

## Complexity reduction of RBF multiuser detector for DS-CDMA using a genetic algorithm

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**Abstract:** The optimal receiver for detecting direct sequence code division multiple access (DS-CDMA) signals suffers from computational complexity that increases exponentially with the number of users. Several suboptimal multiuser detectors (MUDs) have been proposed to overcome this problem. Due to the nonlinear nature of the decision boundary of the optimal receiver, it is known that nonlinear receivers outperform linear receivers. Radial basis function (RBF) MUD is a nonlinear suboptimal receiver that can perfectly approximate this decision boundary and it needs no training since it is fully determined when the spreading codes of all users and the channel impulse response (CIR) are known. However, the RBF MUD suffers from structural complexity since the number of hidden nodes (center functions) in its structure increases exponentially with the number of users. In this study, we propose a new method to minimize the number of center functions of the RBF MUD using a genetic algorithm (GA) and the least mean squares (LMS) algorithm. With simulations performed in AWGN and multipath channels it is shown that the proposed method immensely reduces the complexity of the RBF MUD with a negligible performance degradation.

**Key words:** DS-CDMA, radial basis function multiuser detector (RBF MUD), genetic algorithm (GA)

### 1. Introduction

Code division multiple access (CDMA) is a spread spectrum-based access method that has played a significant role in cellular and personal communication systems in the last decade. This access method assigns unique spreading codes to different users, allowing multiple users to communicate simultaneously using the same frequency band. Spread spectrum communication has become popular due to its advantages, including jamming and interference resistance, signal hiding, good multipath performance, secure communications, and improved spectral efficiency over other access methods [1]. Of the many spread spectrum-based multiple access schemes available, the most widely used one is the direct sequence CDMA (DS-CDMA). In DS-CDMA, the transmitter multiplies each user's information bits by a unique signature waveform.

The conventional detector (CD) for DS-CDMA passes the received signal through a bank of filters matched to the user's unique signature waveform, signs the output, and decides on the information bits. Here, each user is treated separately as a signal and the others are considered as interference or noise. This interference is commonly called multiple access interference (MAI). In single user detection, due to MAI, there is a problem called the "near-far effect", which refers to the case where users near the receiver supply more power to the

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receiver than those far from it [2]. Thus, several power control techniques are proposed to overcome the near-far effect [3].

There is a second approach, called joint multiuser detection, where information of multiple users is used jointly to detect the information of a particular user. Here, MAI is treated as a part of the information rather than noise [4]. Optimum multiuser detection offers superior performance over CD in terms of near-far resistance with the cost of computational complexity, which increases exponentially with the increasing number of users. In a real-life CDMA system, there is a very large number of users, which makes the optimum detector (OD) impractical and very expensive to implement. Thus, many researchers have tried to develop suboptimal detectors with reasonable computational complexities, near-far resistance, and performances close to that of the OD. Decorrelating detector (DECD) is one of these suboptimal detectors that is linear and near-far resistant and it has a computational complexity proportional to the number of users [5, 6]. DECD introduces performance improvement over the conventional detector in terms of MAI but it also introduces a problem called noise enhancement [2]. Moreover, almost all linear suboptimal receivers perform badly with short spreading codes due to the ill-conditioned empirical correlation matrix. Several blind algorithms that utilize regularization [7] to overcome this problem have been proposed [8, 9, 10].

It is known that nonlinear receivers outperform linear receivers since the optimal decision boundary in DS-CDMA is nonlinear [11]. The multistage detector (MSD) is a nonlinear detector that improves each stage's estimate by subtracting the estimate of the MAI obtained by the previous stage [2, 12]. Performance of an MSD can reach that of the OD but it highly depends on the initial estimate, which is usually provided by CD or DECD.

In addition to the aforementioned traditional suboptimal MUDs (CD, DECD, and MSD), many other suboptimal detectors that utilize a genetic algorithm [13], neural networks [7], and machine learning [7, 14] have been proposed.

Genetic algorithm (GA) is an alternative method for the solution of highly nonlinear problems and it mimics the natural selection and survival of the fittest [13, 15, 16, 17, 18]. A GA-based multiuser detector (MUD) was first proposed in [19]. In this work, initial values of the possible user bit sequences are obtained from other detectors, like CD or DECD, and good initial guesses are required for the detector to achieve a good performance. The GA-based receiver that is proposed in [20] is assisted by a local search algorithm that improves the initial guess of the population. In this work, simulations are done under the assumption that the channel impulse response (CIR) is known for the multipath case. A receiver that uses the GA to jointly estimate both the CIR and the transmitted data bits for all users in a synchronous DS-CDMA system is introduced in [21] and this work is applied to the asynchronous case in [22]. In both studies, the proposed detectors can reach the single user performance. However, since the input of the GA-based detectors in [21, 22] is assisted by  $U$  matched filters, where  $U$  is the number of users in the channel, the receivers are not perfectly near-far resistant. In [23] it was shown that embedding the MSD algorithm into the GA as a genetic operator improves the performance of GA-based receivers and provides faster convergence. Authors of [24] made several modifications to the classical GA in terms of parent selection and mutation operation; then they applied this modified GA to the MUD problem in DS-CDMA. It was shown in the paper that the performance of the modified GA MUD is better than that of the classical GA MUD. In [25] a novel  $\mu$ -GA with a very low complexity is proposed for both DS-CDMA multiuser detection and CIR estimation. A control on the diversity of the population is achieved by an entropy-guided method that adjusts the probabilities of the genetic operators at each iteration. The performance of the detector is close to that of the single user detector.

The authors of [26] designed a nonlinear MUD based on neural networks. In [26], 2 structures employing multilayer perceptrons (MLPs) were proposed for the demodulation of spread spectrum signals in Gaussian channels. Nonlinear detector structures based on MLPs or polynomial series may provide near-optimum performance but they also suffer from high complexity [27, 28]. Several other machine learning-based nonlinear receivers use generative modeling [29, 30, 31, 32] and discriminative modeling [26, 27, 28, 33, 34].

