

Multireference TDOA-based source localization

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Abstract: Time difference of arrival (TDOA)-based methods usually achieve the most accurate source localization in passive systems. In these methods, one of the sensors is assigned as the reference and the TDOA of the others with respect to the reference is measured. The source location is estimated by processing these TDOAs. In this paper, a multireference maximum likelihood (ML)-Taylor algorithm is proposed in order to decrease the source localization error of the TDOA methods. In the proposed algorithm, TDOAs are measured assuming 2 different reference sensors, and then the ML objective function of this multireference TDOA algorithm is derived and solved using Taylor series linearization. The developed method is evaluated regarding a 2-dimensional space with some different but deterministic sensor locations. Monte Carlo simulation is used to evaluate the proposed solution. The simulation results show that the proposed method has better performance relative to the other traditional TDOA-based algorithms.

Key words: Source localization, multireference TDOA, Taylor series, maximum likelihood

1. Introduction

Source localization is an important issue in human communications, sensor networks, electronic warfare, etc. In most cases, multiple sensors are placed in signal emission paths and the location of the source is estimated by processing the received signals [1].

Source localization systems can be divided into active and passive ones. In active source localization, the sensors have actual information about the emitted signal. This information includes the time of emission, signal frequency, etc., and usually, a central node in these systems emits a signal and the echo back from the target is received in all of the other nodes. This echo is used to estimate the time of arrival (TOA) of the signal. The measured TOAs at different sensors are used to determine the target location [2,3]. In passive source localization, no exact information exists about the signals characteristics, and the processing needed for the location estimation is performed solely by comparing the received signal at different sensors.

In the literature, several methods for passive source localization have been presented. These methods include received signal strength [4], time difference of arrival (TDOA) [3,5], angle of arrival [6,7], and Doppler frequency difference of arrival (FDOA) [8,9]. As was described in [10], the TDOA method is usually superior to the other competing ones.

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The TDOA localization is a 2-step process. In the first step, one of the sensors is assigned as the reference sensor and the TDOAs of the nonreference sensors relative to the reference are calculated. In the second step, some mathematical manipulations are done over these TDOAs to find the true location of the emitting source. In 2-dimensional space, each TDOA measurement defines a hyperbola whose foci points are located at 2 corresponding sensors. The location of the source can be determined as the intersection of multiple hyperbolas. Due to the nonlinear equations involved, the solution of the problem is not an easy task. Furthermore, the nonlinear hyperbolic equations become inconsistent as the TDOA measurements are corrupted by noise.

Several algorithms are proposed to solve the hyperbolic equations. The least squares (LS) method is one of most famous ones [11]. In this algorithm, the hyperbolic equations are transformed to linear form $Ax = b$, where the system matrix A and the data vector b are given, and the location vector x is unknown. The linear form is solved using the LS method [12]. Weighted LS (WLS) is another algorithm in which the noise covariance matrix is used as a weight matrix in the basic LS solution [2]. The LS and WLS are less accurate methods compared to more recently developed ones. Moreover, the WLS method needs the noise covariance matrix, which may not be available in some situations. Chan and Ho [13] used 2 WLS solutions with different weight matrices to achieve a more accurate solution. However, the proposed method generates 2 sets of solutions, and only 1 set is correct. The constrained WLS is another algorithm for TDOA-based source localization [14]. This method gives an optimal solution, but requires a priori knowledge of the TDOA measurement noises, which is usually unavailable in practical applications.

The maximum likelihood (ML) is one of the most accurate solution methods for nonlinear equations [15]. In the ML method, a nonlinear objective function should be minimized. This nonlinear objective function usually is very complicated. Thus, in [16], the Taylor series was proposed to convert this nonlinear function to a linear approximate one. It was shown that the proposed solution results in good performance in passive source localization.

All of the methods mentioned up to this point use one reference sensor to calculate the TDOAs, and they solve hyperbolic location equations based on a single reference. However, one can assign multiple sensors as the references to perform the source localization in order to improve the performance of the basic TDOA solutions. In this case, one set of TDOA measurements is the result of the first reference sensor and another set of TDOAs is the result of the second reference sensor, etc. It should be mentioned that different sets of measurements are not independent; they do not even contain any more new information than that of the first set. However, because of the nonlinear nature of the equations, as will be shown in the next section, these dependent measurements result in a reduced localization error. The multiple-reference LS method suggested in [17] has higher accuracy compared to the single-reference method.

