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Unscented transformation-based probabilistic optimal power flow for modeling the effect of wind power generation

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Abstract: The unprecedented increasing penetration of distributed energy resources, mainly harvesting renewable energies, is a direct result of environmental concerns. These types of energy resources bring about more uncertainties in the power system and, consequently, necessitate probabilistic analyses of the system performance. This paper develops a new approach for probabilistic optimal power flow (P-OPF) by adapting the unscented transformation (UT) method. The heart of the proposed method lies in how to produce the sampling points. Appropriate sample points are chosen to perform the P-OPF with a high degree of accuracy and less computational burden in comparison with the features of other existing methods. Another salient feature of the UT technique is its capability in modeling the correlated uncertain input variables. This attribute is very desirable in the accommodation of wind generations, which likely have a correlation regarding the geographical proximity. In order to examine the performance of the proposed method, 2 case studies are conducted and the results are compared with those of other existing methods, such as Monte Carlo simulation and 2-point estimation methods. The case studies are the Wood and Wollenberg 6-bus and the IEEE 118-bus test systems. A comparison of the results justifies the effectiveness of the proposed method with regards to the accuracy and execution time.

Key words: Locational marginal price, probabilistic optimal power flow, uncertainty modeling, unscented transformation, wind turbine generator

1. Introduction

The integration of a significant amount of renewable energies (REs) such as wind power into the power systems causes crucial operational challenges that stem from uncertainties associated with the REs. In such power systems, a deterministic optimal power flow (OPF) evaluation cannot reveal the state of system accurately; therefore, probabilistic evaluation is of significant interest. Computation of probabilistic OPF (P-OPF) is one of the major requirements in power system planning and operation. The goal of P-OPF is to incorporate the impact of uncertain input variables and determine the propagation of uncertainties over the output parameters. A deterministic OPF study requires specific values for the loads, generation, and network conditions. On the contrary, in restructured power systems including intermittent and variable energy resources like REs, the uncertainty of the power system's OPF is not ignorable. In system expansion planning, it is desirable to assess the bus voltages and line flows for a range of the load and generation conditions. Performing P-OPF study gives system planning engineers a better feel of future system conditions and will provide more confidence in making judgments concerning investments. Performing OPF for every possible or probable combination of loads,

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generating units' conditions, and network topology are impractical because of the extremely large computational effort required; therefore, probabilistic methods with an acceptable accuracy as well as a tractable computation are needed for the system studies.

To comprehend the power system uncertainties, probabilistic techniques have been used since the early 1970s [1]. To date, many probabilistic methods have been proposed to study the uncertainty problem in engineering systems. These methods can be classified into 2 categories: simulation and analytical methods. Monte Carlo simulation (MCS) is the most widely used simulation method. MCS provides more accurate results but its execution might be extremely time-consuming, and this makes this method unattractive in real work problems. In order to reduce the computational effort, analytical methods were proposed, whose summary review is as follows. The problem of economic dispatch was considered as a probabilistic problem in [2], where the authors used a Gram–Charlier series to represent the probability density function (PDF) of the system's uncertain loads. Based on the same Gram–Charlier series technique, in [3] and [4], a more general approach to account for the uncertainties associated with all of the OPF variables was proposed. The method is based on linearization and series expansion that have some difficulties and application problems. In [5–7], the authors developed a new approach in probabilistic studies based on the point estimation method. The 2-point estimation method (2PEM) was proposed for P-OPF study in [8], which can obtain the results with an acceptable level of accuracy. The computation time associated with the method is exactly proportional to the number of uncertain variables, and this feature makes the method impractical for even medium-scale problems.

The main advantage of the analytical methods mentioned above is to avoid cumbersome computer simulations. In contrast, these methods impose more assumptions and complex mathematical algorithms. Some approaches presented in the reviewed literature use linearization algorithms that have several shortcomings, including the following [9]:

1. The existence of the Jacobian matrix is a prerequisite for the linearization. In addition, calculating the Jacobian matrix could be a complex or error-prone process.
2. Linearization transformation would be only reliable if the error propagation is well approximated by a linear function.

For some analytical methods, the execution time is either proportional or exponential with respect to the number of uncertain variables. This can diminish the superiority of the analytical methods against the simulation ones in large systems. To overcome such a drawback, new analytical methods for probabilistic nonlinear systems are exceedingly necessary and must have the following properties:

1. An acceptable level of accuracy.
2. A reasonable execution time not strongly dependent on the number of uncertain variables.
3. Easy to implement.

Effectively, to realize the properties, a new method for nonlinear transformation of the means and covariances of the output variables in the filters and estimators was proposed in [10]. The method was developed to address the deficiencies of linearization by providing a more direct and explicit mechanism for transforming the mean and covariance information. Unscented transformation (UT) is novel and has shown good performance in nonlinear transformations and state estimators [9]. The UT method has found a number of applications in high-order and nonlinear coupled systems, including navigation systems for high-speed road vehicles [11,12], public transportation systems [13], data assimilation systems [14], and underwater vehicles [15]. In these applications, the UT method proves high accuracy levels and is significantly faster than an MCS approach.

None of analytical approaches reviewed above, except for the UT method, are able to tackle correlated variables. Moreover, some of them have critical shortcomings stemming from the linearization. Accordingly, having a computationally efficient but still accurate method that can handle correlated variables is of significant interest. The main contribution of this paper is to develop a new methodology for P-OPF problems by modifying the UT method applied to power systems, taking into account different kinds of uncertain variables, e.g., load and wind power generation. The modification reduces the computational burden while keeping the accuracy at a satisfactory level. To this end, at commencement, the uncertainties are modeled with the appropriate distribution functions; thereafter, the proposed modified UT method is tailored to the problem. In continuation, the P-OPF methodology is applied on the test systems, including 6-bus and 118-bus power systems. To ensure the effectiveness of the proposed method as well as the correctness of the results, the obtained results are compared with those of other currently used methods with respect to both the accuracy and execution time criteria. Simulation evidence verifies the applicability of the developed P-OPF method, particularly in large-scale systems where the conventional methods suffer from computational burden.