Radial basis function multiuser detector (RBF MUD) was originally introduced in [27] and further investigated in [35, 36, 37, 38]. An RBF MUD needs no training since it is fully determined when the spreading codes of all users and the CIR are known. This is a realizable downlink scenario since a simple and yet efficient way of detecting the CIR is to send a pilot tone to the receiver. The main drawback of the RBF MUD is its structural complexity. When the number of users is large, the RBF MUD becomes impractical since its structural complexity increases exponentially with the number of users. A preprocessing method was proposed in [35] to reduce the complexity of the RBF MUD and the resultant RBF MUD was named a preprocessing-based (PPB) receiver. This work was further investigated in [39]. Performance analysis of RBF MUD and PPB RBF MUD for ULTRA-TDD was reported in [37] and it was shown that these receivers achieve low bit error rates (BERs) even for time-variant multipath propagation channels like pedestrian and vehicular environments. The number of neurons in the hidden layer of a radial basis function network (RBFN) may become excessive, even equal to the number of training samples due to the training process. This problem spawned an area of research on optimization of RBFN structures. One particular tool to optimize the structures of RBFNs is GA. The common approach is representing the network as a string and optimizing the structure by applying genetic operators to these strings [40]. In [41] a different method is introduced where, instead of each string representing a network, the whole population represents one network. Many methods that aim to optimize the RBFN with GA are well documented in [40]. A method for reducing the number of neurons (centers) in the hidden layer of RBF MUD using GA is proposed in [38]. By discarding the low-contribution centers, the complexity of the receiver is reduced from  $O(2^{LU})$  to  $O(P)$ , where  $U$  is the number of users,  $L$  is the number of taps in the multipath fading channel, and  $P$  is the GA's population size. However, the performance of the receiver highly depends on the training set and BER reaches an error floor very quickly with the increasing SNR rates.

In this paper, a novel method that reduces the number of centers in RBF MUD using GA on AWGN channels proposed by the authors in [46] is detailed and its extension to the multipath channels is discussed. Instead of eliminating low-contribution centers as was done in [35], our method starts with a small subset of centers whose members are randomly selected from the set of all possible centers. Then the location of each center in DS-CDMA space is allowed to change in every dimension at each iteration by applying the genetic operators. This new method also searches for the best variance value for each center. It starts with an initial value that is equal to noise variance; then it optimizes the variance of each center simultaneously as it searches for the best center location. Although the resultant RBF structure has significantly fewer centers in comparison with  $2^U$ , it can still reach close to the single user performance. Simulations in both additive white Gaussian noise (AWGN) and multipath fading channels show that the flexibility of the location and variance of each center function enable the resultant structure to represent the DS-CDMA space almost perfectly, and make the proposed GA assisted RBF MUD achieve near-optimum performance. The receiver can easily be implemented with an additional overhead of a short training sequence.

This paper is organized as follows. Brief explanations of RBFN and GA are presented in Section 2, while DS-CDMA channel model and structure of the RBF MUD and generation of the super-center vectors are given in Section 3. Section 4 describes the proposed method and gives the definition of the GA assisted RBF MUD.

Computer simulation methods and the results are presented in Section 5. Finally concluding remarks are given in Section 6.

## 2. Preliminaries

The following 2 subsections provide basic information about tools that have been used in our work. First, the structure and operation of the RBFN will be summarized and then some basic terminology about GA will be introduced.

### 2.1. Radial basis function networks

A radial basis function network (RBFN) is a type of neural network that uses radial basis functions as the activation function. Originally, the RBFN was developed for data interpolation in a multidimensional space [42, 43] and it has been used in a wide range of areas like time series prediction, function approximation, control theory, and communications.

An RBFN consists of 3 layers as shown in Figure 1. The input layer connects the network to the environment, while the second layer applies a nonlinear transformation from the input space to the hidden space. In most applications, the hidden space has a higher dimension than the input space. The output layer sums the outputs of the basis functions after suitable weighting. The output of the RBFN is defined with the following equation:

$$y = \sum_{i=1}^N w_i \phi(\|\mathbf{x} - \mathbf{c}_i\|) \quad (1)$$

where  $\mathbf{x}$  is the input vector,  $w_i$  is the weight of the  $i^{th}$  basis function output's path,  $N$  is the number of neurons in the hidden layer, and  $\phi(\cdot)$  is a radially symmetric function with  $\mathbf{c}_i$  as its center. Hence, the vector  $\mathbf{c}_i$  is usually called the *center*. The most common basis function used in the RBFN's is the Gaussian kernel given as:

$$\phi(\zeta) = \exp\left(-\frac{\zeta^2}{2\sigma^2}\right) \quad (2)$$

where  $\sigma^2$  is the variance that controls the radius of the influence of the basis function, and  $\zeta^2$  is the Euclidean distance between the input vector and the center vector.

Many methods have been proposed to determine the parameters of an RBFN. The most common method is grouping the training samples with  $k$ -means algorithm, selecting the centers from the means, and using the least mean squares (LMS) algorithm to determine the weights in the third layer.

### 2.2. Genetic algorithm

Genetic algorithm (GA) [13, 44] is a stochastic search method based on the laws of natural selection, biological evolution, and genetics that operates as an entirely different optimization procedure among other optimization methods (calculus-based techniques, enumerative techniques, etc.) In general, a basic GA consists of 3 operations: *selection*, *genetic operation*, and *replacement*. Figure 2 shows the flow diagram of a simple GA.