In this paper, we have proposed a new multiple-reference TDOA-based algorithm named the multireference ML-Taylor (MRMLT) algorithm. In the proposed algorithm, the ML estimation of the multireference TDOA method is calculated and the nonlinear objective function of the multireference ML is solved using Taylor series linearization. The proposed algorithm is investigated assuming 2-dimensional space, but with some mathematical reformation, the algorithm can be extended to 3-dimensional space as well.

The rest of the paper is organized as follows. In Section 2, the modeling for source localization based on the TDOA method is introduced. The MRMLT method is explained in Section 3. The simulation results are presented in Section 4, and finally, the paper and the results are concluded in Section 5.

2. TDOA modeling

In this section, we review the TDOA model for source localization in 2-dimensional space. Let $\mathbf{p} = [x_s \ y_s]^T$ and $\mathbf{r}_i = [x_i \ y_i]^T, i = 1, 2, \dots, M$ be the coordinates of the source and the i th sensor, respectively. The signal propagation delay between the source and the i th sensor is equal to:

$$\tau_i = \frac{d_i}{c} = \frac{\sqrt{(x_s - x_i)^2 + (y_s - y_i)^2}}{c}, \tag{1}$$

where c is the propagation velocity and d_i is the distance between the source and the i th sensor. Assuming the first sensor as the reference sensor, the TDOA of the i th sensor with respect to the reference sensor is:

$$\tau_{i1} = \tau_j - \tau_1 = \frac{\sqrt{(x_s - x_i)^2 + (y_s - y_i)^2}}{c} - \frac{\sqrt{(x_s - x_1)^2 + (y_s - y_1)^2}}{c} \tag{2}$$

The range DOA (RDOA) is obtained by multiplying the TDOA with the signal velocity:

$$d_{i1}(\mathbf{p}) = c\tau_{i1} = \sqrt{(x_s - x_i)^2 + (y_s - y_i)^2} - \sqrt{(x_s - x_1)^2 + (y_s - y_1)^2} \tag{3}$$

Usually, the measured TDOAs are corrupted by some noise. In this case, the measurement of the TDOA can be written as:

$$\hat{d}_{i1} = d_{i1}(\mathbf{p}) + n_{i1}, \quad i = 2, 3, \dots, M \tag{4}$$

Here, $\hat{d}_{i1}(\mathbf{p})$ is the i th measurement of the RDOA and n_{i1} is the i th measured noise. In Eq. (4), x_s and y_s are unknowns and should be estimated. There are $M - 1$ hyperbolic location equations in the form of Eq. (4). Thus, the minimum value of M is 3. As was mentioned earlier, several algorithms can be used for solving these equations [15,16].

3. Proposed MRMLT algorithm

In this section, we assume that in place of one unique reference sensor, R reference sensors exist. The TDOAs between the other sensors with respect to each of these reference sensors are measured and then these TDOAs are manipulated in order to estimate the source's true location

Suppose that $\{1, 2 \dots R\}$ is the list of reference sensors. The measured RDOA of the multireference sensors can then be written as:

$$\hat{d}_{ir} = d_{ir}(\mathbf{p}) + n_{ir}, \quad i = R + 1, R + 2, \dots, M, \quad r = 1, 2, \dots, R \tag{5}$$

The vector form of above equation is:

$$\hat{\mathbf{d}} = \mathbf{d}(\mathbf{p}) + \mathbf{n} \tag{6}$$

$$\hat{\mathbf{d}} = \begin{bmatrix} \hat{d}_{(R+1)1} \\ \hat{d}_{(R+2)1} \\ \vdots \\ \hat{d}_{M1} \\ \vdots \\ \hat{d}_{(R+1)R} \\ \hat{d}_{(R+2)R} \\ \vdots \\ \hat{d}_{MR} \end{bmatrix} \quad \mathbf{d}(\mathbf{p}) = \begin{bmatrix} d_{(R+1)1}(\mathbf{p}) \\ d_{(R+2)1}(\mathbf{p}) \\ \vdots \\ d_{M1}(\mathbf{p}) \\ \vdots \\ d_{(R+1)R}(\mathbf{p}) \\ d_{(R+2)R}(\mathbf{p}) \\ \vdots \\ d_{MR}(\mathbf{p}) \end{bmatrix} \quad \mathbf{n} = \begin{bmatrix} n_{(R+1)1} \\ n_{(R+2)1} \\ \vdots \\ n_{M1} \\ \vdots \\ n_{(R+1)R} \\ n_{(R+2)R} \\ \vdots \\ n_{MR} \end{bmatrix} \quad (7)$$

Assuming Gaussian-distributed noise the conditional power density function of the RDOAs is:

$$f_{\mathbf{d}}(\hat{\mathbf{d}}/\mathbf{p}) = \frac{1}{\sqrt{2\pi\Sigma_{\mathbf{n}}}} e^{-\frac{1}{2}(\hat{\mathbf{d}}-\mathbf{d}(\mathbf{p}))^T \Sigma_{\mathbf{n}}^{-1}(\hat{\mathbf{d}}-\mathbf{d}(\mathbf{p}))} \quad (8)$$

Here, $\Sigma_{\mathbf{n}}$ is the noise covariance matrix. According to the above probability density functions, the maximum likelihood estimation of the problem is:

$$\hat{\mathbf{p}}_{ML} = \arg \max_{\mathbf{p}} f_{\mathbf{d}}(\hat{\mathbf{d}}/\mathbf{p}) = \arg \min_{\mathbf{p}} (\hat{\mathbf{d}} - \mathbf{d}(\mathbf{p}))^T \Sigma_{\mathbf{n}}^{-1} (\hat{\mathbf{d}} - \mathbf{d}(\mathbf{p})) \quad (9)$$

Therefore, the following ML objective function should be minimized in order to achieve the ML solution of the problem:

$$g(\mathbf{p}) = (\hat{\mathbf{d}} - \mathbf{d}(\mathbf{p}))^T \Sigma_{\mathbf{n}}^{-1} (\hat{\mathbf{d}} - \mathbf{d}(\mathbf{p})) \quad (10)$$

Here, we use the Taylor series expansion to minimize the ML objective function. Hence, first, the RDOA function is explained as:

$$\mathbf{d}(\mathbf{p}) = \mathbf{d}(\mathbf{p}_0) + \nabla_{\mathbf{p}}^T \otimes \mathbf{d}(\mathbf{p})|_{\mathbf{p}=\mathbf{p}_0} (\mathbf{p} - \mathbf{p}_0) + \nabla_{\mathbf{p}}^{2T} \otimes \mathbf{d}(\mathbf{p})|_{\mathbf{p}=\mathbf{p}_0} (\mathbf{p} - \mathbf{p}_0)^2 + \dots \quad (11)$$

Assuming a \mathbf{p} value adequately close to \mathbf{p}_0 , the third and higher portions of the above equations can be ignored, resulting in:

$$\mathbf{d}(\mathbf{p}) \simeq \mathbf{d}(\mathbf{p}_0) + \nabla_{\mathbf{p}}^T \otimes \mathbf{d}(\mathbf{p})|_{\mathbf{p}=\mathbf{p}_0} (\mathbf{p} - \mathbf{p}_0) \quad (12)$$

where

$$\nabla_{\mathbf{p}}^T \otimes \mathbf{d}(\mathbf{p}) = \begin{bmatrix} \frac{x-x_{R+1}}{r_{R+1}} - \frac{x-x_1}{r_1} & \frac{y-y_{R+1}}{r_{R+1}} - \frac{y-y_1}{r_1} \\ \frac{x-x_{R+2}}{r_{R+2}} - \frac{x-x_1}{r_1} & \frac{y-y_{R+2}}{r_{R+2}} - \frac{y-y_1}{r_1} \\ \vdots & \vdots \\ \frac{x-x_M}{r_M} - \frac{x-x_1}{r_1} & \frac{y-y_M}{r_M} - \frac{y-y_1}{r_1} \\ \vdots & \vdots \\ \frac{x-x_{R+1}}{r_{R+1}} - \frac{x-x_R}{r_R} & \frac{y-y_{R+1}}{r_{R+1}} - \frac{y-y_R}{r_R} \\ \frac{x-x_{R+2}}{r_{R+2}} - \frac{x-x_N}{r_R} & \frac{y-y_{R+2}}{r_{R+2}} - \frac{y-y_R}{r_R} \\ \vdots & \vdots \\ \frac{x-x_M}{r_M} - \frac{x-x_R}{r_R} & \frac{y-y_M}{r_M} - \frac{y-y_R}{r_R} \end{bmatrix} \quad (13)$$