The rest of the paper is organized as follows. Section 2 introduces the original UT method and the modified method. In Section 3, the procedure of uncertainty modeling in P-OPF is given. Section 4 describes the case studies, and then the obtained results are presented for each case study. Finally, a comparison of the results and the concluding remarks are given in Sections 5 and 6, respectively. Additionally, an Appendix that presents more technical details about the used wind turbines is included.

2. UT method

2.1. General UT method

The UT method is a credible method for calculating the statistics of an output random variable undergoing a nonlinear transformation. It builds on the fact that it is easier to approximate a probability distribution than an arbitrary nonlinear function. It refers to the demerits of linearization by providing a more direct and explicit mechanism for transforming mean and covariance information [9]. The heart of the method lies in how to produce appropriate samples of the input variables to maintain enough information about the input variable PDF. Assume that $\mathbf{X}$ is a vector of n-dimensional random variables with a mean $\bar{\mathbf{X}} = \mathbf{m}$ and covariance $\mathbf{P_{XX}}$, and the other random variable $\mathbf{Y}$ relates to $\mathbf{X}$ through a nonlinear function as:

$$\mathbf{Y} = f(\mathbf{X}) \tag{1}$$

where f can be a set of nonlinear functions. With the UT method, the mean and covariance of the output variables $\bar{\mathbf{Y}}$ and $\mathbf{P_{YY}}$ can be obtained through the following steps [9].

Step 1: Use Eqs. (2) through (4) to obtain $2n+1$ sample points.

$$\mathbf{x}^0 = \mathbf{m} \tag{2}$$

$$\mathbf{x}^i = \mathbf{m} + (\sqrt{(n+\lambda)\mathbf{P_{XX}}})_i, \quad i = 1, 2, ..., n \tag{3}$$

$$\mathbf{x}^{i+n} = \mathbf{m} - (\sqrt{(n+\lambda)\mathbf{P_{XX}}})_i, \quad i = 1, 2, ..., n \tag{4}$$

Step 2: Use Eqs. (5) through (8) to calculate the weight associated with each $\mathbf{x}$.

$$W^0 = \frac{\lambda}{(\lambda+n)} \tag{5}$$

$$W^i = \frac{1}{2(\lambda + n)}, \quad i = 1, 2, ..., n \tag{6}$$

$$W^{i+n} = \frac{1}{2(\lambda + n)}, \quad i + n = n + 1, ..., 2n \tag{7}$$

It should be noted that the associated weights must meet the following condition:

$$\sum_{i=0}^{2n} W^i = 1, \tag{8}$$

where $(\sqrt{(n+\lambda)\mathbf{P_{XX}}})_i$ is the ith row or column of the matrix square root of $(n+\lambda)\mathbf{P_{XX}}$, and $\lambda = \alpha_u^2(n + k_u) - n$. Here, α_u is a scaling factor; k_u is another parameter of the method and can be chosen as $k_u = 3 - n$ if x is Gaussian [12].

The matrix square root of the positive definite matrix $\mathbf{P}$ means that a matrix $\mathbf{A} = \sqrt{\mathbf{P}}$ exists such that $\mathbf{P} = \mathbf{A}\mathbf{A}^{\mathbf{T}}$. The matrix square root should be calculated using numerically efficient and stable methods such as the Cholesky decomposition.

Step 3: Institute each sample point through the nonlinear function to yield the set of transformed sample points as:

$$\mathbf{y^i} = f(\mathbf{X}^i). \tag{9}$$

Figure 1 shows the principle of the UT method [9].

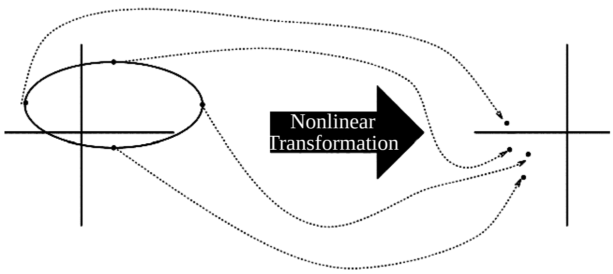

Figure 1. The principle of the UT method [9].

Step 4: Calculate the mean and covariance of the output variable $\mathbf{Y}$ using Eqs. (10) and (11), respectively.

$$\bar{\mathbf{Y}} = \sum_{i=0}^{2n} \mathbf{W}^i \mathbf{Y}^i \tag{10}$$

$$\mathbf{P_{YY}} = \sum_{i=0}^{2n} \mathbf{W}^i (\mathbf{Y}^i - \bar{\mathbf{Y}})(\mathbf{Y}^i - \bar{\mathbf{Y}})^T \tag{11}$$

Although the UT method has an apparent similarity to particle filters, it has several basic differences from them. First, the sample points are not chosen randomly; they are selected in such a way that they have a given mean and covariance. The second difference is that the associated weights on the selected points do not have to lie in the range of [0, 1]. The weights can have positive or negative values but, to provide an unbiased estimate, they must obey the situation in Eq. (8), for which the sum of all of the associated weights must be equal to unity. It is important to recognize that the basic UT algorithm is conceptually very simple and easy to apply [10].

2.2. Modified UT method

The execution time of the UT method directly depends on the dimensions of the uncertain input variables. For the cases with many uncertain input variables, this feature may decrease the merits of the UT method against the others in large-scale systems; accordingly, the modified UT (MUT) method, with much faster executions, would be highly desired. The numerical evidences obtained here support the effectiveness of the MUT method in the application at hand from the computation speed viewpoint, while keeping the accuracy at a reasonable grade. The procedure of the MUT method is as follows.

Step 1: Set n, instead of the dimension of the input variable vector, to a sufficiently small even number. Therefore, $2n+1$ evaluations are needed to achieve the final results. Based on our experience, setting $n = 2$ is a proper choice in Gaussian distributions.

