

Multiobjective weighted sum approach model reduction by Routh–Padé approximation using harmony search

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Abstract: A new weighted-sum multiobjective approach is investigated for order reduction based on Routh–Padé approximation, in which the harmony search algorithm is used to optimize the reduced-order model’s parameters. In this method, apart from minimizing the errors between a set of subsequent time moments/Markov parameters of the system and those of the model, the error between the singular values of the reduced-order system and those of the original system is minimized. The Routh criterion is applied for specifying the stability conditions. The stability condition is then considered as a constraint in the optimization problem. To present the ability of the proposed method, 3 test systems are reduced. The results obtained show that the proposed approach performs well.

Key words: Routh–Padé approximation, harmony search algorithm, multiobjective, order reduction, stability constraints

1. Introduction

Model reduction of a high-order system is an important problem in analysis, as well as in controller synthesis of a practical system. In view of this, many methods are available in the literature for order reduction in the time and frequency domains [1–18].

Among the reported methods in the literature, a frequency-domain method is known as Padé approximation, in which $2r$ time moments of the high-order transfer function $G_n(s)$ (n th-order) are fully retained in a low-order model $G_r(s)$ (r th-order) to ensure the steady-state response approximation. Since the Padé approximation does not guarantee the stability of the reduced-order model, several methods, such as Routh approximation [19–21], the Mihailov stability criterion [22], Hurwitz polynomial approximation [23], and the stability equation method [24], have been used.

Furthermore, to ensure the initial time response approximation, Shamash [25] considered the effect of including Markov parameters (M_1, M_2, \dots) along with time moments. In [26], a different procedure was presented to obtain Routh–Padé approximation.

In recent decades, evolutionary techniques such as particle swarm optimization and the genetic algorithm have been used for order reduction of systems [27–30]. In these approaches, the reduced-order model’s parameters are achieved by minimizing a fitness function, which is often the integral square error (ISE), integral absolute error, H_2 norm, or H_∞ norm [31–33].

In this study, a new method based on Routh–Padé approximation is investigated for order reduction.

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In this method, first, the system is expanded around $s = 0$ and $s = \infty$ to get the first r time moments and Markov parameters. Using the concept of moment matching and the harmony search (HS) algorithm, the unknown coefficients are determined. To get a better result, a multiobjective criterion is used based on the weighted sum approach. Apart from minimizing the errors between a set of subsequent time moments/Markov parameters of the system and those of the model, the error between the singular values of the frequency response of the reduced-order system and those of the original system is minimized. To satisfy the stability, the Routh criterion is applied [34], where the Routh criterion is stated in optimization problems as constraints. Therefore, the optimization problem is converted to a constrained optimization problem. To show the accuracy of the proposed method, 2 systems are reduced by the proposed method and compared with the typical Padé approximant. Furthermore, to compare the proposed method with the suggested method in [26], the test system in [26] is adopted as a third example and the reduced system in [26] is compared with the reduced system obtained by the proposed method in this paper.

To give a proper background, the HS algorithm is briefly explained in Section 2. The proposed method is explained in Section 3. The ability of the proposed approach is shown in Section 4 through 3 examples, and the paper is concluded in Section 5.

2. HS algorithm

HS is based on a natural musical performance that searches for a perfect state of harmony. In general, the HS algorithm works as follows [35,36]:

Step 1. Initialization: Define the objective function and decision variables. Input the system parameters and the boundaries of the decision variables. The optimization problem can be defined as:

Minimize $f(x)$ subject to $x_{iL} < x_i < x_{iU}$ ($i = 1, 2, \dots, N$), where x_{iL} and x_{iU} are the lower and upper bounds for the decision variables.

The parameters of the HS algorithm are also specified in this step. They are the harmony memory size (HMS) or the number of solution vectors in the harmony memory (HM), HM considering rate (HMCR), distance bandwidth, pitch adjusting rate (PAR), and the number of improvisations (K) or stopping criterion, where K is the same as the total number of function evaluations.

