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Implementation of an AC model for transmission expansion planning considering reliability constraints

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Abstract: In this paper, a hybrid heuristic methodology that employs fuzzy logic for solving the AC transmission network expansion planning (AC-TEP) problem is presented. An enhanced constructive heuristic algorithm aimed at obtaining a significant quality solution for such complicated problems considering contingency is proposed. In order to indicate the severity of the contingency, 2 performance indices, namely the line flow performance index and voltage performance index, are calculated. An interior point method is applied as a nonlinear programming solver to handle such nonconvex optimization problems, while the objective function includes the costs of the new transmission lines as well as the real power losses. The performance of the proposed method is examined by applying it to the well-known Garver system for different cases. The simulation studies and result analysis demonstrate that the proposed method provides a promising way to find an optimal plan. Obtaining the best quality solution shows the capability and the viability of the proposed algorithm in AC-TEP.

Key words: Transmission expansion planning, reliability, hybrid heuristic algorithm, nonconvex optimization, fuzzy decision making

1. Introduction

The objective of a transmission expansion and planning problem is to determine where, how many, and when new devices, such as transmission lines, transformers, and other related equipment, must be added to an existing network in order to make its operation viable for a predefined planning horizon at minimum cost. The benchmark network of the base year, the candidate lines, the power generation, and the power demand of the planning horizon associated with the investment constraints are the major data for such a problem. The transmission network expansion planning (TEP) problem has been studied extensively in the literature since the early 1970s [1] and it is still an active research area, where a broad recent literature review can be addressed in [2]. Most of these studies employ only simplified DC models, while the AC network modeling has been proposed in new studies [3,4]. In fact, TEP is usually divided into the following: long-term (up to 20 years), in which large transmission interconnections associated with new energy sources will be considered; medium-term (up to 10 years), in which more details of the interconnections incorporating alternatives for the regional systems will be determined; and short-term (up to 5 years), where the final adjustments are made regarding the alternatives previously chosen, such as the reactive compensation, while the information from the

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system operation is included in the planning process.

Generally in TEP, a steady-state analysis is usually performed using simplified models, such as a linearized power flow model, DC model, or transportation model. Transportation models, hybrid models, linear disjunctive models, and DC models, among others, have been used to achieve the primary topology in the first stage [5]. In a subsequent stage, the expanded network will be checked for other operational constraints. A DC model TEP problem can be solved both by classical optimization methods [6] and metaheuristic techniques, such as simulated annealing [7], genetic algorithms [8], tabu search [9], and a greedy randomized adaptive search procedure [10]. It should be noted that the use of a DC model in TEP has the following disadvantages: a) The difficulty of taking into account the power losses in the initial phase of planning and b) it is frequently necessary to reinforce an expansion plan obtained via a DC model satisfying operational AC conditions. In short-term planning, the steady-state studies use an AC model in order to accurately assess the real power losses and to facilitate reliability as well as security. It can be said that the use of an AC model in the first phase is incipient, while there are few technical papers on the subject.

In this paper, a reliability-constrained AC-TEP is introduced with the following advantages: incorporating the determination of precise transmission real power losses and the possibility of carrying out contingency as well as security analysis. Unlike the basic TEP problem that addresses only the adequacy criteria, additional criteria such as security, reliability, and efficiency can be included, against which the plans have to be subsequently validated. If such additional criteria are not satisfied in the planning phase, they may emerge as barriers in the operational phase. For example, an (N-1) contingency criterion as a security criterion requires that the planned system should be able to operate adequately in the event of the outage of each component in the network. The literature on expansion planning considering security constraints is limited, where some contributions can be addressed in [11,12]. In fact, they employ only a simplified DC model, while this paper proposes a methodology for incorporating an (N-1) contingency criterion in AC-TEP. On the other hand, 2 kinds of performance indices, the so-called line flow performance index (PI_L) and voltage performance index (PI_V) are deployed, which reflect the degree of severity of the contingencies. Using these performance indices, the numbers of branches that must be added considering the line outages are decreased while facilitating the network security. Here, for each network proposal (topology), a combined constructive heuristic algorithm (CHA) with a fuzzy system [13] is first employed to find the best configuration, while it will be treated as a basis network for the contingency study. The algorithm provides high quality solutions with the use of fuzzy decision making, which is based upon nondeterministic criteria. To analyze a contingency using an (N-1) criterion, an enhanced CHA (ECHA) is introduced, associated with the contingency severity, to achieve the best network expansion that guarantees the network reliability as well as security. The advantage of the proposed ECHA is that it is simple to understand, while it is significantly robust with less computational effort. One of the difficulties that may appear when employing the AC model, using ECHA, is the necessity to deal with nonlinear programming (NLP). An interior point method (IPM) has been reformulated and adapted to solve such a nonlinear problem. In fact, the IPM has been widely used to solve problems like the optimal power flow for large-scale systems [14], load ability maximization [15], voltage stability analysis [16], and security-constrained economic dispatch [17]. The IPM can be employed to solve TEP as a NLP problem that should be solved in each step of the ECHA. In fact, the IPM provides a better computational performance for large-scale problems than other classical approaches.

The outline of the paper is as follows: Section 2 describes a general mathematical model for the AC-TEP problem. The performance indices for contingency ranking are discussed in Section 3. The solution methodology is presented in Section 4. In Section 5, the fuzzy system that used in this paper is discussed. The ECHA and solution methodology are presented in Section 6, while, in Section 7, different case studies are simulated. Section 8 provides an economic analysis, and finally, the concluding remarks are given in Section 9.

