrix. The matrix may have 1 of the 3 different structures, depending on the number of robots and the number of tasks.

If $m > n$ , the matrix is $m \times m$ . $m - n$ rows with all 0 values are augmented to the bottom of the matrix.

If $m = n$ , the matrix is $m \times m$. No additional row or column is required.

If $m < n$ , the matrix is $n \times n$. $n - m$ columns with all 0 values are augmented to the right of the matrix.

An example of a bid matrix is given in Figure 7a. In this example, the number of tasks is 5 and the number of robots in the team is 6. A positive value at the position $(i, j)$ represents the bid of robot $j$ to task $i$. There are 2 all-0 rows in the matrix. The 2nd row represents the 2nd task and no bids were received for this task. The last row is an augmented 0 row because $m > n$. The Hungarian algorithm assigns tasks using the minimum entries of the input matrix. However, the assignments of this study must correspond to the maximum entries of the bid matrix because the objective is to maximize the utility of the robot team. Therefore, the matrix is transformed by subtracting all of the nonzero entries from the greatest entry of the matrix. In the example, the greatest entry is 6.5 and all of the nonzero values are subtracted from 6.5. Additionally, it is necessary to avoid undesired assignments. For this purpose, a very large number, ($Big\ M$) is inserted in undesirable assignment places. The transformed matrix of the example bid matrix is given in Figure 7b. The transformed matrix is then used as the input of the Hungarian algorithm.

$$\begin{bmatrix} 0 & 1.3 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 6.5 & 0 & 5 & 0 & 0 & 0 \\ 1.2 & 0 & 3 & 4 & 0 & 0 \\ 3.4 & 2 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**Figure 7a.** Example bid matrix.

$$\begin{bmatrix} M & 5.2 & M & 4.5 & M & M \\ M & M & M & M & M & M \\ 0 & M & 1.5 & M & M & M \\ 5.3 & M & 3.5 & 2.5 & M & M \\ 3.1 & 4.5 & M & M & M & 1.5 \\ M & M & M & M & M & M \end{bmatrix}$$

**Figure 7b.** Transformed matrix.

For the sample matrix, assignments (task-robot) 1-2, 3-1, 4-4, and 5-6 are obtained using the Hungarian algorithm and the total utility for the given assignments is 16.8.

## 6. Applications

Simulations are carried out to show the effectiveness of the proposed method. Each producer generates 30 tasks during a simulation. Each producer randomly generates approximately the same number of tasks of each type (cleaning, carrying, and monitoring). At an instant, the number of announced tasks by a producer is between 2 and 6. A producer is allowed to bid for the self-announced tasks. Therefore, a producer of a task may become a consumer of the same task. In the simulations, the task and robot team combinations of Table 1 are used. The simulations are repeated 20 times for each bid calculation method. The results of the 3 bid calculation methods are compared in terms of the percentage of allocated tasks, percentage of allocated tasks versus the sensitivity and priority of the tasks, and the energy consumption.

**Table 1.** Multirobot systems with respect to the robot teams and tasks.

| Multirobot systems used in the simulations | | |
|---|---|---|
| Tasks | Robot team | |
| | Homogeneous | Heterogeneous |
| Homogeneous | System1 | System2 |
| Heterogeneous | System3 | System4 |

