Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2014) 22: 132 – 142

doi:10.3906/elk-1207-55

Adaptive feedback control for linear singular systems

Azita AZARFAR[1,*], **Heydar Toossian SHANDIZ**[1], **Masoud SHAFIEE**[2]

[1]Department of Electrical and Robotics Engineering, Shahrood University of Technology, Shahrood, Iran
[2]Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran

Received: 17.07.2012 • **Accepted:** 31.10.2012 • **Published Online:** 20.12.2013 • **Printed:** 20.01.2014

Abstract: An adaptive state feedback control approach for linear descriptor (or singular) systems is investigated in this paper, which makes the closed-loop dynamics of this class of systems regular, impulse-free, and stable. The proposed adaptive method is based on the Lyapunov theorem to ensure the admissibility of the descriptor system and it also provides the state tracking with the desired reference trajectory. First, a control law is generated to stabilize the equilibrium point of origin, and, after that, the method is extended to a state tracking controller. The presented controller not only gets the stable output tracking asymptotically but is also fairly simple compared to the existing methods in the literature proposed for the control problem of descriptor systems. Simulation results on controlling a sample descriptor system are presented to demonstrate the effectiveness of the proposed approach in this paper.

Key words: Singular system, adaptive feedback control, state tracking control, admissible singular system

1. Introduction

Singular systems have attracted interest in recent years because of special features that are not found in classical systems, such as impulse terms in state response, nonproperness of the transfer function, or noncausality between the input and output. Descriptor systems are more precise models for the realization of different practical systems, such as robotics, mechanical systems, or electrical circuits [1,2]. Due to the importance of descriptor systems, many control approaches have been extended successfully in the last 3 decades. The complex nature of this type of system causes many difficulties in the control strategy. In addition to being stable, it is necessary for the descriptor systems to be regular and impulse-free. A system that poses these 3 characteristics is known as an admissible system. The control objective is therefore making the system admissible.

Most of the control approaches that have been studied so far are in the fields of optimal control and robust control [3–6]. The other control methods, such as state feedback [7–9], intelligent control methods [10,11], and Lyapunov-based approaches [12–14], have also been extended to date. However, there are only a few papers on adaptive control of descriptor systems. The adaptive mechanism during the adaptive control can adjust the controller for a system with structural and parametric uncertainty to improve the system performance.

In this paper, an adaptive state feedback controller is designed to ensure that the closed-loop system is admissible and provides a state tracking ability. The controller is constructed from a state feedback approach with adaptive gains and a mechanism to adjust the feedback gains. Based on the Lyapunov stability theorem, we prove that the proposed controller can control the descriptor system to the fixed point of origin and, with some extensions, it is possible to apply the result to a desired set point or tracking problem. The proposed controller is fairly simple and easy to apply.

*Correspondence: azita.azafar@gmail.com

The rest of the paper is organized as follows. The problem formulation and some preliminaries are introduced in Section 2. After that, in Section 3, the adaptive control strategy is extracted. A simulation on a sample descriptor system is done and the results are presented in Section 3.1. In Section 4, the proposed method is extended to solve the tracking problem of singular systems. Section 5 concludes the paper.

2. Problem formulation and preliminaries

Consider the following general single-input descriptor system:

$$E\dot{x}\left(t\right)=Ax\left(t\right)+Bu\left(t\right), \tag{1}$$

where $x \in R^n$ is the vector of the system states and $u \in R$ is the control input. The matrices $B \in R^{n\times 1}, A, E \in R^{n\times n}$ are constant system matrices. The matrix E can be singular (Rank (E) $<$ n). For the unforced descriptor system of

$$E\dot{x}\left(t\right)=Ax\left(t\right), \tag{2}$$

the following definitions are introduced.

Definition 1

1. System (2) is regular if $|sE-A|$ is not identically 0 $\left(|.|=\det(.)\right)$. The regularity of a descriptor system ensures that x (t) has a unique solution [2].
2. System (2) has an impulse-free response if $\deg|sE-A| = rank\left(E\right)$ [2].
3. System (2) is stable if all of the roots of $|sE-A|=0$ are in the open left-half of the complex plane [2].
4. System (2) is called admissible if it is regular, stable, and impulse-free [15].