2. AC-TEP mathematical model

The mathematical model for the AC-TEP problem can be formulated as the following optimization system:

$$\min v = c^T n + K_e P_{Loss} D \tag{1}$$

$$s\ t.,$$

$$P(V,\theta,n) - P_G + P_D = 0, \tag{2}$$

$$Q(V,\theta,n) - Q_G + Q_D = 0, \tag{3}$$

$$\underline{P_G} \leq P_G \leq \overline{P_G}, \tag{4}$$

$$\underline{Q_G} \leq Q_G \leq \overline{Q_G}, \tag{5}$$

$$\underline{V} \leq V \leq \overline{V}, \tag{6}$$

$$(N + N^0)S^{from} \leq (N + N^0)\overline{S}, \tag{7}$$

$$(N + N^0)S^{to} \leq (N + N^0)\overline{S}, \tag{8}$$

$$0 \leq n \leq \overline{n}. \tag{9}$$

$$N\ \ Integer\ and\ \theta\ \ unbounded$$

Eq. (1) is the objective function related to the investment costs of the new transmission lines and the costs of the real power losses. Eqs. (2) and (3) represent the AC power flow equations considering a variable vector n, which is the number of circuits (lines and transformers). The limits for the real power and reactive power are represented by Eqs. (4) and (5), respectively, while Eq. (6) shows the voltage magnitude limit. The line flow limits, megavolt amperes (MVA), are represented by Eqs. (7) and (8). The lower and upper values for the voltage limits are assumed as 95% and 105%, respectively, and $k_e D$ is set to 1. The total real power loss to be minimized is defined by Eq. (10).

$$P_{Loss} = \sum [g_{ij}(V_i^2 + V_j^2 - 2V_i V_j \cos(\theta_{ij})] \tag{10}$$

The elements of vectors $P(V,\theta,n)$ and $Q(V,\theta,n)$ are calculated using Eqs. (11) and (12), respectively.

$$P_i(V,\theta,n) = V_i \sum_{j \in N_B} V_j [G_{ij}(n) \cos\theta_{ij} + B_{ij}(n) \sin\theta_{ij}] \tag{11}$$

$$Q_i(V,\theta,n) = V_i \sum_{j \in N_B} V_j [G_{ij}(n) \sin\theta_{ij} - B_{ij}(n) \cos\theta_{ij}] \tag{12}$$

The bus admittance matrix elements (G and B) are:

$$G = \left\{ \begin{array}{l} G_{ij}(n) = -(n_{ij} g_{ij} + n_{ij}^0 g_{ij}^0) \\ G_{ii}(n) = \sum\limits_{j \in \Omega_l} (n_{ij} g_{ij} + n_{ij}^0 g_{ij}^0) \end{array} \right\}, \tag{13}$$

$$B = \left\{ \begin{array}{lcl} B_{ij}(n) & = & -(n_{ij} b_{ij} + n_{ij}^{0} b_{ij}^{0}) \\ B_{ii}(n) & = & b_i^{sh} + \sum\limits_{j \in \Omega_l} [n_{ij}(b_{ij} + b_{ij}^{sh}) \\ & & +n_{ij}^{0}(b_{ij}^{0} + (b_{ij}^{sh})^{0})] \end{array} \right\}. \tag{14}$$

Note that in Eqs. (13) and (14) the possibility of a different transmission line or transformer being added in parallel with an existing one in the base case is considered, although the equivalent circuit parameters may be different. It should be notified that off-nominal transformer taps are not considered and in this case both the transmission lines and the transformers have similar equivalent circuits. It must be mentioned that the present model does not consider the phase shifters. Element (ij) of vectors S^{from} and S^{to} in Eqs. (7) and (8) is given by

$$S_{ij}^{from} = \sqrt{(P_{ij}^{from})^2 + (Q_{ij}^{from})^2}, \tag{15}$$

$$P_{ij}^{from} = V_i^2 g_{ij} - V_i V_j (g_{ij} \cos\theta_{ij} + b_{ij} \sin\theta_{ij}), \tag{16}$$

$$Q_{ij}^{from} = -V_i^2 (b_{ij}^{sh} + b_{ij}) - V_i V_j (g_{ij} \sin\theta_{ij} - b_{ij} \cos\theta_{ij}), \tag{17}$$

$$S_{ij}^{to} = \sqrt{(P_{ij}^{to})^2 + (Q_{ij}^{to})^2}, \tag{18}$$

$$P_{ij}^{to} = V_j^2 g_{ij} - V_i V_j (g_{ij} \cos\theta_{ij} - b_{ij} \sin\theta_{ij}), \tag{19}$$

$$Q_{ij}^{to} = -V_j^2 (b_{ij}^{sh} + b_{ij}) + V_i V_j (g_{ij} \sin\theta_{ij} + b_{ij} \cos\theta_{ij}). \tag{20}$$

The aforementioned variables in Eqs. (15) to (20) represent the operating state of a feasible solution, in which a feasible investment proposal is defined through a specified value n that may include several feasible operational states.