In GA, the population consists of a group of chromosomes where each of them represents a candidate solution to the problem. A chromosome is a string of numbers; usually it is a vector of binary digits. The initial population may be generated randomly or manually if there is an initial guess about the solution. At each

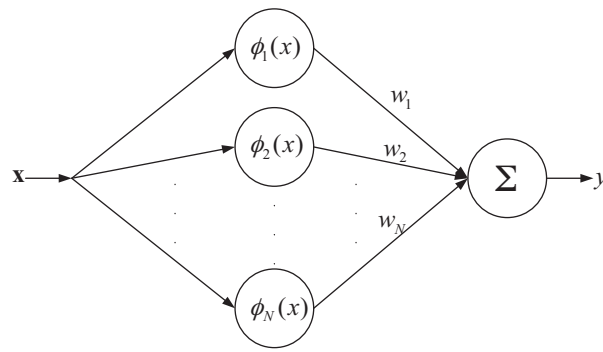


Figure 1. Architecture of a radial basis function network

iteration, all of the chromosomes are evaluated and their fitness values are calculated. According to their fitness values, a probability of selection is assigned to each of them. A particular group of chromosomes (parents) are selected and genetic recombination (crossover) is applied to pairs of parents to generate offsprings. Some of the offsprings are mutated with a pre-defined probability and a new population whose chromosomes would be the parents of the next generation is created.

The GA cycle ends when a desired criterion is satisfied. This criterion may be defined as the number of generations and/or a desired fitness value. Due to this simulated evolution, the chromosome with the best fitness value in the final population can become a highly evolved solution to the problem.

### 3. Definitions

In this section, channel models and detector structures are defined for both RBF MUD for AWGN channel and RBF MUD for the multipath channel.

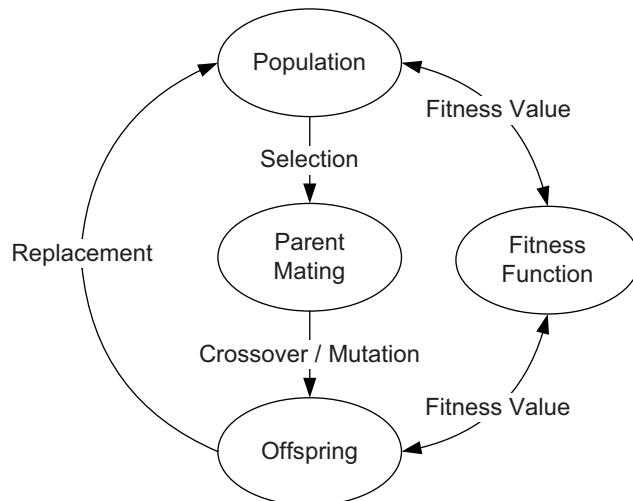


Figure 2. Simple genetic algorithm cycle

### 3.1. RBF MUD for AWGN channel

#### 3.1.1. Channel model

Our system consists of  $U$  independent users. DS-CDMA signals transmitted by these users are assumed to be bit and chip synchronous. Each user transmits DS-CDMA signals with equal power, which is normalized to 1. The modulation scheme of the system is BPSK. The equi-probable data bit, which is transmitted by user  $u$  in the  $k$  bit interval, will be denoted by  $D_u(k)$  and is either +1 or -1. The unique spreading code of length  $N$ , which is assigned to user  $u$ , will be denoted by  $\mathbf{S}_u$  and each chip in the spreading code will be denoted by  $S_{u,n}$ ,  $n = 1, 2, \dots, N$  and is either +1 or -1. The received signal at chip rate in the presence of AWGN is given by:

$$y(kN + n) = \sum_{u=1}^U D_u(k)S_{u,n} + g(kN + n) \quad (3)$$

where  $g(kN + n)$  is the added noise component with the variance  $\sigma_n^2 = N_0/2$ , and  $N_0/2$  is the double-sided noise power spectral density.

Since the user's transmitted bits are synchronized, we may write the vector representation of chip level expression  $y(kN + n)$  of the received signal by

$$\mathbf{y}(k) = [y(kN + 1) \quad y(kN + 2) \quad \dots \quad y(kN + N)]^T \quad (4)$$

This vector representation provides better understanding of how the RBF MUD operates. Vector  $\mathbf{y}(k)$  is supplied to the input layer of the RBF MUD at symbol rate.

#### 3.1.2. Structure of the detector

The RBF MUD needs a set of  $M$  basis functions (centers) as shown in Figure 3. The basis function used in the RBFN is the Gaussian kernel:

$$\phi_m(\mathbf{y}(k)) = \exp\left(-\frac{\|\mathbf{y}(k) - \mathbf{c}_m\|^2}{2\sigma^2}\right) \quad (5)$$

where  $\mathbf{c}_m$ ,  $m = 1, 2, \dots, M$  are the center vectors of length  $N$  in AWGN,  $M$  is the number of center vectors that are introduced by the RBF MUD for each  $2^U$  possible received signal, and  $U$  is the number of users in the Gaussian channel. Since the vector set  $\mathbf{c}_m$ ,  $m = 1, 2, \dots, M$  contains all of the possible received signal vectors,  $\mathbf{y}(k)$ , these centers are also called *super-centers*. Variance of the Gaussian center function,  $\sigma^2$ , equals variance of the added noise component,  $\sigma_n^2$ .

The output layer of the RBF MUD consists of linear weights denoted by  $w_{m,u}$ ,  $m = 1, 2, \dots, M$ . The outputs of the center functions are linearly weighted by  $w_{m,u}$ , summed up, and fed into a sign operator, resulting in the detected symbol for user  $u$ ,  $\hat{D}_u$ :

$$\hat{D}_u(k) = \text{sgn}\left(\sum_{m=1}^M w_{m,u}\phi_m(\mathbf{y}(k))\right) \quad (6)$$

where  $\mathbf{y}(k)$  is the vector of length  $N$  containing the DS-CDMA signal of  $U$  users distorted by AWGN. The weights,  $w_{m,u}$ , in the output layer of the RBF MUD are chosen from the *code matrix*. Generation of the *code*

