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Chaos control of single time-scale brushless DC motor with sliding mode control method

Yılmaz UYAROĞLU,* Barış CEVHER
Department of Electrical and Electronics Engineering, Faculty of Engineering, Sakarya University, 54187 Serdivan, Turkey

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Abstract: In this paper, the sliding mode control (SMC) scheme of single time-scale brushless DC motor (BLDCM) is investigated. The SMC method consists of 2 sections. To simplify the directive of the stability of the controlled single time-scale BLDCM in the sliding mode, first a special type of PI switching surface is adopted. Second, the SMC controller is obtained to guarantee the occurrence of the PI switching surface. The effectiveness of the theoretical analysis is evaluated by numerical simulations. Thus, the numerical results are used to show the verification and trustworthiness of the proposed method.

Key words: Brushless DC motor, sliding mode control, chaos, nonlinear system

1. Introduction

Chaos is defined as an aperiodic long-time behavior arising in a deterministic dynamical system that exhibits a sensitive dependence on initial conditions [1]. A key element of deterministic chaos is the sensitive dependence of the trajectory on the initial conditions. This, the basic characteristic of chaotic behavior, is due to the internal structure of the systems. However, chaotic behavior may lead to undesirable effects and may need to be controlled.

Many researchers have endeavored to find new ways to suppress and control chaos more efficiently. So far, many researchers have presented different types of controllers and control methods, e.g., linear state error feedback control [2,3], sinusoidal state error feedback control [4], variable substitution (or replacing variable) control [5,6], variable structure control [7,8], nonlinear feedback control [9], active control [10], and adaptive control [11] have been successfully applied to chaotic systems.

In recent years, brushless motors have been used as a viable choice for motion control applications, such as in electric propulsion, robotics, or aerospace. The advantages of brushless motors when compared with conventional DC motors have caused increasing interest. This interest was provided with the elimination of the physical contact between the mechanical brushes and commutators. Among the numerous types of brushless motors, the brushless DC motor (BLDCM) is the one with the highest potential in high-performance applications. Therefore, BLDCMs are widely used in industrial applications. Hemati, Ge and Chang, and Ge et al. [12–15] used the bifurcation theory to study BLDCMs. Their studies showed that these kinds of machines experience chaotic oscillations. These undesirable chaotic oscillations need to be eliminated. Sliding mode control (SMC) is especially preferred by many researchers due to its capability to tolerate disturbances and

*Correspondence: uyaroglu@sakarya.edu.tr

dynamic model uncertainties [16–23]. Thus, the SMC controller is designed as a nonlinear controller to eliminate the undesirable chaotic oscillations. Herein, the control of the chaotic system of a BLDCM is investigated by SMC terms.

This paper presents the chaos control of a single time-scale BLDCM by means of SMC. This work is organized as follows. In Section 2, the mathematical model of a single time-scale BLDCM is given. In Section 3, the chaos control of a single time-scale BLDCM chaotic system is investigated. In Section 4, numerical simulations are provided to confirm the validity of the method. Finally, in Section 5, the conclusions are given.

2. Mathematical model of single time-scale BLDCM

A BLDCM is an electromechanical system. The equations for the electrical and mechanical dynamics of a BLDCM were described by Hemati [12] and Ge and Chang [13]. The system equations for the BLDCM are transformed via Park's transformation and take the following form:

$$\left.\begin{array}{l} \dot{i}_q = \frac{1}{L_q}\left[-Ri_q - nw\left(L_d i_d + k_t\right) + v_q\right] \\ \dot{i}_d = \frac{1}{L_q}\left[-Ri_d - nwL_d i_q + v_q\right] \end{array}\right\}, \tag{1}$$

and the electromagnetic torque can be rewritten as follows:

$$T\left(i_q, i_d\right) = n\left[k_t i_q + \left(L_d - L_q\right) i_q i_d\right], \tag{2}$$

where i_q, i_d are the quadrature axis and direct axis currents; v_q, v_d are the quadrature axis and direct axis voltages; L_q, L_d are the fictitious inductance on the quadrature axis and direct axis; R is the winding resistance; n is the number of permanent pole pairs; and $k_t = \sqrt{3}/2k_e$, where k_e is the permanent magnet flux constant.