Step 2. Initialize the HM. The HM is a memory location where all of the solution vectors (sets of decision variables) are stored. The initial HM is randomly generated in the region $[x_{iL}, x_{iU}]$ ($i = 1, 2, \dots, N$). This is done based on the following equation:

$$x_i^j = x_{iL} + rand() \times (x_{iU} - x_{iL}) \quad j = 1, 2, \dots, HMS, \quad (1)$$

where $rand()$ is a random from a uniform distribution of $[0, 1]$.

Step 3. Improvise a new harmony from the HM. Generating a new harmony x_i^{new} is called improvisation, where it is based on 3 rules: memory consideration, pitch adjustment, and random selection. First of all, a uniform random number r_1 is generated in the range $[0, 1]$. If r_1 is less than the HMCR, the decision variable x_i^{new} is generated by the memory consideration; otherwise, x_i^{new} is obtained by a random selection. Next, each decision variable x_i^{new} will undergo a pitch adjustment with a probability of PAR if it is produced by the memory consideration. The pitch adjustment rule is given as follows:

$$x_i^{new} = x_i^{new} \pm r_1 \times bw. \quad (2)$$

Step 4. Update HM. After generating a new harmony vector x^{new} , the HM will be updated. If the fitness of the improvised harmony vector $x^{new} = (x_1^{new}, x_2^{new}, \dots, x_N^{new})$ is better than that of the worst harmony, the worst harmony in the HM will be replaced with x^{new} and become a new member of the HM.

Step 5. Repeat steps 3 and 4 until the stopping criterion (maximum number of improvisations K) is met.

3. The proposed model reduction method

Consider a stable single-input single-output (SISO) system described by the transfer function of order n as follows:

$$G(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_n}{s^n + b_1 s^{n-1} + b_2 s^{n-2} + \dots + b_n}, \quad (3)$$

where a_i and b_i are constants.

The objective is to obtain a reduced model of order r , where r is smaller than n , such that the principal and important specifications of the full-order system are retained in the reduced-order model. This reduced-order system is presented as follows:

$$G_r(s) = \frac{c_1 s^{r-1} + c_2 s^{r-2} + \dots + c_r}{s^r + d_1 s^{r-1} + d_2 s^{r-2} + \dots + d_r}, \quad (4)$$

where c_i and d_i are unknown constants.

To obtain the reduced model by the Padé approximants method, the full-order and reduced systems are expanded around $s = 0$ [Eqs. (5) and (7)] and $s = \infty$ [Eqs. (6) and (8)].

$$G_n(s) = t_1 + t_2 s + \dots + t_n s^{n-1} + \dots \quad (5)$$

$$G_n(s) = M_1 s^{-1} + M_2 s^{-2} + \dots + M_n s^{-n} + \dots \quad (6)$$

$$G_r(s) = \hat{t}_1 + \hat{t}_2 s + \dots + \hat{t}_r s^{r-1} + \dots \quad (7)$$

$$G_n(s) = \hat{M}_1 s^{-1} + \hat{M}_2 s^{-2} + \dots + \hat{M}_r s^{-r} + \dots \quad (8)$$

Based on the Padé approximants and using the concept of moment matching, to retain the important characteristic of the original system, the first r time moments and the first r Markov parameters of the full-order system must be the same as the first r time moments and the first r Markov parameters of the reduced-order system. It should be noted that the time moments contribute to the steady-state response and Markov parameters to the initial time response.

Therefore, to find the best parameters in Eq. (4), the HS algorithm is applied. To generate the optimal solution, the following fitness function is minimized:

$$J^* = \sum_{i=1}^r \left(|t_i - \hat{t}_i| + |M_i - \hat{M}_i| \right). \quad (9)$$

Since the proposed approach must guarantee the stability of the reduced system, the Routh criterion is applied for specifying the stability conditions as follows.