is the gradient matrix. Replacing the Taylor series expansion into Eq. (6), we have:

$$\mathbf{n} = \hat{\mathbf{d}} - \mathbf{d}(\mathbf{p}) \simeq \hat{\mathbf{d}} - \mathbf{d}(\mathbf{p}_0) + \nabla_{\mathbf{p}}^T \otimes \mathbf{d}(\mathbf{p})|_{\mathbf{p}=\mathbf{p}_0} (\mathbf{p} - \mathbf{p}_0) \quad (14)$$

We can rewrite Eq. (14) in a linear form as below:

$$\mathbf{A}\Delta\mathbf{p} = \mathbf{b} \quad (15)$$

where

$$\mathbf{A} = \nabla_{\mathbf{p}}^T \otimes \mathbf{d}(\mathbf{p})|_{\mathbf{p}=\mathbf{p}_0}, \quad \Delta\mathbf{p} = \mathbf{p} - \mathbf{p}_0, \quad \mathbf{b} = \hat{\mathbf{d}} - \mathbf{d}(\mathbf{p}_0) \quad (16)$$

Now the ML objective function can be reformed as:

$$g(\mathbf{p}) = (\mathbf{b} - \mathbf{A}\Delta\mathbf{p})^T \Sigma_n^{-1} (\mathbf{b} - \mathbf{A}\Delta\mathbf{p}) \quad (17)$$

Using the LS solution and ML objective function, the following solution is reached:

$$\Delta\mathbf{p} = (\mathbf{A}^T \Sigma_n^{-1} \mathbf{A})^{-1} \mathbf{A}^T \Sigma_n^{-1} \mathbf{b} \quad (18)$$

Replacing \mathbf{A} , \mathbf{b} , and $\Delta\mathbf{p}$ in Eq. (18), and assuming a diagonal noise covariance matrix, we have:

$$\mathbf{p} = \mathbf{p}_0 + (\Phi_{\mathbf{p}}^T(\mathbf{p}_0)\Phi_{\mathbf{p}}(\mathbf{p}_0))^{-1} \Phi_{\mathbf{p}}^T(\mathbf{p}_0)(\hat{\mathbf{d}} - \mathbf{d}(\mathbf{p}_0)) \quad (19)$$

where

$$\Phi_{\mathbf{p}}(\mathbf{p}_0) = \nabla_{\mathbf{p}}^T \otimes \mathbf{d}(\mathbf{p})|_{\mathbf{p}=\mathbf{p}_0}. \quad (20)$$

Finally, the following iterative function can be used to find the true solution:

$$\mathbf{p}^{k+1} = \mathbf{p}^k + (\Phi_{\mathbf{p}}^T(\mathbf{p}^k)\Phi_{\mathbf{p}}(\mathbf{p}^k))^{-1} \Phi_{\mathbf{p}}^T(\mathbf{p}^k)(\hat{\mathbf{d}} - \mathbf{d}(\mathbf{p}^k)) \quad (21)$$

The multireference LS (MRLS) method can be used as the initial guess in the above iterative solution as:

$$d_{ir}(\mathbf{p}) + d_r(\mathbf{p}) = d_i(\mathbf{p}) \quad (22)$$

By squaring both sides of Eq. (22), the equation is reordered as:

$$d_{ir}^2(\mathbf{p}) + d_r^2(\mathbf{p}) + 2d_{ir}(\mathbf{p})d_r(\mathbf{p}) = d_i^2(\mathbf{p}) \quad (23)$$