3. Contingency study

A transmission network should be secure against any possible line outage, which is usually studied by implementing an (N-1) criterion. Some of the outages may result in system constraint violations, such as load bus voltages outside of their permissible limits and transmission line overloads. Flows on transmission lines are typically constrained by thermal limits and sometimes by stability considerations on long lines. In light of these constraints, the system performance may be quantitatively evaluated in terms of indices reflecting the severity of out-of-limit voltage values or line overloads resulting from a particular contingency (Eqs. (7) to (9)). For defining the system performance indices, the constraints on the load bus voltage and the line flows are treated as soft constraints, i.e. the violation of these constraints, if not excessive, may be tolerated for short periods of time. The system performance indices are defined as a penalty function to severely penalize any violation of the bus voltage constraints and/or line flow constraints. There are 2 widely used performance indices, namely the PI_v and PI_L. The voltage performance index quantifies the system's deficiency due to out-of limit bus voltages, as defined by Eq. (21) [18]:

$$PI_V = \sum_{i=1}^{NB} \left(\frac{W_{Vi}}{2m} \right) \left(\frac{(|V_i| - |V_i^{sp}|)}{\Delta V_i^{Lim}} \right)^{2m}, \tag{21}$$

where: $|Vi|\,|V_i|$ is the voltage magnitude at bus i, $|V_i^{sp}|$ is the rated voltage magnitude at bus i, ΔV_i^{Lim} is the maximum permissible voltage deviation limit, m is the exponent of the penalty function (preferably $m =$

2), NB is the number of buses in the system, and$W_{vi}W_{Vi}$ is the real nonnegative weighting factor (preferably, $W_{Vi}=1$). It can be said that any contingency load flow with voltage levels outside of ΔV_i^{Lim} yields a high value of the index PI_V. When all of the voltage level deviations from the rated voltage are within $\Delta V_i^{Lim}\Delta V_i^{Lim}$, the voltage performance index PI_V is small. The PI_V index measures the severity of the out-of-limit bus voltages, and for a set of contingencies this index provides a direct value of comparing the relative severity for different outages on the system voltage profile.

Another performance index for quantifying the extent of the line overloads is defined in terms of the MVA line flow limit by Eq. (22):

$$PI_L=\sum_{L=1}^{NL}\left(\frac{W_{Li}}{2m}\right)\left(\frac{(S_L)}{S_L^{Lim}}\right)^{2m}, \tag{22}$$

where$S_L S_L$ is the MVA flow of line l,$S_L^{Lim}S_L^{Lim}$ is the maximum line flow capacity of line L, and NL is the number of lines in the system. W_{Li} is a real nonnegative weighting factor (preferably $W_{Li}=1$), n is the exponent of the penalty function (preferably $m=2$). The performance index PI$_L$ contains all of the line flows normalized by their limits. These normalized flows are raised to an even power (by setting $m=1;\ 2\ldots$); thus, the use of the absolute magnitude of the flows is avoided. When all of the line flows are within their limits, PI$_L$ has a small value, while it increases when there are line overloads. This index provides a measure of the severity of the line overloads for a given state in a power system. The total performance index is a linear combination of 2 indices, as in Eq. (23):

$$PI_{Total}=\alpha\frac{PI_V}{PI_V^{max}}+\beta\frac{PI_L}{PI_L^{max}}, \tag{23}$$

α and β are weighting factors.

4. Solution methodology

The proposed solution methodology considering the contingency is ECHA to perform the decision for the network reinforcement-added lines. First, a CHA is employed for finding a good network topology for the contingency analysis. The CHA chooses the required line numbers via an iterative manner to obtain a good quality solution satisfying the operational constraints. The simplest type of this algorithm was used by Garver [2] to find a good quality solution using the transportation model. The CHA may be considered a step-by-step search of a good quality solution in a complex multiobjective optimization problem. In the case of the TEP problem, in each step important circuits are added to the system (transmission line or transformer). The circuits to be added are chosen using a prespecified sensitivity index (SI). The major concern of the CHA is based on this index. Numerically, the circuit performance according to the topology is used, such as the objective of such indices. Some of the major shortcomings have been addressed in [19]. Most of the constructive algorithms perform a greedy search, which rarely results in good solutions for realistic systems. In this paper, we propose an algorithm that not only deals with the deficiencies of the common constructive algorithms, but also makes the heuristic more flexible using fuzzy decision making. The iterative process finishes when a feasible solution, generally good quality, is found. The sensitivity index chosen is extracted from the fuzzy decision maker system. To do that, in each step of the CHA, the circuits that are to be added to the system are identified by the following output of the fuzzy system. In each step of the CHA, the current topology must be updated. The circuits of the base (initial) topology and the circuits added in the iterative process form the current topology. One main characteristic of the CHA is that the strategy identifies the most important circuit in terms of the investment and operational constraints. The proposed CHA is meant to solve the TEP problem without a contingency and is explained briefly in the following:

1) Assume an initial topology to provide the current topology.

2) Solve an NLP problem for the current topology. *If* the solution of NLP offers a feasible solution with the actual additions, *stop*, and *then* go to 4; *otherwise*, continue to 3.

3) Use the SI index (*fuzzy system)* to identify the most attractive circuit that can be added to the system. Update the current topology by adding the chosen circuits and *then* go to 2.

4) Arrange the circuits added in a cost-wise decreasing order. Solving an NLP problem, verify if it is possible to remove the circuit still having a feasible solution. If yes, remove the circuit; otherwise, the circuit must be maintained.

Repeat the process, simulating the outage of all of the circuits. The remaining added circuits represent the solution of the TEP. In the above CHA, an NLP problem must be solved in 2 and 4. Figure 1 shows the flowchart summarizing the CHA.

