

Minimal controller synthesis algorithms with output feedback and their generalization

Ata SEVİNÇ*

Department of Electrical and Electronics Engineering, Faculty of Engineering, Kırıkkale University, Kırıkkale, Turkey

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Abstract: This paper proposes several improvements in the minimal controller synthesis algorithms, which were developed for a class of nonlinear systems with uncertainties. The major proposition is that only the output feedback is enough to control some nonlinear systems without an observer while the existing algorithms require the complete state feedback. Next, the extended version and the parameter identification technique of the minimal controller synthesis algorithm are combined in a single method estimating some more parameters for some known forms of nonlinearities. This is applicable with or without the improvement of removing the need for using the complete state feedback. It is proven that the scaling factors in the algorithms can be variable and different for each component. The algorithms are applicable to some noncanonical forms of systems. All the propositions and some of the existing algorithms are generalized in a single form with some options. A speed-sensorless DC motor control simulation is also included with quite exaggerated noisy conditions and the results verify that the proposed modifications make the minimal controller synthesis algorithms much more efficient model reference adaptive control algorithms for certain types of systems. The proposed method, which does not use the speed measurement, yields fewer ripples than the existing method, which uses the speed measurement. In addition, the proposed method accurately estimates the armature resistance and inductance while the existing method fails to do this.

Key words: Output feedback control, identification and control methods, DC motor control

1. Introduction

The minimal controller synthesis (MCS) algorithm was developed by Stoten and Benchoubane [1–5] from Landau's model reference adaptive controller [6]. There are a few versions of this algorithm: the basic MCS algorithm; the extended MCS (EMCS) algorithm [4], which can deal with larger and any kinds of disturbances; and the MCS identification (MCSID) algorithm [5], which can estimate system parameters under disturbance-free conditions. These MCS algorithms do not require any information about the system parameters except the signs of the input coefficients; however, they require full information about the system state in the phase-variable canonical form. Some other improvements have been proposed in literature. The decentralized MCS algorithm [3] was developed to control interconnected subsystems composing a large-scale system using their local information without requiring communication among individual controllers. Hodgson and Stoten [7] showed how to apply the MCS algorithm to a plant with an unknown order but a known relative degree. Thawar [8] modified the algorithm by adding a term of reference model state multiplied by another gain matrix

*Correspondence: a.sevinc@kku.edu.tr

to the control in order to enhance the robustness of the controlled system against disturbances. However, all of these algorithms still require the complete state feedback. In most cases, the state vector is not completely measurable. It can be calculated by taking derivatives of the measured output, but this is not practical as the noise will increase in the derivatives. For this reason, the existing MCS algorithms are not suitable for systems consisting of high-order blocks in the canonical form. Therefore, almost all the MCS applications in literature use reference models with relative degree of 1 or 2 (e.g., [9–13]).

This paper solves this problem with a modification to the MCS algorithm for a class of systems. With this modification, the controller uses only the output feedback, i.e. a lower number of measurements, and becomes quite insensitive to the measurement noise without an observer. This paper also proposes a generalization combining the EMCS and MCSID algorithms for some known forms of nonlinearities estimating some more parameters, which extends the applicability area of the MCSID algorithm and gives smoother results than the EMCS algorithm. The generalization is applicable either with the state feedback or output feedback types. It is shown that the algorithms are applicable for some noncanonical forms, which also extends the applicability area. It is also shown that scaling factors can be variable and different for each component, which allows us to adjust them according to different error levels in transient and steady states, resulting in a more accurate and smoother control.

This paper concentrates on 3 versions of the MCS algorithms: the basic MCS, EMCS, and MCSID algorithms. Their state feedback types are briefly given in the next section. In Section 3, modification for the output feedback type and the generalization are introduced with their proofs and some remarks. Simulation results presented in Section 4 show the success and practical value of the proposals on a quite exaggerated noisy DC motor control example. Even though the state feedback type uses a rotational sensor, the output feedback type is speed-sensorless, yet the latter controls the motor more accurately and can estimate some of the motor parameters while the former fails in estimations.

2. Original MCS algorithms (state feedback type)

The MCS approach considers the plants in the following form:

$$\dot{x}(t) = Ax(t) + Bu(t) + d(t, x), \quad (1)$$

where $u(t) \in \mathbb{R}^q$ and $x(t) \in \mathbb{R}^n$ are the input and state of the system respectively; A and B are constant matrices with proper dimensions; and $d(t, x)$ denotes any unmodeled terms, nonlinearities, external disturbances, and parameter variations. The reference model is given by:

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t), \quad (2)$$

where $r(t) \in \mathbb{R}^q$ is a reference signal, A_m and B_m are constant matrices with proper dimensions, and $x_m \in \mathbb{R}^n$ is the state of the reference model, which is the desired state vector for the actual system; therefore, $r(t)$ and $x_m(t)$ are assumed to be bounded and A_m is assumed to be stable. The MCS algorithms also assume that both the plant and reference models are in the phase-variable canonical form [14], where the matrices are in special forms as:

$$A = \begin{bmatrix} A_{11} & \dots & A_{1q} \\ \vdots & & \vdots \\ A_{q1} & \dots & A_{qq} \end{bmatrix}_{n \times n}, \quad B = \begin{bmatrix} B_1 & 0 & \dots & 0 \\ 0 & B_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & B_q \end{bmatrix}_{n \times q}, \quad d(t, x) = \begin{bmatrix} d^1(t, x) \\ d^2(t, x) \\ \vdots \\ d^q(t, x) \end{bmatrix}_{n \times 1},$$

where

$$d^i(t, x) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ d_{n_i}^i(t, x) \end{bmatrix}_{n_i \times 1}, \quad A_{ii} = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \dots & 0 & 1 \\ -a_{ii1} & -a_{ii2} & \dots & \dots & -a_{iin_i} \end{bmatrix}_{n_i \times n_i}, \quad B_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_{ii} \end{bmatrix}_{n_i \times 1},$$

and dimensions of A_{ij} and B_i are $n_i \times n_j$ and $n_i \times 1$ respectively for $i = 1, \dots, q; j = 1, \dots, q$ with $\sum_{i=1}^q n_i = n$.

