

Direct adaptive fuzzy sliding mode decoupling control for a class of underactuated mechanical systems

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Abstract: Motivated by the dynamic characteristics of underactuated mechanical systems with 2 degrees of freedom, a decoupling adaptive fuzzy sliding mode decoupling controller (DAFSMDC) is presented in this paper. By exploiting the universal approximation property of fuzzy logic systems and the sliding mode control method, this paper proposes a new decoupling strategy of the system into 2 second-order subsystems and introduces an adaptive control algorithm that guarantees the convergence of both subsystems. Since fuzzy systems are used to approximate an unknown ideal controller, the adjustable parameters of the used fuzzy systems are updated using a gradient descent algorithm that is designed to minimize the error between the unknown ideal controller and the fuzzy controller. Based on Lyapunov stability theory, proofs and conditions are then given to ensure the stability of the closed-loop system. Two examples are provided to illustrate the effectiveness and potential of the DAFSMDC technique for the stabilization of underactuated mechanical systems.

Key words: Underactuated mechanical system, sliding mode control, fuzzy systems, adaptive control, overhead crane, beam and ball

1. Introduction

During recent decades, an essential effort with several works has been devoted to underactuated mechanical systems (UMSs) [1–3]. These systems are characterized by a number of actuators less than the number of degrees of freedom (DOF) to be controlled. Furthermore, compared to fully actuated mechanical systems, UMSs present many advantages, including lower numbers of actuators, lightening of the system, and reduction of the cost [3]. Lately, there has been extensive and remarkable research effort into the control of UMSs and several classifications and papers including modeling, stability, and controllability issues have been discussed, focusing on linear control, optimal control, adaptive control, and nonlinear control theories. Moreover, because of the certain need to deal with the presence of uncertainties in real-life control systems, a robust control theory has been introduced for UMS control.

Sliding mode control (SMC) is an important robust control approach that was developed using a systematic scheme based on a sliding surface and Lyapunov's stability theory [4–6]. Recently, it has been widely applied to control of nonlinear systems [7–9]. Mainly, the effectiveness of maintaining the stability and consistent performances inherent to SMC has influenced many researchers to adopt such methods [10–12]. The main

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feature of SMC is that the system's response remains insensitive, to a certain extent, to modeling inaccuracies and disturbances [7,8].

During the last 2 decades, there has been significant progress in the area of adaptive control design of nonlinear systems [7,13,14]. In a general case, it is difficult to perfectly model a nonlinear system by known nonlinear functions; therefore, the problem of controlling nonlinear systems with incomplete model knowledge remains a challenging task. As a model-free design method, fuzzy control has found extensive applications for complex and ill-defined plants [14–18]. One major feature of fuzzy logic is its ability to express the amount of ambiguity in human thinking [16]. This ability is driven accordingly by fuzzy membership functions and fuzzy rules. However, it is sometimes difficult to find the matched membership functions and fuzzy rules for some plants, or the need may arise to tune the controller parameters if the plant dynamics change. In the hope of overcoming this problem, based on the universal approximation theorem and online learning ability of fuzzy systems, several stable adaptive fuzzy control schemes have been developed to incorporate the expert knowledge systematically [14,16,17]. The stability analysis in such schemes is performed using the Lyapunov approach. Conceptually, there are 2 distinct approaches that have been formulated in the design of a fuzzy adaptive control system: direct and indirect schemes. The direct scheme uses fuzzy systems to approximate unknown ideal controllers, while the indirect scheme uses fuzzy systems to estimate the plant dynamics and then synthesizes a control law based on these estimates [16,17,19].

Nowadays, there has been a lot of research on the design of fuzzy logic controllers based on the SMC scheme, referred to as fuzzy SMC (FSMC) [20–24]. FSMC, which is an integration of fuzzy logic systems (FLSs) and SMC, provides a simple way to design fuzzy logic control systematically. The main advantage of FSMC is that the control system can achieve asymptotic stability. Recently, a fuzzy sliding mode (FSM) decoupling control design method was proposed to achieve the decoupling performance of a class of nonlinear coupled systems [6]. However, UMSs represent a challenging class of coupled nonlinear systems and the problem of adaptive fuzzy control of UMSs presents more difficulties because of the coupling that exists between the control input and the outputs. In the literature, many researchers have incorporated a decoupled approach into the fuzzy adaptive control design for those systems by considering plant uncertainties, which were studied in [25–28]. In the papers mentioned above, the adjustable parameters of the fuzzy systems are updated by an adaptive law based on a Lyapunov approach, i.e. the parameter adaptive laws are designed in such a way as to ensure the convergence of a Lyapunov function. However, for an effective adaptation, it is more judicious to directly base the parameter adaptation process on the identification error between the unknown function and its adaptive fuzzy approximation [29].

This paper presents a direct decoupling adaptive fuzzy control scheme for a class of UMSs with 2 DOF. The proposed adaptation idea is based on the results in [29]. In this direct approach, since fuzzy systems are used to approximate unknown ideal controllers, the adjustable parameters of the used fuzzy systems are updated using a gradient descent algorithm that is designed to minimize the error between the unknown ideal controller and the fuzzy controller. Basically, the control scheme is extracted from the decoupling of the UMSs into 2 second-order subsystems, and then 2 sliding surfaces are constructed through the state variables of the decoupling subsystems. Hence, this paper is organized as follows: the problem formulation and preliminaries on fuzzy systems are given in Section 2. The direct decoupled adaptive fuzzy controller and a proof of the stability results are presented in Section 3. Section 4 is devoted to the simulation results of the proposed strategy applied to the overhead crane and beam-and-ball systems. Finally, Section 5 concludes the paper.