In the investigation of the stability of descriptor systems, many results in the sense of Lyapunov stability have been derived. For descriptor systems, the control objective is to make the system admissible. In [16], Zhang presented a theorem to ensure the system admissibility, as follows:

Theorem 1 System (2) is regular, impulse-free, and asymptotically stable if and only if there exists a matrix P satisfying the following equations:

$$\left\{\begin{array}{l} P^TA+A^TP=-Q \\ E^TP=P^TE\geq 0 \end{array}\right., \tag{3}$$

where the matrix Q is positive definite. The proof is found in [16]. We use Theorem 1 to extract the control law.

3. Controller design

Our goal in this paper is to find an adaptive state feedback controller

$$u=\theta x, \tag{4}$$

where θ is a $1\times n$ vector, and a mechanism to adjust θ such that the closed-loop system

$$E\dot{x}\left(t\right)=(A+B\theta)x\left(t\right) \tag{5}$$

is admissible.

To solve this problem, first of all, we consider some assumptions for the singular system.

Assumption 1 For the singular system (1), we can find a matrix P that satisfies the following condition:

$$E^T P = P^T E \geq 0. \tag{6}$$

Assumption 2 Knowing matrix P from Assumption (1), we assume that there will exist a θ^* with appropriate dimensions such that all of the eigenvalues of the following matrix Q are positive:

$$-Q = P^T(A + B\theta^*) + (A + B\theta^*)^T P, \tag{7}$$

where θ^* is also unknown and A and B are the system matrices.

Considering the above assumption, we propose our result in the following theorem.

Theorem 2 For system (1), we use the adaptive state feedback controller (4), where θ is an adaptive vector with the adaption law

$$\dot{\theta}^T = -\gamma x x^T P^T B, \tag{8}$$

in which γ is a positive scalar that is known as the adaption rate and P is a matrix that satisfies Eq. (6). Next, the closed-loop systems (5) and (8) will be admissible.

Proof Assume that we choose the control law as $u^* = \theta^* x$, while θ^* satisfies (7). Hence, based on Theorem 1, the closed-loop system (E, A^*) is stable and admissible where:

$$A^* = A + B\theta^*. \tag{9}$$

Hence, the control input u^* is our desired control law. Now we choose $u = \theta x$, and the control objective is that the control input u tends to u^*. In other words, the vector θ should approach θ^*.

Knowing this, with the rewriting of (5) using (9), we reach the closed-loop system dynamics as follows:

$$E\dot{x} = A^* x + B(\theta - \theta^*)x. \tag{10}$$

From Eq. (10), it is clear that if vector θ is equal to θ^*, then system (10) would be admissible. To minimize the parametric error and to stabilize the system, we consider the following positive Lyapunov function:

$$V = x^T E^T P x + \frac{1}{\gamma}(\theta - \theta^*)(\theta - \theta^*)^T. \tag{11}$$

Differentiating V, we get:

$$\dot{V} = \dot{x}^T E^T P x + x^T P^T E \dot{x} + \frac{2}{\gamma}(\theta - \theta^*)\dot{\theta}^T. \tag{12}$$

Using Eq. (10) in Eq. (12) results in:

$$\dot{V} = x^T[P^T A^* + A^{*T} P]x + 2x^T P^T B(\theta - \theta^*)x + \frac{2}{\gamma}(\theta - \theta^*)\dot{\theta}^T. \tag{13}$$

Next, based on Eqs. (7) and (9), we get:

$$\dot{V} = -x^T Qx + 2(\theta - \theta^*)[xx^T P^T B + \frac{1}{\gamma}\dot{\theta}^T]. \tag{14}$$

To let $\dot{V}$ be negative, we choose the adaption law as in Eq. (8), which results in:

$$\dot{V} = -x^T Qx. \tag{15}$$

As we know that Q is positive definite, it is easy to see that the closed-loop system is admissible based on Theorem 1. □

Needless to say, the proposed controller is fairly simple compared to the existing methods in the literature proposed for the control problem of descriptor systems. The adaptive state feedback control strategy is summarized in Figure 1. A simulation is done to demonstrate the performance of the proposed method for controlling a singular system to a fixed point of origin. The simulation is explained shortly. After that, in Section 4, we extend this method for state tracking control.