The first $n_i - 1$ rows of A_{ij} for $i \neq j$ are zero. Similarly, the reference model of Eq. (2) is also defined in the same form and the same dimensions except for the disturbance:

$$A_m = \begin{bmatrix} A_{m,11} & \dots & A_{m,1q} \\ \vdots & & \vdots \\ A_{m,q1} & \dots & A_{m,qq} \end{bmatrix}_{n \times n}, \quad B_m = \begin{bmatrix} B_{m,1} & 0 & \dots & 0 \\ 0 & B_{m,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & B_{m,q} \end{bmatrix}_{n \times q}.$$

A_m and B_m are in form of A and B , respectively, and nonzero elements of $B_{m,i}$ are denoted by $b_{m,ii}$. In addition, parameters in A and B are assumed to be unknown, except for the signs of b_{ii} parameters.

For the above form of matrices, due to the phase-variable canonical form, there exist unique matrices $A_B \in \mathbb{R}^{q \times n}$ and $B_B \in \mathbb{R}^{q \times q}$, and a unique vector $d_B(t, x) \in \mathbb{R}^q$ such that:

$$B_m A_B = A_m - A, \tag{3}$$

$$B_m B_B = B, \tag{4}$$

$$B_m d_B(t, x) = d(t, x). \tag{5}$$

Using the notation $X_{i\bullet}$ for the i th row of any X matrix, it can be seen by inspection that the solutions are:

$$A_B = \begin{bmatrix} \frac{1}{b_{m,11}}(A_m - A)_{i\bullet} \\ \frac{1}{b_{m,22}}(A_m - A)_{i\bullet} \\ \vdots \\ \vdots \\ \frac{1}{b_{m,qq}}(A_m - A)_{i\bullet} \end{bmatrix}, \tag{6}$$

$$B_B = \text{diag} \left(\frac{b_{11}}{b_{m,11}}, \frac{b_{22}}{b_{m,22}}, \dots, \frac{b_{qq}}{b_{m,qq}} \right), \tag{7}$$

$$d_B(t, x) = \text{diag} \left(\frac{d^1(t, x)}{b_{m,11}}, \frac{d^2(t, x)}{b_{m,22}}, \dots, \frac{d^q(t, x)}{b_{m,qq}} \right). \tag{8}$$

The original MCS algorithms, which are based on the state feedback, use the unique positive definite solution P of

$$P A_m + A_m^T P = -Q \tag{9}$$

for a desired $Q \geq 0$ to construct

$$C_e = B_m^T P \quad . \quad (10)$$

This means that the triple $\{A_m, B_m, C_e\}$ satisfies the hyperstable linear block condition [15]. C_e will be used in the gain formulae with

$$y_e = C_e e, \quad (11)$$

$$e(t) = x_m(t) - x(t). \quad (12)$$

In the original algorithms, y_e is not considered as the output error since the complete state vector is assumed to be accessible.

2.1. Basic MCS algorithm

The MCS input consists of feedback and feedforward parts:

$$u = K_x(t)x + K_r(t)r, \quad (13)$$

where $K_x(t)$ and $K_r(t)$ are the feedback and feedforward gain matrices, respectively, and they are calculated as:

$$K_x(t) = \int \alpha y_e x^T d\tau + \beta y_e x^T, \quad (14)$$

$$K_r(t) = \int \alpha y_e r^T d\tau + \beta y_e r^T, \quad (15)$$

where α and β are desired scalars satisfying

$$B_B \alpha > 0 \text{ and } B_B \beta \geq 0. \quad (16)$$

The integrals are indefinite since any initial gain matrices are acceptable.

It was shown in [2] that if the disturbance $d(t, x)$ term is assumed in the form of

$$d(t, x) = \delta A(t) x + \delta B(t) u, \quad (17)$$

where nonzero parts of $\delta A(t)$ and $\delta B(t)$ are slowly varying and in the same structures as $(A_m - A)$ and B , respectively, then the error with the basic MCS algorithm approaches zero asymptotically provided that conditions in Eq. (16) are satisfied.

2.2. MCS identification algorithm

It was also shown in [5] that the MCS algorithm can be used to estimate parameters in A and B matrices if there is no disturbance, i.e. $d(t, x) = 0$. This is called the MCS identification (MCSID) algorithm. However, the control signal u must be within its saturation limits for a proper convergence of the parameter estimates and the reference signal r must be persistently exciting, i.e. it must have a sufficiently large number of frequency components. r has sufficient frequency spectra if it contains at least $n + q$ distinct frequencies. The estimations for the parameter matrices B and A are then given in the steady state by

$$\hat{B} = B_m K_r^{-1}, \quad (18)$$

$$\hat{A} = A_m - \hat{B} K_x, \quad (19)$$

respectively, assuming that $|K_r| \neq 0$.

2.3. EMCS algorithm

In another study [4], the MCS algorithm was extended to include some disturbances other than that in the form of Eq. (17). In this extension, which is called the EMCS algorithm, the control is:

$$u = K_x(t)x + K_r(t)r + N \text{ sign } (y_e),$$

where $K_x(t)$ and $K_r(t)$ are the same as given by Eq. (14) and Eq. (15), respectively; $\text{sign } (y_e)$ is a q -vector consisting of the signs of corresponding elements of y_e , and

$$N = \text{diag } (N_1, N_2, \dots, N_q),$$

such that

$$\frac{b_{ii}}{b_{m,ii}} N_i \geq \max |d_i(t, x)| \quad ; \quad i = 1, \dots, q. \tag{20}$$

Therefore, bounds that are greater than or equal to $\max |d_i(t, x)|$ for $i = 1, \dots, q$ should be known to apply to the EMCS algorithm. The parameter estimations of Eq. (18) and Eq. (19) are not valid in the EMCS algorithm.

Since the switching function $\text{sign } (y_e)$ causes chattering, its effect can be made smoother using a quasi-switching function like $\frac{y_{ei}}{|y_{ei}| + \xi_i}$ instead of $\text{sign } (y_{ei})$, where ξ_i is a small positive constant for $i = 1, \dots, q$ [4].