### 6.1. Properties of robots and tasks

Three types of tasks and 6 robots are used in the simulations. The tasks are cleaning a room, carrying an object from one room to another, and monitoring a room for a specified time interval. For heterogeneous tasks, each task may have a certain level of the 2 features. The first feature is the sensitivity of the task. For example, carrying a fragile object has a higher sensitivity than carrying a nonfragile object. Since carrying a fragile object needs more attention, the robot should have more sensors and may spend more time to complete the task compared to a robot that carries a nonfragile object. In the simulations, 2 sensitivity levels (low and high) are used. The percentage of high-sensitivity tasks is 30% of all tasks. The second feature is the priority of the task. Three priority levels, low, normal, and high, are used. The percentage of normal-priority, low-priority, and high-priority tasks is 60%, 20%, and 20% of all of the tasks, respectively. For homogeneous tasks, all of the tasks are of low priority and low sensitivity. Additionally, 2 different robot teams are used in the simulations. One is a homogeneous robot team, in which each member of the team has all of the sensing resources. The second robot team is a heterogeneous one. The tasks and the robots of the heterogeneous team that are able to perform each type of task are given in Table 2. The $\surd$ sign represents that the robot can perform the task. Clean1 and Clean2 represent the clean task with low and high sensitivity, respectively. The Carry and Monitor tasks are divided using the same notation to represent the tasks. The resource requirement of each type of task is given in Table 3.

**Table 2.** Tasks and robots.

| Tasks | Robots | | | | | |
|---|---|---|---|---|---|---|
| | $MR_1$ | $MR_2$ | $MR_3$ | $MR_4$ | $MR_5$ | $MR_6$ |
| Clean1 (Cl1) | √ | X | X | √ | X | X |
| Carry1 (Cr1) | X | √ | X | X | √ | X |
| Monitor1 (M1) | X | √ | √ | X | √ | √ |
| Clean2 (Cl2) | √ | X | X | X | X | X |
| Carry2 (Cr2) | X | √ | X | X | √ | X |
| Monitor2 (M2) | X | X | √ | X | X | X |

**Table 3.** Resource requirement of each task.

| Tasks | Requirements | | | |
|---|---|---|---|---|
| | Sonar | Laser | Camera | Gripper |
| Clean1 (Cl1) | √ | X | X | X |
| Carry1 (Cr1) | X | X | X | √ |
| Monitor1 (M1) | X | √ | X | X |
| Clean2 (Cl2) | √ | X | √ | X |
| Carry2 (Cr2) | X | √ | X | √ |
| Monitor2 (M2) | X | √ | √ | X |

The price of a task is calculated by considering the worst-case conditions. In the distance-based method, the price is calculated for a robot that is located at the farthest location from the task. In the time-based method, the price is calculated using the slowest robot of the team and it is assumed that this robot is located at the farthest location from the task. In the energy-based method, the price is calculated by assuming that the highest-capacity robot is at the farthest location from the task. Calculating the price of the task in this manner ensures that all of the available robots in the environment can bid for the task. For a high-sensitivity or higher-priority task, the price is multiplied by a constant that is greater than unity. This increases the price of the task. Because of its higher price, a high-feature task becomes more attractive than other tasks for the consumers. Each consumer calculates the cost of a task using its own parameters, current state, and only the required resources to perform the task.

In the simulations, Pioneer 3-DX robots are used as the robot platform. A robot may have a SICK LMS200 laser rangefinder, Canon PTZ VC-C4 camera, and a gripper. Motion and sonar power are calculated using Eqs. (5) and (6), respectively. The value of the parameters in these equations are $a = 0.6\,m/s^2$, $g = 9.8\,m/s^2$, $\mu = 0.02$, $P_l = 0.25\,W$, $c_0 = 0.51$, $c_1 = 0.0039$, and $f_s = 40\,Hz$. The weight of the laser rangefinder, the camera, the gripper, and the Pioneer 3-DX Robot Platform with batteries is 4.5 kg, 0.375 kg, 1.125 kg, and 9 kg, respectively. The power consumed by the laser [33], the camera [34], and the gripper [35] were obtained from the official sites of the manufacturers of these devices. The power consumption of the laser, the camera, and the gripper is 20, 12, and 12 W, respectively. Additionally, the power consumption of the microcontroller and the computer is 4.6 and 12 W, respectively [32]. For the homogeneous robot team systems (System1 and System3) all of the robots have the same average velocity (0.2 m/s). The other parameters of the robots for all of the systems and the average velocity for System2 and System4 used in the simulations are given in Table 4.

