

A methodology for memristance calculation

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Abstract: A memristor is a newly found fundamental circuit element whose behavior can be predicted using either the charge-dependent function called memristance or the flux-dependent function called memductance. Therefore, it is important to find the memristance or memductance function of a memristor. To the best of our knowledge, there is no methodology describing how to obtain the memristance function or memristor characteristic in the literature for this purpose as of yet. In this work, a methodology is suggested to find the memristance or memductance functions. The methodology suggests first doing several experiments with a memristor using a square-wave signal to acquire data and then using an algorithm inspired by the experience on ionic memristors reported in the literature to obtain its memristance and memductance functions. The methodology is applied to calculate the memristance function and memristor characteristic of a memristor emulator. Justifications for this method are also given.

Key words: Memristor modeling, modeling methodology, memristor emulator, memristive systems, memristance calculation

1. Introduction

A memristor is a new fundamental circuit element that dissipates power and has a memory. It was theoretically claimed to have existed by Chua in 1971 [1]. The notion of memristor was extended to systems called memristive systems in 1976 [2]. For a long time, the memristor was only seen in some theoretical papers [3–6]. An HP research team reported that they found the missing memristor [7]. However, the TiO₂ memristor voltage depends not only on the memristor charge but also on its current [7,8]. Only in small currents does it behave as the circuit element memristor that Chua described in 1971 [1]. In fact, if the concept of memristive systems in [2] is considered, it is a memristive system whose memristance depends on both the current and charge. Despite discussions among researchers about whether the memristor defined by Chua's paper in 1971 was found or not, seeking new kinds of memristors, their nonlinear modeling or new application areas have become new research areas [8–22].

Memristor models with current dependency or nonlinear dopant drift also exist in the literature [14,18,20,23]. However, the TiO_2 memristor model with a linear drift speed is still commonly used in the literature [14,15]. To the best of our knowledge, there is no methodology in the literature describing how to do an experiment for the calculation of the memristance function and obtain the memristor flux-charge characteristic. In [24], Williams described the difficulty experienced in understanding and figuring out how to model a TiO_2 memristor. In this work, a methodology is given to fill the need. To model a memristor, we need to calculate its memristance. The

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memristance described by Chua is:

$$M(q) = \frac{d\lambda}{dq} = \frac{v(t)}{i(t)}.$$
(1)

Its memristance must be obtained as a function of the memristor charge. How can we obtain it? One method might be applying a DC or AC voltage and measuring its current and voltage, and then by taking the integration of its current, its charge or memristance, as a function of it, can be calculated. However, there is the problem of the initial charge. When the initial charge is different than zero, for each different value of the memristor's initial charge, we will obtain a different charging characteristic. How can this problem be solved? We should make sure that the initial charge is zero or it is equal to a value already known. The problem can be overcome by making use of the memristor hysteresis reported in the literature, since TiO_2 or all other types of ionic memristors have a hysteresis phenomenon when they are excited by AC current. In ionic memristors, when the current flows in one direction, their memristance decreases and, in the opposite direction, their memristance increases. When a certain amount of charge flows through them, they reach their minimum or maximum memristance values. The TiO₂ memristor modeling experience reported in the literature is used as a starting point in this paper.

A memristor emulator is also used for the experiments in this work since memristor emulators are commonly used in experiments to prove the concepts in the literature [1,25–31]. It is assumed that the memristor emulator used in this work is able to mimic a memristor, which has charge or flux dependency with a saturation mechanism. Our methodology consists of doing an experiment feeding the memristor emulator with a squarewave voltage source by allowing full saturation or full resistive switching to occur in an alternance, and then postprocessing the data considering the saturated and unsaturated regions with the least-squares method. The justifications for the methodology are also given in the related sections.

The paper is arranged as follows. In Section 2, the Williams' TiO_2 memristor model with a linear dopant speed is explained, and in Section 3, the memristor emulator used in this paper is introduced. In Section 4, the experiments are done to acquire the necessary data. In Section 5, using least squares, the memristor charge-flux characteristic and memristance formula are derived. In Section 6, how to repeat the process for the memductance calculation is described. The results are summarized in the conclusion.