For the switching gains, an adaptive scheme [16] was proposed with positive adaptation weights (γ_i) as:

$$N_i = \int \gamma_i y_{ei}(t) dt. \tag{21}$$

3. Modifications and generalization of the algorithms

3.1. The MCS algorithm with output feedback

The main proposition in this paper is to use the reference model state instead of the actual state feedback for a class of systems. This requires further changes in the algorithm. First, with the output definition

$$y = C_e x, \tag{22}$$

y_e in Eq. (11) is assumed to be the output error as:

$$y_e = C_e x_m - y = C_e e. \tag{23}$$

However, while the MCS algorithms with state feedback assign C_e for a desired $Q \geq 0$ such that the triple $\{A_m, B_m, C_e\}$ satisfies the hyperstable linear block condition given by Eqs. (9) and (10), the proposed MCS algorithms with output feedback assume a priori that the triple $\{A, B_m, C_e\}$ satisfies this condition. The major proposition in this paper is to use

$$u = K_{x_m}(t)x_m + K_r(t)r \tag{24}$$

with

$$K_{x_m}(t) = \int \alpha y_e x_m^T d\tau + \beta y_e x_m^T, \tag{25}$$

and the same $K_r(t)$ as in Eq. (15).

Since the proposed MCS algorithms use only the accessible variables, the need for the complete state information is removed if the triple $\{A, B_m, C_e\}$ satisfies the hyperstable linear block condition given by Eqs. (9) and (10) instead of $\{A_m, B_m, C_e\}$.

3.2. Generalization of the MCS algorithms

Apart from the MCS algorithms with the output feedback, this paper includes some other propositions:

(i) Combining the MCSID and EMCS algorithms for some known form of nonlinearities identifying their coefficients.

(ii) Usage of different and variable α and β values for each gain component.

(iii) Application to some noncanonical form of systems.

All these algorithms for both the state feedback and output feedback types can be generalized in a single form.

3.2.1. Theorem

The system of Eq. (1) follows Eq. (2) with stable error dynamics if the first $n_i - 1$ rows of A_{ii} and A_{ij} blocks can consist of any known quantities provided that the same quantities are used in the corresponding positions of corresponding $A_{m,ii}$ and $A_{m,ij}$ blocks and

$$d_B(t, x) = \theta \eta(t, x), \tag{26}$$

where $\eta(t, x) \in \mathfrak{R}^l$ consists of locally Lipschitz functions in x , and $\theta \in \mathfrak{R}^{q \times l}$ is the matrix of unknown but slowly varying coefficients, by applying the generalized MCS (GMCS) input:

$$u = K_x^*(t)x^* + K_r(t)r + K_\eta(t)\eta(t, x^*) + s(y_e), \tag{27}$$

where x^* and $K_x^*(t)$ are x and $K_x(t)$ for the state feedback type but x_m and $K_{x_m}(t)$ for the output feedback type, respectively; $s(y_e)$ is a q -vector whose i th component is a switching function $N_i \text{sign}(y_{ei})$, and N_i values can be selected according to either Eq. (20) or the adaptive scheme of Eq. (21) as stated at the end of Section 2.3.

$$(K_x^*(t))_{ij} = \int \alpha_{ij} y_{ei} x_j^* d\tau + \beta_{ij} y_{ei} x_j^*; \quad i = 1, \dots, q; \quad j = 1, \dots, n \tag{28}$$

$$(K_r(t))_{ij} = \int \alpha_{i,(j+n)} y_{ei} r_j d\tau + \beta_{i,(j+n)} y_{ei} r_j; \quad i, j = 1, \dots, q \tag{29}$$

$$(K_\eta(t))_{ij} = \int \alpha_{i,(j+n+q)} y_{ei} \eta_j d\tau + \beta_{i,(j+n+q)} y_{ei} \eta_j; \quad i = 1, \dots, q; \quad j = 1, \dots, l \tag{30}$$

α_{ij} and β_{ij} coefficients can be variable and different for each gain component provided that $(b_{ii}/b_{m,ii})\beta_{ij}$ values must be nondecreasing and

$$(b_{ii}/b_{m,ii})\alpha_{ij} > 0 \text{ and } (b_{ii}/b_{m,ii})\beta_{ij} \geq 0 \tag{31}$$

for $i = 1, \dots, q; \quad j = 1, \dots, n + q + l$.

In addition, if a quasi-switching function $\frac{y_{ei}}{|y_{ei}|+\xi_i}$ is used instead of $\text{sign}(y_{ei})$, i.e. if Eq. (27) is replaced with

$$u_i = (K_{xm}^*x^* + K_r r + K_\eta \eta(t))_i + N_i \frac{y_{ei}}{|y_{ei}| + \xi_i}; \quad i = 1, \dots, q, \tag{32}$$

where ξ_i is a sufficiently small positive constant for $i = 1, \dots, q$, and r contains at least $p = n + q + l$ distinct frequencies, then as $t \rightarrow \infty$

$$\hat{\theta} = -B_B K_\eta, \tag{33}$$

$$\hat{B} = B_m K_r^{-1}, \text{ and} \tag{34}$$

$$\hat{A} = A_m - \hat{B} K_x^* \tag{35}$$

approach θ , B , and A , respectively.