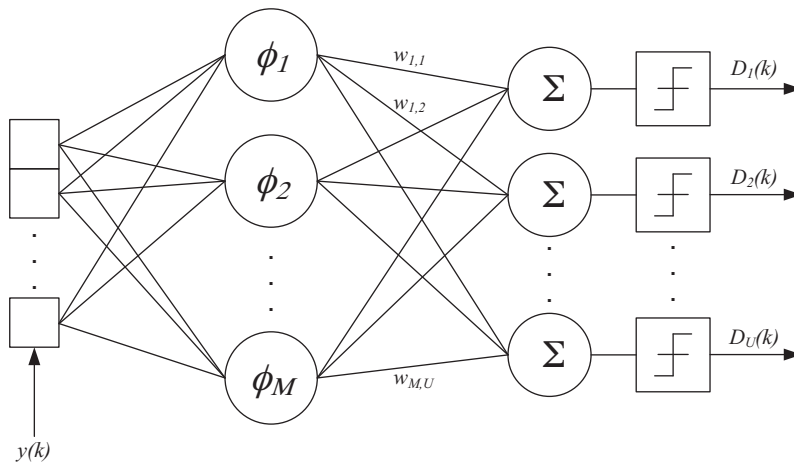


Figure 3. Structure of the radial basis function multiuser detector (RBF MUD)

matrix, which comprises all combinations of all users, and *super-center matrix* which has super-center vectors as its rows, will be explained in the following section.

### 3.1.3. Generation of super-center and code matrices

The super-center matrix,  $\mathbf{C}$ , contains all possible received DS-CDMA signals of  $U$  users in AWGN channel, and is derived using the formula:

$$\mathbf{C} = \mathbf{D}\mathbf{S} \tag{7}$$

where  $\mathbf{S}$  is the  $U \times N$  matrix, comprising the spreading codes of length  $N$  of all  $U$  users and expressed as follows:

$$\mathbf{S} = [\mathbf{s}_1^T \quad \mathbf{s}_2^T \quad \cdots \quad \mathbf{s}_{U-1}^T \quad \mathbf{s}_U^T]^T \tag{8}$$

where  $\mathbf{S}_u$  is the  $N \times 1$  spreading code vector of user  $u$ .

In Eq. (7)  $\mathbf{D}$  is the  $M \times U$  code matrix, which contains all possible bit combinations as its rows where  $M = 2^U$  and is expressed as follows:

$$\mathbf{D} = \begin{bmatrix} -1 & -1 & \cdots & -1 \\ -1 & -1 & \cdots & +1 \\ \vdots & \vdots & \ddots & \vdots \\ +1 & +1 & \cdots & -1 \\ +1 & +1 & \cdots & +1 \end{bmatrix} \tag{9}$$

Thus, Eq. (7) can be written in the expanded form:

$$\mathbf{C} = \mathbf{D}\mathbf{S} = \begin{bmatrix} -\mathbf{s}_1^T - \mathbf{s}_2^T - \cdots - \mathbf{s}_{U-1}^T - \mathbf{s}_U^T \\ -\mathbf{s}_1^T - \mathbf{s}_2^T - \cdots - \mathbf{s}_{U-1}^T + \mathbf{s}_U^T \\ \dots \\ +\mathbf{s}_1^T + \mathbf{s}_2^T + \cdots + \mathbf{s}_{U-1}^T - \mathbf{s}_U^T \\ +\mathbf{s}_1^T + \mathbf{s}_2^T + \cdots + \mathbf{s}_{U-1}^T + \mathbf{s}_U^T \end{bmatrix} \tag{10}$$

Each row in Eq. (10) represents a center vector of the RBF MUD and the weight  $w_{m,u}$  in Eq. (6) must be selected from the  $m^{th}$  row and  $u^{th}$  column of matrix  $\mathbf{D}$  [27].

### 3.2. RBF MUD for the multipath channel

#### 3.2.1. Channel model

It can be seen from Figure 4 that a number of  $L - 1$  head chips of a sequence in the multipath environment is affected by the previous transmitted sequence, and a number of  $L - 1$  tail chips will affect the next transmitted sequence. This problem is called inter-chip interference (ICI) and in commercial CDMA systems RAKE receivers [45] are used in combating the ICI.

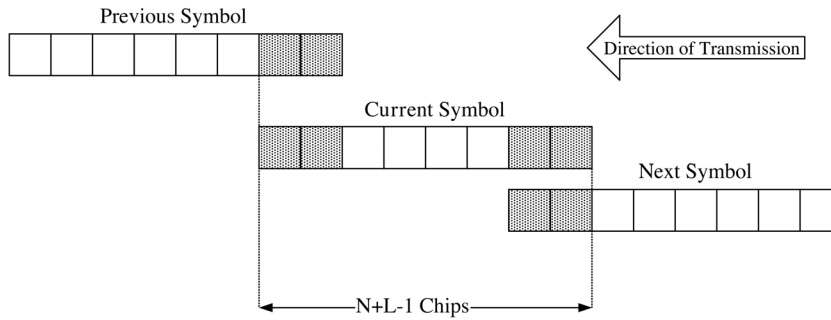


Figure 4. Inter-chip interference in multipath channel

It is possible to model the multipath channel using a finite impulse response (FIR) structure with  $L$  taps [2]. In conventional CDMA systems, the base station transmits a pilot tone and the receiver estimates the channel response by monitoring this tone. Let the channel be a stationary  $L$  tap with the impulse response  $H_{ch}(z) = h_1 + h_2z^{-1} + \dots + h_Lz^{-L+1}$ ; then the received signal at chip rate becomes

$$y(kN + n) = h_1 \sum_{u=1}^U D_u(k) S_{u,n} + h_2 \sum_{u=1}^U D_u(k) S_{u,n-1} + \dots + h_L \sum_{u=1}^U D_u(k) S_{u,n-L+1} + g(kN + n) \quad (11)$$

where  $g(kN + n)$  is the added noise component with the variance  $\sigma_n^2 = N_0/2$ , and  $N_0/2$  is the double-sided noise power spectral density. The vector representation of chip level expression  $y(kN + n)$  of the received signal becomes

$$\mathbf{y}(k) = [y(kN - L + 2) \quad \dots \quad y(kN + 1) \quad y(kN + 2) \quad \dots \quad y(kN + n)]^T \quad (12)$$

where  $\mathbf{y}(k)$  is a vector of length  $N + (L - 1)$ .