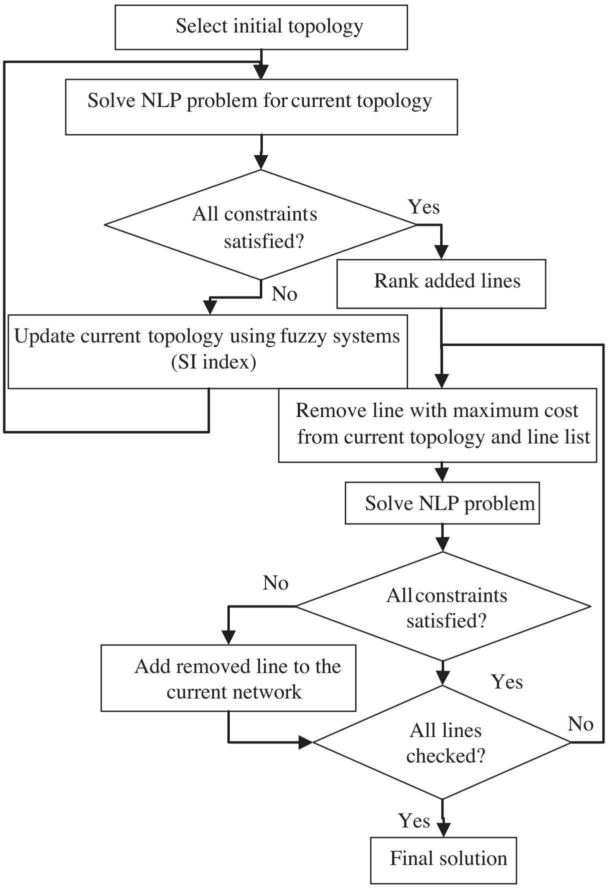

Figure 1. Flowchart summarizing the CHA.

5. Fuzzy systems

Fuzzy systems are rule-based systems in which a set of so-called fuzzy rules represent a decision maker mechanism to adjust the effects of certain system motivations. The aim of fuzzy systems is normally to replace a skilled human operator with a fuzzy rule-based system [13]. The fuzzy decision maker system provides an algorithm that can convert the linguistic control strategy based on expert knowledge into an automatic control strategy.

5.1. Fuzzification and membership function

Fuzzification is the assigning of a linguistic value, defined by a relatively small number of membership functions, to a variable, which also means mapping from the crisp domain into the fuzzy domain. As explained earlier, 2 separate inputs are created from the n_{ij} and the cost of the line in the path i–j and the branching as an output variable. The next step is to determine the shape and number of the membership functions. For this particular design, numerous membership functions were tried and the functions under consideration proved the most promising. For the inputs, 5 membership functions are used for the fuzzification: very large (VL), large (L), medium (MD), small (SM), and tiny (TY), which are represented by the vectors of the interval [0 ... 1]. The sets SM and L have been modeled as a triangular membership function. The sets TY, MD, and VL have been modeled as a trapezoidal membership function. Figure 2 shows the input membership functions and the details can be found in [13].

The output variable branching has 2 fuzzy sets: not branching (NB) and branching (B). The universe of discourse of the variable value of n_{ij} is defined in the real numbers between 0 and 1. The input variable cost of the line in path i–j is in interval [0 ... max $\{c_{ij}\}$] and the output variable branching is the real numbers between 0 and 1. For the outputs, 2 membership functions are used for the defuzzification. The membership functions consist of 2 opposite sigmoid functions, as shown in Figure 3.

The designed rule consists of 25 fuzzy rules, shown in Table 1.

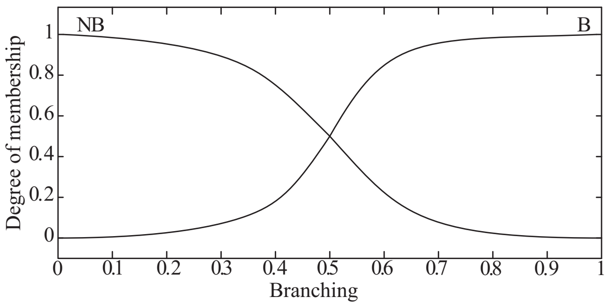

Figure 2. Input membership functions.

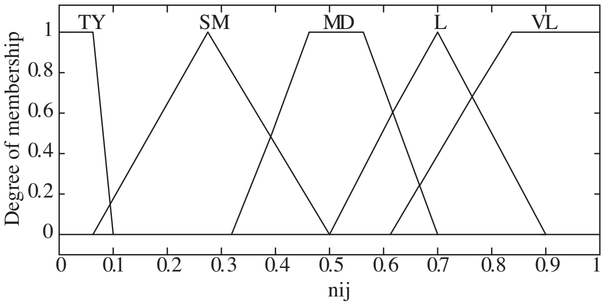

Figure 3. Output membership functions.

Table 1. Fuzzy rules.

	Cost of the line in the path i–j				
Value of n_{ij}	TY	SM	MD	L	VL
TY	B	NB	NB	NB	NB
SM	B	B	NB	NB	NB
MD	B	B	B	NB	NB
L	B	B	B	B	NB
VL	B	B	B	B	B

The individual contribution coming from each rule depends on the membership functions, the shape of the membership functions, and the type of operators used in the inference mechanism, such as IF-THEN rules, aggregation method, and implication method. In the current design, the centroid method is used for defuzzification. The max function is used for both implications of the inputs, as well as for the aggregation of the individual fuzzy outputs, and the overall fuzzy output is then defuzzified to obtain a crisp output.