3.2.2. Proof of Theorem

Lemma 1 *Given a stable matrix $G \in \mathbb{R}^{n \times n}$, strictly positive coefficients γ_{ij} s, and nonnegative and nondecreasing coefficients λ_{ij} s for $i = 1, \dots, q; j = 1, \dots, p$; $H \in \mathbb{R}^{n \times q}$, $F \in \mathbb{R}^{q \times n}$, and an arbitrary vector function $\omega(t) : [t_0, \infty) \rightarrow \mathbb{R}^p$ whose elements are bounded and piece-wise continuous, the equilibrium state of the set of differential equations*

$$\left. \begin{aligned} \dot{e} &= Ge + HK\omega \\ \dot{\Phi}_{ij} &= -\gamma_{ij} y_{ei} \omega_j; \quad i = 1, \dots, q; \quad j = 1, \dots, p \end{aligned} \right\} \tag{36}$$

with

$$\left. \begin{aligned} y_e &= Fe \\ K_{ij} &= \Phi_{ij} - \lambda_{ij} y_{ei} \omega_j; \quad i = 1, \dots, q; \quad j = 1, \dots, p \end{aligned} \right\} \tag{37}$$

is stable and $y_e(t) \rightarrow 0$ as $t \rightarrow \infty$ if the triple $\{G, H, F\}$ satisfies the hyperstable linear block condition given by Eqs. (9) and (10), i.e. a symmetric positive definite matrix P exists such that

$$\left. \begin{aligned} G^T P + PG &= -Q \\ PH &= F^T \end{aligned} \right\} \tag{38}$$

for some matrix $Q \geq 0$. Furthermore, if $\omega(t)$ is sufficiently rich, containing at least p distinct frequencies, $\Phi(t) \rightarrow 0$ as $t \rightarrow \infty$, where $\Phi(t)$ is the matrix of Φ_{ij} .

Proof of Lemma 1 Let us define the following Lyapunov function candidate:

$$V = e^T P e + \sum_{i=1}^q \sum_{j=1}^p \frac{1}{\gamma_{ij}} \Phi_{ij}^2, \tag{39}$$

where $P = P^T > 0$. Then:

$$\begin{aligned} \dot{V} &= e^T (G^T P + PG) e + 2 (e^T P H) (K\omega) - \sum_{i=1}^q \sum_{j=1}^p 2\Phi_{ij} y_{ei} \omega_j \\ \dot{V} &= e^T (G^T P + PG) e + 2 (e^T P H) (K\omega) - 2y_e^T \Phi \omega. \end{aligned} \tag{40}$$

By Eq. (37) and Eq. (38), $e^T PH = e^T F^T = y_e^T$. Then:

$$\begin{aligned} \dot{V} &= -e^T Qe + 2y_e^T (K - \Phi) \omega \\ \dot{V} &= -e^T Qe - \sum_{i=1}^q \sum_{j=1}^p 2\lambda_{ij} y_{ei}^2 \omega_j^2 \leq 0. \end{aligned}$$

Hence, the equilibrium state is uniformly stable and $e(t)$ and $\Phi(t)$ are bounded if $e(t_0)$ and $\Phi(t_0)$ are bounded. Since $\omega(t)$ is bounded, from [17], $e(t) \rightarrow 0$ (and $y_e(t) \rightarrow 0$) as $t \rightarrow \infty$. This, in turn, yields from Eqs. (36) and (37) that:

$$\lim_{t \rightarrow \infty} \dot{\Phi}(t) = 0, \tag{41}$$

$$\lim_{t \rightarrow \infty} K = \Phi, \tag{42}$$

$$\lim_{t \rightarrow \infty} K\omega(t) = \lim_{t \rightarrow \infty} \Phi\omega(t) = 0. \tag{43}$$

If $\omega(t)$ is sufficiently rich [17],

$$\lim_{t \rightarrow \infty} \Phi(t) = \lim_{t \rightarrow \infty} K = 0 \tag{44}$$

due to Eqs. (41)–(43). It should be noted that e and K are bounded with no constraints on $\omega(t)$. The boundedness of $\|\omega(t)\|$, however, assures the convergence of e to zero. Further, the richness of $\omega(t)$, which means that it contains at least p distinct frequencies [17], results in K converging to zero. It is also noted that the lemma is valid even if the λ_{ij} values are variable. γ_{ij} s can be variable, too, provided that they never decrease, since variable γ_{ij} s bring the term $\sum_{i=1}^q \sum_{j=1}^p \frac{-\dot{\gamma}_{ij}}{\gamma_{ij}^2} \Phi_{ij}^2$ in the expression of \dot{V} . \square

This lemma includes some developments of the first lemma in [18]. This is of multiple-input, multiple-output form and K consists of proportional and integral terms here, while [18] used single-input, single-output form with K consisting only of an integral term. In addition, variable coefficients are also considered here.

Lemma 2 Let us add another bounded term, $-H\mu$, into the first equation of Eq. (36) as

$$\dot{e} = Ge + HK\omega - H\mu,$$

where $\mu(t) : [t_0, \infty) \rightarrow \mathbb{R}^p$ is an arbitrary vector function whose elements are bounded and piece-wise continuous. If

$$\text{sign}(\mu_i) = \text{sign}(y_{ei}), \quad i = 1, \dots, q, \tag{45}$$

the results of Lemma 1 still hold, with the exception that $\Phi(t)$ may not approach zero as $t \rightarrow \infty$. That is, Eqs. (42)–(44) are not valid.

Proof of Lemma 2 Using the same Lyapunov function candidate as Eq. (39), \dot{V} is obtained with an additional term to Eq. (40):

$$\dot{V} = e^T (G^T P + PG) e + 2(e^T PH) (K\omega) - 2y_e^T \Phi\omega - 2e^T PH\mu,$$

where the additional term

$$-2e^T PH\mu = -2y_e^T \mu \leq 0$$

due to the conditions in Eq. (45). The sign of the rest of \dot{V} is not affected by the modification. Therefore, the results of Lemma 1 still hold, except for Eqs. (42)–(44), which fail now since $K\omega$ has been replaced by $K\omega - \mu$.