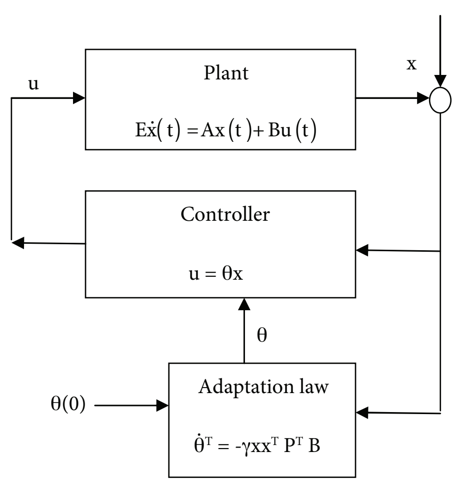

Figure 1. Adaptive state feedback control formulation.

3.1. Simulation results

To illustrate the effectiveness of the proposed controller, a simulation on controlling a sample descriptor system is performed. Consider a continuous descriptor system (1) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ -2 & 1 & 0 \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 \\ 3 & -1 & 0 \\ 4 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}. \tag{16}$$

The state's initial values are set to $x_1(0) = -2.2, x_2(0) = 4.1, x_3(0) = -1$. Next, the following control input is applied to the system:

$$u = \theta x = [\theta_1\theta_2\theta_3]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \dot{\theta}^T = -\gamma xx^T P^T B, P = \begin{bmatrix} 0.7029 & 0.9404 & 0.9001 \\ 0.4581 & 0.8770 & -0.4500 \\ -0.5948 & -0.5674 & 0.9001 \end{bmatrix}, \tag{17}$$

where $\gamma = 0.9$ is the adaption rate and matrix P satisfies Eq. (6). The initial value of vector θ is set to $[0.2, 1, -2.8]$. The state response is shown in Figure 2. We can see that the states quickly approximate 0, which verifies the validity of the proposed controller. Figure 3 illustrates the convergence of the adaptive parameters of vector θ. It can be seen that vector θ tends to a fixed value. The final value of θ that is considered to be an estimation of θ^* is [0.3800−5.2173−5.7965]. The control input is presented in Figure 4, which is smooth and limited. From all of the results, it is easy to see that the controller renders the closed-loop system stable. From Definition 1, it is clear that the closed-loop system with the final value of vector θ is an admissible one.

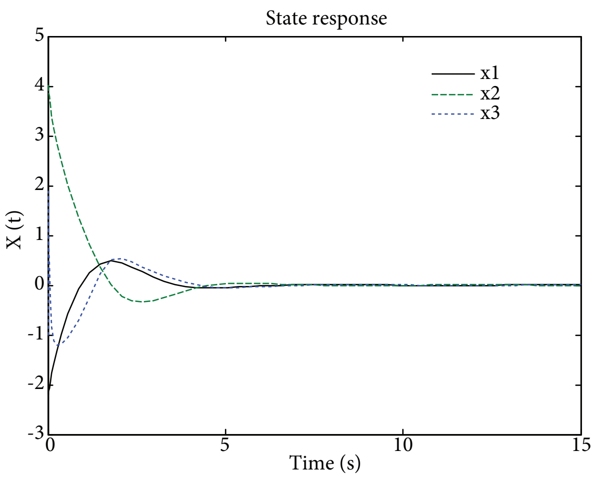

Figure 2. State variables of the closed-loop system.