The system equations are transformed to a compact form through a single time-scale transformation [14,15]. Hence, the equations in compact forms, with a greatly reduced number of parameters, were obtained by Ge et al. [15]. After this transformation by Ge et al. [15], the system equations of the BLDCM take the following form:

$$\left.\begin{array}{l} \dot{x}_1 = v_q - x_1 - x_2.x_3 + \rho.x_3 \\ \dot{x}_2 = v_d - \delta.x_2 + x_1.x_3 \\ \dot{x}_3 = \sigma\left(x_1 - x_3\right) + \eta.x_1.x_2 - T_L \end{array}\right\}, \tag{3}$$

where $v_q = 0.168$, $\rho = 60$, $v_d = 20.66$, $\delta = 0.875$, $\eta = 0.26$, $T_L = 0.53$, and $\sigma = 4.55$.

In Figure 1, the phase portraits of the BLDCM system, given in Eq. (3), are presented. Figure 1a shows $x_1 - x_3$ and Figure 1b shows $x_1 - x_2$. In Figure 2, the time responses of the state variables of the BLDCM system are given.

3. SMC design for chaos control of a single time-scale BLDCM

The proposed BLDCM chaotic system is described in Eq. (3). Thus, the controlled chaotic system of the single time-scale BLDCM is attained as follows:

$$\left.\begin{array}{l} \dot{x}_1 = v_q - x_1 - x_2.x_3 + \rho.x_3 + u_1 \\ \dot{x}_2 = v_d - \delta.x_2 + x_1.x_3 + u_2 \\ \dot{x}_3 = \sigma\left(x_1 - x_3\right) + \eta.x_1.x_2 - T_L + u_3 \end{array}\right\}, \tag{4}$$

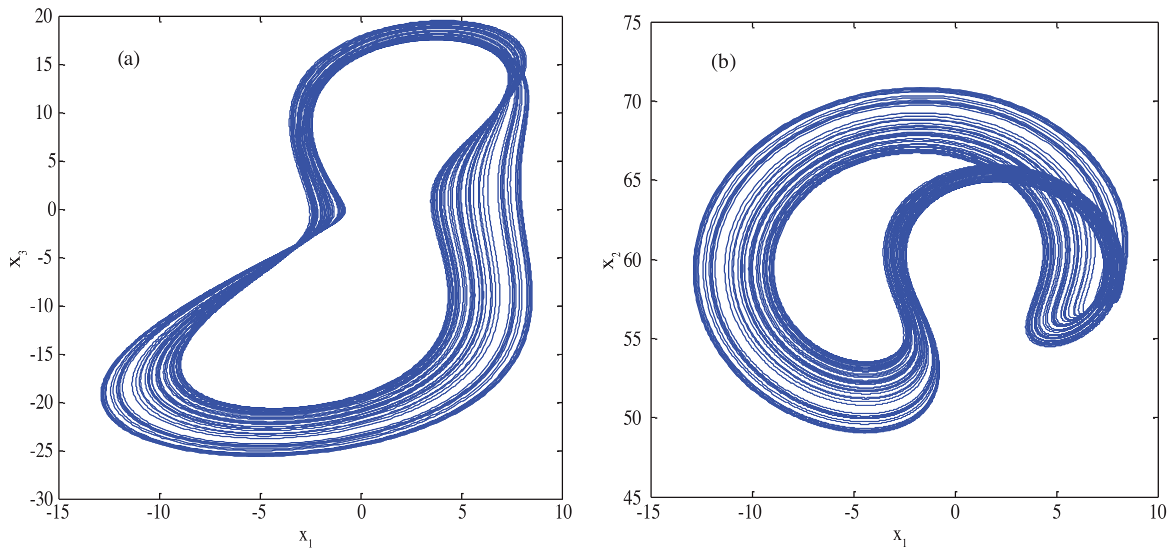

Figure 1. σ = 4.55 for the phase portraits of the uncontrolled BLDCM system: a) $x_1 - x_3$ and b) $x_1 - x_2$.