Replacing $d_r(\mathbf{p})$ we have:

$$d_{ir}^2(\mathbf{p}) + (x_s - x_r)^2 + (y_s - y_r)^2 + 2d_{ir}(\mathbf{p})d_r(\mathbf{p}) = (x_s - x_i)^2 + (y_s - y_i)^2 \quad (24)$$

Eq. (24) can be expanded as:

$$d_{ir}(\mathbf{p})d_r(\mathbf{p}) + (x_i - x_r)x_s + (y_i - y_r)y_s = 1/2 (d_{ir}^2(\mathbf{p}) - x_i^2 - x_r^2 + y_i^2 - y_r^2) \quad (25)$$

The linear form of Eq. (25) is:

$$\mathbf{B}\mathbf{q} = \mathbf{c} \quad (26)$$

where

$$\mathbf{B} = \begin{bmatrix} x_{R+1} - x_1 & y_{R+1} - y_1 & d_{(R+1)1}(\mathbf{p}) \\ x_{R+2} - x_1 & y_{R+1} - y_1 & d_{(R+2)1}(\mathbf{p}) \\ \vdots & \vdots & \vdots \\ x_M - x_1 & y_M - y_1 & d_{M1}(\mathbf{p}) \\ \vdots & \vdots & \vdots \\ x_{R+1} - x_R & y_{R+1} - y_R & d_{(R+1)R}(\mathbf{p}) \\ x_{R+2} - x_R & y_{R+2} - y_R & d_{(R+2)R}(\mathbf{p}) \\ \vdots & \vdots & \vdots \\ x_M - x_R & y_M - y_R & d_{MR}(\mathbf{p}) \end{bmatrix} \quad (27)$$

$$\mathbf{q} = \begin{bmatrix} x_s \\ y_s \\ d_r(\mathbf{p}) \end{bmatrix} \quad (28)$$

$$\mathbf{c} = \begin{bmatrix} 1/2 \left(d_{(R+1)1}^2(\mathbf{p}) - x_{R+1}^2 - x_1^2 + y_{R+1}^2 - y_1^2 \right) \\ 1/2 \left(d_{(R+2)1}^2(\mathbf{p}) - x_{R+2}^2 - x_1^2 + y_{R+2}^2 - y_1^2 \right) \\ \vdots \\ 1/2 \left(d_{M1}^2(\mathbf{p}) - x_M^2 - x_1^2 + y_M^2 - y_1^2 \right) \\ \vdots \\ 1/2 \left(d_{(R+1)R}^2(\mathbf{p}) - x_{R+1}^2 - x_R^2 + y_{R+1}^2 - y_R^2 \right) \\ 1/2 \left(d_{(R+2)R}^2(\mathbf{p}) - x_{R+2}^2 - x_R^2 + y_{R+2}^2 - y_R^2 \right) \\ \vdots \\ 1/2 \left(d_{MR}^2(\mathbf{p}) - x_M^2 - x_R^2 + y_M^2 - y_R^2 \right) \end{bmatrix} \quad (29)$$

In the above equation, we can use the measured RDOA as the real RDOA in \mathbf{B} . The linear equation of Eq. (26) can be solved by the LS method. Next, this answer is used to initialize the proposed ML-Taylor method iteration in Eq. (21).

4. Simulation results

In this section, the simulation results are presented to compare the performance of the proposed algorithm with the previously suggested methods. In the simulations, we implement the proposed algorithm with $R = 2$ reference sensors. The root mean square error (RMSE) is used as the measure to compare the accuracy of the different methods. The RMSE of the proposed algorithm is compared with the RMSE of the LS [11], multiple-reference LS [17], single-reference Taylor algorithms [16], and CramerRao lower band (CRLB) [12]

In all of the simulations, 6 sensors are placed at $(0, 0)$, $(15, 0)$, $(-5, 10)$, $(-5, -10)$, $(5, 10)$, and $(5, -10)$. All of the values are scaled in kilometers. The noise values are generated using the Gaussian probability density function. For each noise variance, the results are extracted based on 10,000 independent trials.

Based on the distance between the source and the sensors, we divided our simulations into 3 categories: far-field, medium-field, and nearfield.