2. TiO_2 memristor model with a linear dopant drift speed

Until now, the most explicit memristor model was given by Stanley Williams of HP Labs. It is a first-order memristance model, i.e. it has a linear charge dependency or a linear dopant drift speed. The TiO₂ memristor is actually more complex than the first-order memristor model that Williams' team presented [3]. The first-order model is very easy to analyze and is still commonly used in the literature [14,15,19,31–34]. A TiO₂ thin-film memristor topology is shown in Figure 1. It consists of TiO₂ sandwiched between platinum contacts. The TiO₂ region has oxygen vacancies defining its circuit characteristic. The TiO₂ memristor principle can be explained using the equivalent circuits in Figure 2. When a positive voltage is applied, as shown in Figure 1, oxygen ions start diffusing within the TiO₂. If the TiO₂ is fully doped with oxygen ions, its memristance becomes maximum and equal to R_{OFF} . When the current starts flowing from the doped to the undoped region, its resistance or memristance decreases. It continues decreasing until the TiO₂ is fully doped and stays constant at this minimum value. When the current starts flowing from the undoped and stays constant at this maximum value. If the doped region has a length of w, the memristance as a function of diffusion length w,

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M(w), becomes equal to the total resistance of the doped and undoped regions:

$$M(w) = R_{ON}\frac{w}{D} + R_{OFF}\frac{D-w}{D}.$$
(2)

According to Williams' team, the diffused charge is proportional to the diffusion length when the memristor has a constant cross-section. Therefore, they were able to give a memristance formula as a function of the physical dimensions and parameters for the first time in the literature. In [24], Williams mentioned difficulties such as the memristor polarity that he experienced when modeling the memristor. As a result of his assumption, the TiO_2 memristor memristance is linearly dependent on the memristor charge and his memristance formula is:

$$M(q) = R_{ON} \frac{q}{q_{sat}} + R_{OFF} \frac{q_{sat} - q}{q_{sat}}$$
(3)

or

$$M(q) = M_o - Kq, (4)$$

where $M_O = R_{OFF}$ is the resistance if the memristor region is fully undoped or the maximum memristance, R_{ON} is the resistance if the memristor region is fully doped, K is the charge coefficient of the memristor, and q_{sat} is the maximum doped charge or the maximum memristor charge.



Figure 1. TiO_2 memristor topology.

Figure 2. The TiO_2 memristor and its equivalent circuit.

Considering saturation, at w = D or $q = q_{sat}$:

$$M(q_{sat}) = R_{ON} = M_o - Kq_{sat} = M_{SAT},$$
(5)

and at q = 0 or w = 0, i.e. there is no doped region:

$$M(0) = R_{OFF}.$$
(6)

A memristance formula with the mobility of oxygen ions can be found in [7].

3. The memristor emulator

A memristor emulator is an electronic circuit that mimics a memristor. More information on memristor emulators can be found in [25–32]. The memristor emulator whose schematic is given in Figure 3 is used in the experiments. The memristor emulator is shown in Figure 4. When it is excited by a sinusoidal voltage source, it has a zero-crossing pinched hysteresis loop, as a memristor must have, as shown in Figure 5. At high frequencies, the emulator starts behaving as a resistor, as shown in Figure 6. The emulator is clearly able to mimic memristive behavior. Therefore, the emulator can be used in the suggested experiment to show the usefulness of the methodology.



Figure 3. Emulator schematic.



Figure 5. Memristor emulator hysteresis curve, which shows that the emulator is able to mimic memristor characteristics.



Figure 4. Emulator circuit.



Figure 6. Memristor emulator behavior at high frequencies.

4. Method of finding the memristor parameters and the experiment

Now we can use the knowledge of the TiO_2 memristor model to do a minimum number of experiments and to postprocess the data for modeling similar ionic memristors in this and the following sections as long as memristors have no other dependency except the charge or flux, in addition to inherently having a saturation mechanism. If a DC voltage is applied to a memristor long enough, the memristor saturates, i.e. its memristance would take minimum or maximum values. The delayed switching time, which is the time needed for saturation under a constant DC voltage, was investigated in [29]. The pulse alternance length must be a bit higher than the delayed switching time. As shown in Figures 7 and 8, for a constant voltage excitation, the current waveform has 2 regions, transient and steady-state, i.e. the unsaturated and saturated regions.



Figure 7. Unsaturated (transient) and saturated regions when excited with a positive DC voltage when memristance decreases in value and saturates.

Figure 8. Unsaturated (transient) and saturated regions when excited with a negative DC voltage when memristance increases in value and saturates.