2. Problem formulation and preliminaries

2.1. Problem formulation

This work focuses on the design of a fuzzy control algorithm for a class of underactuated systems with 2 DOF given by the following dynamic equations [1,2]:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = f_1(\underline{x}) + b_1(\underline{x})u(t) \\ \dot{x}_3(t) = x_4(t) \\ \dot{x}_4(t) = f_2(\underline{x}) + b_2(\underline{x})u(t) \end{cases}, \quad (1)$$

where $\underline{x} = (x_1(t), x_2(t), x_3(t), x_4(t))^T$ is the state variable vector, $u(t)$ is the control input, and $f_1(\underline{x})$, $f_2(\underline{x})$, $b_1(\underline{x})$, and $b_2(\underline{x})$ are the uncertain smooth nonlinear functions. In the remainder of this paper, the time variable is omitted for abbreviation reasons. However, without loss of generality, the following assumptions are considered.

Assumption 1 The state vector \underline{x} is available for measurement.

Assumption 2 Each control gain $b_i(\underline{x})$ is finite, nonzero, and of a known sign for each \underline{x} . It is assumed that the sign of $b_i(\underline{x})$ does not change, and without loss of generality, this sign can be taken as positive. In addition, the functions $b_i(\underline{x})$ are unknown and bounded, i.e. $0 < b_{\min} \leq b_i(\underline{x}) \leq b_{\max}$.

The system of Eq. (1) can be viewed as 2 subsystems with a second-order canonical form including the states (x_1, x_2) and (x_3, x_4) , for which we define the following pair of sliding surfaces:

$$S_1 = \dot{x}_1 + \lambda_1 \tilde{x}_1 = x_2 + \lambda_1 \tilde{x}_1, \quad (2)$$

$$S_2 = \dot{x}_3 + \lambda_2 \tilde{x}_3 = x_4 + \lambda_2 \tilde{x}_3, \quad (3)$$

where $\tilde{x}_1 = x_1 - x_{1d}$, $\tilde{x}_3 = x_3 - x_{3d}$ (x_{1d} and x_{3d} are constant desired values), and λ_1 and λ_2 are positive constants. Next, from Eqs. (2) and (3), it follows that:

$$\dot{S}_1 = f_1 + b_1 u + \lambda_1 x_2, \quad (4)$$

$$\dot{S}_2 = f_2 + b_2 u + \lambda_2 x_4. \quad (5)$$

If $f_i(\underline{x})$ and $b_i(\underline{x})$ are known, using the equivalent control law of each subsystem, we can obtain:

$$u_{eq1} = -\frac{f_1 + \lambda_1 x_2}{b_1}, \quad (6)$$

$$u_{eq2} = -\frac{f_2 + \lambda_2 x_4}{b_2}. \quad (7)$$

The control objective is to design an adaptive fuzzy controller so that the overall system is stabilized and the outputs, x_1 and x_3 , are forced to follow the desired values, x_{1d} and x_{3d} . Since the control laws in Eqs. (6) or (7) cannot ensure the control objective because they are designed to only stabilize the corresponding subsystem, as an obvious concept, the total control law should include some parts of the control law of each subsystem or a sliding surface should be defined as a combination of the sliding surfaces of the 2 subsystems.

2.2. Fuzzy logic systems

In this subsection, the FLS is briefly described. The basic configuration of the FLS [15] includes a fuzzy base that consists of a collection of fuzzy IF-THEN rules, which can be written as:

$$R_l : \text{if } x_1 \text{ is } A_1^l \text{ and } \dots \text{ } x_n \text{ is } A_n^l \text{ then } y \text{ is } B^l. \tag{8}$$

The FLS performs a mapping from $U = U_1 \times \dots \times U_n \subseteq R^n$ to R , where the input vector is $\underline{x} = [x_1, \dots, x_n]^T \in R^n$ and the output variable is $y \in R$. A_i^l and B^l are labels of the input and output fuzzy sets, respectively. Let $i = 1, 2, \dots, n$ denote the number of inputs for the FLS, and let $l = 1, 2, \dots, m$ denote the number of the fuzzy IF-THEN rules.

The output of the fuzzy system with a fuzzy rule base as in Eq. (8), product inference engine, singleton fuzzifier, and center average defuzzifier can be expressed as follows [16,17]:

$$y(\underline{x}) = \frac{\sum_{l=1}^m \left(\prod_i^n \mu_{A_i^l}(x_i) \right) y^l}{\sum_{l=1}^m \prod_i^n \mu_{A_i^l}(x_i)}, \tag{9}$$

where $\mu_{A_i^l}(x_i)$ is the membership function of the linguistic variable x_i , and y^l represents a crisp value for which the membership function $\mu_{B^l}(x_i)$ reaches its maximum value (usually we assume $\mu_{B^l}(y^l) = 1$).

By introducing the concept of fuzzy basis functions (FBFs) [16,17], the fuzzy output in Eq. (9) can be rewritten in the following compact form:

$$y(\underline{x}, \theta) = \underline{\xi}^T(\underline{x}) \theta, \tag{10}$$

where $\theta = [y^1, y^2, \dots, y^m]^T$ is the parameter vector and $\underline{\xi} = [\xi_1, \xi_2, \dots, \xi_m]^T$ is a set of FBFs defined as:

$$\xi^l(\underline{x}) = \frac{\prod_i^n \mu_{A_i^l}(x_i)}{\sum_{l=1}^m \prod_i^n \mu_{A_i^l}(x_i)}. \tag{11}$$

The fuzzy system in Eq. (10) is assumed to be well defined, such that $\sum_{l=1}^m \prod_i^n \mu_{A_i^l}(x_i) \neq 0$ for each x . In this paper, it is assumed that the structure of the fuzzy system and the FBF parameters are properly specified in advance by the designer. This means that the designer’s decision is needed to determine the structure of the fuzzy system (that is, to determine the relevant inputs, number of membership functions for each input, membership function parameters, and number of rules), and the consequent parameters should be calculated by the appropriate learning algorithms. The Gaussian-type membership function is used for $A_i^l(\cdot)$ in this paper and it is given as [14]:

$$\mu_{A_i^l}(x_j) = e^{-0.5(\omega_s x_j - \omega_c)^2 \omega_d^2}, \tag{12}$$

where the parameter ω_c represents the center value and the parameter ω_d denotes the reciprocal value of the deviation from the center. The input x_j is scaled by the parameter ω_s .