Application of the lemmas to the algorithms

Subtracting Eq. (1) from Eq. (2), the error dynamics equation can be written as

$$\dot{e} = A_m(x_m - x) + (A_m - A)x + B_m r - B(K_x x + K_r r) - d(t, x)$$

for the MCS input with state feedback, or as

$$\dot{e} = A(x_m - x) + (A_m - A)x_m + B_m r - B(K_{x_m} x_m + K_r r) - d(t, x)$$

for the MCS input with output feedback. If $d(t, x)$ is in form of Eq. (17), it can be considered within unknown terms of A and B and both equations can be combined as

$$\dot{e} = A^* e + B_m \begin{bmatrix} (A_B - B_B K_x^*) & (I - B_B K_r) \end{bmatrix} \begin{bmatrix} x^* \\ r \end{bmatrix}.$$

If $d(t, x)$ has an additional part in the form of Eq. (26), the error dynamics for the GMCS input including the switching function of the EMCS become:

$$\begin{aligned} \dot{e} = B_m \begin{bmatrix} (A_B - B_B K_x^*) & (I - B_B K_r) & -\theta - B_B K_\eta \end{bmatrix} \begin{bmatrix} x^* \\ r \\ \eta(t, x^*) \end{bmatrix} \\ + A^* e + B_m \{ \theta (\eta(t, x^*) - \eta(t, x)) - B_B N \text{sign}(y_e) \}, \end{aligned} \tag{46}$$

where A^* is A_m for the state feedback type but A for the output feedback type. The term $\theta (\eta(t, x^*) - \eta(t, x))$ in Eq. (46) disappears for the state feedback type but plays the role of $-d_B(t, x)$ in the EMCS algorithm for the output feedback type. If there is no extra parameter θ to identify, $l = 0$ is substituted for the dimensions of θ , K_η , and $\eta(\cdot, \cdot)$. That is, those terms vanish with their dimensions in that case. If the EMCS algorithm is out of consideration, then $N = 0$ is substituted.

Now, by Lemma 2, substituting A^* , B_m , C_e , $(b_{ii}/b_{m,ii})\alpha_{ij}$, and $(b_{ii}/b_{m,ii})\beta_{ij}$ for G , H , F , γ_{ij} , and λ_{ij} , respectively, ($i = 1, \dots, q$; $j = 1, \dots, n + q + l$), and assigning

$$\mu = \{ B_B N \text{sign}(y_e) - \theta (\eta(t, x^*) - \eta(t, x)) \},$$

$$K = \begin{bmatrix} (A_B - B_B K_x^*) & (I - B_B K_r) & -\theta - B_B K_\eta \end{bmatrix},$$

and

$$\omega = \begin{bmatrix} x^* & r & \eta(t, x^*) \end{bmatrix}^T$$

forces e to approach zero in the steady state. However, this does not force K to approach zero if $s_i(y_{ei}) = N_i \text{sign}(y_{ei})$ values are used for the i th component of the switching function in Eq. (27). Instead, $K\omega - \mu$ approaches zero and the switching function $s(y_e)$ causes chattering, preventing μ from going to zero.

If Eq. (32) is used instead of Eq. (27), then there always exist sufficiently small ξ_i values to ensure $\mu \rightarrow 0$, since $\eta(t, x)$ is locally Lipschitz in x and x approaches x^* even with the switching function $s(y_e)$. Adding the

quasi-switching functions speeds up the convergence of the state estimations as in the EMCS algorithm without causing chattering and allows the parameter estimation because

$$K\omega - \mu \rightarrow K\omega \rightarrow 0$$

in the steady state. The parameter estimation result of Lemma 1 is then also applicable. That is, if r contains $p = n + q + l$ (the dimension of ω) distinct frequencies, $K \rightarrow 0$ as $t \rightarrow \infty$, which means Eq. (33), Eq. (34), and Eq. (35) approach θ , B , and A , respectively.

3.3. Remarks

(i) A priori assumption that the triple $\{A, B_m, C_e\}$ satisfies the hyperstable linear block condition in the output feedback type is rarely more restrictive than that the triple $\{A_m, B_m, C_e\}$ satisfies the condition in the state feedback type for the same C_e , as practical systems such as electric motors and electric circuits are usually self-stable. Their stability problem usually arises with the error dynamics when they are controlled to follow a reference model.

(ii) The main restriction in the output feedback type is the lack of the ability to assign C_e freely as in the state feedback type. However, since it is more restrictive to have completely accessible states, the applicability area of the output feedback type will be wider than that of the state feedback type unless an observer is used.

(iii) Normally, looking at the stability proof of the MCS algorithms, one might expect that the convergence rate is mainly determined by $\lambda_{\min}\{Q\}/\lambda_{\max}\{P\}$; however, this ratio is usually limited with small values for the phase-variable canonical form of stable A_m or A matrices. For example, the ratio cannot be greater than 1 for 2×2 matrices. Therefore, it has very little importance on the convergence rate. This fact reduces the advantage of freely choosing P or Q to determine C_e in the existing MCS algorithms over the proposed ones, where C_e is fixed and hence P and Q are not arbitrary.

(iv) It is observed along a lot of simulations in this study that the convergence rate is mainly determined by the difference between A_m and A in both the state feedback and output feedback types of the MCS algorithms. The closer A_m is to A , the faster the convergence. Therefore, there is not a remarkable difference between the convergence rates of both types when both are applicable. However, the additional gain matrix K_η in the GMCS algorithm speeds up the convergence if used. The GMCS algorithm also introduces some benefits as stated in Section 3.2, which are valid for both types.

(v) The state feedback type algorithms are superior to the output feedback types only when A is unstable, where the output feedback type is not applicable. In the other cases where the state feedback type is applicable, the output feedback type is also applicable and advantageous to the state feedback type because it is less affected by the noise. If the triple $\{A, B_m, C_e\}$ satisfies the hyperstable linear block condition given by Eqs. (9) and (10), the main advantages of the output feedback type are removal of the need for complete state feedback and noise rejection.

(vi) The disadvantage of a noncanonical form application is that some of the parameter values must be known. It is advantageous especially when the transfer function is strictly positive real, but transformation into the phase-variable canonical form is prevented since derivatives of the input appear. An example is given in the next section.