#### 3.2.2. Structure of the detector

The structure of the RBF MUD for the multipath channel is same as that of the RBF MUD for the AWGN channel as shown in Figure 3. The basis function used is again the Gaussian kernel

$$\phi_m(\mathbf{y}(k)) = \exp\left(-\frac{\|\mathbf{y}(k) - \mathbf{c}_m\|^2}{2\sigma^2}\right) \quad (13)$$



where  $\mathbf{c}_m$ ,  $m = 1, 2, \dots, M$  are the center vectors of length  $N + (L - 1)$ ,  $N$  is the length of the spreading sequences,  $L$  is the number of taps of the multipath channel,  $M$  is the number of center vectors that are introduced by the RBF MUD for each  $2^{3U}$  possible received signal, and  $U$  is the number of users in the channel.

### 3.2.3. Generation of super-center and code matrices

In order to construct the super-center matrix for the multipath channel, the  $L$ -tap impulse response  $H_{ch}$  of the channel has to be known at the detector. As discussed before, we have to deal with ICI in a multipath environment. It is possible to realize the convolution of the spreading sequences with the channel impulse response using matrix algebra in order to combat ICI while constructing the RBF MUD that operates in the multipath environment.

The super-center matrix for the multipath channel is defined in [33] as

$$\mathbf{C}^{MP} = \mathbf{S}^{MP} \mathbf{H}^T \quad (14)$$

where  $\mathbf{H}$  is an  $(N + L - 1) \times 3N$  Toeplitz matrix constructed using the CIR vector  $H_{ch}$ , and  $N$  is the length of the spreading sequence. The first  $N - L + 1$  column in  $\mathbf{H}$  is zero. The  $\mathbf{S}^{MP}$  in Eq. (14) is the Hadamard product of extended code matrix  $\mathbf{D}^{MP}$  and  $U \times 3N$  matrix comprising the spreading sequences of length  $N$  of all  $U$  users for the previous, current, and next symbols; thus

$$\mathbf{S}^{MP} = \mathbf{D}^{MP} \bullet [\mathbf{S} \ \mathbf{S} \ \mathbf{S}] \quad (15)$$

where  $\mathbf{S}$  is defined in Eq. (8) and  $\bullet$  is the Hadamard product operator. The extended code matrix  $\mathbf{D}^{MP}$  is a  $2^{3U} \times 3U$  matrix containing all possible bit combinations of previous, current, and next symbols for the  $U$  users as its rows and it can be partitioned into 3 sub-matrices in order to simplify the notation

$$\mathbf{D}^{MP} = [\mathbf{D}_P \ \mathbf{D}_C \ \mathbf{D}_N] \quad (16)$$

where  $\mathbf{D}_P$ ,  $\mathbf{D}_C$ , and  $\mathbf{D}_N$  are  $2^{3U} \times U$  matrices representing the previous, current, and next code matrices respectively. By substituting Eqs. (8) and 16 into Eq. (15) we have

$$\begin{aligned} \mathbf{S}^{MP} &= [\mathbf{D}_P \ \mathbf{D}_C \ \mathbf{D}_N] \bullet [\mathbf{S} \ \mathbf{S} \ \mathbf{S}] \\ &= \begin{bmatrix} \mathbf{D}_P & \begin{bmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_U^T \end{bmatrix} & \mathbf{D}_N \end{bmatrix} \bullet \begin{bmatrix} \mathbf{S} & \mathbf{S} & \mathbf{S} \\ \begin{bmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_U^T \end{bmatrix} & \begin{bmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_U^T \end{bmatrix} & \begin{bmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_U^T \end{bmatrix} \end{bmatrix} \end{aligned} \quad (17)$$

Each row in the matrix  $\mathbf{C}^{MP}$  represents a center vector of the RBF MUD for the multipath channel and the weight  $w_{m,u}$  in Eq. (6) must be selected from the  $m^{th}$  row and  $u^{th}$  column of matrix  $\mathbf{D}_C$  [27].

## 4. Method

### 4.1. Definition of the problem

The RBF MUD uses all the centers in the super-center matrix  $\mathbf{C}$  in Eq. (10) to achieve the optimum performance [35]. However, the number of rows in matrix  $\mathbf{C}$  (which is also the number of centers in the RBF MUD) is equal to  $2^U$ , where  $U$  is the number of users in the channel. When the number of users in the channel is large,

the structure of RBF MUD gets too complicated since its number of centers increases exponentially with the number of users,  $U$ . Thus, a need for structure optimization of this detector arises.

The method proposed in [38] gives the idea that it is possible to represent the DS-CDMA space by less than  $2^U$  basis functions. The problem is to find the best center locations and variation values of the basis functions. Another problem is determining the minimum number of centers to be used. These optimization problems can be solved by a GA. In other words, the structure of the RBF MUD can be optimized using a GA. In this paper a method that reduces the number of centers of RBF MUD by optimizing its center locations and variations is proposed. The method uses GA as the optimization tool. The optimized RBF MUD will be named as “GA assisted RBF MUD” or shortly “GA RBF MUD” hereafter.

#### 4.2. GA Assisted RBF MUD

In the traditional RBF MUD, there are  $M = 2^U$  centers for the AWGN case, and  $M = 2^{3U}$  centers for the multipath case. However, each center has a variance that is equal to the variance of the added noise,  $\sigma_n^2$ . In GA assisted RBF MUD, the center vectors are not chosen from the set of super-center vectors. Furthermore, the variance of each center can take any value.

It is possible to reduce the number of centers of RBF MUD by allowing the center locations to be anywhere in the DS-CDMA space and setting different variance values to different basis functions. By doing so, the RBF MUD can cover the whole space with a smaller number of centers, with a negligible performance degradation in terms of BER. In the GA RBF MUD the center vectors of the proposed RBF MUD are not chosen from the set of super-centers:

$$\mathbf{c}'_k \neq \mathbf{c}_m; \quad k = 1, 2, \dots, K; \quad m = 1, 2, \dots, M \quad (18)$$

where  $\mathbf{c}'_k$  is the  $k^{th}$  center vector of the GA RBF MUD and  $\mathbf{c}_m$  is the  $m^{th}$  super-center vector of the traditional RBF MUD.