3. Controller design

3.1. Decoupling SMC

If the plant dynamics are known, i.e. $f_i(\underline{x})$ and $b_i(\underline{x})$ are completely known, the overall ideal SMC input is given by:

$$u^* = u_{eq1} - b_1^{-1} (K \operatorname{sgn}(S_1) + QS_1 + \beta(S_1 - S_2)), \tag{13}$$

where K , Q , and β are strictly positive design parameters and $\operatorname{sgn}(\cdot)$ is the standard sign function defined for the surface S_1 .

Effectively, when we select the control input as $u = u^*$, Eq. (5) simplifies to:

$$\dot{S}_1 = -K \operatorname{sgn}(S_1) - QS_1 - \beta(S_1 - S_2). \tag{14}$$

To guarantee that an ideal sliding motion takes place from any initial conditions after the sliding surface is reached, the following inequalities must be satisfied:

$$K > \max(|\beta(S_1 - S_2)|), \tag{15}$$

and, consequently, we have $S_1 \rightarrow 0$ as $t \rightarrow \infty$.

According to Fillipov, the system's motion on the sliding surface can be given in an interesting geometric interpretation, as an 'average' of the system's dynamics on both sides of the surface [7]. The dynamics while in sliding mode can then be written as $\dot{S}_1 \approx 0$. Furthermore, when $S_1 \rightarrow 0$, we have $\operatorname{sgn}(S_1) \approx S_1$, and then Eq. (14) becomes $\dot{S}_1 \approx -(K + Q + \beta)S_1 + \beta S_2$, or in the Laplace p-domain we can write:

$$S_1(p) = \frac{\beta}{(K + Q + \beta) + p} S_2(p). \tag{16}$$

Lemma 1 For any 2 continuous and derivable functions ϕ and g , if the following relation holds:

$$\dot{\phi} = -\gamma(\phi - g), \text{ with } \gamma \gg 1, \tag{17}$$

then:

$$\lim_{t \rightarrow \infty} \phi = g. \tag{18}$$

Proof In the Laplace p-domain, Eq. (17) can be written as $\phi(p) = \frac{\gamma}{p+\gamma}g(p) = \frac{1}{1+(1/\gamma)p}g(p)$. Then, if $\gamma \gg 1$, we have $1/\gamma \rightarrow 0$ and $\phi(p) \approx g(p)$. □

Therefore, in the time domain one has $\phi(t) \approx g(t)$. This means that for $\gamma \gg 1$ and a function $g(t)$ continuous, derivable, and band-limited, we have $\lim_{t \rightarrow \infty} \phi = g$.

Since in Eq. (16) the transfer function $\beta/(p + (K + Q + \beta))$ is stable with $(K + Q + \beta) \gg 1$, then using lemma 1, the fact that $S_1 \rightarrow 0$ implies that $S_2 \rightarrow 0$.

However, due to the fact that $f_1(\underline{x})$ and $b_1(\underline{x})$ are unknown, the ideal control law in Eq. (13) is usually difficult to obtain. To overcome this problem, we propose using adaptive fuzzy systems to construct this ideal controller. On the other hand, the error between the fuzzy controller and the ideal controller will be used to update the free parameters of the fuzzy controller.

Next, in the following, we will show how to develop a direct adaptive fuzzy SMC to approximate the unknown part of the total control through a rule adaptation and then construct the correct control to guarantee the system's stability. The proposed FSMC has an online self-tuning fuzzy rule without the trial-and-error process to find the appropriate consequent parameters of the fuzzy rules. In addition, to improve the convergence performance of the proposed control law, an adaptive tuning method of the coupling factor β is proposed. The update law of β will be based on the descent gradient method.

3.2. Direct adaptive fuzzy sliding mode decoupling control

To develop the control law, we assume that the unknown ideal controller in Eq. (13) can be approximated using a fuzzy system in the form of Eq. (11) as the following:

$$u^* = \xi^T(\underline{S})\theta^*, \tag{19}$$

where $\underline{S} = [S_1, S_2]^T$, $\xi(\underline{S})$ is a FBF vector assumed to be suitably specified by the designer.

Because the optimal vector θ^* is unknown, let us consider its estimate θ instead to construct the adaptive control:

$$u = \xi^T(\underline{S})\theta. \tag{20}$$

Consider now the approximation error between both controllers u^* and u as:

$$e_u = u - u^*. \tag{21}$$

Clearly, the error e_u represents the actual deviation between the unknown function u^* and the online fuzzy approximator in Eq. (20). Next, using Eqs. (19) and (20), Eq. (21) becomes:

$$e_u = \xi^T(\underline{S})\theta - u^* = \xi^T(\underline{S})\tilde{\theta}, \tag{22}$$

where $\tilde{\theta} = \theta - \theta^*$ is the parameter estimation error vector. The optimal parameter vector θ^* is defined as:

$$\theta^* = \arg_{\theta} \min \left[\sup_{S \in \Omega_s} (u - u^*) \right]. \tag{23}$$

Indeed, we assume that the used fuzzy system satisfies the universal approximation property on the compact set Ω_S , which is assumed to be large enough that the variable \underline{S} remains inside it under closed-loop control.

The following theorem summarizes the main results of this paper.

Theorem 1 Consider a class of underactuated systems given by Eq. (1) and design the sliding surfaces as in Eqs. (2) and (3). Suppose that Assumptions 1 and 2 hold and consider the control law given by Eq. (19) with the following parameter update law:

$$\dot{\theta} = -\eta_u \xi(\underline{S}) \left(\dot{S}_1 + K \operatorname{sgn}(S_1) + QS_1 - w \right), \tag{24}$$

where

$$w = -\beta(S_1 - S_2), \tag{25}$$

and K, Q, β , and η_u are the positive design parameters. Next, all of the signals in the closed-loop system will be bounded and the sliding surfaces given by Eqs. (3) and (4) will converge asymptotically to 0.

Proof Step 1. Stability analysis of S_1 .