Step 2: Use Eqs. (12) through (14) to obtain $2n+1$ sample points.

$$\mathbf{x}^0 = \mathbf{m} \tag{12}$$

$$\mathbf{x}^i = \mathbf{m} + (\sqrt{(n+\lambda)\mathbf{P_{XX}}})_i, \quad i = 1, 2, ..., n \tag{13}$$

$$\mathbf{x}^{i+n} = \mathbf{m} - (\sqrt{(n+\lambda)\mathbf{P_{XX}}})_{i+n}, \quad i = 1, 2, ..., n \tag{14}$$

It must be noted that here we decompose the covariance matrix to obtain $\mathbf{P} = \mathbf{A}\mathbf{A}^\mathbf{T}$, in which $\mathbf{A}_{k\times 2n}$ has k rows and $2n$ columns.

Step 3: Use Eqs. (5) through (8) to calculate the weight associated with each $\mathbf{x}$.

Step 4: Institute each sample point through the nonlinear function to yield the set of transformed sample points as Eq. (9).

Step 5: Calculate the mean value of the output variable using Eq. (10) as before, but in order to calculate the covariance, use Eq. (15).

$$\mathbf{P_{YY}} = \sum_{i=0}^{2n} \mathbf{W}^i(\mathbf{Y}^i - \bar{\mathbf{Y}})(\mathbf{Y}^i - \bar{\mathbf{Y}})^T + \frac{(1+\beta-\alpha_u^2)}{n+1} \sum_{t=0,1,...,n} (\mathbf{y}^{2t} - \bar{\mathbf{Y}})(\mathbf{y}^{2t} - \bar{\mathbf{Y}})^T \tag{15}$$

Eq. (15) is a modified version of the proposed equation in [9] for the covariance calculation. The value of β for a Gaussian distribution is 3 [9]. In the proposed procedure, the number of problem evaluations is independent of the number of uncertain variables; however, it can obtain the results with an acceptable level of accuracy.

It must be emphasized that the UT method is able to take into account the correlation between uncertain input variables. This is also true in the case of the MUT method.

3. Uncertainty in OPF

3.1. Uncertain parameters

An OPF problem attempts to optimize an objective function by adjusting a set of control variables subject to certain physical and operational constraints. As the power industry is being reregulated, together with the unprecedented development of new technologies during the last decades, the importance of OPF for power system optimal dispatch has been increased considerably. Very often, OPF is addressed as a deterministic optimization problem with fixed model parameters and input variables. However, many random disturbances or uncertain factors exist within the power system operation due to measurement errors, forecasting errors, or outages of the system elements. The availability of power generation is another uncertain factor in power systems; specifically, recent effort is dedicated to the increasing penetration of renewable generations, such as wind and solar energy, which have uncertain and intermittent output power. These uncertainties introduce errors into the OPF solutions; therefore, probabilistic analyses must be carried out. In this study, the focus is on the uncertainties associated with the load and wind power generation.

3.2. Uncertainty modeling

One of the most important power system uncertainties refers to the load forecast. It varies as a function of time, weather, regulation, electricity price, and peoples' income, among other factors. In this paper, the load is modeled through a normal distribution function with μ, the mean value equal to the base load, and σ, the standard deviation (SD) equal to $\pm 5\%$ of the mean. The system loads are distributed among the PQ buses with specified real and reactive load demands.

In order to model the wind power generation uncertainty, some buses are assumed to have integrated wind farms with uncertain output power. Wind speed varies both in time and place and its PDF is assumed as either Gaussian or Weibull in the reported literature [16]. In this paper, for the sake of simplicity, the wind speed is modeled with a normal distribution. However, as implied in [9], modified versions of the UT method have been developed that can handle any type of PDFs The simplified modeling of the wind turbine generator (WTG)'s output power uncertainty is summarized as follows. This model was chosen to simply test the proposed method and illustrate a possible use of the UT method in general applications.

Step 1: The wind speed is modeled with an appropriate PDF. It can describe the wind speed's behavior and the occurrence probability of each wind speed.

Step 2: In order to assess the uncertainties, the problem is evaluated several times to cover at least the most important or probable conditions. For modeling the uncertainty of the WTG's output power, the wind speed samples are generated in each evaluation by an appropriate manner.

Step 3: The generated wind speed samples are converted to the wind turbine's output power using the wind speed–power curve through Eq. (16). The power generation of wind turbines is strongly dependent on the wind speed. The relationship between the power generation and wind speed may follow different shapes; however, the linear relation adequately fits the experimental data [17]. Since the input data of the UT method is the power output of a given wind turbine, it is clear that there would be no restriction in applying the nonlinear curves of wind speed conversion. However, for the sake of simplicity, the following linear model, Eq. (16), is assumed here to convert the wind speed data into the corresponding electric power output quantities. Moreover, this model is able to appropriately reflect the uncertainty in the wind speed data.

$$\begin{aligned} &P_{WTG} = 0 \text{ if } v \leq V_i \text{ or } v \geq V_o \\ &P_{WTG} = P_r \tfrac{v-V_i}{V_r-V_i} \text{ if } V_i < v < V_r \\ &P_{WTG} = P_r \text{ if } V_r \leq v < V_o \end{aligned} \tag{16}$$

Step 4: The wind farm's output power is modeled as a negative load in the corresponding bus (in this study, it is assumed that the wind farm's power factor is kept at a 0.85 lag).

3.3. Adaptation of the MUT method into the P-OPF problem

The main goal of the P-OPF study is to determine the state of the system as a function of the uncertain input variables. This matter can be stated as Eq. (1).