The algorithms in the proposed approach are coded in C++ and tested using the maps of the first floor of the Eskişehir Osmangazi University Electrical and Electronic Laboratory Building (Figure 8) and the experimental environment in the Artificial Intelligence and Robotic Laboratory (Figure 9). On the first floor, there are 18 rooms, and in the experimental environment, there are 7 rooms. In these environments, 6 mobile robots $(MR_j, j = 1, \ldots, 6)$ are operated. Initially, the robots are in the rooms indicated by their names inside rectangles. The dimensions of the first floor and experimental environment are $52 \times 15$ m and $16 \times 18$ m, respectively.

**Table 4.** Parameters of the robots.

| Parameters | Robots | | | | | |
|---|---|---|---|---|---|---|
| | $MR_1$ | $MR_2$ | $MR_3$ | $MR_4$ | $MR_5$ | $MR_6$ |
| Initial charge (%) | 20% | 10% | 40% | 60% | 50% | 30% |
| Max charge (kW s) | 300 | 300 | 300 | 300 | 300 | 300 |
| Average velocity (m/s) | 0.25 | 0.2 | 0.3 | 0.2 | 0.3 | 0.25 |
| Grabbing time (s) | 0 | 5 | 0 | 0 | 5 | 0 |
| Dropping time (s) | 0 | 5 | 0 | 0 | 5 | 0 |

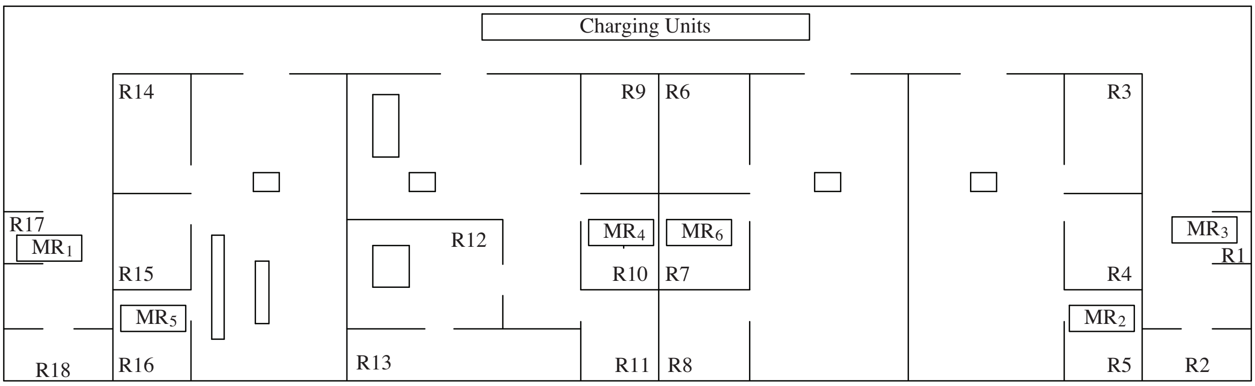


**Figure 8.** The first floor.

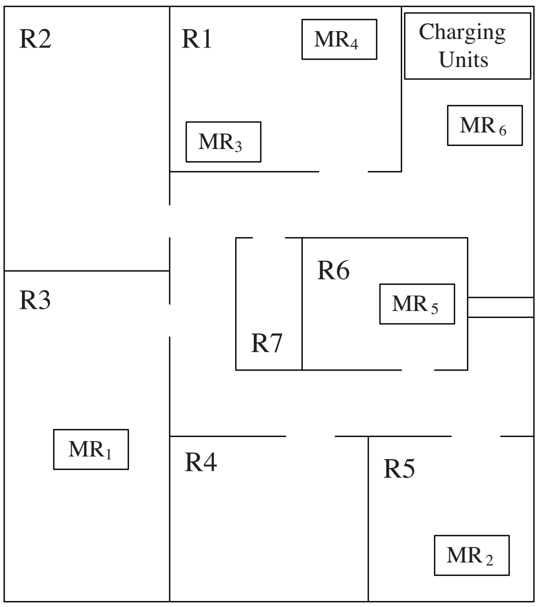


**Figure 9.** Experimental environment.

### 6.2. Performance analysis of the bid calculation methods for the 4 systems

The main purpose of the task allocation approach is to assign all of the tasks to robots with maximum utility or minimum cost. This can be accomplished if the robot team has enough robots to perform all of the tasks. However, in real applications, this requirement generally is not satisfied. In this case, the following questions arise: a) What percentage of the total number of tasks can be allocated? and b) How could the features of the tasks be handled? These questions indicate that a task-based performance analysis of the task allocation approaches is important.