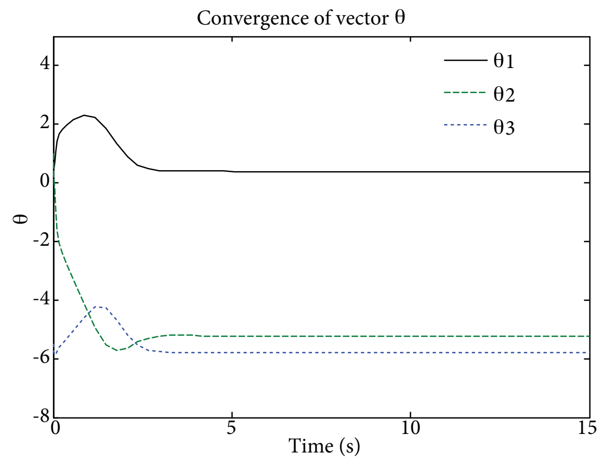

Figure 3. Convergence of the adjustable parameters (vector θ).

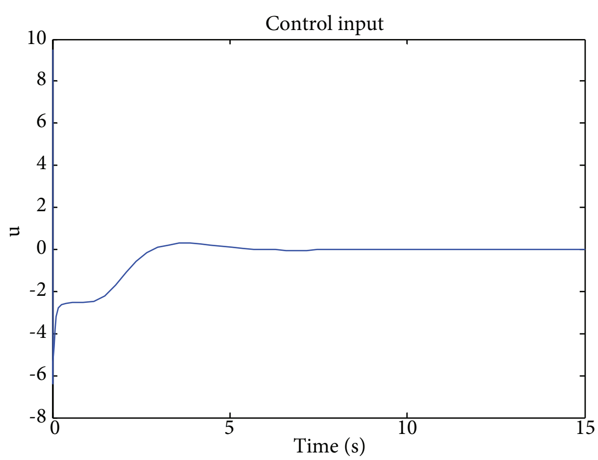

Figure 4. Control input.

4. State tracking control problem

In the previous section, the controlling problem of a singular system to a fixed point 0 was investigated. We will extend the results to achieve a state tracking controller, because in more practical applications, we need the system to track a desired trajectory. A reference trajectory should be smooth enough to have continued derivatives of any order required in the calculation. It should also satisfy the system dynamic equations. Thus, we introduce a reference trajectory x_d that satisfies the condition below.

$$E\dot{x}_d(t) = Ax_d(t) + Bu_d(t) \tag{18}$$

Hence, by choosing a determined x_d, we can calculate the reference input u_d as follows:

$$u_d(t) = (B^T B)^{-1} B^T (E\dot{x}_d - Ax_d), \tag{19}$$

where u_d is the nominal control input, which we name the tracking cost. u_d is obtained offline and it only depends on the system parameters and reference trajectory.

Next, a proposed controller,

$$u = u_d - \theta(x_d - x), \tag{20}$$

is employed using u_d and $\theta(x_d - x)$ as the tracking error feedback. Now the objective is to find an adaption mechanism to adjust the vector θ so that the states of the closed-loop system

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{21}$$

can track the desired trajectory x_d.

Our results of solving the tracking problem for linear singular systems are summarized in the following theorem.

Theorem 3 For system (1), using the adaptive state feedback controller (20), where θ is an adaptive vector with the adaption law:

$$\dot{\theta}^T = -\gamma e e^T P^T B, \tag{22}$$

in which γ is a positive scalar, P is a matrix that satisfies Eq. (6), and e is the tracking error, as follows:

$$e = x_d - x, \tag{23}$$

where x_d, u_d satisfies (18).

Next, the closed-loop systems (21) and (22) will asymptotically track the desired trajectory x_d.

Proof Subtracting the closed-loop dynamics (21) from the desired model (18) results in:

$$E(\dot{x}_d - x) = A(x_d - x) + B\theta(x_d - x). \tag{24}$$

Using Eq. (23) we have:

$$E\dot{e} = Ae + B\theta e. \tag{25}$$

Adding and deducting the statement $B\theta^* e$ in Eq. (25) results in:

$$E\dot{e} = (A + B\theta^*)e + B(\theta - \theta^*)e. \tag{26}$$

Using (9) in (26), the error dynamic will be:

$$E\dot{e} = A^* e + B(\theta - \theta^*)e. \tag{27}$$

That is very similar to (10). Hence, using a similar Lyapunov function as in Theorem 2:

$$V = e^T E^T P e + \frac{1}{\gamma}(\theta - \theta^*)(\theta - \theta^*)^T. \tag{28}$$

We can end in a corollary of Theorem 2, where if we choose the adaption mechanism (22), the derivative of the Lyapunov function (28) would be negative. Hence, the tracking error tends to 0 and, finally, we can conclude that the system states asymptotically track the desired trajectory, x_d. □

Two simulations on the different singular systems were done to demonstrate the effectiveness of the proposed controller to track a reference trajectory.