In the original definition of RBF MUD [27] and in related works [35, 37, 38], the variations of all the centers has been chosen to be constant and equal to the noise variance,  $\sigma_n^2$ :

$$\sigma_m^2 = \sigma_n^2 \quad \forall m, \quad m = 1, 2, \dots, M \quad (19)$$

where  $M = 2^U$  is the number of centers and  $U$  is the number of users in the channel. The variances of Gaussian basis functions of the GA RBF MUD are different from each center:

$$\sigma_i^2 \neq \sigma_j^2, \quad i \neq j, \quad 1 \leq i < j \leq K \quad (20)$$

This flexibility of the proposed method lets a center represent more than one super-center, which leads to a considerable amount of performance increase in terms of structural complexity, especially when the number of users,  $U$ , is large.

#### 4.3. Structure Optimization of RBF MUD for the AWGN Channel

The optimization procedure starts with a randomly selected small subset of super-center vector set as initials. In other words, the  $K \times N$  matrix  $\mathbf{C}'$  is generated by selecting from the rows of  $M \times N$  matrix  $\mathbf{C}$  in Eq. (10). Initial variances of centers are set to be equal to the noise variance,  $\sigma_n^2$ , since this is the case for the original RBF MUD. Then center vector locations and variances of each center function are optimized to get better BER's by using a GA.

Each member of the population is formed as follows:

$$\mathbf{I}_{p,i} = [c'_{1,i} \quad c'_{2,i} \quad \cdots \quad c'_{K,i} \quad \sigma_{1,i}^2 \quad \sigma_{2,i}^2 \quad \cdots \quad \sigma_{K,i}^2] \quad (21)$$

where  $p = 1, 2, \dots, P$  is the member index,  $P$  is the population size, and  $i$  is the generation number. Each member of a chromosome is selected to be a real number. As can be seen in Eq. (21), each member of the population represents a different RBF MUD structure. At each generation of the GA, the RBF MUD structures defined by each member of the population are formed up and tested with an input set of  $10^4$  samples. The fitness function of each member is defined as:

$$f = 1 - BER \quad (22)$$

where BER is the bit error rate of the RBF MUD whose structure is defined by the associated member. Thus, the GA minimizes BER while maximizing the fitness function. At each iteration, the members with the best fitness function are selected. Then GA operators like crossover and mutation are applied to these members and a new and evolved population is generated.

Since the optimized locations of the center vectors are different from the locations of the super-center vectors, the weight values in the output layer of the RBF MUD,  $w_m$  for  $m = 1, 2, \dots, M$  cannot be determined from the columns of the code matrix  $\mathbf{D}$ . Thus, the weight values are calculated by using the least mean squares (LMS) algorithm for each member at each iteration of the GA. When the algorithm terminates, the member with the best BER is selected to be the centers and variances of the RBF MUD and associated weights calculated by the LMS become the weights of the output layer of the RBF MUD.

#### 4.4. Structure optimization of RBF MUD for the multipath channel

The method for optimizing the RBF MUD structure in the multipath channel is similar to the one in AWGN. It is possible to generate  $K \times (N + L - 1)$  matrix  $\mathbf{C}'$  again by randomly selecting from the rows of  $M \times (N + L - 1)$  matrix  $\mathbf{C}^{MP}$  in Eq. (14).

Since the dimension of the input space is greater than that of AWGN, it is expected that the GA will require a larger population size, and number of iterations. Again, the weight values are calculated by using the LMS algorithm for each member at each iteration of the GA.

## 5. Simulations

### 5.1. AWGN channel

A DS-CDMA system with 20 users having Walsh spreading codes of length 32 in a nondispersive channel distorted by AWGN is simulated. A number of  $10^4$  equi-probable bits for training and  $10^7$  bits for testing are generated for each user. As explained in Section III, code matrix  $\mathbf{D}$  and super-center matrix  $\mathbf{C}$  are generated for  $U = 20$  and  $N = 32$ . Each member (chromosome) in the initial population of GA is formed as follows:

$$\mathbf{I}_{p,0} = [c'_{1,0} \quad c'_{2,0} \quad \cdots \quad c'_{K,0} \quad \sigma_{1,0}^2 \quad \sigma_{2,0}^2 \quad \cdots \quad \sigma_{K,0}^2] \quad (23)$$

where  $\mathbf{c}_{x,0}$ ,  $x = 1, 2, \dots, K$ , vectors are selected randomly from the rows of super-center matrix  $\mathbf{C}$ . Each row of matrix  $\mathbf{C}$  is selected only once in the same chromosome. Initial variance values in Eq. (23),  $\sigma_{y,0}^2$ ,  $y = 1, 2, \dots, K$ , are set to be equal to the variance of the added noise component,  $\sigma_n^2$ .

Not only the transmitted data of a selected user but also the transmitted data of all the users in the channel are detected in both the train and test stages of our simulations. This is done by updating the weights of the RBF MUD, which are calculated at each iteration of the training stage and at the beginning of the testing stage by the LMS algorithm for that particular user.

The parameters that may have significant effect on the performance of the GA RBF MUD are tested in the simulations. These parameters are as follows: number of centers used in the structure, initial population, number of generations produced by the GA, mutation probability of the GA and population size (number of members in the population) of the GA. RBF MUD and GA parameters used in the tests are presented in the Table. Note that in the Table in all cases, population type is double vector, i.e. elements of the chromosomes are real numbers with double precision; the function that selects parents of crossover and mutation children is stochastic and its distribution is uniform; and the algorithm that is used to create crossover children is scattered. The function that produces mutation children is stochastic and its distribution is uniform in 1 of the 4 cases and it is Gaussian in the rest. Effects of these parameters on the performance of the RBF MUD will be explained in the following subsections in the light of the simulation results.