During the simulations, the price, cost, and bid of a task are determined in terms of meters, seconds, and kilowatt seconds for the distance-based, time-based, and energy-based methods, respectively. In order to compare the bid calculation methods with respect to price, cost, and utility, all of these parameters must be in the same unit, such as kilowatt seconds. For this reason, after the simulations, the cost of each task for the distance-based and time-based methods is converted into kilowatt seconds using the power model of the P3-DX mobile robot. Similarly, the price of each task is converted into kilowatt seconds using the power model of the P3-DX mobile robot and multipliers for the priority and sensitivity levels. The results given in the following Tables and Figures for the price, cost, utility, and percentage of the allocated tasks are the averages of 20 runs.

#### 6.2.1. Homogeneous tasks

In the homogeneous-task simulations, low-priority/low-sensitivity tasks are considered. In this case, allocating a low- or high-capacity robot to a task would not make a significant difference because robots use only the required devices to perform the allocated task and the energy consumption is calculated using these devices. Therefore, the advantages of the energy-based method do not become visible for System1 and System2. As a result, although energy-based, distance-based, and time-based methods could generate different robot-task allocations, the total utility of the robot team for all of the methods would be almost equal. The average total price, total cost, and total utility of the allocated tasks for System1 and System2 in both environments are given in Tables 5 and 6, respectively. As can be seen, the price, cost, and utility of the allocated tasks are almost the same for all 3 bid calculation methods in the same environments. The energy consumption of a high-capacity robot is greater than that of a low-capacity robot because it carries extra devices. Therefore, the utilities of the task allocations of each method are greater for System2 than the utilities obtained for System1 in the same environments.

**Table 5.** Average total price, total cost, and total utility of the allocated tasks for System1 (kW s).

| Environment | Method | Price | Cost | Utility |
|---|---|---|---|---|
| First floor | Energy | 408.80 | 335.20 | 73.60 |
| | Time | 417.76 | 345.69 | 72.07 |
| | Distance | 410.22 | 338.59 | 71.63 |
| Experimental | Energy | 303.80 | 266.62 | 37.18 |
| | Time | 308.94 | 272.25 | 36.69 |
| | Distance | 313.71 | 277.86 | 35.85 |

For System1, the energy-based method calculates the cost and price of a task by considering the required resources and the completion time. Therefore, small differences may occur between the utilities of clean, carry, and monitor tasks. Similarly, in the time-based method, there are small differences between the utilities of the tasks. However, the distance-based method takes into account only the distance traveled by the robot in the cost and price calculations, and there is no difference between the utilities of the tasks. Robots are allocated to tasks depending on the differences between the utilities. Since the energy-based and time-based bid calculation methods calculate different utilities for different types of tasks, the percentage of the allocated tasks would be different in these methods. However, in the distance-based method, a similar percentage of the allocated tasks is expected. Figure 10 shows the results of 2 different environments [first floor (FF) and experimental environment (EE)], where it can be seen that the percentage of the allocated tasks is different for the energy-based and time-based methods, but it is almost the same for the distance-based method.