The input vector $\mathbf{X}$ is written as:

$$\mathbf{X} = \begin{bmatrix} \mathbf{P_D} \\ \mathbf{Q_D} \\ \mathbf{P}_{wind} \\ \mathbf{Q}_{wind} \\ \cdots \end{bmatrix}, \tag{17}$$

and the output vector $\mathbf{Y}$ is stated as:

$$\mathbf{Y} = \begin{bmatrix} \delta \\ \mathbf{V} \\ \mathbf{P_G} \\ \mathbf{Q_G} \\ \cdots \end{bmatrix}. \tag{18}$$

Implementation of the MUT approach to the P-OPF study requires taking the following steps.

Step 1: The uncertain input variables are represented by appropriate PDFs as normal. The set of inputs will have a vector of means $\mathbf{m}$ and a covariance matrix $\mathbf{P_{XX}}$.

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ \cdots \\ m_k \end{bmatrix} \tag{19}$$

$$\mathbf{P_{XX}} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1k} \\ P_{21} & P_{22} & \cdots & P_{2k} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ P_{k1} & P_{k2} & \cdots & P_{kk} \end{bmatrix} \tag{20}$$

Step 2: Obtain the sample points from Eqs. (12) through (14).

Step 3: The nonlinear function is applied to each sample point, in turn, to yield a cloud of transformed points. Here, the nonlinear function is the OPF.

Step 4: Eqs. (10) and (15) are used to calculate the statistics of the output variables.

4. Case studies

Two case studies, namely the Wood and Wollenberg 6-bus and the IEEE 118-bus test systems, are studied using the MUT method and the obtained results are discussed. It has been argued before that the MUT method can consider the correlation between the uncertain input variables, e.g., between wind power generations or between loads in different buses. Wind speeds at proportionately nearby wind farms can be actually correlated and this matter may cause a large/little power generation by the wind farms simultaneously, which can severely affect the energy price in some buses. Similarly, this matter may happen for loads in some buses. However, in each case study, a subsidiary case in which the correlation between the uncertain input variables is considered is examined to demonstrate the functionality of the method in this situation.

The proposed method was implemented on a Dell Inspiron 1420 system with a 2-GHz processor and 2 GB of RAM using the MATLAB optimization toolbox [18].

4.1. The Wood and Wollenberg 6-bus system

This system has 6 buses, 3 generators, and 11 transmission lines. The technical data of this system were taken from [19]. It is such a simple system that the correctness of the results can be verified intuitively, and some interesting features of the proposed method can additionally be revealed by this network. Three case studies with different assumptions are carried out below.

4.1.1. Case 0: The base case

The base case is assumed to have a wind farm at bus 5, whose detailed information is given in Table 1. Note that the parameters given in Table 1 are associated with the Weibull distribution of the wind speed. Additionally, more detailed information about the used wind turbines is given in the Appendix.

Table 1. Wind farm information for Case 0.

Parameter	Quantity
No. of WTGs	8
μ (m/s)	6.13
σ (m/s)	3.146

As mentioned before, an OPF problem attempts to optimize an objective function (here the total production cost) by adjusting a set of control variables subject to a certain physical and operational constraints. With this knowledge, the proposed method was applied to this case to find the P-OPF solution assessing the effect of the uncertainty of the input variables. Table 2 lists the obtained results for this case study, including the mean and the SD of some of the output variables.

Table 2. Obtained results using the MUT method for Case 0.

Parameter	Mean	SD
Total cost ($)	3115.5	86.6
Total loss (MW)	6.8681	1.0326
LMP-bus1 ($/MWh)	12.5217	0.2357
LMP-bus2 ($/MWh)	11.5077	0.2096
LMP-bus3 ($/MWh)	11.8406	0.0757
LMP-bus4 ($/MWh)	16.5907	3.5981
LMP-bus5 ($/MWh)	13.0190	0.4953
LMP-bus6 ($/MWh)	12.1838	0.0158
Generation-bus1 (MW)	79.9882	22.1084
Generation-bus2 (MW)	66.0681	11.7917
Generation-bus3 (MW)	67.9897	5.1026

In Table 2, the results indicate that for this case, the uncertain output variables can be modeled with suitable distribution functions with the mean value and SD given here. Here, the number of samplings is 5, and Table 3 gives the transformed sample points for some of the selected output variables, i.e. total production cost and losses, along with their associated weights.

Based on Tables 2 and 3, although the mean value of the total active losses is 6.8681 MW, in some combinations, this parameter has values of about 6.3856 MW (7% less than the mean value) and 8.7079 MW (27% more than the mean value) due to the system uncertainties. The occurrence probability of these extreme values is rare, but they may happen. Similar variations exist in the case of the production cost. The maximum cost is for a situation in which the load is at its maximum level and the power produced by the wind farm is at its minimum level. Another important output of the OPF is the locational marginal prices (LMPs) at the generation and demand buses. Briefly, it is known that the LMP at a given bus is the incremental operation cost of the system due to the increase of the demand by 1 MWh. The value of the LMP can be broken down into 3 components, as the marginal energy price, marginal loss price, and marginal congestion price [20]. Figure 2 shows the transformed sample points of the LMP at different buses for this case study.

Table 3. Transformed sample points for Case 0.

Parameter weights	Associated (1e3$)	Cost (MW)	Losses
Sample1	0.659	3.0794	6.4414
Sample2	0.08525	3.2689	8.7079
Sample3	0.08525	3.2689	8.7078
Sample4	0.08525	3.0868	6.6580
Sample5	0.08525	3.0712	6.3856

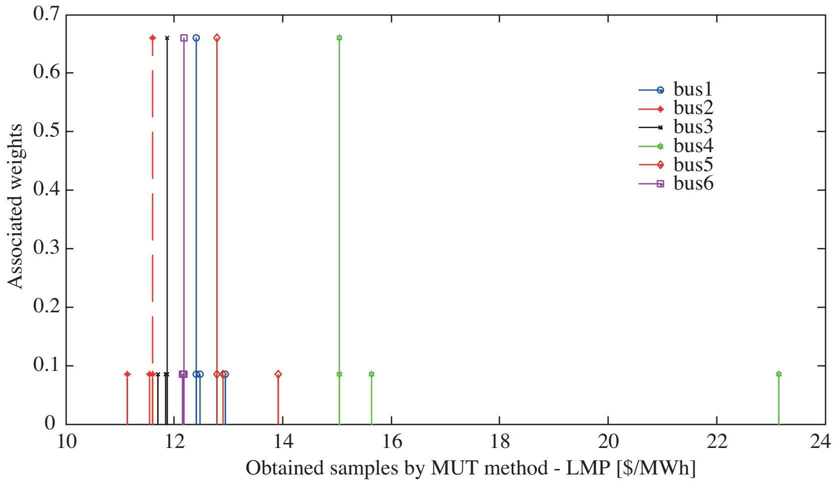

Figure 2. The obtained samples from the LMP at different buses.