6. Enhanced constructive heuristic algorithm

By including the 2 indices discussed above, an ECHA is proposed here to solve the TEP problem considering a contingency that is explained in the following:

1) Derive an initial topology for the ECHA from the CHA.

2) Calculate the PI_V and PI_L for each line by removing all of the lines one by one from the initial topology and rank the lines in descending order according to their performance indices.

3) Remove lines according to the ranking list.

4) Solve the NLP problem for all of the line outages from the current topology. *If* a feasible solution with the actual additions is found *stop*, and go to 3; *otherwise*, continue to the next step.

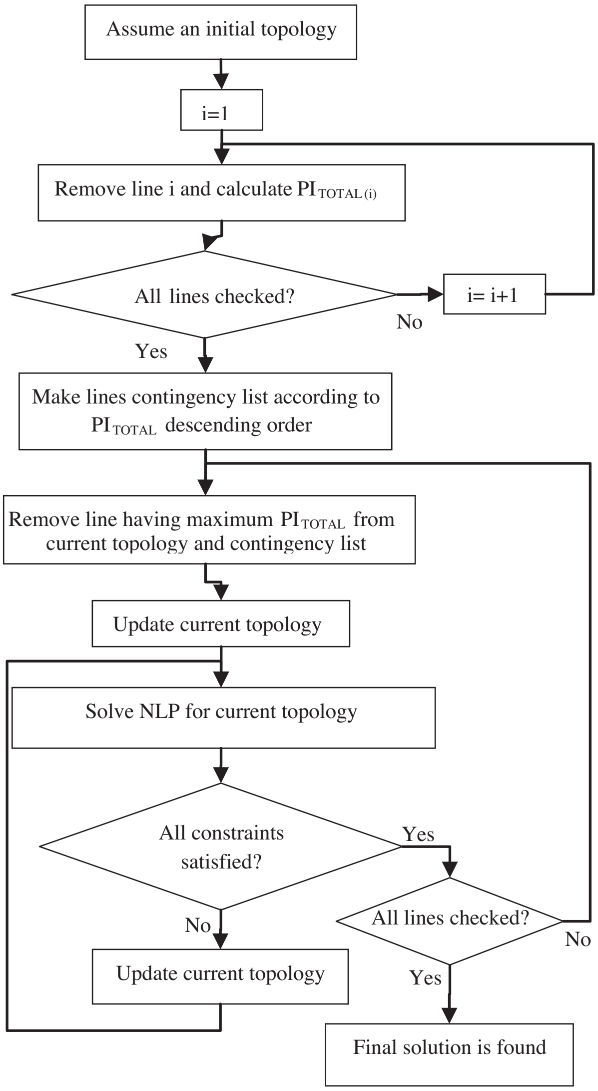

Figure 4. Flowchart summarizing the ECHA.

5) Use the fuzzy system (SI) to identify the most attractive circuit (line) that can be added to the system. Update the base topology with the addition of the chosen circuit and *then* go to 5.

Figure 4 shows flowchart summarizing the ECHA.

7. Illustrative tests

The proposed ECHA is implemented in some illustrative tests, where different cases were simulated using the Garver system. This system has 6 buses and 15 branch candidates, the total demand is 760 MW and 152 MVAr, and a maximum of 5 lines can be installed into each branch [3]. Four different studies have been carried out, while 2 different base cases have been used. Base case 1 is with the base topology proposed by Garver and Base case 2 is without the base topology proposed by Garver. For each case, 2 tests are managed, the TEP with and without a contingency, where in all of the tests generation rescheduling is considered. All of the tests have been carried out using a personal computer with an Intel$^{(R)}$ Core^{TM} 2 CPU8300, a 2.4-GHz processor, and 2.0 GB of memory.

Test 1. *Garver system considering Base case 1:*

In this case, the CHA converges after solving 9 NLPs with a line investment of US$160,000,000, where the following lines are added: $n_{2-6} = 2, n_{3-5} = 2$, and $n_{4-6} = 2$, as is shown in Figure 5. For this case, the active power loss is assumed as 12.117 MW. The results presented in the example illustrate better performance of a TEP. It can be noted that in the AC model, the voltages must be constrained and the real and reactive losses are precisely computed. In fact, in the results presented, circuits were added to enforce the voltage limits.