If the memristor's initial charge is not zero, the transient region of the current would be different for each value of the initial charge, as shown in Figure 9. We could make saturation occur in one direction and then, by switching polarity, we could make the memristor emulator saturate in the opposite direction. As an alternative, instead of using a DC voltage, we could just use a square-wave voltage for that purpose, as shown in Figure 10, since saturation would occur again in the experimental system shown in Figure 10 with an alternance or a period long enough. In this methodology, the memristor emulator is excited by a square-wave voltage source for parameter extraction experiment(s).



Figure 9. Effect of the initial charge on the profile of memristor current when the memristance decreases in value and saturates.

Figure 10. Experimental circuit.



Figure 11. Flowchart for the experimental method.

The algorithm given in Figure 11 can be used to do the needed experiment. The memristor emulator is excited by a square-wave voltage. Preferably, low voltages should be used considering the power dissipation in this method. Its current and voltage waveforms are captured from a digital oscilloscope and are shown in Figures 12–15. In this memristor calculation methodology, low frequencies should be used considering the data acquisition speed, because at high frequencies, the memristor starts behaving as a resistor and its charge does not vary much. The frequency is too high or the alternance is too low for the application of the method for the current and voltage waveforms shown in Figures 12 and 13, since the emulator behaves as a resistor. Now we can make use of the saturation phenomenon by adjusting the pulse length long enough to let saturation occur. The method can be used for the waveforms shown in Figures 14 and 15, since the waveforms show both the saturated and unsaturated regions. Now curve-fitting can be applied to the acquired data in the next section.





Figure 12. When the frequency is high, the memristance cannot switch from a low to high or high to low resistance (memristance) state.

Figure 13. At a frequency high enough, the memristance starts behaving as a resistor.



Figure 14. Saturation phenomena.



Figure 15. Positive alternance voltage saturates the memristor with on-state memristance and negative alternance voltage saturates it with off-state memristance.

5. Memristance calculation and least square curve-fitting

The memristance can easily be calculated using the experimental data:

$$M(q) = \frac{d\lambda}{dq} = \frac{v(t)}{i(t)} = \frac{V_{dc}}{i(t)}.$$
(7)

Since the memristor is driven to saturation in an alternance, the current of the square-wave voltage response is separated into 2 regions, the unsaturated and saturated regions. In the saturated region, the current becomes constant, since the memristance is either minimum or maximum. If the memristor current is maximum:

$$i(t) = i_{ON} = \frac{V_{dc}}{R_{ON}} = \frac{V_{dc}}{M_{SAT}}.$$
 (8)

If the memristor current is minimum:

$$i(t) = i_{OFF} = \frac{V_{dc}}{R_{OFF}} = \frac{V_{dc}}{M_O}.$$
(9)

Therefore, the minimum and maximum memristances are:

$$R_{ON} = M_{SAT} = \frac{V_{dc}}{i_{ON}} \tag{10}$$

and

$$R_{OFF} = M_O = \frac{V_{dc}}{i_{OFF}}.$$
(11)

Next, in the unsaturated region, the memristor memristance is calculated as a function of time using Eq. (1), by dividing the memristor voltage by the memristor current. The memristor charge is calculated by integrating the memristor current with respect to time.

$$q(t) = \int_{t=0}^{t} i(t)dt.$$
 (12)

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The memristor charge value at which saturation occurs is:

$$q_{sat} = \int_{t=0}^{t=\tau_{sat}} i(t)dt.$$
(13)

Remembering $i(\tau_{sat}) = i_{OFF}$, τ_{sat} , the time at which current takes its extremum value can be found. τ_{sat} is also the time point between the saturated and unsaturated regions if only one alternance is considered. Now the memristor memristance can be drawn as a function of the memristor charge, as shown in Figure 16.