4.1. Simulations

To illustrate the adaptive feedback control scheme proposed in this paper, 2 simulations were performed. In the first simulation, the controller is applied to the sample system introduced by (16), and in the second, we use the proposed method for the state tracking of a DC motor modeled by singular systems.

4.1.1. Sample singular system

In this section, a simulation of the tracking control of the sample singular system is done. Consider the system introduced by (16), with the same initial values for x, θ. The objective is that the states of the system track the following desired trajectory:

$$x_d = [1 - \cos(t), \sin(t), -1 - \sin(t)]. \tag{29}$$

Applying the control law (20) and the adaption mechanism (22) to the system above results in the graphs shown in Figures 5–8. In this simulation, matrix P is selected as:

$$p = \begin{bmatrix} 1.1882 & 1.2846 & -1.8363 \\ 0.2321 & 0.7336 & 0.9182 \\ -0.0534 & -0.2967 & -1.8363 \end{bmatrix}, \tag{30}$$

which satisfies (6). The state responses of tracking the reference trajectory are shown in Figure 5, which shows the performance of the proposed controller. The tracking error is shown in Figure 6. It is clear that the error is acceptable and tends to 0. Figure 7 shows the convergence of the adaptive parameters of vector θ. It is obvious that vector θ tends to fixed values. The final value of θ is $[-1.6511 -1.11633.4305]$, which is considered to be an estimation of our ideal $\theta^*_.$ It is easy to observe that matrices P and θ^* satisfy (7). The control input is smooth and limited and is presented in Figure 8. The results show that the tracking error converges to 0 while adjusting gains adaptively.

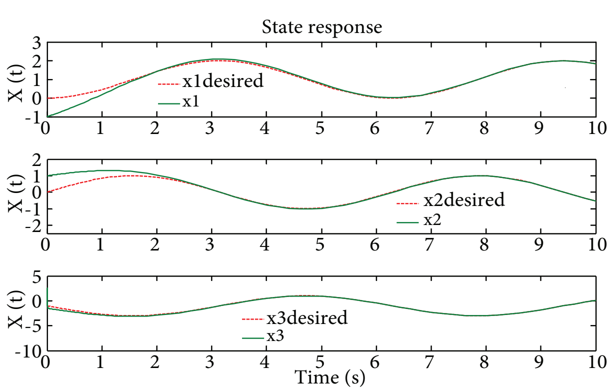

Figure 5. States and desired trajectories.

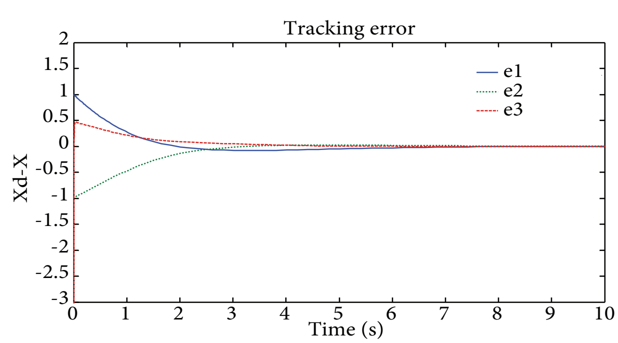

Figure 6. Tracking error.