**Table 6.** Average total price, total cost, and total utility of the allocated tasks for System2 (kW s).

| Environment | Method | Price | Cost | Utility |
|---|---|---|---|---|
| First floor | Energy | 396.63 | 314.23 | 82.40 |
| | Time | 399.32 | 315.64 | 83.68 |
| | Distance | 392.61 | 311.69 | 80.92 |
| Experimental | Energy | 314.08 | 265.45 | 48.63 |
| | Time | 313.58 | 264.70 | 48.88 |
| | Distance | 320.26 | 270.80 | 49.46 |

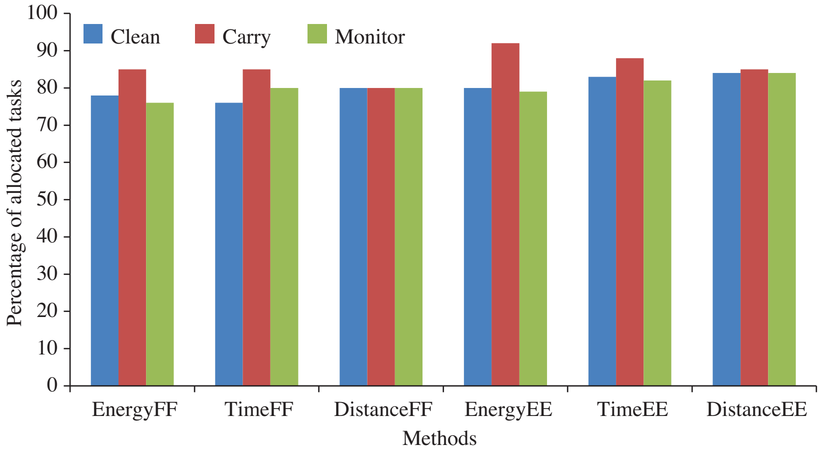


**Figure 10.** Average of the percentage of the allocated tasks for System1.

The percentage of allocated tasks of a task type is directly proportional to the number of robots that can perform this task. The bid calculation method would not increase the percentage of the allocated task when the number of robots for a given task is very limited. Figure 11 shows the average of percentage of allocated tasks for System2 in both environments. Due to the limited resources (laser, camera, gripper, etc.) of the robot team, the robot-task allocations in all 3 methods are similar to each other and the percentage of allocated tasks shows similar characteristics for the same environments.

### 6.2.2. Heterogeneous tasks

In System3 and System4, tasks could have low/high-priority and/or low/high-sensitivity. The advantages of the energy-based method come into sight in these systems because the tasks are heterogeneous. In both systems, the energy-based method behaves as a filter by allocating high-capacity robots in the team to high-priority and/or high-sensitivity tasks. As a result, the robot team exploits scarce resources to perform high-featured tasks. The average of total price, total cost, and total utility of the allocated tasks for System3 and System4 are given in Tables 7 and 8, respectively. The utility column shows that the energy-based method generates more efficient robot-task allocations than the other 2 methods for both environments and systems.