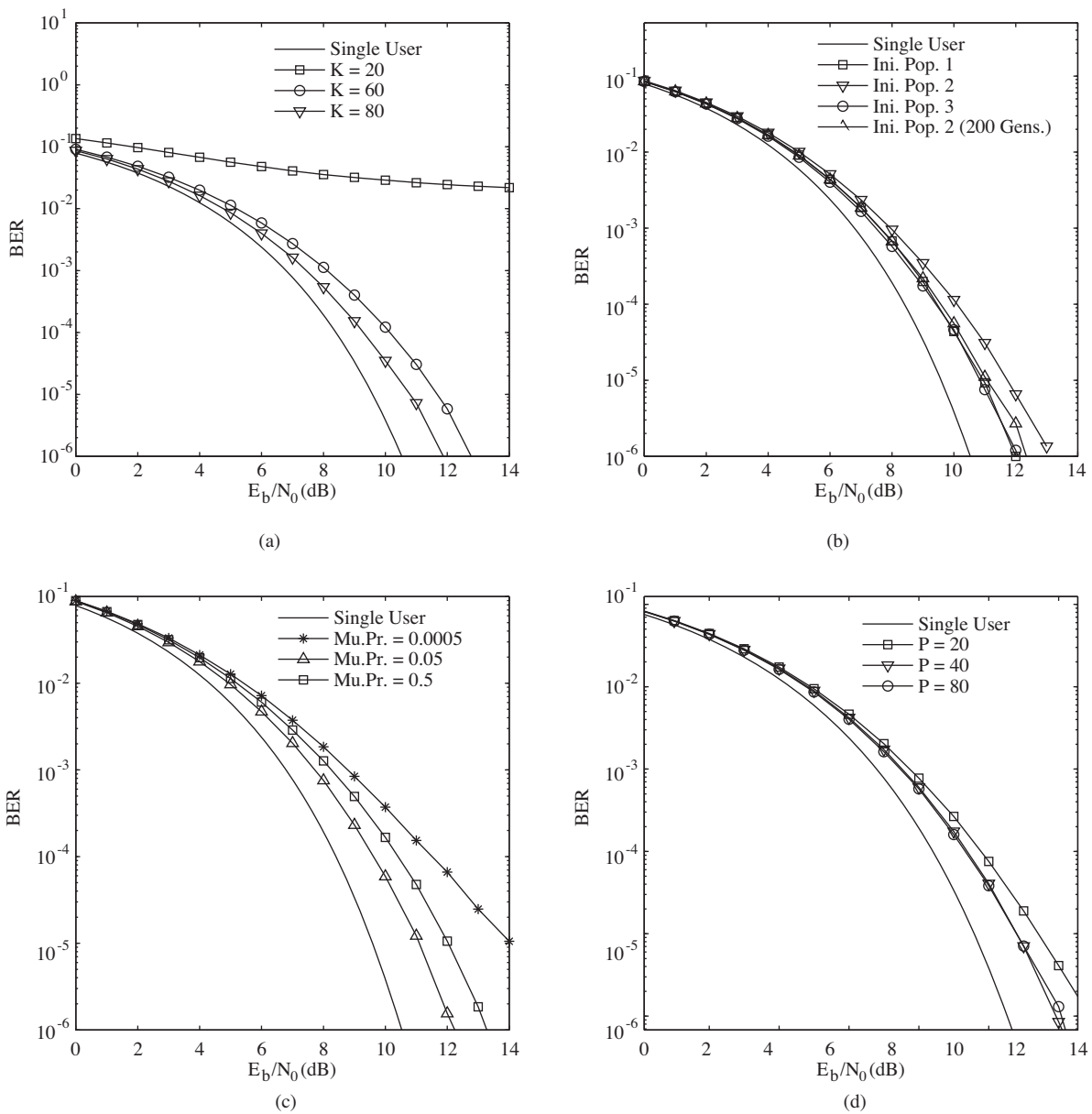
**Table.** Genetic algorithm parameters used in different tests.

	Test for Number of Centers	Test for Number of Generations	Test for Mutation Probability	Test for Population Size
Number of Centers	20, 60, and 80	40	40	40
Population Type	Double Vector	Double Vector	Double Vector	Double Vector
Population Size	40	40	40	20, 40, and 60
Selection Function	Stochastic	Stochastic	Stochastic	Stochastic
Selection Function	Uniform Gaussian	Uniform Gaussian	Uniform Uniform	Uniform Gaussian
Crossover Function	Scattered	Scattered	Scattered	Scattered
Mutation Probability	Shrink:0.75; Scale:0.5	Shrink:0.75; Scale:0.5	0.001, 0.1, and 0.5	Shrink:0.75; Scale:0.5
Crossover Probability	0.8	0.8	0.8	0.8
Number of Generations	50	50 and 200	50	50

### 5.1.1. Number of centers

Three GA assisted RBF MUD's with different number of centers were simulated. Numbers of centers are selected to be 20, 60, and 80. In order to operate at optimum performance, an RBF MUD needs  $2^{20}$  centers to support 20 users. Thus, increasing the number of centers would improve the performance of the receiver. *BER* versus  $E_b/N_0$  plot for the GA assisted RBF MUD having a different number of centers is given in Figure 5a, where  $K$  is the number of centers of the GA RBF MUD. As can be seen in Figure 5.a,  $K = 80$  gives the best performance, where this result meets the theoretical assumption stating the performance improvement due to the increase in the number of centers. When  $K = 20$ , the receiver is unable to provide a near-optimum performance. However, due to the flexible structure of the proposed receiver, even in the case of 20 centers, its BER performance is about 1 dB better than the performance of the receiver proposed in [35], which forms an RBF MUD by selecting the most influential super-centers as the centers.

The RBF MUD needs  $2^{20}$  centers to operate, while the method proposed in this work reduces this number to 80. The complexity reduction ratio is a considerable amount, about 1/13,000 on a rough calculation.



**Figure 5.** Simulations results of (a) test for number of centers, (b) test for initial population, (c) test for mutation probability, (d) test for population size in the AWGN channel.