4.1. Far-field source

In the far-field simulation, the source is located at (40, 90). In the first simulation, the performance of the 3 algorithms is compared for a timing error range of 3 ns to 10 ns. The results of this comparison are shown in Figure 1, indicating that the RMSE of the proposed MRMLT is less than those of the others. For example, for a timing error of 7 ns, the RMSE of the singlereference LS, 2reference LS, and single-reference Taylor are 435 m, 334 m, and 376 m, respectively. These are greater than the proposed MRMLT’s RMSE, which is 312 m, resulting in an improvement of at least 22 m. The results of the first experiment show that the proposed algorithm’s improvement for a timing error of 7 ns is 22 m for the 2reference LS and 66 m for the singlereference Taylor algorithm.

In the second simulation, the first experiment is repeated for a timing error range of 50 ns to 80 ns. The results of this experiment are shown in Figure 2. It is indicated that the RMSE of the proposed MRMLT algorithm is closer to the CRLB when compared to the single-reference LS, 2-reference LS, and single-reference Taylor algorithms.

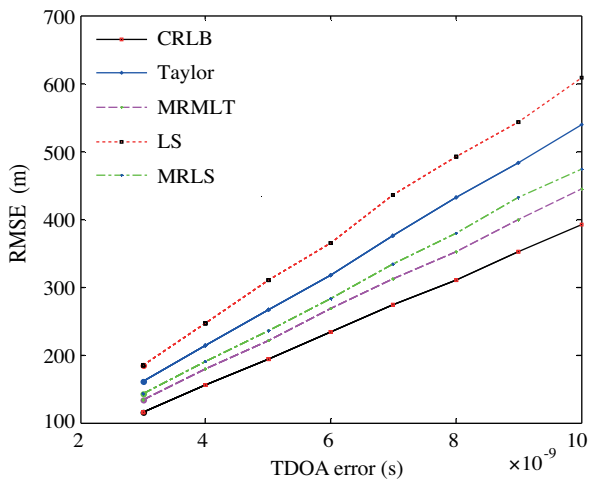


Figure 1. RMSE of the methods for a timing error in the range of 3 ns to 10 ns in the far-field category.

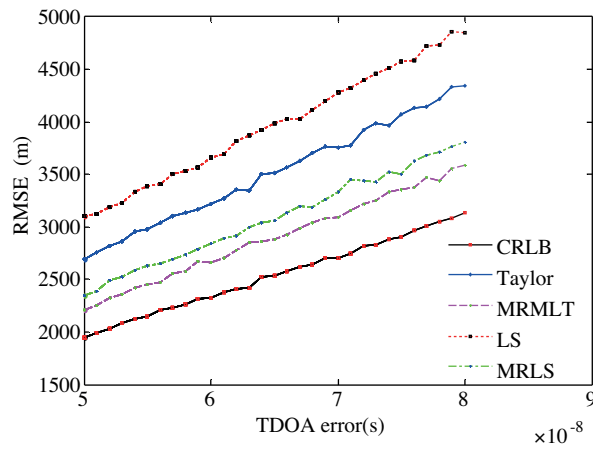


Figure 2. RMSE of the methods for a timing error in the range of 50 ns to 80 ns in the far-field category.

Figure 3 shows the performance comparison of the algorithms for a timing error range of 80 ns to 100 ns. The results are similar to those of previous simulations, which shows that the proposed algorithm has a lower RMSE compared to the other algorithms, due to the utilization of 2 references in the TDOA measurement in the proposed algorithm.

4.2. Medium-field source

In the medium-field category, the source is located at (20 50), which is a medium-respect array dimension. In the first simulation of this category, the performance of the algorithms is compared for the TDOA timing errors in the range of 50 ns to 80 ns. The results of these simulations are shown in Figure 4. It is shown that the RMSE of the proposed algorithm is less than that of the others. Comparing the results in Figure 4 with those of the far-field category, it can be shown that the RMSE of all of the algorithms in the medium-field simulations is less than that of the far-field case. This is true in all of the TDOA-based algorithms.