Substituting Eq. (13) into the right-hand side of Eq. (4) and recalling that $b_1 u = b_1 u^* + b_1 (u - u^*)$, then one has:

$$\begin{aligned}\dot{S}_1 &= f_1 + \lambda_1 x_2 + b_1 u^* + b_1 e_u \\ &= b_1 [-K b_1^{-1} \text{sgn}(S_1) - Q b_1^{-1} S_1 - b_1^{-1} \beta (S_1 - S_2) + u_{eq1}] - b_1 u_{eq1} + b_1 e_u . \\ &= b_1 e_u - K \text{sgn}(S_1) - Q S_1 - \beta (S_1 - S_2)\end{aligned}\quad (26)$$

Finally we have:

$$\dot{S}_1 = b_1 e_u - K \text{sgn}(S_1) - Q S_1 + w. \quad (27)$$

Now, consider a quadratic cost function that measures the discrepancy between the ideal controller and the actual fuzzy controller, defined as:

$$J(\theta) = \frac{1}{2} b_1 e_u^2 = \frac{1}{2} b_1 (\xi^T(\underline{S}) \theta - u^*)^2. \quad (28)$$

We use the gradient descent method to minimize the cost function in Eq. (28) with respect to the adjustable parameters θ . Accordingly, by applying the gradient method [7,29], the minimizing trajectory $\theta(t)$ is generated by the following differential equation:

$$\dot{\theta} = -\eta_u \nabla_{\theta} J(\theta). \quad (29)$$

Clearly, from Eq. (28), the gradient of $J(\theta)$ with respect to θ is:

$$\nabla_{\theta} J(\theta) = \frac{\partial J(\theta)}{\partial \theta} = \xi(\underline{S}) b_1 e_u. \quad (30)$$

Therefore, the gradient descent algorithm can be written as:

$$\dot{\theta} = -\eta_u \xi(\underline{S}) b_1 e_u. \quad (31)$$

However, because of the unavailability of b_1 and u^* , the adaptive law in Eq. (31) cannot be performed. Next, Eq. (27) will be used to overcome this design inconvenience.

Clearly, from Eq. (27), the term $b_1 e_u$ is available and it is given as:

$$b_1 e_u = \dot{S}_1 + K \text{sgn}(S_1) + Q S_1 - w. \quad (32)$$

Eq. (31) then becomes:

$$\dot{\theta} = -\eta_u \xi(\underline{S}) \left(\dot{S}_1 + K \text{sgn}(S_1) + Q S_1 - w \right). \quad (33)$$

Let the Lyapunov function candidate be defined as:

$$V = \frac{1}{2} S_1^2 + \frac{1}{2\eta_u} \tilde{\theta}^T \tilde{\theta}. \quad (34)$$

Now, using Eq. (33), the time derivative of V along the dynamics in Eqs. (27) and (33) is given as:

$$\begin{aligned}\dot{V} &= S_1 \dot{S}_1 + \frac{1}{\eta_u} \tilde{\theta}^T \dot{\tilde{\theta}} \\ &= S_1 (b_1 e_u - K \text{sgn}(S_1) - Q S_1 + w) - \tilde{\theta}^T \xi(\underline{S}) b_1 e_u . \\ &= -S_1 (K \text{sgn}(S_1) - w) - Q S_1^2 + S_1 b_1 e_u - b_1 e_u^2\end{aligned}\quad (35)$$

Let us now use the following inequality:

$$S_1 b_1 e_u \leq \frac{1}{2} b_1 e_u^2 + \frac{1}{2} b_1 S_1^2. \quad (36)$$

Next, one can obtain the following from Eq. (35):

$$\begin{aligned} \dot{V} &\leq -S_1 (K \operatorname{sgn}(S_1) - w) - Q S_1^2 + \frac{1}{2} b_1 S_1^2 - \frac{1}{2} b_1 e_u^2 \\ &\leq -S_1 (K \operatorname{sgn}(S_1) - w) - Q S_1^2 + \frac{1}{2} b_{\max} S_1^2 - \frac{1}{2} b_1 e_u^2. \\ &\leq -S_1 (K \operatorname{sgn}(S_1) - w) - (Q - \frac{1}{2} b_{\max}) S_1^2 - \frac{1}{2} b_1 e_u^2 \end{aligned} \quad (37)$$

It is easy to find from Eq. (37) that for $Q \geq 1/2 b_{\max}$ and $K > \max(|w|) + \kappa, \kappa > 0$, we have:

$$\dot{V} \leq -\kappa |S_1|. \quad (38)$$

This guarantees the boundedness of S_1 and $\tilde{\theta}$. In addition, using Barbalat's Lemma, the sliding surface S_1 can be shown to be asymptotically stable, i.e. $S_1 \rightarrow 0$ as $t \rightarrow \infty$. \square

Step 2. Stability analysis of S_2 .

This step is devoted to the stability analysis of S_2 . From Eq. (27), we have:

$$\dot{S}_1 = b_1 e_u - K \operatorname{sgn}(S_1) - Q S_1 - \beta (S_1 - S_2). \quad (39)$$

On the other hand, on the average we have $\operatorname{sgn}(S_1) \approx S_1$, as $t \rightarrow \infty$. Next, Eq. (39) yields:

$$\dot{S}_1 = b_1 e_u - (K + Q + \beta) S_1 + \beta S_2. \quad (40)$$

By assuming that the adaptation process converges and e_u is very small, then Eq. (40) yields:

$$\dot{S}_1 = -(K + Q + \beta) S_1 + \beta S_2. \quad (41)$$

Consequently, using Lemma 1, the convergence of S_1 implies the convergence of S_2 to 0.

3.3. Direct adaptive fuzzy sliding mode decoupling control with adaptive β

In the previous subsection, we considered a constant design parameter $\beta > 0$. Since there is no systematic way for selecting this parameter, we propose in this subsection to consider the parameter β as a free parameter and design an adaptive law to improve the controller performance. Intuitively, such a procedure may speed up convergence and reduce the large transients that may occur when β is chosen to be constant.