The denominator of the reduced-order model that is presented by Eq. (4) can be shown as [26]:

$$s^r + h_1 s^{r-1} + (h_2 + h_3 + \dots + h_r) s^{r-2} + h_1(h_3 + h_4 + \dots + h_r) s^{r-3} + [h_2(h_4 + h_5 + \dots + h_r) + h_3(h_5 + h_6 + \dots + h_r) + h_4(h_6 + h_7 + \dots + h_r) + \dots], \tag{10}$$

which is constructed by taking the coefficients of the first 2 rows of the Routh array with the elements of its first column given by:

$$1, h_1, h_2, h_1 h_3, h_2 h_4, h_1 h_3 h_5, \dots, h_{1+k} h_{3+k} \dots h_{r-2} h_r, \tag{11}$$

where k is equal to 1 for the even r and k is equal to 0 for the odd r .

Comparing the entries of the first row with $1, d_2, d_4, \dots$ and those of the second row with d_1, d_3, d_5, \dots , the relations defined in Eq. (12) are obtained:

$$\begin{aligned} d_1 &= h_1 \\ d_2 &= (h_2 + h_3 + \dots + h_r) \\ d_3 &= h_1(h_3 + h_4 + \dots + h_r) \\ &\vdots \\ d_r &= (h_{1+k} h_{3+k} h_{r-2} h_r) \end{aligned} \tag{12}$$

By substituting the above relations in the reduced-order model's denominator, Eq. (10) is achieved.

Therefore, the necessary and sufficient condition for all of the poles of the reduced system to be strictly in the left-half plane is:

$$\begin{aligned} h_1 &> 0 \\ h_2 &> 0 \\ &\vdots \\ h_r &> 0 \end{aligned}, \tag{13}$$

and subsequently

$$\begin{aligned} d_1 &> 0 \\ d_2 &> 0 \\ &\vdots \\ d_r &> 0 \end{aligned}. \tag{14}$$

Therefore, to have a stable reduced system, the reduced-order model's parameters are determined by minimizing Eq. (9) subject to Eq. (14). In other words, the reduced-order model is obtained by minimizing the following fitness function:

$$J^* = \sum_{i=1}^r \left(|t_i - \hat{t}_i| + |M_i - \hat{M}_i| \right) \tag{15}$$

subject to $d_j > 0$ *for* $j = 0, 1, \dots, r$

This method not only guarantees the stability conditions but also is fast for finding the reduced-order model. To get a better result, another objective function is added to the objective function defined in Eq. (15). The error between the singular values of the reduced-order system and those of the original system is minimized. It should be noted that the singular values give better information about the gains of the plant and the maximum singular value is very useful in terms of the frequency-domain performance and robustness. Thus, the singular values of the full-order system in the frequency domain should be the same as the singular values of the reduced-order

system. For this, the singular values of the full and reduced systems are sorted in descending order and the l first singular values with larger values are compared. Therefore, the following objective function is minimized:

$$J = w_1 \sum_{i=1}^r \left(|t_i - \hat{t}_i| + |M_i - \hat{M}_i| \right) + w_2 \sum_{i=1}^l |\sigma_i - \hat{\sigma}_i|, \quad (16)$$

subject to $d_j > 0$ *for* $j = 0, 1, \dots, r$

where w_1 and w_2 are the weights. Since 2 different types of objectives are considered simultaneously in Eq. (16), a balancing factor should be included (by w_1 and w_2) to weight these objectives. The reduced-order model that is achieved by this method tries to retain the important characteristic of the original system.

4. Simulation and results

To assess the efficiency of the proposed approach, it has been applied on 3 test systems. To obtain the reduced-order system, a step-by-step procedure is given below for test system 1.

Test system 1: The first test system is a system of order 6, as follows [37]:

$$G(s) = \frac{2s^5 + 3s^4 + 16s^3 + 20s^2 + 8s + 1}{2s^6 + 33.6s^5 + 155.94s^4 + 209.46s^3 + 102.42s^2 + 18.3s + 1}. \quad (17)$$

Using the proposed method, the reduced system is obtained as follows:

Step 1: First the full-order system is expanded around $s = 0$ and $s = \infty$.