The memristor can be modeled as either charge- or flux-dependent [1]. In this section, the memristor is modeled as charge-dependent. If the memristor is not saturated, the memristance is a function of the memristor charge, which is an integration of the memristor current. In [32], it is shown that the Taylor series can be used to represent the memristance function. In this case, using the Maclaurin series, the memristance as a function of the memristor charge, q, becomes:

$$M(q) = M_0 + \sum_{k=1}^{\infty} \frac{\partial M^k(0)}{\partial q^k} \frac{q^k}{k!}.$$
(14)

If we take only the first n terms and ignore the remainder, the memristance is:

$$M(q) = M_o + K_1 q + K_2 q^2 + \dots + K_n q^n.$$
(15)

Now, the least square curve-fitting method can be used for calculation of the coefficients of the Maclaurin series of the memristance. First, both linear and quadratic curve-fitting are done, but it has been found that a quadratic curve-fitting provides a much better fit. The quadratic memristance function is:

$$M(q) = aq^2 + bq + c. \tag{16}$$

The quadratic coefficients are: $a = -3.0301e8 \ \Omega/C^2$, $b = -5.709e6 \ \Omega/C$, and $c = 32339 \ \Omega$. The memristance calculated from the experimental data and curve-fitted memristance is shown in Figure 16. The flux-charge characteristics of some conceptual and TiO₂ memristors are given in [1,7,24]. To obtain the characteristic, the memristor (emulator) flux is found by the integration of its memristance function with respect to the memristor charge.

$$\lambda = \int_{0}^{q} M(q) dq = aq^{3}/3 + bq^{2}/2 + cq$$
(17)

The memristor flux versus the memristor charge characteristic is shown in Figure 17.

6. Memductance calculation

As mentioned previously, a memristor can also be modeled as flux-dependent [24]. The same process done in the previous section can be repeated for finding the memductance of a memristor, which is reciprocal of memristance and a function of the memristor flux. The memductance is reciprocal to memristance and is given as a function of memristor flux [1]:



Figure 16. Memristance vs. memristor charge.

Figure 17. Memristor flux vs. memristor charge.

$$\psi(\lambda) = \frac{i}{V} = \frac{1}{M(q)}.$$
(18)

If a memductance function is to be found, the following steps should be taken. The memristor flux is:

$$\lambda(t) = \int_{t=0}^{t} v(t)dt.$$
(19)

The memristor flux value at which saturation occurs is:

$$\lambda_{sat} = \int_{t=0}^{t=\tau_{sat}} v(t)dt.$$
(20)

Within the interval memristor unsaturated, the memductance can be assumed as:

$$\psi(\lambda) = \psi_o + K_1 \lambda + K_2 \lambda^2 + \dots + K_n \lambda^n = \frac{i}{V}.$$
(21)

The maximum memductance is:

$$\psi_{sat} = \frac{1}{M_{sat}}.$$
(22)

Moreover, the minimum memductance is:

$$\psi_O = \frac{1}{M_O}.\tag{23}$$

By repeating a similar process as that done for the memristance function and using the same voltage and current data, a memductance function can also be calculated using the least-squares method.

7. Conclusion

A methodology was given to find memristor characteristics. A memristor characteristic is assumed to be independent of a memristor current. As a result, memristance is calculated as a function of memristor charge obtained using the experimental memristor voltage and current. In this methodology, first, the experimenter starts with an experiment at a frequency at which he can see both the hysteresis and saturation phenomenon in the same experiment for the given memristor. Next, the saturated and unsaturated regions are found, and then the desired memristance and memductance functions are calculated using the least squares, easily and quickly. Nonlinear curve-fitting or nonparabolic functions such as logarithmic or exponential may also be used to describe memristors by taking inspiration from their physical characterization.

This methodology can help newcomers doing research on memristors to make the charge- and fluxdependent models of ionic memristors quickly. Using microcontrollers, a digital signal processor (DSP), or data acquisition systems with the methodology can automatize characterization of the memristors and may decrease precious research time. The algorithm in Figure 11 can be used as a starting point for that purpose. Using a microprocessor, a DSP, or a data acquisition system, the methodology can be easily adopted to test memristive/memristor (memory) chips for measurements or for doing statistics to see their parameter variations.

The effect of temperature or current on the memristor characteristic was ignored throughout this study. Therefore, memristance was assumed to be temperature-independent. When there is current dependency or temperature dependency, it describes a higher order memristive system, which can be described by the concept of the memristive systems developed by Chua and Kang in 1976 [2], not the ideal circuit element memristor, which was described in 1971 [1]. If temperature dependency of a memristor or memristive system is to be investigated, the method can be easily modified by taking the temperature as a new parameter for this purpose.

For current-dependent memristors, more sophisticated models are needed. The effect of the excitation voltage or the memristor current on the memristance function can also be investigated by modifying this method for more circuit variables and using the least squares again.

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