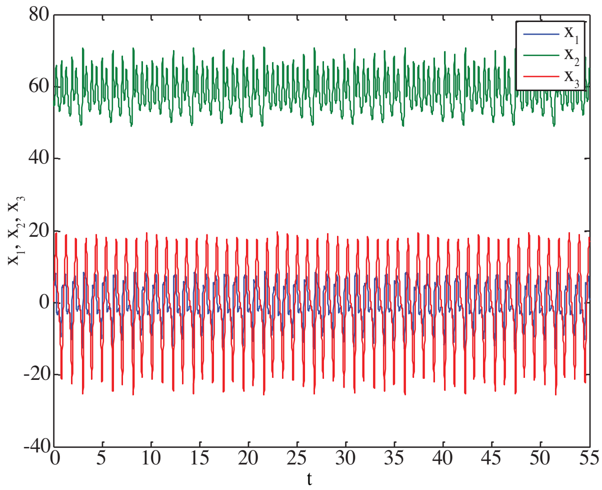

Figure 2. σ = 4.55 for the time series of the uncontrolled BLDCM system.

where u_1, u_2, and u_3 are control signals.

$$e = x - x_d, \tag{5}$$

where $e = [e_1 e_2 e_3]^T$ is the tracking error vector. The error dynamics may be written as below:

$$\dot{e} = \dot{x} - \dot{x}_d = Ax + Bg + Bu - \dot{x}d, \tag{6}$$

where A is the system matrix, B is the control matrix, and g represents the system nonlinearities plus the parametric uncertainties in the system. The control problem is to get the state $\mathrm{x} = [\mathrm{x}_1\ \mathrm{x}_2\ \mathrm{x}_3]^T$ to track a specific time-varying state $\mathrm{x}_d = [\mathrm{x}_{d1}\ \mathrm{x}_{d2}\ \mathrm{x}_{d3}]^T$ in the presence of nonlinearities.

$$A = \begin{bmatrix} -1 & 0 & \rho \\ 0 & -\delta & 0 \\ \sigma & 0 & -\sigma \end{bmatrix}; \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad g = \begin{bmatrix} v_q - x_2.x_3 \\ v_d + x_1.x_3 \\ -T_L + \eta.x_1.x_2 \end{bmatrix}$$

Now, a time varying proportional plus integral (PI) sliding surface $s(\mathrm{e},\ \mathrm{t}) \in \mathrm{R}^3$ is defined by the scalar equation $s = s(\mathrm{e},\ \mathrm{t})$ as:

$$s = Ke - \int_0^t K\left(A - BL\right)e(\tau)d\tau, \tag{7}$$

where $K \in R^{3\times 3}$, which must satisfy $\det(KB) \neq 0$, is a gain matrix, and $L \in R^{3\times 3}$, which must have a stable A-BL, is a gain matrix; namely, the eigenvalues $\lambda_{i(i=1,2,3)}$ of the matrix A-BL are negative ($\lambda_i| < 0$). It is well known that when the system operates in the sliding mode, the sliding surface and its derivative must satisfy $s = \dot{s} = 0$ [24,25]. The equations may be written as below:

$$\dot{s} = KBg + KBLe + KBu + KAx_d - K\dot{x}_d = 0. \tag{8}$$

Since KB is nonsingular, the equivalent control in the sliding mode is given by:

$$u_{eq} = -[\hat{g} + Le] - (KB)^{-1}[KAx_d - K\dot{x}_d], \tag{9}$$

where g is not exactly known but is guessed as $\hat{g}$, and the estimation error on g is presumed to be restricted by some known function G, such that $\|g - \hat{g}\| \leq G$. In addition, it reveals that the stability of the systems in the sliding motion can be guaranteed just by selecting an appropriate matrix L using any pole assignment method. To ensure the achievement of reaching the condition indicated in Eq. (8), a control law is proposed as:

$$u = u_{eq} - (KB)^{-1}\left[\varepsilon + \|KBG\|\right]sign(s), \tag{10}$$

where $\varepsilon > 0$.