Step 2: A singular value plot of the frequency response of the full-order system is calculated and sorted in descending order, and the first l terms of the singular values are considered, where l is set to be 25.

Step 3: A desired fixed structure for the reduced-order model is considered. Suppose a second-order approximant is required, as follows:

$$G_r(s) = \frac{c_1s + c_2}{d_1s^2 + d_2s + d_3}, \quad (18)$$

where c_i and d_i are unknown parameters of the reduced-order model.

Step 4: To obtain the unknown parameters, the HS algorithm is applied. The goal of the optimization is to find the best parameters for $G_r(s)$. Therefore, each harmony is a d -dimensional vector, in which $d = 5$ (unknown parameters). The HMS is selected to be 6, and the HMCR and evaluation number are set to be 0.9 and 500, respectively.

The initial HM is randomly generated, where each harmony is a solution for $G_r(s)$. For each harmony (or each $G_r(s)$), the time moments, Markov parameters, and singular values are calculated and then evaluated using the objective function defined by Eq. (16), searching for the harmony associated with J_{best} , in which w_1 and w_2 are selected to be 0.7 and 0.3, respectively.

When the stopping criterion (maximum evaluation number) is met, the following solution is obtained:

$$G_{Multi \ Objective}(s) = \frac{7.80016s + 0.81849}{87.58712s^2 + 12.43314s + 0.81657}. \quad (19)$$

Now, the full-order system is reduced by the typical Padé approximant [considering Eq. (15)] using the HS algorithm. A similar procedure as above is repeated, except for the calculation of the singular values in Steps 2 and 4. Finally, the following system is obtained:

$$G_{typical \ pad\acute{e}}(s) = \frac{6.87815s + 1.09228}{89.54625s^2 + 12.96860s + 0.99732}. \quad (20)$$

The step response of the original system and the obtained reduced models by the proposed method and the typical Padé approximant are shown in Figure 1, which shows that the achieved result from the proposed method is much closer to the original system with respect to the typical Padé approximant.

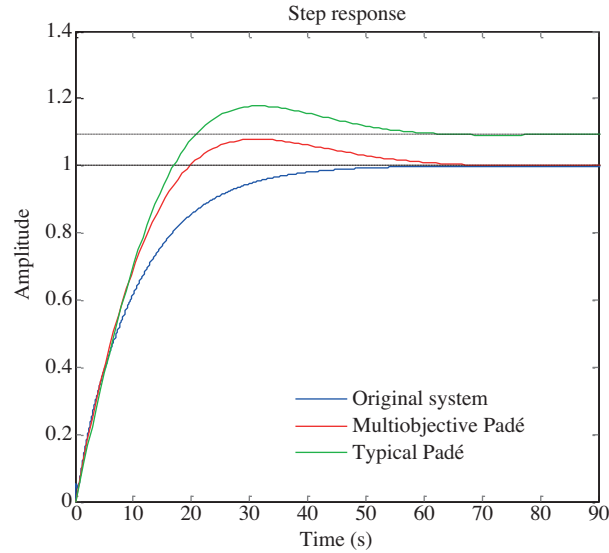


Figure 1. Step response of the full-order and reduced-order model by the proposed method (multiobjective Padé) and typical Padé approximant for test system 1.

Furthermore, the specifications of the proposed method, such as the maximum overshoot, rise time, settling time, steady-state value, ISE, H_2 norm, and H_∞ norm, are compared with the full-order system and the typical Padé approximant and are shown in Table 1. Moreover, the H_∞ norm of the error between the step responses of the full- and reduced-order models ($e = |y - y_r|$) is given in Table 1, which shows that the proposed method performs better than the typical Padé approximant.