It is clear that the variations in the LMP at different buses are mainly due to the load and generation uncertainties, and these variations exist at all of the buses, but to different degrees. Investigating the problem load flow reveals that no congestion occurred in the network, and so it is apparent that increase in the LMP is due to a high load and low generation level, while its decrease is due to a low load and high generation level. The LMP variation at buses 4 and 5 is much more than that in the other buses. Although it is expected that the LMP at bus 5, in which the wind farm is located, will have the greatest SD values, the results of Table 2 and also Figure 2 show another thing, i.e. the LMP at bus 4 has the greatest SD and mean values. In order to find the reason for this observation, one may go to each of the evaluations and see what happens in depth. Below, we address the first sample point, i.e. the $\mathbf{x}^0 = \mathbf{m}$ evaluation, and discuss the solution.

In the first OPF evaluation, the sampled sample points and their transformations are given in Table 4. It must be recognized that the generation at bus 5 is due to the wind farm existence in that bus. In the first evaluation, the values of the uncertain variables are kept at their means. Here, the uncertain variables are the loads at buses 4, 5, and 6 and the wind farm power generation at bus 5. It is assumed that the output power of the wind farm is modeled with a negative load, and so the net of the load at bus 5 equals 63.9 MW, which is the lowest load among the PQ buses. Since no congestion occurred at this network, the LMP at the PQ buses is proportional to the loss term. Hence, the LMP at bus 6 has the smallest value among the PQ buses. The loads at buses 4 and 6 are equal, but the LMP at bus 4 has a greater value because the lines that connect bus 6 to the generating units have lower impedances than the lines connecting bus 4 to the generating units. The difference of the LMP at the PQ and generation buses is due to the loss term; therefore, the LMP at bus 4 is greater than that at bus 6.

Table 4. First combinations for the selected and transformed sample points.

Bus	Load (MW)	Generation (MW)	LMP ($/MWh)
Bus1	0	69.05	12.405
Bus2	0	71.27	11.6
Bus3	0	69.88	11.86
Bus4	70	0	15.021
Bus5	70	6.1	12.78
Bus6	70	0	12.185

Table 5. Obtained results for Case 1.

Parameter	Mean	SD
Total cost ($)	3089.3	100.6
Total loss (MW)	6.753	1.133
LMP-bus1 ($/MWh)	12.5	0.2485
LMP-bus2 ($/MWh)	11.51	0.223
LMP-bus3 ($/MWh)	11.84	0.0656
LMP-bus4 ($/MWh)	16.58	3.99
LMP-bus5 ($/MWh)	13.02	0.6048
LMP-bus6 ($/MWh)	12.186	0.0116
Generation-bus1 (MW)	78.348	23.13
Generation-bus2 (MW)	66.253	12.56
Generation-bus3 (MW)	68.07	4.427

Figure 3 shows the obtained samples of each unit's production for Case 0. The generating unit at bus 1 has much more variation compared to the others, since it is the marginal generating unit. In order to illustrate the correctness of the method as well as to fulfill the sensitivity analysis, 2 subsidiary cases are investigated in the following text.

4.1.2. Case 1: The correlated input variables

As previously stated, the proposed method is able to take into account the correlation between the uncertain input variables. This correlation can impose a more rigorous operating condition and, consequently, more severe variations in the LMPs are expected. The correlation quantity between variables may have positive or negative values. A positive correlation means that any increment or decrement in one variable causes the other to increase or decrease, respectively. The magnitude of these effects is proportional to the correlation degree. A negative correlation exists whenever an opposite direction appears in the varieties of 2 uncertain parameters. Consider the case in which the load at buses 4 and 5 are correlated by about +30% and the load at bus 4 is correlated with the wind farm generation by about +25%, or mathematically expressed as:

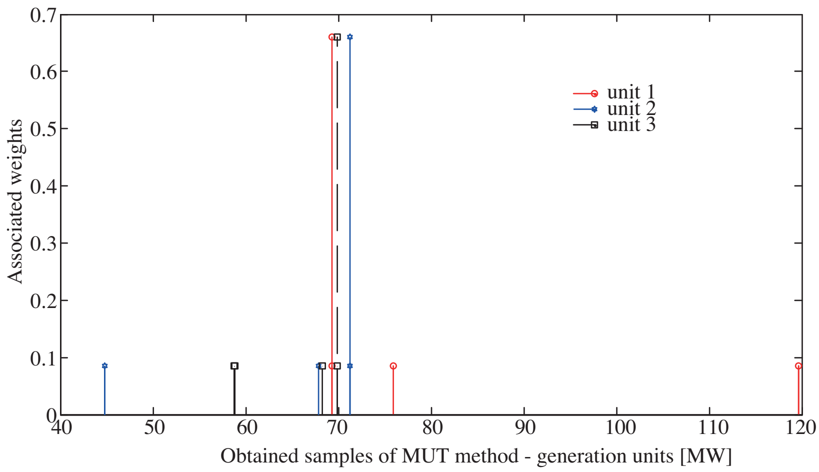

Figure 3. The obtained samples of each generating unit's production.

$$\mathbf{P}_{load4,load5} = \mathbf{P}_{load5,load4} = 1, \tag{21}$$

$$\mathbf{P}_{load4,wind-power} = \mathbf{P}_{wind-power,load4} = 1. \tag{22}$$

The solution of the P-OPF study by the proposed method is given in Table 5.