Proposition 1 For a small positive constant σ , the gradient adaptive law

$$\begin{cases} \dot{\beta} = \eta_\beta K \operatorname{sgn}(S_1) (S_1 - S_2), \text{ if } : S_1^2 + S_2^2 \geq \sigma \\ 0, \text{ otherwise} \end{cases} \quad (42)$$

retains all of the asymptotic stability and convergence properties of the adaptive law in Eq. (24) established in case of a constant coupling parameter and gives the best performance.

Proof It follows from the previous proof that the use of the decoupled direct adaptive law in Eq. (19) with the parameter adaptation law in Eq. (24) yields:

$$\dot{S}_1 = b_1 e_u - K \operatorname{sgn}(S_1) - Q S_1 + w. \quad (43)$$

In addition, it was shown previously that under the condition $\beta > 0$, we ensure the asymptotic converge of all sliding surfaces. Nevertheless, it is adequate to design a limit of adaptation of β to speed up the attraction of the convergence rates of the whole system within a global set given as:

$$\Omega_\sigma = \{S_1, S_2 : S_1^2 + S_2^2 \leq \sigma\}. \quad (44)$$

For different values of β , we have different convergence behaviors, varying from slow to fast. Next, it is worthwhile to find the best value of β that gives the suitable convergence rate. As a result, we use the gradient descent method with respect to the parameter β by minimizing the following cost function:

$$J_\beta(\beta) = \frac{1}{2} \left(\dot{S}_1 + Q S_1 + b_1 e_u + \beta (S_1 - S_2) \right)^2. \quad (45)$$

Next, by applying the gradient method [15,16], the minimizing optimal coupling parameter β is generated by the following differential equation:

$$\dot{\beta} = -\eta_\beta \nabla_\beta J_\beta(\beta). \quad (46)$$

Clearly, from Eq. (45), the gradient of $J_\beta(\beta)$ with respect to β is:

$$\nabla_\beta J_\beta(\beta) = \frac{\partial J_\beta(\beta)}{\partial \beta} = (S_1 - S_2) \left(\dot{S}_1 + Q S_1 + b_1 e_u + \beta (S_1 - S_2) \right). \quad (47)$$

Thus, by substituting Eq. (47) into Eq. (46), the gradient descent algorithm is given by:

$$\dot{\beta} = -\eta_\beta (S_1 - S_2) \left(\dot{S}_1 + Q S_1 + b_1 e_u + \beta (S_1 - S_2) \right). \quad (48)$$

Finally, recalling Eq. (27) and substituting in Eq. (48) yields:

$$\dot{\beta} = \eta_\beta K \operatorname{sgn}(S_1) (S_1 - S_2). \quad (49)$$

It is more reasonable to update the parameter β only when the signals S_1 and S_2 are large and switch off the adaptation when these signals are small. Accordingly, the modified update law can be given as:

$$\begin{cases} \dot{\beta} = \eta_\beta K \operatorname{sgn}(S_1) (S_1 - S_2), & \text{if } : S_1^2 + S_2^2 \geq \sigma \\ 0, & \text{otherwise} \end{cases}. \quad (50)$$

□

Remark 1 It is worth noting that the parameter updating law in Eq. (24) is not implementable in the case that the derivative of S_1 is not available. However, a discrete implementable version of Eq. (24) can be obtained by rewriting Eq. (24) as:

$$\frac{\theta(t) - \theta(t - \Delta t)}{\Delta t} = -\eta_u \xi(\underline{S}) \left(\frac{S_1(t) - S_1(t - \Delta t)}{\Delta t} + K \operatorname{sgn}(S_1(t)) + Q S_1(t) + \beta (S_1(t) - S_2(t)) \right) \quad (51)$$

where Δt is a small positive constant.

By assuming that Δt is small enough, the discrete implementable version of Eq. (24) is given as follows.

$$\theta(t) = \theta(t - \Delta t) - \eta_u \xi(\underline{S}) ((1 + (Q + \beta) \Delta t) S_1(t) - S_1(t - \Delta t)) + K \operatorname{sgn}(S_1(t)) \Delta t - \beta S_2(t) \Delta t \quad (52)$$

Remark 2 In order to remedy the control discontinuity in the boundary layer, the sign function $\operatorname{sgn}(S_1)$ all through this paper is replaced by a saturation function of the following form:

$$\operatorname{sat}(x) = \begin{cases} \operatorname{sgn}(x), & \text{if } |x| \geq 1 \\ x, & \text{if } |x| < 1 \end{cases} \quad (53)$$

4. Simulation results

In this section, we test the proposed direct adaptive fuzzy sliding control scheme for the stabilization of 2 different UMSs: the overhead crane and beam-and-ball systems.

4.1. Overhead crane

In this subsection, we apply our proposed adaptive controller for an underactuated overhead crane system (Figure 1). The control objective of the overhead crane is to move the trolley to its destination and complement the antiswing of the load at the same time.

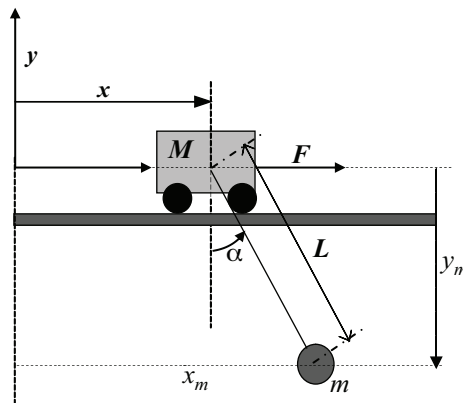


Figure 1. Overhead crane system.

For simplicity, in this paper, the following assumptions are made:

- a) The trolley and the load can be regarded as point masses.
- b) The friction force that may exist in the trolley and the elongation of the rope due to the tension can be neglected.
- c) The trolley moves along the rail and the load moves in the $x-y$ plane.