### 5.1.2. Initial population and number of generations

The performance of the GA depends on the choice of initial population. The GA provides better performance if the algorithm is started with an initial population whose members are close to the solution to the optimization problem. In our case, the members in the population represent the center vectors and variance values of the basis functions that form the RBF MUD. Thus, the method would converge to an optimum structure faster if the initial population were set close to the final structure. Hence, we start the GA with an initial population whose members are formed by randomly selecting the centers from a number of  $2^{20}$  super-centers. In Figure 5b,

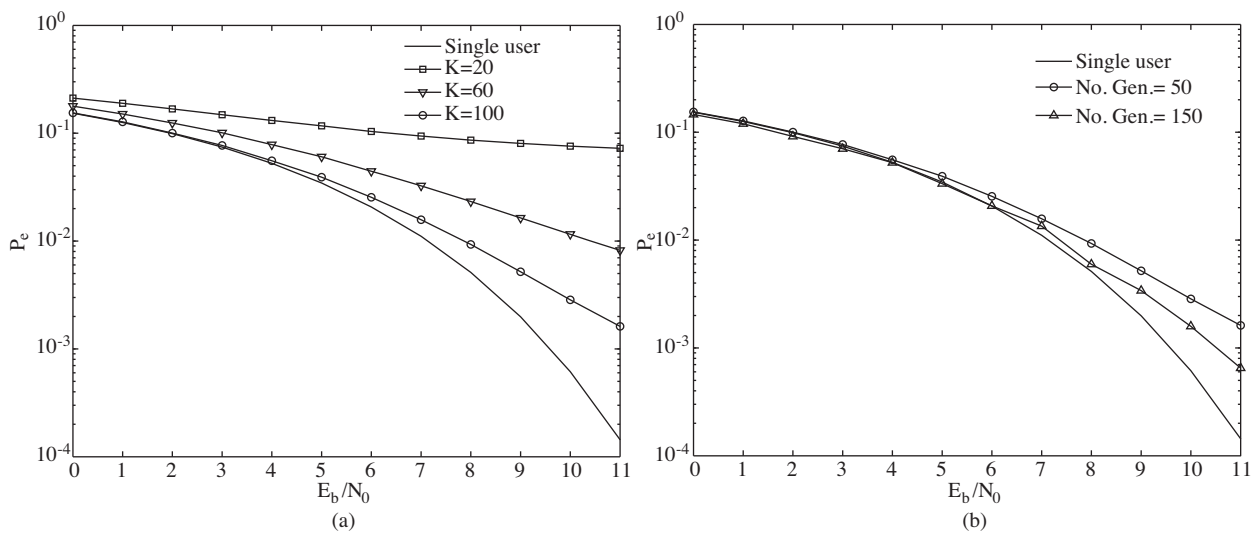
BER versus  $E_b/N_0$  plots are shown for the RBF MUD's, which are optimized by the GA starting with different initial populations. In this test, the GA is terminated when the number of generations reaches 50. It is seen in Figure 5b that starting with the first and third initial populations has led the GA to generate RBF MUD's that have performances identical to each other and better than that of the RBF MUD that was generated by the GA started with the second initial population. However, starting with the same initial population and terminating the GA at 200 generations instead of 50, the resultant optimized RBF MUD provided the identical performance with the other RBF MUD's. Thus, the effect of the initial population on the performance of the resultant RBF MUD can be eliminated by letting the algorithm generate more populations.

**5.1.3. Mutation probability**

The recommended mutation probability range in the literature [13] is  $10^{-3} - 10^{-2}$ . Since our strings (members in the population) are represented by real numbers and no encoding is used, the space that is needed to be scanned is real valued and there are an infinite number of locations for a center to be located. Hence, we would need a high mutation probability to search the space effectively. According to Figure 5c, the GA with a mutation probability close to the upper recommended limit, 0.05, generated an RBF MUD with the best performance. A probability that is less than the recommended lower limit, 0.0005, or a probability that is much greater than the upper limit, 0.5, ends up with an RBF MUD of worse performance.

**5.1.4. Population size**

In the GA, increasing the number of members in the population to be evolved decreases the probability of the algorithm being stuck at local maxima. In this test, populations of different sizes are generated and optimized via the GA. The algorithm is stopped at the same number of generations for each population. Observing Figure 5d, we may conclude that a GA evolving a population of greater size would generate an RBF MUD with better performance.



**Figure 6.** Simulation results in multipath channel. (a) Effect of number of centers. (b) Effect of number of iterations. Increasing the number of iterations provides better results as expected.

## 5.2. Multipath channel

The number of centers for the RBF MUD supporting 10 users in a 3-tap multipath fading channel are reduced to 20, 60, and 100. It can be seen in Figure 6a that the proposed method reduced the number of centers from  $2^{30}$  to 100 with a slight performance degradation. When  $K = 20$  and  $K = 60$  the detector is unable to provide a near-optimum performance. If we consider the case for  $K = 100$ , the complexity reduction ratio is a considerable amount, i.e. about  $1/10^7$  on a rough calculation. Performance loss in the new detector is about 1 dB for a BER of  $10^{-3}$ .

In Figure 6b the bit error rate performances of 2 RBF MUD's where one of them is generated after 50 iterations of GA and the other is generated after 150 iterations are shown. We may conclude that the performance of the optimized RBF MUD may be increased further if we let the GA run more iterations.

Note that due to the structural complexity of the RBF MUD, a time-invariant multipath channel is used in the simulations with the channel impulse response given as

$$H_{ch}(z) = 0.3482 + 0.8704z^{-1} + 0.3482z^{-2} \quad (24)$$

## 6. Conclusion

We proposed a new method to reduce the complexity of the RBF MUD by minimizing the number of center functions using a GA. By determining the optimal values of the centers and the variances of the radial basis functions through the GA, we managed to reduce the complexity of the RBF MUD from  $O(2^U)$  to  $O(K)$ , where  $K$  is a predetermined number of centers, at the expense of negligible performance degradation compared to the single user receiver. Increasing the number of centers, population size, and number of generations will make the performance of the GA assisted RBF MUD approach the optimum performance of the traditional RBF MUD, which has a complexity of  $O(2^U)$ .

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