4. Application to a speed-sensorless DC motor-generator set

Let us consider 2 separately excited DC machines coupled to each other with their shafts as shown in Figure 1. One of them, working as a motor, is fed by the control voltage $u = v_a$, and the other, working as a generator,

feeds a resistive load, R_L . The purpose of the control is to keep the load voltage at its set value even if the load changes. The generator's armature and load resistance values are assumed to be known. Since the speed is proportional to their total voltage, which is the induced electromotive force (EMF), without losing the applicability for this purpose, the shaft speed at its set value can be an alternative purpose of the control. The latter purpose is chosen in this simulation because it also shows the speed-sensorless control performance of the proposed algorithm. No measurement is taken from the generator side because such a measurement would be a kind of tachogenerator measurement, which is not allowed in speed-sensorless operations. The excitation currents are kept constant in the simulation, and hence the models of the motor and generator are in the same form as a DC servo motor model [19]. For the motor, this model is

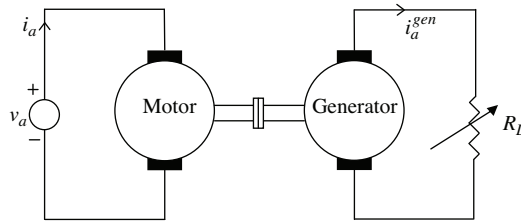


Figure 1. Armature circuits of a speed-sensorless separately excited DC motor-generator set.

$$\begin{bmatrix} \dot{\omega}_r \\ \dot{i}_a \end{bmatrix} = \begin{bmatrix} -f/J & K_t/J \\ -K_t/L_a & -R_a/L_a \end{bmatrix} \begin{bmatrix} \omega_r \\ i_a \end{bmatrix} + \begin{bmatrix} -1/J \\ 0 \end{bmatrix} T_L + \begin{bmatrix} 0 \\ 1/L_a \end{bmatrix} v_a \quad (47)$$

where ω_r , i_a , and v_a are rotor speed, armature current, and armature voltage, respectively; T_L is load torque; R_a and L_a are armature resistance and inductance; K_t is the back EMF constant, which equals the torque constant; f is the total friction constant of the motor and generator including the magnetic friction; and J is the total inertia of the motor and generator. Using similar symbols with the superscript “gen” for the generator, the electrical load of the generator is normally observed by the motor as

$$T_L = K_t^{gen} i_a^{gen} \quad (48)$$

There is no voltage source in the generator's armature circuit but the induced EMF is $K_t^{gen} \omega_r$, which produces the armature current

$$i_a^{gen} = \frac{K_t^{gen} \omega_r}{R_a^{gen} + R_L} \quad (49)$$

if the transient effect of L_a^{gen} is ignored. The load torque observed by the motor then becomes

$$T_L = \frac{(K_t^{gen})^2}{R_a^{gen} + R_L} \cdot \omega_r \quad (50)$$

Being proportional to the speed, this torque is in the form of magnetic friction, and as assuming f also includes its coefficient $\frac{(K_t^{gen})^2}{R_a^{gen} + R_L}$, we can get rid of the T_L/J term from the model. Now, assuming the parameters K_t^{gen} , R_a^{gen} , R_L , K_t , f , and J are known, the model takes a form to which the MCS algorithms can be applied. A reference model of the same form can be given as:

$$\begin{bmatrix} \dot{\omega}_{r m} \\ \dot{i}_{a m} \end{bmatrix} = \begin{bmatrix} -f/J & K_t/J \\ -K_t/L_{a m} & -R_{a m}/L_{a m} \end{bmatrix} \begin{bmatrix} \omega_{r m} \\ i_{a m} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/L_{a m} \end{bmatrix} r \quad (51)$$

where the subscript “ m ” denotes the reference model parameters and variables that can be different from those of the actual system. The MCS algorithms can now be applied with:

$$A_m = \begin{bmatrix} -f/J & K_t/J \\ -K_t/L_{am} & -R_{am}/L_{am} \end{bmatrix}, \quad x_m = \begin{bmatrix} \omega_{rm} \\ i_{am} \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 \\ 1/L_{am} \end{bmatrix},$$

$$A = \begin{bmatrix} -f/J & K_t/J \\ -K_t/L_a & -R_a/L_a \end{bmatrix}, \quad x = \begin{bmatrix} \omega_r \\ i_a \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/L_a \end{bmatrix}. \quad (52)$$

In this example, the MCS algorithm with state feedback assumes that both ω_r and i_a are accessible while the MCS with output feedback assumes that ω_r is inaccessible; hence:

$$y = i_a = C_e x \text{ with } C_e = [0 \quad 1]. \quad (53)$$

Both $\{A_m, B_m, C_e\}$ and $\{A, B_m, C_e\}$ satisfy the hyperstable linear block condition given by Eqs. (9) and (10). The purpose of the MCS algorithm is to make the actual system state follow the reference model state. However, to keep the reference model speed at its set value ω_r^* , the reference model input r is produced by a PID rule [20] as

$$r = \frac{K_D}{J} (-K_t i_{am} + f \omega_{rm}) + K_P (\omega_r^* - \omega_{rm}) + \int K_I (\omega_r^* - \omega_{rm}) dt, \quad (54)$$