Using the Euler–Lagrange principle, we can obtain the following dynamic model for the overhead crane system [26]:

$$\begin{aligned} x : (m + M)\ddot{x} + mL(\ddot{\alpha} \cos \alpha - \dot{\alpha}^2 \sin \alpha) &= F \\ \alpha : \ddot{x} \cos \alpha + L\ddot{\alpha} + g \sin \alpha &= 0 \end{aligned} \quad (54)$$

where M and m are the masses of the trolley and the load respectively, $x_3 = r$ is the horizontal displacement, α is the sway angle of the load, g is the gravitation, and L is the length of the suspension rope. In summary, based on the system form in Eq. (2), we obtain f_1, f_2, b_1 , and b_2 as:

$$\begin{cases} f_1 = \frac{mL\dot{\alpha}^2 \sin \alpha + mg \sin \alpha \cos \alpha}{M + m \sin^2 \alpha} \\ b_1 = \frac{1}{M + m \sin^2 \alpha} \\ f_2 = -\frac{(M + m)g \sin \alpha + mL\dot{\alpha}^2 \sin \alpha \cos \alpha}{(M + m \sin^2 \alpha)L} \\ b_2 = -\frac{\cos \alpha}{(M + m \sin^2 \alpha)L} \end{cases} \quad (55)$$

To synthesize the decoupling adaptive FSM decoupling controller (DAFSMDC), a fuzzy system in the form of Eq.

$$(10) \text{ is used to generate the control signals and the fuzzy system has } Z = \left[S_1 + \int_0^\infty \beta (S_1 - S_2) dt, \dot{S}_1 + \beta (S_1 - S_2) \right]$$

as the input, and for each input variable, $Z_j, j = 1, 2$, we have 3 Gaussian membership functions defined as:

$$\mu_{F_j^1}(Z_j) = \exp\left(-0.5 \left(\frac{Z_j + 1}{0.25}\right)^2\right), \mu_{F_j^2}(Z_j) = \exp\left(-0.5 \left(\frac{Z_j}{0.25}\right)^2\right), \mu_{F_j^3}(Z_j) = \exp\left(-0.5 \left(\frac{Z_j - 1}{0.25}\right)^2\right).$$

In the proposed DAFSMDC, the inputs are mapped onto a normalized domain of $[-1,1]$ by scaling the input variables of the fuzzy system by a partitioning coefficient φ_j (with $j = 1, 2$) given as $\varphi_i = \max(Z_i)$. Figure 2 illustrates the membership functions for the normalized inputs, which are composed of Gaussian functions with variable means and variances. Nine linguistic values ($N = 3$) are used to construct the control table to provide stable and robust decision rules for the DAFSMDC. For this system, the following values are used [14]: $M = 1$ kg; $m = 0.8$ kg; $L = 0.305$ m; and $g = 9.8$ m/s².

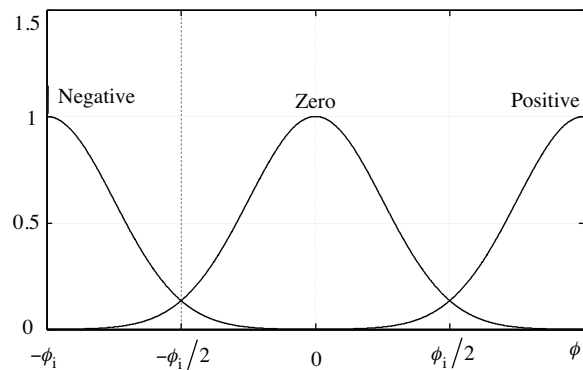


Figure 2. Input fuzzy membership functions.

The design parameters used in this simulation are chosen as follows: $\eta_u = 3.5, \eta_\beta = 0.5, \sigma = 0.05, \lambda_1 = 2, \lambda_2 = 19, K = 14$. The initial conditions of the overhead crane system are $(x_0, \dot{x}_0) = (0, 0); (\alpha_0, \dot{\alpha}_0) = (-\pi/3, 0)$. The objective is to control the trolley to its expected displacement: $(x_d, \alpha_d) = (5, 0)$. Accordingly, $Z(0) = [19\pi/3 - 8, 0]$ and the initial values of the parameter estimates $\theta(0) = 0$ are set equal to 0.

Depending on the value of β , 3 cases are considered in the DAFSMDC: the small, big, and adaptive values were tested as shown in Figures 3 and 4.

The dynamic responses of the position and the angle of the overhead crane system are shown in Figures 3a and 3b. Clearly, the position and the angle converge asymptotically to their desired values for all values of β . The DAFSMDC shows good decoupling performance and ensures the asymptotic stability for all of the state variables of the system. In addition, from Figures 3a and 3b, it appears that the trolley position and crane angle are influenced differently by the linking coefficient β . Indeed, when β increases, the overshoot related to x is more affected than that related to α . In the case of an adaptive β , the plant response is close to that obtained for $\beta = 35$. However, due to its adaptation law, the variation of β is oscillatory, and this leads to a control input that is less smooth than whenever β is constant (Figures 4a and 4b).

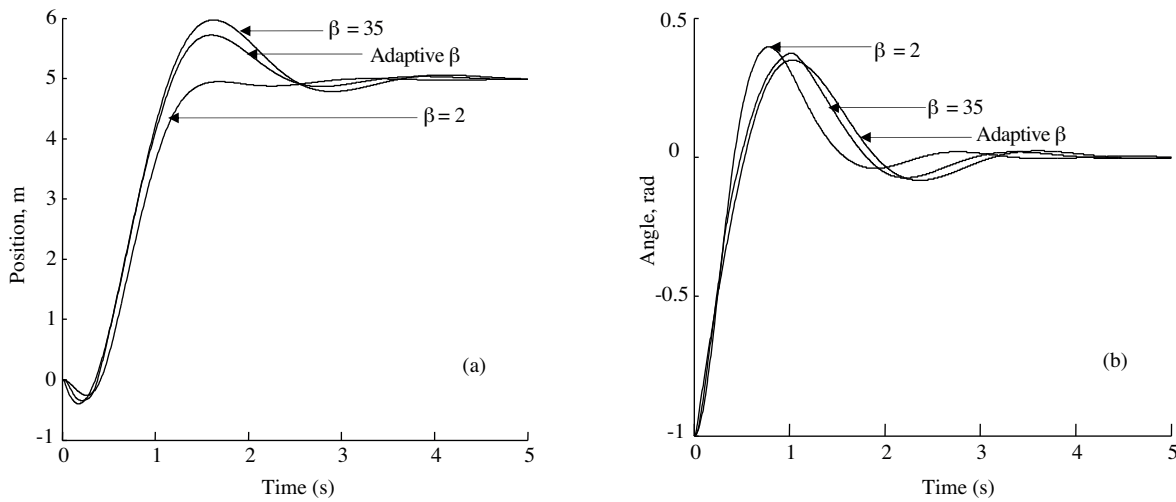


Figure 3. Response of the overhead crane system for different values of β : a) the position of the trolley and b) the angle of the crane.