A comparison of the results with those of Table 2 shows that in the correlated area, the SD of all of the parameters increases. The SD of the total active losses, as well as the SD of the total production cost, is increased. Due to the positive correlation between the wind generation and the load, the mean values of the total cost and the total losses are decreased.

4.1.3. Case 2: The wind farm relocation

Among the most effective parameters on the LMPs are the network configuration and the load variations. As more as a certain bus is connected to the other buses through more low impedance lines, the LMP and its variation are less. Reasonably, in that bus, there are several options to import the required energy from the cheapest areas with less loss and, consequently, less cost. To comprehend this matter, consider the situation in which the wind farm is located at bus 4. The P-OPF solution obtained by the proposed method is given in Table 6.

Table 6. Obtained results for Case 2.

Parameter	Mean	SD
Total cost ($)	3084	87.7
Total loss (MW)	6.5857	0.878
LMP-bus1 ($/MWh)	12.09	0.4464
LMP-bus2 ($/MWh)	11.746	0.2812
LMP-bus3 ($/MWh)	11.9	0.0782
LMP-bus4 ($/MWh)	13.85	3.832
LMP-bus5 ($/MWh)	12.65	0.541
LMP-bus6 ($/MWh)	12.21	0.0315
Generation-bus1 (MW)	61	26.3034
Generation-bus2 (MW)	79.5	15.817
Generation-bus3 (MW)	71.93	5.27

A comparison with the results of Case 0 indicates that the relocation of the wind farm decreases the total cost as well as the mean value of the total losses. The LMP mean value at bus 4 declines because, with the replacement, the net load at bus 4 is decreased. The LMP uncertainty at bus 4 rises because of the wind farm uncertainty. Because a big portion of the load at bus 4 is provided by the generating unit at bus 1, the LMP uncertainty at bus 1 has been enlarged by about 100% and its mean value has been decreased. The mean value of the generation for unit 1 reduces because a portion of load at bus 4 is now satisfied by the wind farm located at this bus; therefore, the generating unit at bus 1 may change its generation due to the wind generation uncertainty, and this matter causes the generation uncertainty at this unit to intensify.

4.2. The IEEE 118-bus test system

This system has 118 buses and 54 generating units, and its technical data are taken from [19]. The base power and base voltage are equal to 100 MVA and 138 kV, respectively.

4.2.1. Case 0: The base case

The base case is assumed to have 4 wind farms, whose information is given in Table 7.

Table 8 summarizes the results obtained by the MUT method, where the LMPs with SD values of greater than 0.15 [$/MWh] and the production of units with a SD of greater than 0.5 [MW] are given.

Table 7. Wind farm information for Case 0.

Parameter farm1	Wind farm2	Wind farm3	Wind farm4	Wind
Bus no.	35	45	82	118
No. of WTGs	12	14	20	10
μ (m/s)	6.13	6.13	7.18	7.18
σ (m/s)	3.146	3.146	2.64	2.64

Referring to the network topology, it is clear that the existence of wind farms in certain areas brings additional uncertainty in those areas and their neighborhood. Figure 4 graphically illustrates this matter. It can be seen that the SD of the LMP in the buses that have wind farms and in the buses close to them is higher than the others. As an example, consider bus 45, in which a wind farm is located. In the single-line diagram, bus 45 is connected to buses 44 and 46. As shown in Table 8 and Figure 4, these 3 buses have uncertain LMPs that are higher than the threshold.

Interestingly, according to Table 8, the generating unit at bus 1 has the largest SD since this unit is the most expensive unit with LMP mean value equal to 40.5267 [$/MWh]. Hence, the unit at bus 1 is mostly the marginal unit and compensates for the system power mismatch induced by the wind farm uncertainty. Unit 36, which has the second largest SD value, has LMP with a mean value equal to 40.1753 [4/MWh], and thus it is the next marginal generating unit after the generating unit at bus 1.

4.2.2. Case 1: The correlated wind farms

Assume that the wind farms in Table 7 have been located at close buses 81, 82, 95, and 96; therefore, the generations of wind farms are correlated with each other. For the sake of simplicity, the powers generated by all of these wind farms are correlated to each other by about +50%. The P-OPF by the MUT method gives the results outlined in Table 9. Note that in this table, the data for the buses with LMP uncertainties greater than 0.2 [$/MWh] and buses with SDs of generation greater than 2 [MW] are shown.

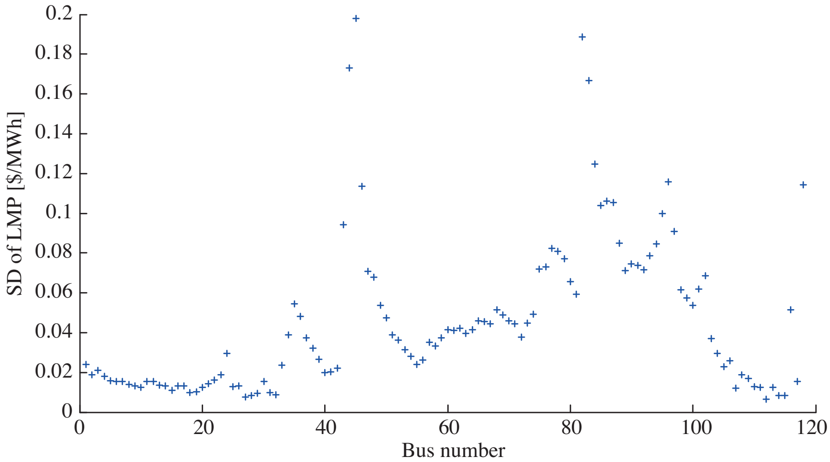

Figure 4. The SD of the LMP at all of the buses for Case 0.