Test system 2: The second system to be reduced is the following ninth-order boiler system [33].

$$\begin{aligned}
 x(t) = & \begin{bmatrix} -0.910 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -4.449 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -10.262 & 571.479 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -571.479 & -10.262 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -10.987 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -15.214 & 11.622 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -11.622 & -15.214 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -89.874 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -502.665 \end{bmatrix} \\
 \times x(t) + & \begin{bmatrix} -4.336 \\ -3.691 \\ 10.141 \\ -1.612 \\ 16.629 \\ -242.476 \\ -14.261 \\ 13.672 \\ 82.187 \end{bmatrix} u(t)
 \end{aligned}$$

$$y(t) = \begin{bmatrix} -0.422 & -0.736 & -0.00416 & 0.232 & -0.816 & -0.715 & 0.546 & -0.235 & -0.080 \end{bmatrix} x(t)$$

Table 1. Comparison of the proposed method with the full-order system and typical Padé approximant for Test system 1.

	Over shoot (%)	Rise time (s)	Settling time (s)	Steady state	ISE	H_2 norm	H_∞ norm	H_∞ norm of error
Original system	0	22.7	40	1	-	0.2740	1	-
Proposed method	7.61	14.3	53	1	0.5377	0.2468	1.05	0.1517
Typical Padé	7.46	15.1	50.4	1.1	2.8887	0.2579	1.13	0.2406

Suppose that a third-order approximant is required:

$$G_r(s) = \frac{a_1 s^2 + a_2 s + a_3}{s^3 + b_1 s^2 + b_2 s + b_3}. \quad (21)$$

Using the proposed method, the obtained reduced system is as follows:

$$G_{Multi\ Objective}(s) = \frac{148.12856s^2 + 4398.96963s + 4725.72521}{s^3 + 29.90996s^2 + 429.17178s + 371.99085}. \quad (22)$$

Moreover, the system is reduced by the typical Padé approximant and the following system is obtained:

$$G_{typical\ pade}(s) = \frac{145.36242s^2 + 4312.31031s + 4701.85734}{s^3 + 23.23900s^2 + 420.38264s + 371.29177}. \quad (23)$$

The step response of the original system and the obtained reduced models by the proposed method and the typical Padé approximant is shown in Figure 2. The simulations confirm that the proposed model's response is very similar to the full-order model's response.

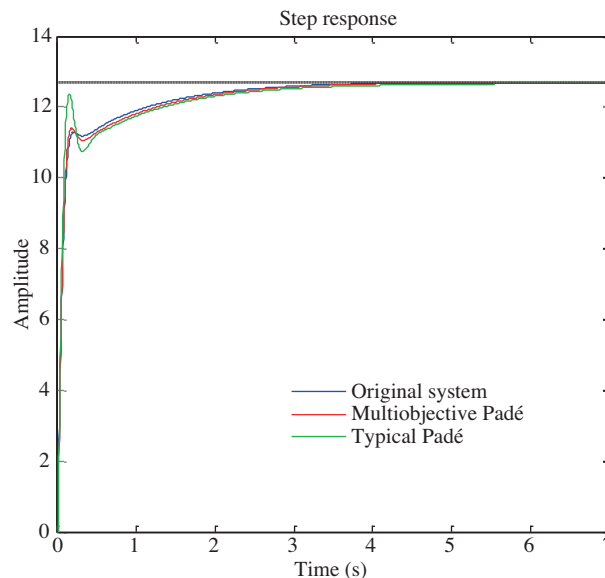


Figure 2. Step response of the full-order and reduced-order model by the proposed method and typical Padé approximant for test system 2.

Once again, the specifications of the proposed method, such as the maximum overshoot, rise time, settling time, steady-state value, ISE, H_2 norm, and H_∞ norm, are compared with the full-order system and the typical Padé approximant and are shown in Table 2. Moreover, the H_∞ norm of the error between the step responses of full- and reduced-order models ($e = |y - y_r|$) is given in Table 2. It is clearly seen that the specifications of the reduced-order model that are achieved by the proposed method are close to the specifications of the original system.