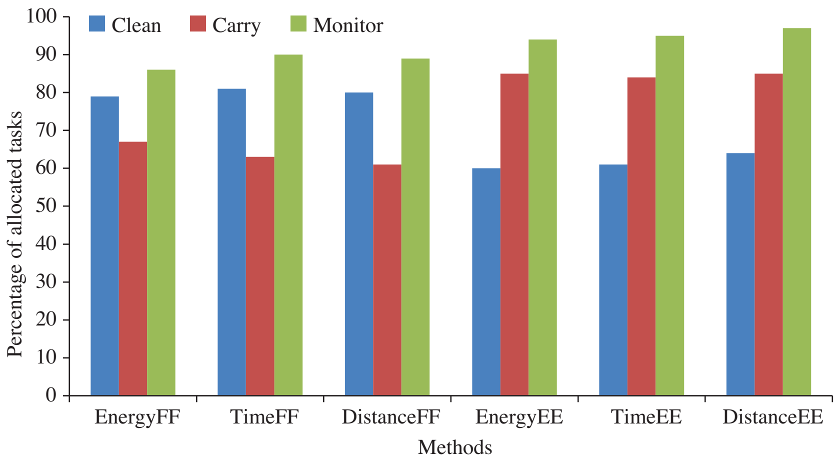


**Figure 11.** Average of the percentage of the allocated tasks for System2.

**Table 7.** Average total price, total cost, and total utility of the allocated tasks for System3 (kW s).

| Environment | Method | Price | Cost | Utility |
|---|---|---|---|---|
| First floor | Energy | 3032.68 | 655.28 | 2377.40 |
| | Time | 2528.26 | 584.84 | 1943.42 |
| | Distance | 2610.52 | 598.26 | 2012.26 |
| Experimental | Energy | 2867.02 | 617.19 | 2249.83 |
| | Time | 2411.70 | 542.03 | 1869.67 |
| | Distance | 2475.38 | 558.86 | 1916.52 |

**Table 8.** Average total price, total cost, and total utility of the allocated tasks for System4 (kW s).

| Environment | Method | Price | Cost | Utility |
|---|---|---|---|---|
| First floor | Energy | 2463.33 | 504.11 | 1959.22 |
| | Time | 2335.95 | 501.18 | 1834.77 |
| | Distance | 2313.53 | 497.34 | 1816.19 |
| Experimental | Energy | 2182.36 | 452.61 | 1729.75 |
| | Time | 2023.68 | 436.57 | 1587.11 |
| | Distance | 1954.40 | 425.32 | 1529.08 |

These 2 systems have different hardware resources. System3 may perform more high-priority and/or high-sensitivity tasks than System4 because it has more hardware resources. In this case, it is expected that System3 results in higher utility than System4 in all of the bid calculation methods for the same environments.

The average of the percentage of the allocated tasks for System3 on the first floor and in the experimental environment is given in Figures 12 and 13, respectively. The letters E, T, and D beside the task type represent the energy-based, time-based, and distance-based bid calculation methods, respectively. In the energy-based method, for both environments, the percentage of allocated high-priority and/or high-sensitivity tasks is significantly higher compared to the allocated lower-priority and low-sensitivity tasks. However, this behavior is not observed in the time-based and distance-based methods.

Figure 14 shows the average of the percentage of the total allocated tasks. As explained for System1, a different percentage of the total allocated tasks is expected for the energy-based and time-based methods. However, in System3, there are high-sensitivity tasks that take a longer time to perform than low-sensitivity tasks. Therefore, the percentage of the total allocated tasks is lower than the percentage of the total allocated tasks in System1.