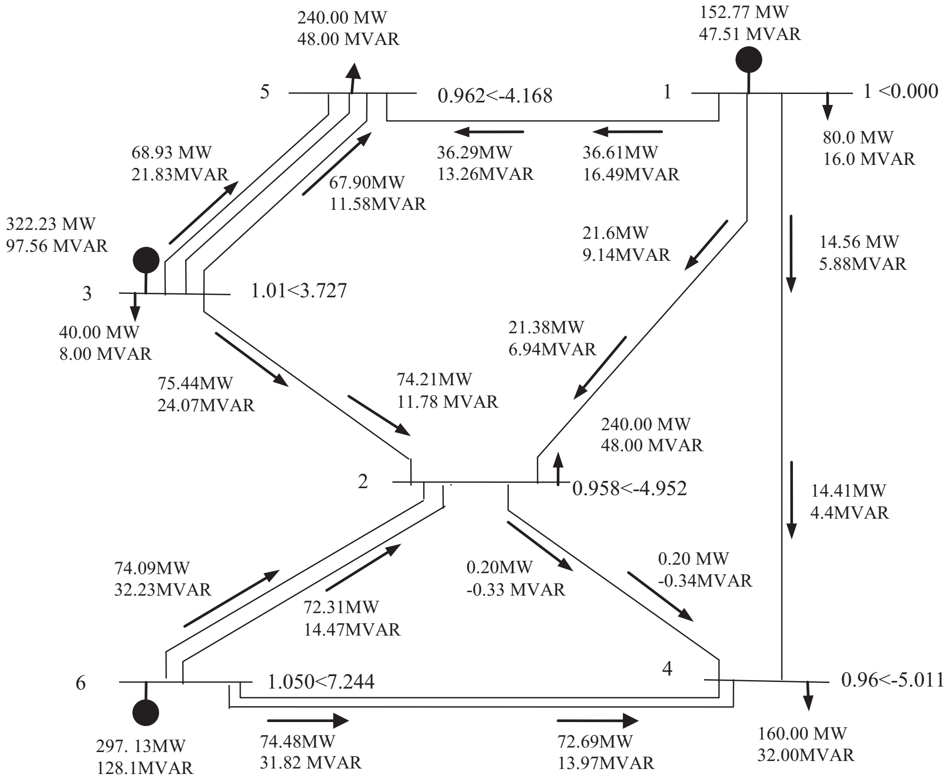

Figure 5. Garver system without the contingency analyzed in Base case 1.

With a superficial analysis (see Figure 5), one can conclude that in branches 3–5 only 2 circuits were enough if the voltage limits are disregarded, such as in the DC model, because of the fact that for each circuit there is a power flow limit of 120 MVA and the total flow in the branch is approximately 144.3 MW. The same analysis can be made in the other results.

Test 2. *Graver system with Base case 2:*

This test is applied to the Garver system without any existing lines and the only generator and load buses are presumed. In this case CHA converges after solving 12 NLPs with line investment US$260 where the following lines are added $n_{1-5} = 1, n_{2-3} = 1, n_{2-6} = 3, n_{3-5} = 2$, and $n_{4-6} = 3$ (Figure 6). The active power loss for this case is 10.987 MW.

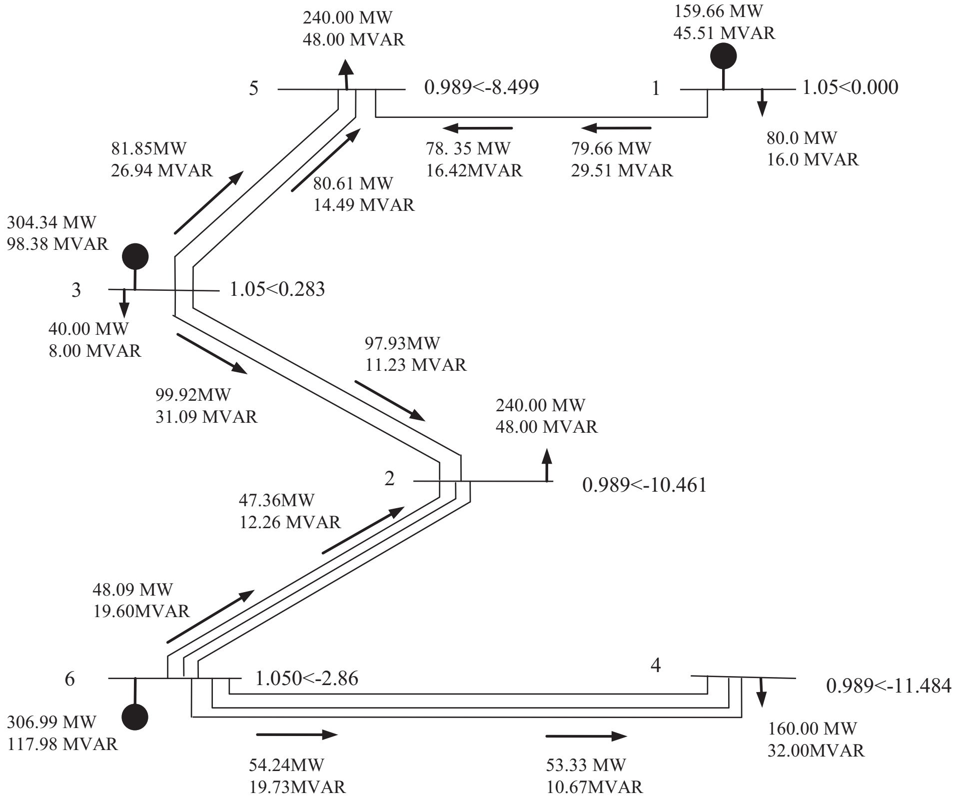

Figure 6. Garver system with the contingency analyzed in Base case 1.

Test 3. *Garver system considering the contingency in Base case 1:*

TEP including the reliability and security constraints via the ECHA is handled in this test. The initial network for this study is the network that is derived from Test 1. After executing the CHA in this system and finding the initial topology, first, according to the contingency list, the lines that have a severe effect on the system performance must be added to the system to reinforce the system against any lines outages; these lines are n_{2-3}, n_{1-5}, n_{4-6}, n_{2-6}, and n_{3-5}. Finally, it is possible that there are some lines that were added unnecessarily, since these lines must be checked in this test, and n_{2-6} is unnecessary, and the obtained result in the line investment is US$230,000,000, where the following lines are added: $n_{2-6} = 2$, $n_{3-5} = 2$, $n_{4-6} = 3$, $n_{1-5} = 1$, and $n_{2-3} = 1$, which is shown in Figure 7. The active power loss for this test is 8.656 MW. Table 2 shows the performance indices and contingency ranking list for this test.