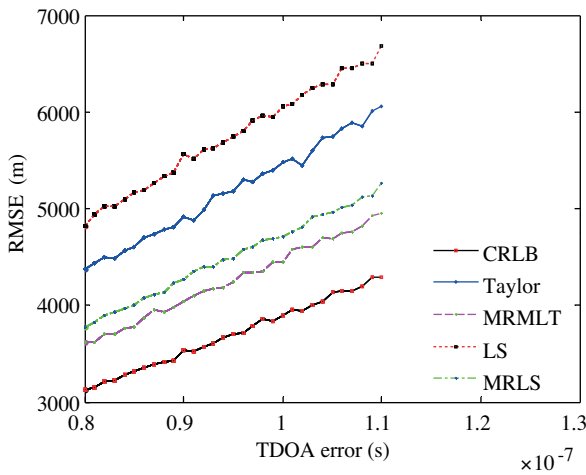


Figure 3. RMSE of the methods for a timing error in the range of 80 ns to 100 ns in the far-field category.

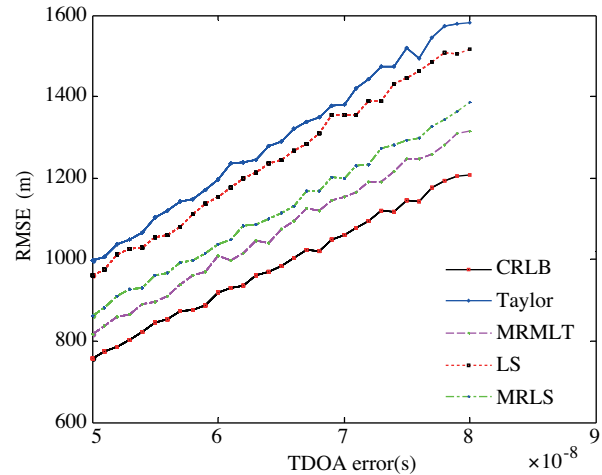


Figure 4. RMSE of the methods for a timing error in the range of 50 ns to 80 ns in the medium-field category.

Figure 5 shows the simulation results of the medium-field for a timing error range of 80 ns to 100 ns. Similar to the other simulations, the RMSE of the proposed algorithm is closer to the CRLB when compared to the other algorithms.

4.3. Near-field source

In the near-field category, the source is located at (5, 5). Figures 6 and 7 show the results when comparing the algorithms' performances for the near-field category for a timing error range of 50 ns to 100 ns. It is indicated that the RMSE of the proposed algorithm is lower than that of the other algorithms.

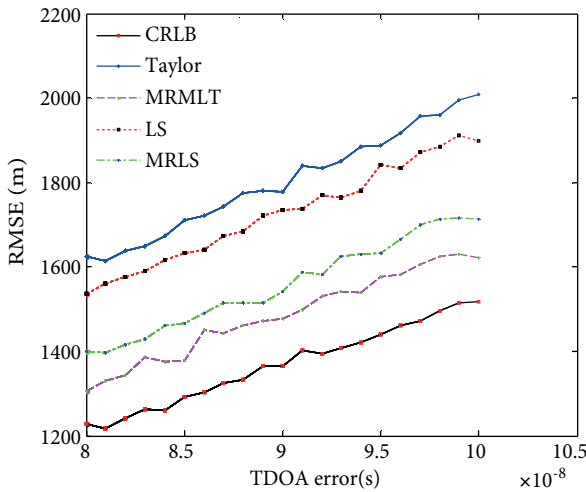


Figure 5. RMSE of the methods for a timing error in the range of 80 ns to 100 ns in the medium-field category.

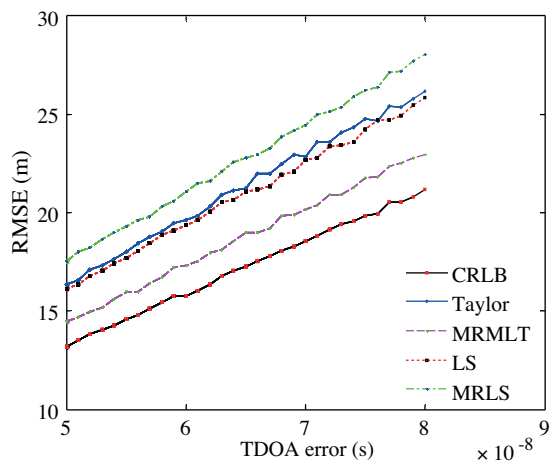


Figure 6. RMSE of the methods for a timing error in the range of 50 ns to 80 ns in the near-field category.