4. Numerical simulations for chaos control of a single time-scale BLDCM

Eq. (3) is rewritten with the numerical values as follows:

$$\left.\begin{aligned} \dot{x}_1 &= 0,168 - x_1 - x_2.x_3 + 60.x_3 \\ \dot{x}_2 &= 20,66 - 0,875.x_2 + x_1.x_3 \\ \dot{x}_3 &= 4,55.\left(x_1 - x_3\right) + 0,26.x_1.x_2 - 0,53 \end{aligned}\right\}, \tag{11}$$

where the A, B, and g matrices are gained as follows:

$$A = \begin{bmatrix} -1 & 0 & 60 \\ 0 & -0.875 & 0 \\ 4.55 & 0 & -4.55 \end{bmatrix}; \quad g = \begin{bmatrix} 0.168 - x_2x_3 \\ 20,66 + x_1x_3 \\ -0.53 + 0.26x_1x_2 \end{bmatrix}; \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Here, the gain matrix K is chosen as K = diag (1, 1, 1), such that KB = diag (1, 1, 1) is nonsingular. The desired eigenvalues of the matrix A-BL are taken as P = [–5 –5.001 –5.0001]. The gain matrix L is found as follows, using the pole placement method:

$$L = \begin{bmatrix} 4 & 0 & 60 \\ 0 & 4,1260 & 0 \\ 4,55 & 0 & 0,451 \end{bmatrix}.$$

As a result, the matrix $K(\ A\text{-}BL\ )$ is computed as $K(\ A\text{-}BL\ )$ = diag (–5, –5.001, –5.0001). The PI switching surfaces are obtained as follows:

$$\left.\begin{array}{l} s_1 = e_1 - \int_0^t 5e_1\,(\tau)\,d\tau \\ s_2 = e_2 - \int_0^t 5.001e_2(\tau)d\tau \\ s_3 = e_3 - \int_0^t 5.0001e_3(\tau)d\tau \end{array}\right\}. \tag{12}$$

For this numerical simulation, the initial points of the system are employed as [$x_1(0)$, $x_2(0)$, $x_3(0)$] = [3.63, 56.02, 0.29]. The constant controller coefficient $\varepsilon$is selected as ε =5. The reference states x_{d1}, x_{d2}, x_{d3} are selected as x_{d1} = x_{d2} = x_{d3} = x_d. Therefore, the control signals may be attained as:

$$\left.\begin{array}{l} u_1 = [-4e_1 + 60e_3 - 59x_d + \dot{x}_d + x_2x_3 - sign\,(s_1)\,(\varepsilon + |x_2x_3 - 0.168|\,) - 0.168] \\ u_2 = [-4.126e_2 + 0.875x_d + \dot{x}_d - x_1x_3 - sign(s_2)(\,(\varepsilon + |x_1x_3 + 20.66|\,-20.66] \\ u_3 = [-0.4501e_3 - 4.55e_1 + \dot{x}_d - 0.26x_1x_2 - sign(s_3)(\,(\varepsilon + |0.26x_1x_2 - 0.53|\,+0.53] \end{array}\right\}. \tag{13}$$

The reference states are taken as x_d = 0, and the state vectors x_1, x_2, and x_3 converge to 0 quickly after the control signals are activated at time t = 0, as shown in Figure 3. Figure 3a shows state vectors x_1, x_2, and x_3, and Figure 3b shows control signals u_1, u_2, and u_3. The reference states are taken as x_d = 1sin(2.4t), and the state vectors x_1, x_2, and x_3 converge to x_d quickly after the control signals are activated at time t = 0, as shown in Figure 4. Figure 4a shows state vectors x_1, x_2, and x_3, and Figure 4b shows control signals u_1, u_2, and u_3.