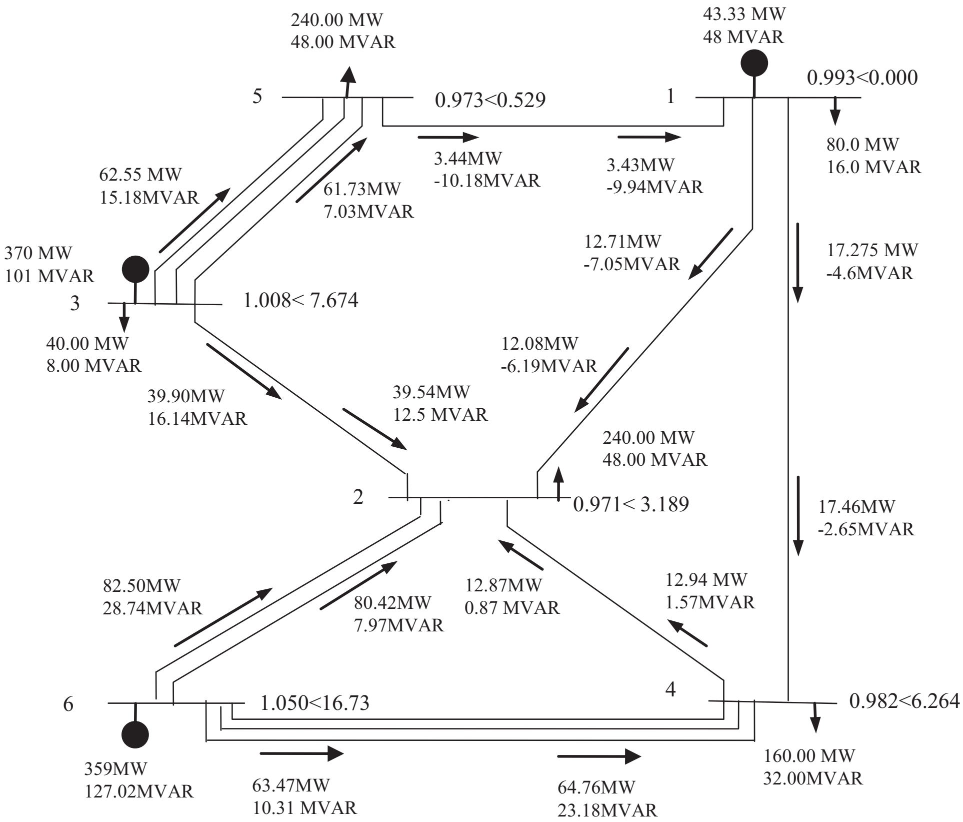

Figure 7. Garver system with the contingency analyzed in Base case 1.

Table 2. Contingency ranking for Base case 1.

Rank	Branch to-from	PI_V	PI_l	PI_{TOTAL}
1	2-3	0.1469	2.4139	1. 280
2	1-5	0.0584	1.3141	0.6863
3	4-6	0.4167	0.4153	0.416
4	2-6	0.1753	0.5061	0.347
5	3-5	0.0664	0.2774	0.1719
6	1-2	0.0535	0.1765	0.115
7	1-4	0.0529	0.1596	0.106
8	2-4	0.0382	0.1556	0.0969

Test 4. *Garver system considering the contingency in Base case 2:*

This test is applied to the Garver system without any existing lines and only the generator and load buses are presumed. TEP including the reliability and security constraints via the ECHA is handled in this test. The initial network for this study is the network that is derived from Test 2. The obtained result in the line investment is US$350,000,000, where the following lines are added: $n_{1-5} = 2$, $n_{2-3} = 2$, $n_{2-6} = 3$, $n_{3-5} = 3$, and $n_{4-6} = 4$, which is shown in Figure 8. The total active power loss is 10.02 MW. Table 3 shows the performance indices and contingency ranking list for this test.