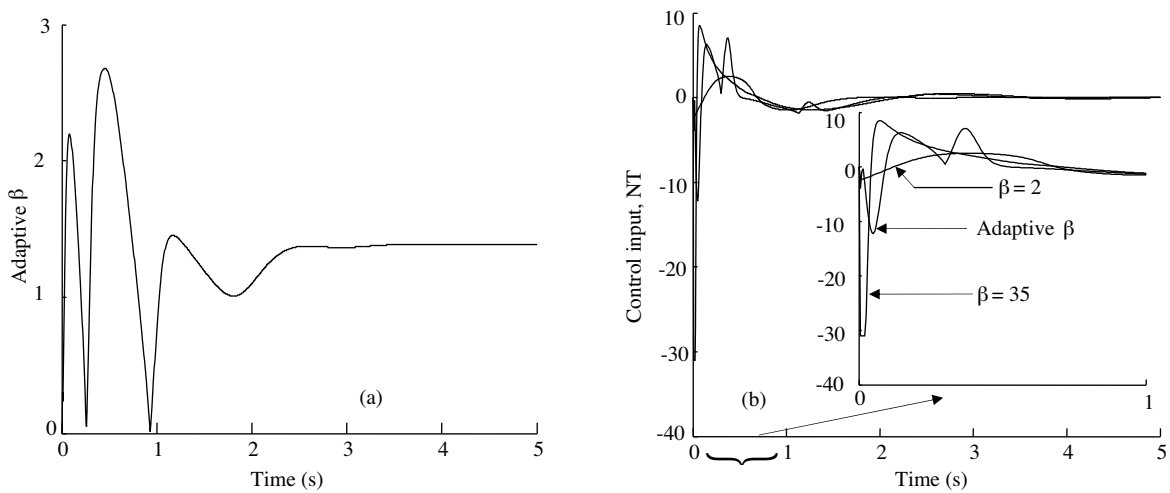


Figure 4. The control input signal and the evolution of parameter β : a) the control input and b) the evolution of parameter β .

The control input is affected by the variation of β . As is illustrated in Figure 4b, an adaptive β yields to the reduction of the input signal amplitude during the transient period. Compared to the case when $\beta = 35$, a reduction of 65% is obtained. Therefore, an optimal design is brought to the DAFSMDC when using an adaptive β .

4.2. Beam and ball

Consider a beam-and-ball system as illustrated in Figure 5. The center of rotation is assumed to be frictionless and the ball is free to roll along the beam. It is required that the ball remains in contact with the beam and that rolling occurs without slipping. The objective is to keep the ball close to the center of the beam and the beam close to the horizontal position.

The mathematical expression of this system is given as follows [30]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u \\ \dot{x}_3 &= x_4 \\ x_4 &= B(x_3x_2^2 - g \sin x_1) \end{aligned}, \tag{56}$$

where $x_1 = \alpha$ is the angle of the beam with respect to the horizontal axis, $x_2 = \dot{\alpha}$ is the angular velocity of the beam with respect to the horizontal axis, $x_3 = r$ is the position of the ball, $x_4 = \dot{r}$ is the velocity of the ball, $B = MR^2 / (J_b + MR^2)$, J_b is the moment of inertia of the ball, M is the mass of the ball, R is the radius of the ball, and g is the gravitation.

The beam-and-ball system poses a challenging stabilization problem, representative of the difficulties introduced by rapidly growing nonlinearities. For the ball, the critical nonlinearity is the centrifugal force $x_3x_2^2$. This force destabilizes when it opposes the controlling gravitational force term $-g \sin x_1$ [30].

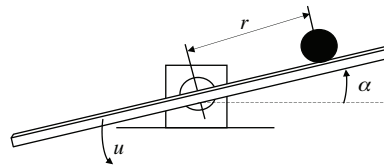


Figure 5. Beam-and-ball system.

The proposed DAFSMDC strategy allows for stabilizing the system and overcomes these drawbacks.

Similar to the previous simulations for the overhead crane, for the design of the controller, the fuzzy system in the form of Eq. (10) is used to generate the control signal with $Z = \left[S_1 + \int_0^\infty \beta (S_1 - S_2) dt, \dot{S}_1 + \beta (S_1 - S_2) \right]$ as the input, and for each input variable Z_j ($j = 1, 2$), 3 Gaussian membership functions are defined as:

$$\mu_{F_j^1}(Z_j) = \exp\left(-0.5 \left(\frac{Z_j + 1}{0.25}\right)^2\right), \mu_{F_j^2}(Z_j) = \exp\left(-0.5 \left(\frac{Z_j}{0.25}\right)^2\right), \mu_{F_j^3}(Z_j) = \exp\left(-0.5 \left(\frac{Z_j - 1}{0.25}\right)^2\right).$$

Moreover, the inputs are mapped onto a normalized domain of $[-1,1]$ by scaling the input variables of the fuzzy system by a partitioning coefficient φ_j (with $j = 1, 2$) given as $\varphi_i = \max(Z_i)$ (see Figure 2).