where ω_r^* is assumed to be piece-wise constant and $\dot{\omega}_{rm}$ is replaced with its equivalent from the reference model state equations. To apply such a PID rule directly to the actual system without using a reference model would not be possible for speed-sensorless operation. The system was controlled with both types of the MCS algorithms in simulations with 0.1 ms time steps on a 48 V, 40 W, and 430 rpm DC motor with $R_a = 7.1 \Omega$, $L_a = 0.44\text{H}$, $K_t = 0.89\text{Nm/A}$, and $J = 0.029 \text{ kg m}^2$ and 2 friction values, 0.009 Nm s/rad for half load and 0.018 Nm s/rad for full load. In order to have a more realistic simulation, noise signals $(0.1 \text{ A}) \cdot \text{randn}(1)$ and $(5.0 \text{ rad/s}) \cdot \text{randn}(1)$ are added to the armature current and speed measurements respectively at every time step, where $\text{randn}(1)$ is MATLAB's scalar Gaussian noise generator with zero mean and unit covariance. Coefficients of $\text{randn}(1)$ are about 11% of the rated values for each measured variable. These are far beyond the measurement noise to meet even in an experimental work and will show us the robustness and practical value of the algorithms. The output feedback type of the MCS algorithm is not affected by the noise on the speed measurement as it is not utilized. The PID gains used for the calculation of r are $K_D = 0.005 \text{ Vs}^2/\text{rad}$, $K_P = 0.4 \text{ Vs}/\text{rad}$, and $K_I = 4 \text{ V}/\text{rad}$. In the simulations, the MCS parameters for both types are chosen as $\alpha_{11} = 1 \Omega \text{ s} \cdot \text{rad}^{-2}$, $\alpha_{12} = 1000 \text{ VA}^{-3} \text{ s}^{-1}$, $\alpha_{13} = 0.1 \text{ V}^{-1} \text{ A}^{-1} \text{ s}^{-1}$, $\beta_{11} = 0.01 \Omega \text{ s}^2 \text{ rad}^{-2}$, $\beta_{12} = 10 \text{ VA}^{-3}$, $\beta_{13} = 0.001 \text{ V}^{-1} \text{ A}^{-1}$, $N = 5 \text{ V}$, and $\xi = 0.01 \text{ A}$. Speed reversal, loading, and unloading tests are shown in the same figures. ω_r^* is kept at the rated speed, 430 rpm = 45 rad/s, except reverse speed intervals $5 \text{ s} < t \leq 10 \text{ s}$ and $15 \text{ s} < t \leq 20 \text{ s}$, when $\omega_r^* = -45 \text{ rad/s}$. As shown in Figures 2–5, convergence rates are almost the same for both types or slightly better for the output feedback type. However, the proposed method's average absolute speed and current errors are about 25% and 60% of the state feedback type's, respectively, in these simulations.

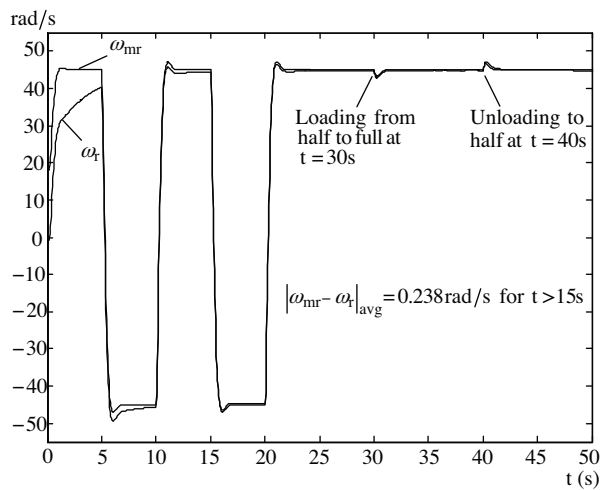


Figure 2. DC motor reference model and actual speeds using the GMCS algorithm with the state feedback.

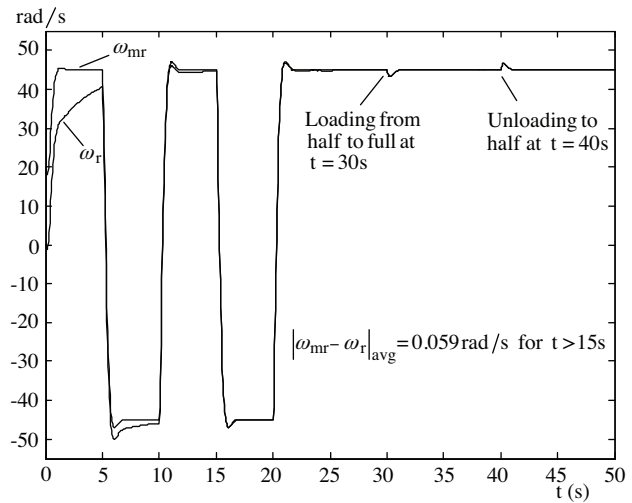


Figure 3. DC motor reference model and actual speeds using the GMCS algorithm with the output feedback.

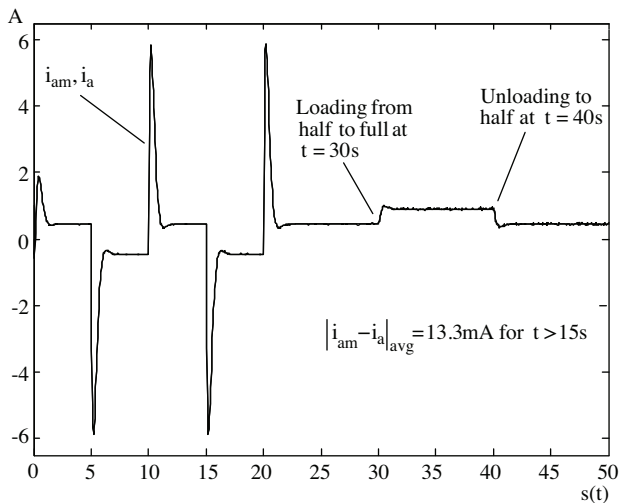


Figure 4. DC motor reference model and actual armature currents using the GMCS algorithm with the state feedback (both seem the same, but the ripples belong to the actual current).

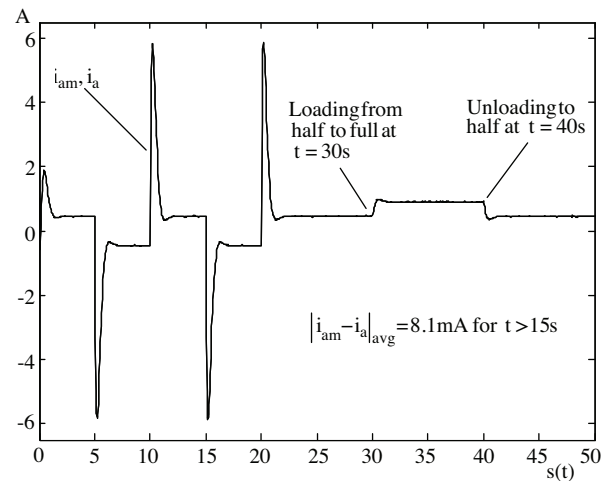


Figure 5. DC motor reference model and actual armature currents using the GMCS algorithm with the output feedback (both seem the same, but the ripples belong to the actual current).