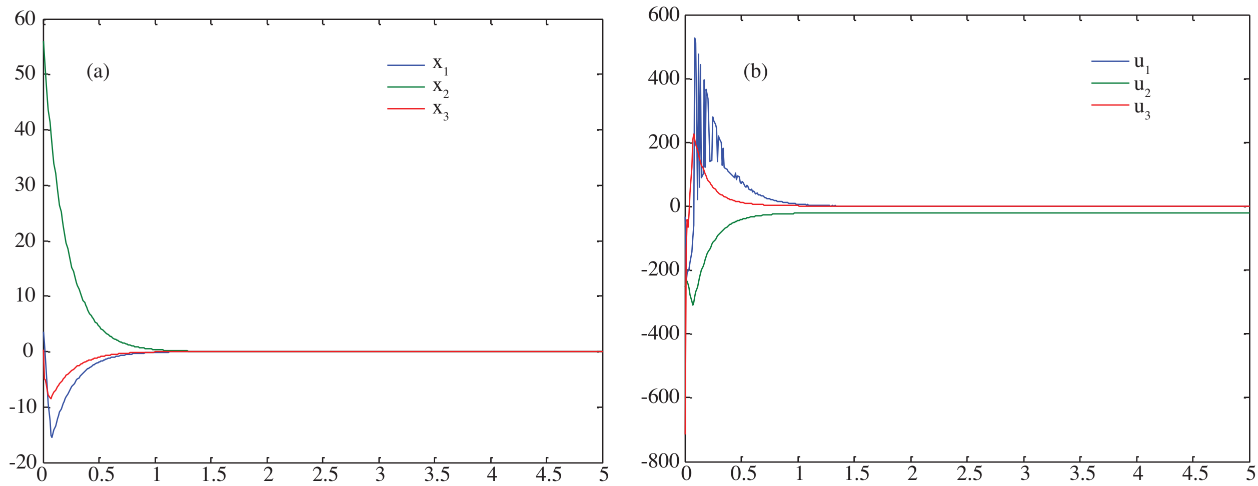

Figure 3. σ = 4.55 and x_d = 0 for the controlled BLDCM system with SMC after t = 0 s: a) time response and b) applied control signals.

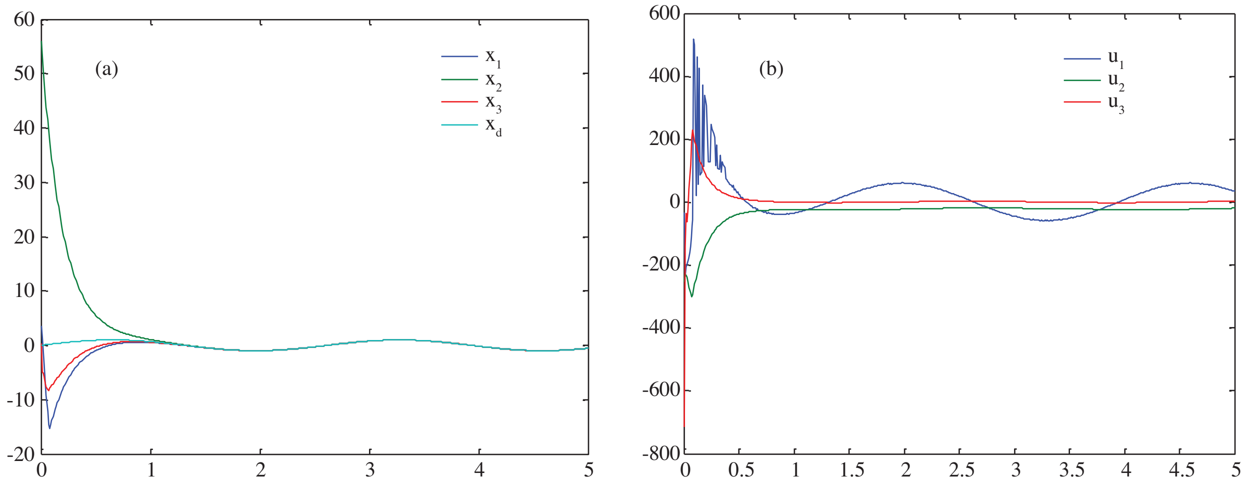

Figure 4. $\sigma = 4.55$ and $x_d = \sin(2.4t)$ for the controlled BLDCM system with SMC after $t = 0$ s: a) time response and b) applied control signals.

5. Conclusions

In this paper, an effective control technique has been suggested to stabilize the chaos of a single time-scale BLDCM chaotic system. A SMC law was applied using a PI switching surface. Hence, it was found that the stability of the error dynamics in the sliding mode is easily ensured by the PI switching surface. The designed SMC controller is rather satisfactory for a nonlinear controller to eliminate the undesirable chaotic oscillations. The related Figures in Figures 3a and 4a show the control of state vectors for different reference states. Figures 3b and 4b show the control signals providing the control of the state vectors. In this paper, the proposed SMC controller can be used in similar DC machines, i.e. a permanent magnet DC motor. Finally, numerical simulations are provided to show the effectiveness of the proposed method. The obtained results are satisfying in view of this.

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