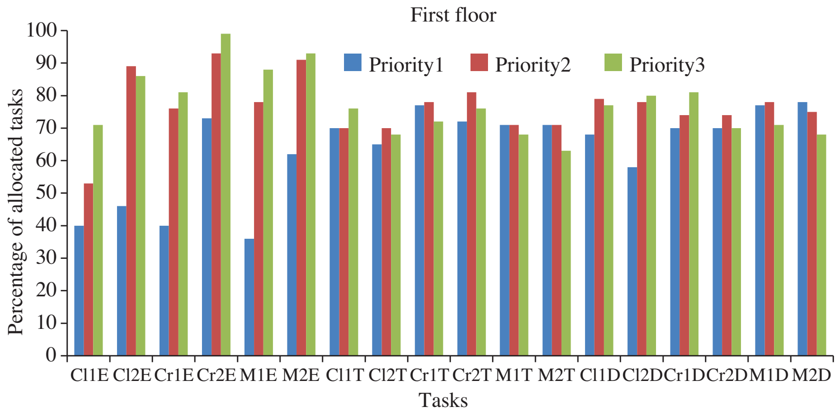


**Figure 12.** Average of the percentage of the allocated tasks for System3 on the first floor.

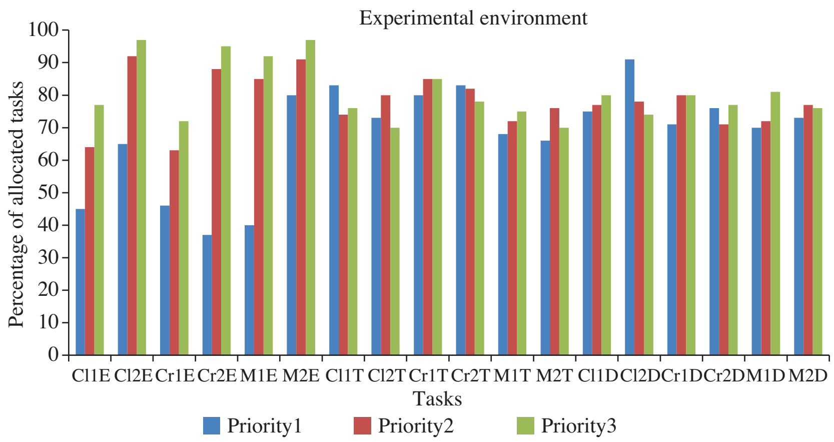


**Figure 13.** Average of the percentage of the allocated tasks for System3 in the experimental environment.

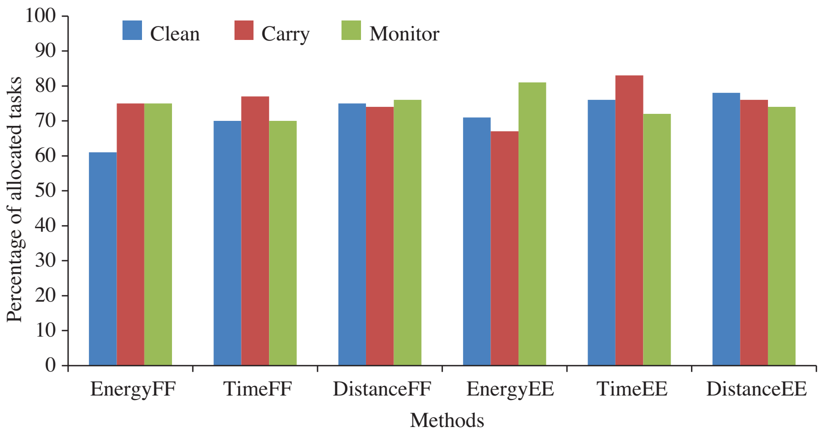


**Figure 14.** Average of the percentage of the total allocated tasks for System3.

The average of the percentage of the allocated tasks for System4 on the first floor and in the experimental environment is given in Figures 15 and 16, respectively. As can be seen, the energy-based method allocates the limited resources to perform high-priority and/or high-sensitivity tasks. As a result, although the heterogeneity of the robot team brings an upper limit for the percentage of the allocated tasks, the energy-based method continues to filter priority levels. Thus, in both environments, the percentages of allocated high-priority tasks are higher compared to the allocated lower-priority tasks. However, this behavior is not observed in the time-based and distance-based methods.