The beam-and-ball parameters are given as $B = 0.7143$, $M = 0.05$ kg; $J_b = 2 \times 10^{-6}$, $R = 0.01$ m; and $g = 9.8$ m/s². The design parameters used in this simulation are chosen as follows: $\eta_u = 3.5, \eta_\beta = 1.2$,

$\sigma = 0.01, \varepsilon_0 = 1, \lambda_1 = 2; \lambda_2 = 6; K = 8$. The initial conditions of the beam-and-ball system are $(x_0, \dot{x}_0) = (5, 0)$; $(\alpha_0, \dot{\alpha}_0) = (\pi/3, 0)$. The objective is to control the ball to its expected displacement, $(x_d, \alpha_d) = (0, 0)$. Accordingly, $Z(0) = [10 - 6\pi/3, 0]$ and the initial values of the parameter estimates $\theta(0)$ are set equal to 0.

The simulation results are done by considering different values of β . The responses of the position and the angle of the ball and beam are shown in Figures 6a and 6b. Obviously, the position and the angle converge asymptotically to their desired values for all values of β . The DAFSMDC shows good decoupling performance and ensures the asymptotic stability for all of the states of the system. In this case, for $\beta = 2$, the overshoot of the ball's position is significant, and with adaptive β , the response is closer to the case with $\beta = 35$, with a fast response time and the overshoot remaining satisfactory (Figures 6a and 6b). The variation of the β coefficient is smooth and it favorably affects the control input, which is also smooth (Figures 7a and 7b). Moreover, it is judicious to mention that the impact of adaptivity is very clear on the control input side, with a reduction of the input signal value. As is shown in Figure 7b, the reduction rate is about 60%, varying from $\beta = 35$ to adaptive β . Clearly, using adaptive β brings an optimal design to the DAFSMDC.

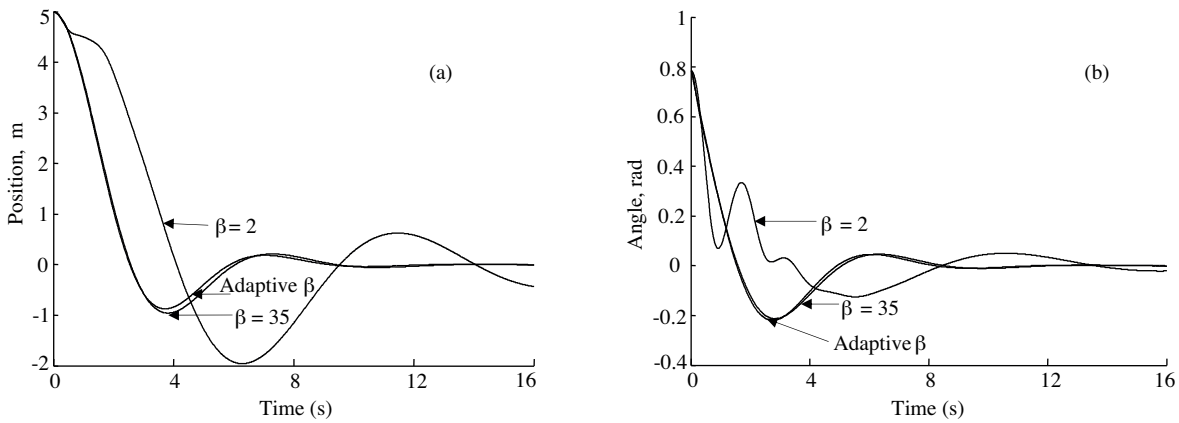


Figure 6. Response of the beam-and-ball system: a) the position of the ball and b) the angle of the beam.

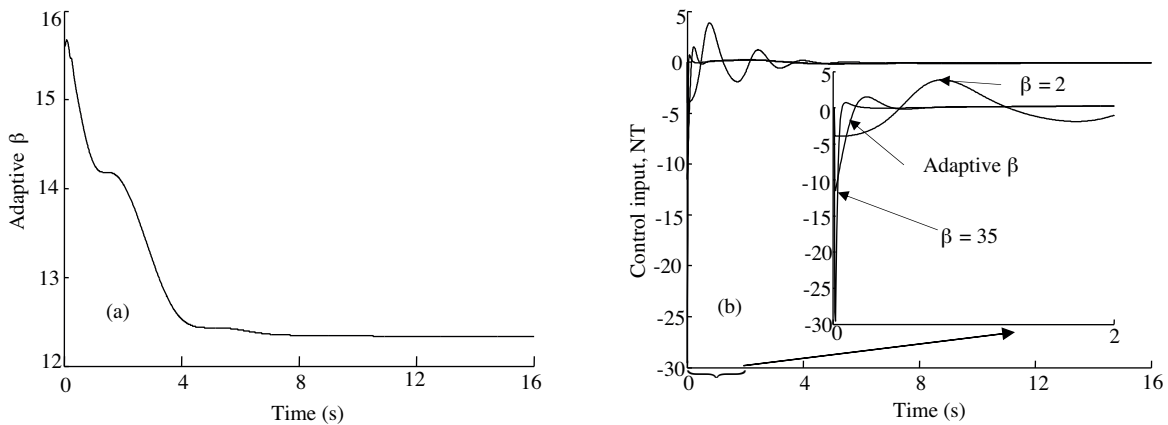


Figure 7. The control input signal and the evolution of parameter β : a) the control input and b) the evolution of parameter β .

5. Conclusion

We have developed a DAFSMDC for a class of UMSs, considering a class of 2 DOF dynamic systems. Indeed, the fuzzy system is directly used to estimate an existent ideal control law. This control law is determined based on the sliding mode methodology, where the used dynamic sliding surface is linked to 2 sliding surfaces involved in the first and second DOF. The adaptive parameters of the fuzzy system are updated based on the gradient descent method by minimizing the quadratic error between the unknown sliding mode controller and the fuzzy controller. The linking parameter influencing the dynamic sliding surface can be taken as constant or adaptive. Its adaptive law is determined based on the gradient descent law by minimizing the quadratic error related to the dynamic sliding surface. We have shown that this control law ensures the stability and convergence of the considered outputs. Its application is carried out for 2 systems: the overhead crane and beam-and-ball systems. The obtained results have revealed that it is possible to obtain satisfactory performances using a constant or adaptive linking parameter despite a lack of information about the system. For further improvements, the generalization of the proposed approach for higher-order underactuated mechanical systems will be considered and will be the scope of potential works.

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