Comparing the simulation results of all 3 categories, it can be seen that in all of the categories and all of the timing error values, the proposed algorithm has the lowest RMSE when compared to the other algorithms. With a decrease in the distance between the source and the sensor array, the RMSE of the algorithms also decreases.

5. Computational load analysis

In Figure 8, the computational loads of 4 different methods are compared. This comparison is based on MATLAB 2009a programs that are executed on an Intel Core 2 Duo CPU with a 2.5 GHz clock and 4 GB of RAM. Each method is executed 10,000 times and the resultant processing time is the one shown in Figure 8

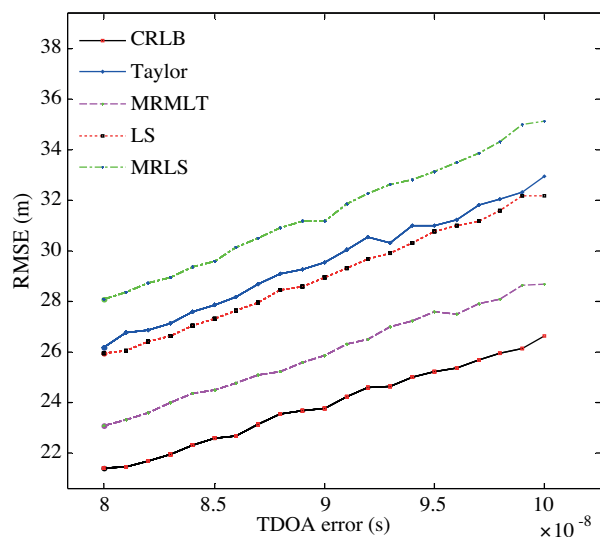


Figure 7. RMSE of the methods for a timing error in the range of 80 ns to 100 ns in the near-field category.

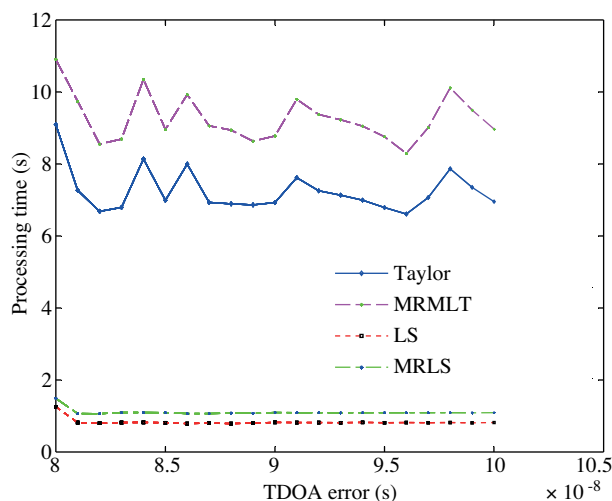


Figure 8. Comparing the processing time for the different algorithms.

As shown in Figure 8, the proposed method needs more processing time compared to the other 3 algorithms. Actually, its processing time is about the same as the Taylor method. It is 7 to 10 times more than that of the MRLS and LS. This processing time is a weakness for our proposed method.

6. Conclusion

In this paper, a new multireference TDOA-based localization method was developed. In this method, 2 different sensors are considered as the reference references. Each reference sensor results in a set of TDOA measurements. These 2 sets of measurements are not independent. However, regarding the nonlinear nature of the TDOA equations, these sets of dependent measurements result in more accurate source localization. The ML-Taylor algorithm was proposed to solve this multireference TDOA-based localization problem. A 2-dimensional space and some deterministic sensor locations were selected to evaluate the performance of the proposed algorithm. The performance was evaluated using Monte Carlo simulation. It was shown that the localization error of the proposed method is superior compared to the single-reference LS, pure ML, and ML-Taylor methods. The results were also compared to the CRLB and it was shown that the results are in the vicinity of the CRLB. The computational load of the proposed method was also compared with the LS and pure Taylor methods and it was shown that while the method needs more computational load, the processing time of the proposed method is no more than 8 to 9 times more than that of the existing methods.

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