It is seen from this example that the effects of the measurement noise on the states are extremely reduced. This result shows that both types of the MCS algorithms can deal with the noise very well even under such an exaggerated noisy condition.

Although the state convergence of the output feedback type is slightly better than that of the state feedback type, the superiority of the output feedback type is obvious in the parameter identification. The output feedback type estimates the armature resistance and inductance very well even under such noisy conditions as shown in Figures 6 and 7. It is usual that convergence times of the parameter estimations are quite longer than those of the states for almost all estimation systems where a high number of parameters and state variables are estimated from each output variable. Although the armature resistance estimation is very noisy, filtering has

no disadvantage since it is not used in the controller. Figure 6 also shows that the filtered value of \hat{R}_a , which is \hat{R}_a^f , is very accurate. \hat{L}_a is already accurate without filtering and its filtered value \hat{L}_a^f is much more accurate, as shown in Figure 7. Both filters are simple low-pass filters with 100 s and 20 s of time constants, respectively.

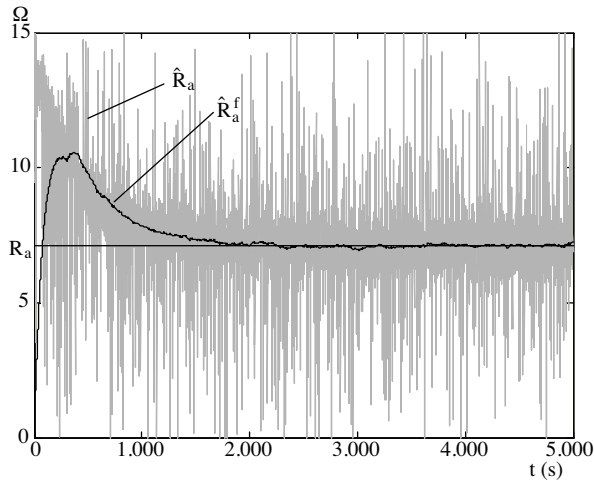


Figure 6. DC motor armature resistance estimation and its filtered value using the GMCS algorithm with the output feedback.

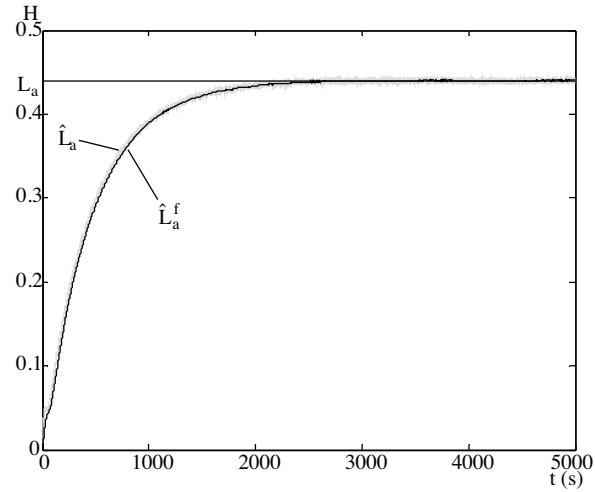


Figure 7. DC motor armature inductance estimation and its filtered value using the GMCS algorithm with the output feedback.

On the other hand, the state feedback type obviously fails to estimate the parameters under such noisy conditions. It is affected by the noise not only in the current measurement but also in the speed measurement differently from the output feedback type. When the state feedback type is simulated without a noise in the speed measurement for another comparison, its parameter estimations fail again, obviously due to the current measurement noise. The state feedback type has been simulated with $(0.01 A) \cdot randn(1)$ noise, which is 10% of that in the output feedback type simulation, in the current measurement and noiseless speed measurement for the last comparison. Then \hat{R}_a and \hat{L}_a get close to the actual values, about 4% and 7%, respectively, but convergence is still not achieved. Those failed results are not given in the figures here. These results clearly show the superiority of the output feedback type over the state feedback type in the GMCS algorithm.

5. Conclusion

In this paper, an important modification to the MCS algorithms has been suggested. The main advantage of the new algorithms is important: they need only the system output measurement, while the previous algorithms needed the complete state measurement. Even if the complete state information is available, this modification is advantageous to the previous algorithms since it makes the algorithms quite insensitive to noise. The modification makes the algorithms inapplicable if the system itself is unstable. However, this is not a serious disadvantage as most systems are already self-stable in practice. In many control applications, the stability problem is with their error dynamics when we want the state to follow a reference trajectory, usually not with the stability of the system itself.

The MCS algorithms require no prior knowledge of the system parameters for canonical forms except the signs of the input coefficients. The MCSID algorithm with output feedback can estimate slowly varying parameters. The EMCS algorithm with output feedback can cope with bounded nonlinearities and arbitrary

disturbances. These features are exactly the same as those of the state feedback type MCS algorithms.

The MCSID and EMCS algorithms for both types have been generalized in a single form with some extra contributions. This combination extends the applicability area of the MCSID algorithm and gives smoother results than the EMCS algorithm. First, more parameters can be identified for some known form of nonlinearities. An additional gain term in this use increases the speed of the state convergence. Second, different and variable scale coefficients can be used for each gain component. That is, the gains can be chosen according to the components' magnitudes and adjusted according to different error levels in transient and steady states, resulting in a more accurate and smoother control. Last, some noncanonical form of systems to which the MCS algorithms are applicable are ascertained. This extends the applicability area of the algorithms. Two lemmas have been introduced and the stability of all the proposed algorithms has been proven with them.

The success of the GMCS algorithm has been shown on a DC motor-generator set model for the output feedback type without a rotational sensor and for the state feedback type with a speed sensor. However, armature resistance and inductance were correctly estimated only with the proposed output feedback type under noisy conditions, where the state feedback type failed in these estimations.

As a result, the propositions in this paper make the MCS algorithms more efficient and flexible than the previous ones for certain types of systems. Some other types of systems may also be controlled in this way, but in order to guarantee the convergence, further investigations should be carried out for them.

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