Table 8. Obtained results using the MUT method for Case 0.

Parameter	Mean	SD
Total cost ($)	1.2873e5	1.1690e3
Total loss (MW)	77.1963	0.3242
LMP-bus44 ($/MWh)	40.9932	0.1731
LMP-bus45($/MWh)	40.7992	0.1979
LMP-bus82 ($/MWh)	38.9852	0.1885
LMP-bus83 ($/MWh)	38.7756	0.1666
Generation-bus1 (MW)	26.8521	1.6320
Generation-bus36 (MW)	10.0747	1.2744
Generation-bus80 (MW)	430.4794	0.7986
Generation-bus89 (MW)	501.3502	1.0984
Generation-bus105 (MW)	4.9053	0.5706

Table 9. Obtained results using the MUT method for Case 1.

Parameter	Mean	SD
Total cost ($)	1.2865e5	0.95e3
Total loss (MW)	77.7380	0.1037
LMP-bus82 ($/MWh)	38.9339	0.2326
LMP-bus83($/MWh)	38.7247	0.2212
LMP-bus95 ($/MWh)	38.7898	0.2548
LMP-bus96 ($/MWh)	38.6979	0.2443
Generation-bus80 (MW)	428.6	2.208
Generation-bus89 (MW)	498.78	3.67
Generation-bus105 (MW)	3.2582	2.2165

Comparing Tables 8 and 9, one can deduce that the uncertainty of the solutions associated with wind farm-integrated buses intensifies due to the correlated uncertainty effect. Figure 5a illustrates the variation of the LMP at all of the buses and Figure 5b portrays the variation of the power generation at the generation buses.

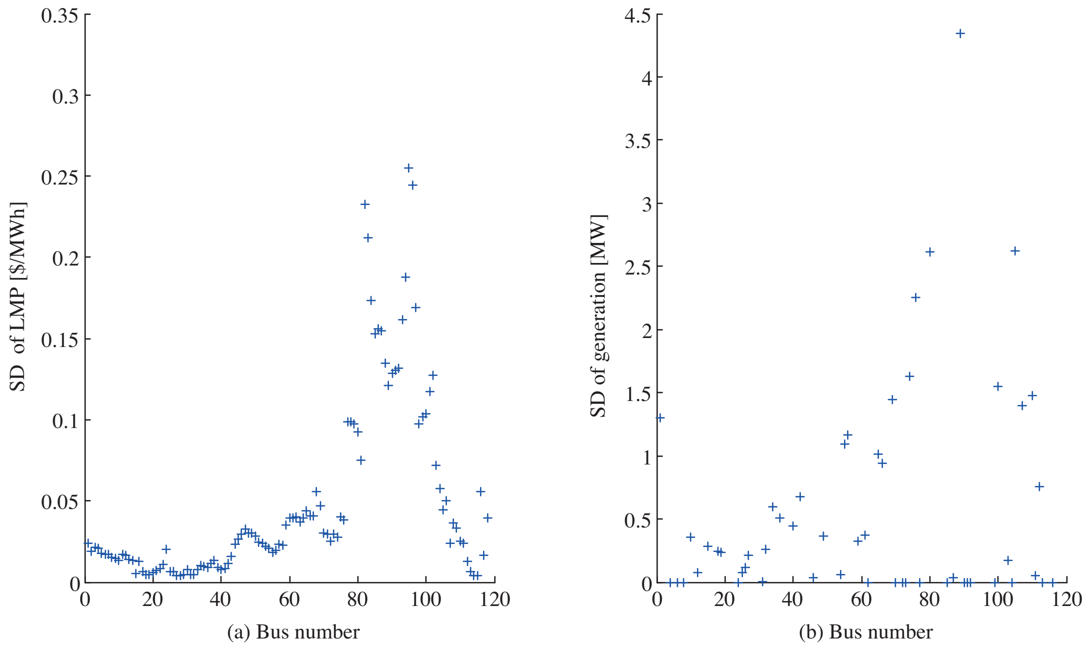

Figure 5. a) SD of the LMP at different buses for Case 1 and b) SD of the generation at different buses for Case 1.

A comparison of Figure 5a with Figure 4 demonstrates that the integration of a set of correlated uncertain distributed energy resources in a given area would dramatically influence the respected parameters, since a stronger correlation exists and the compensating effect is weaker.

5. Comparison with other techniques

To illustrate the effectiveness of the MUT method, the results obtained by the MUT method are compared with those of the MCS and 2PEM techniques, with regard to both accuracy and execution time.

5.1. MCS method

The MCS is a technique that simulates a large number of random situations and calculates the final representative results based on the overall outcomes. This method is often used when the model is complex, is nonlinear, or has many uncertain input variables. A simulation can typically involve over 3000 evaluations of the model, a task that in the past was only practical using supercomputers. Although the MCS method is able to provide accurate results, the computation is in fact relatively time-consuming; therefore, it is not suitable to handle real-time applications.

5.2. 2PEM method

The 2PEM is used to decompose Eq. (1) into several subproblems by taking only 2 deterministic values of each uncertain variable, placed on both sides of its mean value. The deterministic OPF is then run twice for each uncertain variable, once for the value below the mean and once for the value above the mean, while keeping the other variables at their mean values. These 2 points may be symmetric around the mean of a given variable or may be unsymmetrical. Next, each set of the selected sample points undergoes the nonlinear function to obtain the transformed sample points. It must be emphasized that in this method, the number of selected sample points increases as the number of uncertain variables increases.