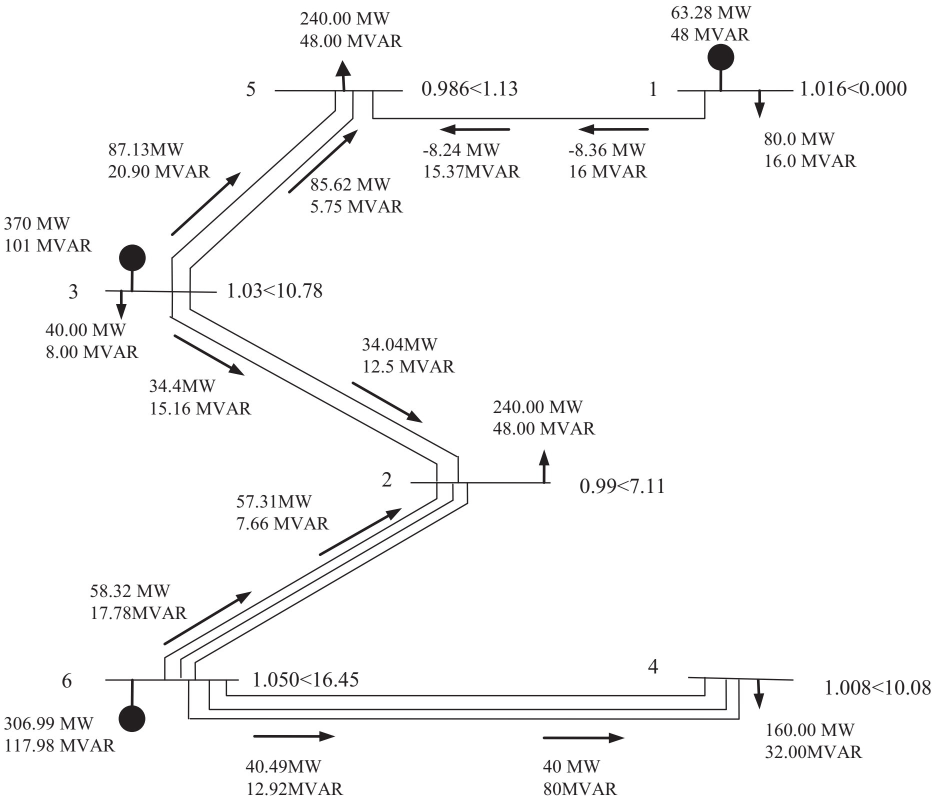

Figure 8. Garver system with the contingency analyzed in Base case 2.

Table 3. Contingency ranking for Base case 2.

Rank	Branch to-from	PI_V	PI_L	PI_{TOTAL}
1	3-5	0.2912	2.2267	1.259
2	2-3	0.0533	1.8833	0.9683
3	4-6	0.0753	0.3002	0.1877
4	2-6	0.0402	0.2826	0.1614
5	1-5	0.0120	0.2239	0.1179

8. Economic evaluation

In 2 tests (1 and 2) the quality of each solution is improved in comparison with the reported results from the literature. In Tests 1 and 2, the cost of the active power losses is decreased, and the lack of studies in the literature did not allow for a comparison of tests 3 and 4. Table 4 shows the total cost including the line investment, the cost of the power losses, the savings, as well as the running time calculated for the different tests. The interesting point in this paper is the large difference between the costs of the TEP with and without the (N-1) contingency. For example, in the Graver system Case 1, the cost of planning without considering the (N-1) contingency is \$110,000,000 and the cost of this system when considering the (N-1) contingency is \$230,000,000. The question that may arise is whether or not considering the (N-1) contingency in planning economically significant. An important point that should be considered is that the obtained solution for the TEP is only for one secure environment and without line outages. In fact, if any possible line outage happens, the transmission grid may not be capable of supporting such produced power, and the lines might be congested. However, when considering the (N-1) contingency in TEP, the transmission network is secure against any possible line outage without any congestion. Therefore, it can be said that considering the (N-1) contingency in TEP has more security and reliability and, according to results, the power losses can be decreased.

Table 4. Test comparisons.

Case method		Cost of the losses ($10^6\$$)	Line cost ($10^6\$$)	Total costs ($10^6\$$)	Saving ($10^3\$$)	Running time (s)
Test 1	PM*	12.117	160	172.117	53	7.6
	[3]**	12.170	160	172.170		
Test 2	PM*	10.987	260	270.987	413	13.4
	[3]**	11.400	260	271.400		
Test 3	PM*	8.656	230	238.656	***	11.3
Test 4	PM*	10.020	350	360.020	***	19.8

*Proposed method.
**The method in [3].
***The lack of bibliography using the AC model did not allow a comparison of this result.

9. Conclusion

A mathematical model and a CHA combined with a fuzzy system that was able to find high quality solutions to solve the TEP problem considering reliability constraints based on an AC model have been presented. An ECHA is proposed to solve such a complicated mathematical model of the expansion planning problem. Two indicators are employed to identify lines that have a more severe impact on the contingency occurrence. The obtained results using the ECHA via implementing 2 introduced indices show a significant performance of the proposed methodology. The real power losses are also decreased significantly, while the economic evaluation presents a considerable amount cost saving. The proposed methodology has a potential capacity for implementation in larger networks. In this regard, further studies need to be performed in order to apply the ECHA to large-scale TEP problems.

Nomenclature

v_0	Investment on new transmission lines
v_1	Total cost of the active power losses and reactive sources
$\mathbf{c}$	Line cost vector
K_e	Converted real power to cost
D	System operating time
θ	Phase angle vector of the buses
P_G	Real power generation vector
P_D	Real power demand vector
Q_D	Reactive power demand vector
$\underline{P}_G$	Minimum limit of the generators real power
$\overline{P}_G$	Maximum limit of the generators real power
P_{Loss}	Total real power loss
$Q_{\mathbf{G}}$	Reactive power generation vector
$\underline{Q}_G$	Minimum limit of the generators reactive power
$\overline{Q}_G$	Maximum limit of the generators reactive power
V	Voltage magnitude vector of the buses
$\underline{V}$	Minimum limit of the voltage
$\overline{V}$	Maximum limits of the voltage
n	New line vector
$\overline{n}$	Maximum number of lines that can be added
N	Diagonal matrices containing vector n
N^0	Diagonal matrices containing the existing lines
S^{from}	Apparent power flow vector “from” the bus
S^{to}	A pparent power flow vector “to” the bus
$\overline{S}$	Maximum apparent power flow vector
g_{ij}	Conductance of the transmission lines between ij
θ_{ij}	Difference in the phase angle between buses i and j
b_{ij}	Susceptance of the transmission line or transformer ij
b_{ij}^{sh}	Shunt susceptance of the transmission line or transformer ij
n_{ij}	Number of lines between bus i and bus j
b_i^{sh}	Shunt susceptance at bus i
Ω_l	Set of all of the load buses
N_B	Set of all of the buses

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