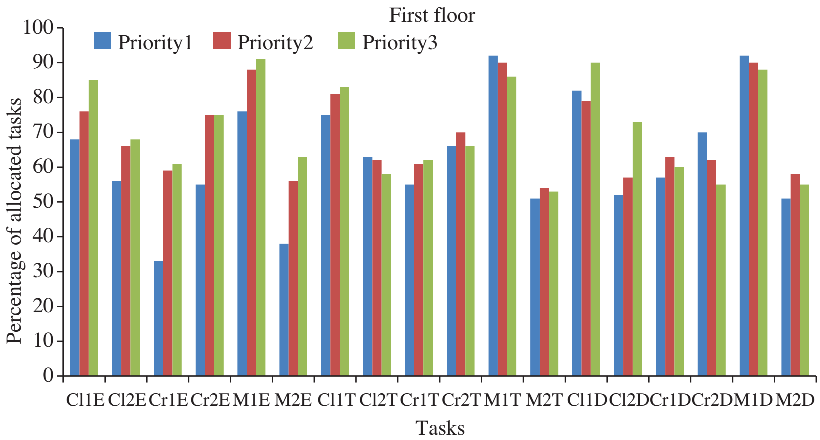


**Figure 15.** Average of the percentage of the allocated tasks for System4 on the first floor.

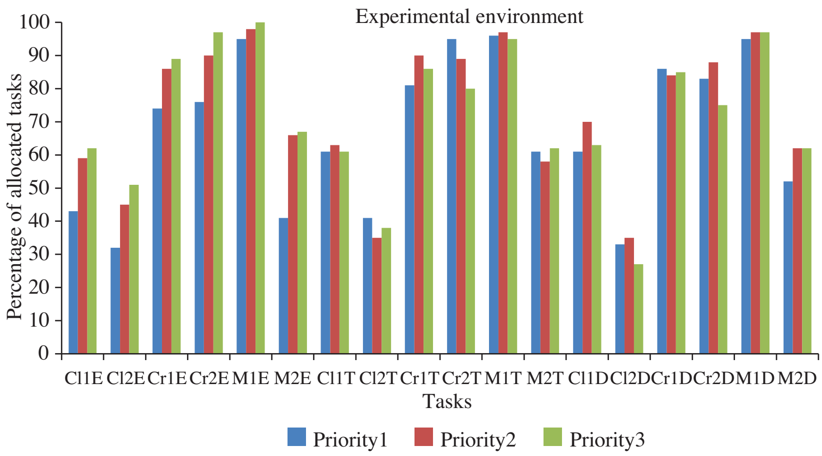


**Figure 16.** Average of the percentage of the allocated tasks for System4 in the experimental environment.

Figure 17 shows the average of the percentage of the total allocated tasks. The results of System4 have a pattern similar to that of the results of System2. However, in System4, there are high-sensitivity tasks that take a longer time to perform than the low-sensitivity tasks. Therefore, the percentage of the total allocated tasks is lower than the percentage of the total allocated tasks in System2.

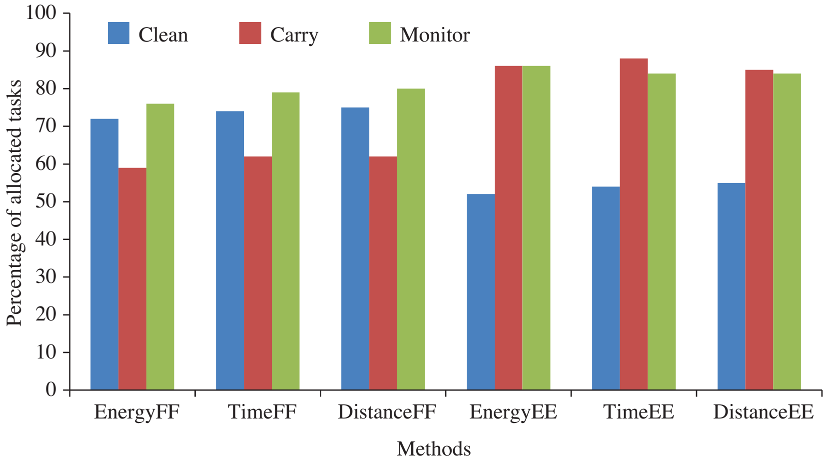


**Figure 17.** Average of the percentage of the total allocated tasks for System4.

## 7. Conclusion

In this paper, we have proposed an energy-based bid calculation method for market-based multirobot task allocation. This study has addressed the ST-SR-IA task allocation problem for heterogeneous robot teams and different types of tasks. The proposed method is compared with the distance-based and time-based bid calculation methods. Simulations show that the robot-task allocation of all of the methods results in similar utility values when single-type and/or same-featured tasks are used. However, for different-type and/or different-featured tasks, the proposed energy-based bid calculation method behaves as a filter to allocate high-skilled robots to high-featured tasks. This filtering property of the proposed method increases the efficiency of the robot team. In the future, the authors plan to apply the energy-based bid calculation method to time-extended MRTA problems. Another extension of this study may be relaxing the requirement that an accepted task must be finished by the robot. Thus, a robot may accept a more profitable task and may give away the task that it is already performing.

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