5.3. Comparison of the results

In order to compare the results, the mean and the SD of output variables obtained by the 2PEM and MUT methods are compared with those corresponding to the MCS, which are considered to be the most accurate. The errors for the mean value and the SD are, respectively, defined as:

$$\varepsilon_\mu = \frac{100\,|(\mu_{MCS} - \mu)|}{\mu_{MCS}} \; [\%], \tag{23}$$

$$\varepsilon_\sigma = \frac{100\,|(\sigma_{MCS} - \sigma)|}{\sigma_{MCS}} \; [\%]. \tag{24}$$

Tables 10, 11, and 12 summarize the comparison of the results obtained by different methods from the viewpoint of the run time and accuracy for the 2 systems, namely the Wood and Wollenberg 6-bus and the 118-bus systems.

Table 10. Run time comparison of both systems.

Method	Run time (s), 6-bus	Run time (s), 118-bus
MCS	1914	16.08e5
2PEM	2.92	2.57e5
MUT	0.5529	1381

Table 11. Accuracy comparison for the 6-bus system.

Method	2PEM	MUT
Cost, ε_μ%	0.11	0.03
Cost, ε_σ%	2.64	13.4
Losses, ε_μ%	0.33	4.67
Losses, ε_σ%	6	17.3

Table 12. Accuracy comparison for the 118-bus system.

Method	2PEM	MUT
Cost, ε_μ%	0.5	0.41
Cost, ε_σ%	12.2	14.58
LMP@8, ε_μ%	1.78	0.017
LMP@ 8, ε_σ%	7	63.18
LMP@35, ε_μ%	0.005	0.05
LMP@ 35, ε_σ%	21.54	14.62
LMP@45, ε_μ%	0.1	0.2
LMP@ 45, ε_σ%	23.25	24.8
LMP@82, ε_μ%	0.05	0.2
LMP@ 82, ε_σ%	21.33	1.2
LMP@118, ε_μ%	0.02	0.1
LMP@ 118, ε_σ%	18.71	16.4

It must be stressed that the computation burden of the general UT method is almost the same as that of the 2PEM. Table 10 shows that the MUT method has significantly reduced the execution time of the problem. It is well recognized that the reduction in the execution time becomes much more sensible as the system becomes larger.

Numerically, from Table 11, it can be seen that the 2PEM is about 649 times faster than the MCS, with a maximum 0.33% error in the mean value. These values for the MUT method are about 3480 times and 4.67%, respectively.

From Tables 10 and 12, it is evident that for the 118-bus system, the 2PEM is about 6.2 times faster than the MCS, with a maximum 1.78% error in the mean value, while these values for the MUT method are about 1230 times and 0.41%, respectively. The simulation results indicate that in the case of the mean values, the MUT results are as accurate as those for the 2PEM, but in case of the SD, the results of the 2PEM have a higher degree of accuracy than the MUT results. It is clear that the computational burden of the 2PEM is directly proportional to the number of uncertain variables, since this method needs exactly 2 runs of the deterministic OPF for each uncertain variable, and this matter decreases the merit of this method in large systems that have many uncertain variables. The computation burden of the MUT method is independent of the number of uncertain variables.

6. Conclusion

In this paper, a fast and accurate method for evaluating the P-OPF problem was proposed. This method selects the appropriate sample points of the uncertain input variables to perform the P-OPF study with a high degree of accuracy. The salient feature of the proposed approach is its extremely short execution time, which makes it applicable and efficient for large-scale practical systems. The proposed method has been successfully tested on 2 standard case studies. The accuracy and computational efficiency of the new method were examined by the numerical results obtained. The existence of uncertain energy resources like wind turbines as well as the system load uncertainty can cause dramatic changes in the LMP at different buses; the severity of these changes depends on the quantity of the load and the generation in each time instant. The correlation between the uncertain input variables can influence the system's parameters, such as LMPs, and this matter can affect the system more and more when the integration of a significant amount of correlated uncertain variables are concentrated in a specific zone of the system. The integration of REs into the network can decrease the mean value of the total costs, but the SD of this parameter will be increased; therefore, the penetration rate of uncertain power generations like REs must be limited in each area.

Appendix

The information about the used wind turbine was taken from the Nordex Company [21].

Parameter	Value
P_r (MW)	2.5
V_r (m/s)	12.5
V_i (m/s)	3
V_o (m/s)	25

Nomenclature

i	Index of the samples, running from 1 to n
k_u	A parameter of the UT method
k	Number of uncertain variables
n	Dimension of the uncertain input variables
$\mathbf{P_D}$	Vector of the load active power (MW)
$\mathbf{P_G}$	Vector of the generator active power (MW)
$\mathbf{P}_{wind}$	Vector of the wind farm generation (MW)
P_{WTG}	Wind turbine generator output power (MW)
P_r	Wind turbine generator rated power (MW)
$\mathbf{P_{XX}}$	Covariance matrix of the input variables
$\mathbf{P_{YY}}$	Covariance matrix of the output variables
$\mathbf{Q_D}$	Vector of the load reactive power (MVAr)
$\mathbf{Q_G}$	Vector of the generator reactive power (MVAr)
$\mathbf{Q}_{wind}$	Vector of the wind farm reactive power consumption (MVAr)

v	Wind speed (m/s)
$\mathbf{V}$	Vector of the bus voltage magnitude (pu)
V_i	Cut-in speed (m/s)
V_o	Cut-out speed (m/s)
V_r	Wind turbine rated speed (m/s)
W^i	Weight associated with the ith sample point
$\mathbf{x^i}$	The ith sample point
$\mathbf{X}$	Vector of the uncertain input variables
$\bar{\mathbf{X}}$	Vector of the mean value for the input variables
$\mathbf{y^i}$	Vector of the transformed sample point of x^i
$\mathbf{Y}$	Vector of the uncertain output variables
$\bar{\mathbf{Y}}$	Vector of the mean value for the output variables
α_u	Scale factor of the UT method
β	Correction factor for the covariance of the UT method
μ	Mean value of the normal distribution
μ_{MCS}	Mean value of the output variable obtained by the Monte Carlo simulation method
σ	Standard deviation of the normal distribution
σ_{MCS}	Standard deviation of the output variable obtained by the Monte Carlo simulation method
δ	Vector of the bus voltage angle (rad)

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