Table 2. Comparison of the proposed method with the full-order system and typical Padé approximant for Test system 2.

	Over shoot (%)	Rise time (s)	Settling time (s)	Steady state	ISE	H_2 norm	H_∞ norm	H_∞ norm of error
Original system	0	0.543	2.28	12.7	-	33.3556	12.7182	-
Proposed method	0	0.612	2.36	12.7	0.0269	34.2032	12.7039	0.3291
Typical Padé	0	0.0918	2.39	12.7	0.3561	38.2471	12.6635	1.64

Test system 3: In [26], a procedure was presented to obtain the Routh-Padé approximation using the Luus-Jaakola algorithm. To compare the proposed method with the Luus-Jaakola algorithm, the system given in [26] is adopted, which is a third-order system:

$$G(s) = \frac{8s^2 + 6s + 2}{s^3 + 4s^2 + 5s + 2}. \quad (24)$$

The above system is reduced by the Luus-Jaakola algorithm to a second-order system as follows:

$$G_{Luss}(s) = \frac{8s + 8.129}{s^2 + 4.307s + 8.129}. \quad (25)$$

Using the proposed method, the obtained reduced system is as below:

$$G_{Multi} = \frac{7.1132s + 4.9186}{s^2 + 3.1094s + 4.9924}. \quad (26)$$

The comparison of the proposed method with the Luus-Jaakola algorithm in [26] is shown by Figure 3 and Table 3, which illustrate a better performance of the proposed method.

Table 3. Comparison of the proposed method with the full-order system and Luus-Jaakola algorithm for Test system 3.

	Over shoot (%)	Rise time (s)	Settling time (s)	Steady state	ISE	H_2 norm	H_∞ norm	H_∞ norm of error
Original system	86.5	0.129	6.74	1	-	3.0368	2.3001	-
Proposed method	92.8	0.137	3.32	0.985	0.0254	2.9859	2.3964	0.1139
Luus-Jaakola method	66.1	0.13	1.71	1	0.1404	2.8937	1.9772	0.3425

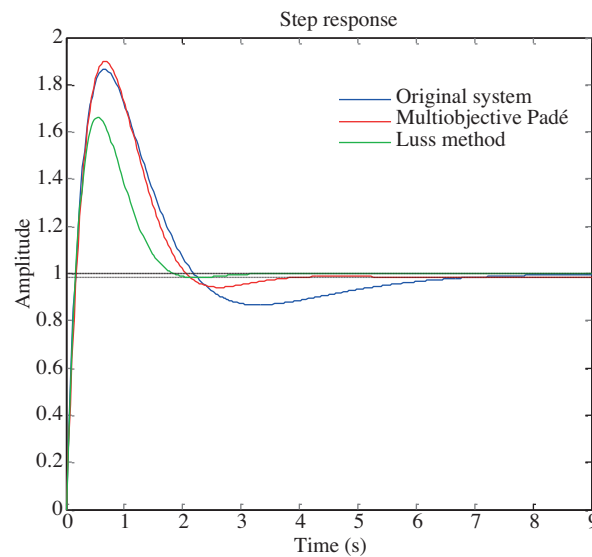


Figure 3. Step response of the full-order and reduced-order model by the proposed method and Luus–Jaakola algorithm for test system 3.

5. Conclusion

A new method based on Routh–Padé approximation was investigated for order reduction. In this method, first, the system was expanded to get the first r time moments/Markov parameters. Using the concept of the time moments/Markov parameters and the HS algorithm, the unknown coefficients were determined. To get a better result, a multiobjective criterion was used based on the weighted sum approach. Apart from minimizing the errors between a set of subsequent time moments/Markov parameters of the system and those of the model, the error between the singular values of the reduced-order system and those of the original system were minimized. To satisfy the stability, the Routh criterion was applied. To present the accuracy and efficiency of the method, 3 systems were reduced by the proposed method. The results obtained showed that the proposed approach has a better accuracy and efficiency with respect to conventional order reduction methods.

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