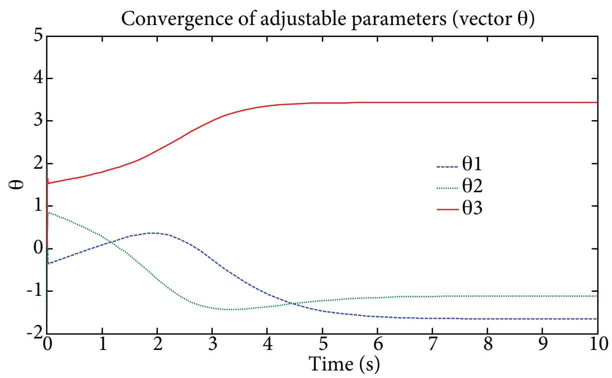

Figure 7. Convergence of the adjustable parameters (vector θ).

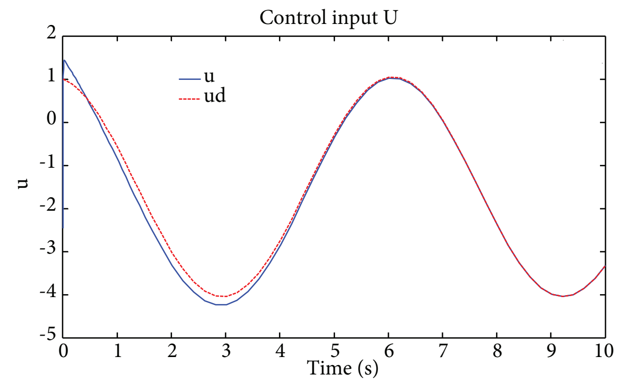

Figure 8. Control input u and nominal input ud.

4.1.2. DC motor

Figure 9 shows a simple DC motor. Since typically La is small, if we neglect the armature inductance, we can write the system equations as [17]:

$$\begin{cases} V_a(t) = Ri_a(t) + K_w\omega(t) \\ J\frac{d\omega}{dt} = K_t i_a(t) - b\omega(t) \end{cases}, \tag{31}$$

where $i_a(t), V_a(t), R$ are, respectively, the armature current, voltage, and resistance. J is the moment of inertia. K_t, K_w are, respectively, the torque and the back electromotive force, developed with a constant excitation flux. The constant b is the damping coefficient and $\omega(t)$ is the angular speed. The model of the motor consists of a differential equation and an additionally algebraic equation, which is the result of Kirchhoff's voltage law in the armature circuit. It is possible to model the DC motor using just one differential equation in ordinary state space, but using the singular model of the motor, we can reach a more precise model, which can also describe the behavior of the armature current. By choosing $x_1(t) = i_a(t), x_2(t) = \omega(t) and u(t) = V_a(t)$, we can rewrite the system equations as:

$$\begin{bmatrix} 0 & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} R & K_w \\ K_t & -b \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \end{bmatrix} u(t), \tag{32}$$

which is in the form of (1). Now we apply the proposed control method to the singular model of the DC motor. The system parameters are defined in the Table. The state's initial values are set to $x_1(0) = 3A, x_2(0) = 500rpm$. A matrix P that satisfies (6) is chosen as below:

$$p = \begin{bmatrix} -0.697 & -0.006 \\ 0 & 0.617 \end{bmatrix}. \tag{33}$$

The control objective is that the states of the motor (armature current and angular speed) track the following desired trajectory:

$$x_d = [(4.7 + 0.93e^{-2t}), (1500 - 1500e^{-2t})]. \tag{34}$$

The control law (20) with an adaption mechanism (22) is applied to the motor system and the results are shown in Figures 10–13. The state responses are displayed in Figure 10 and the tracking errors are shown in Figure

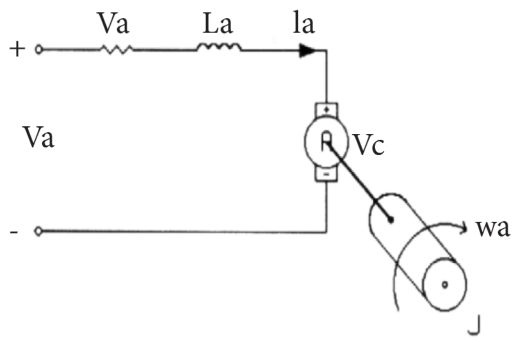

Figure 9. Electrical circuit of the DC motor.

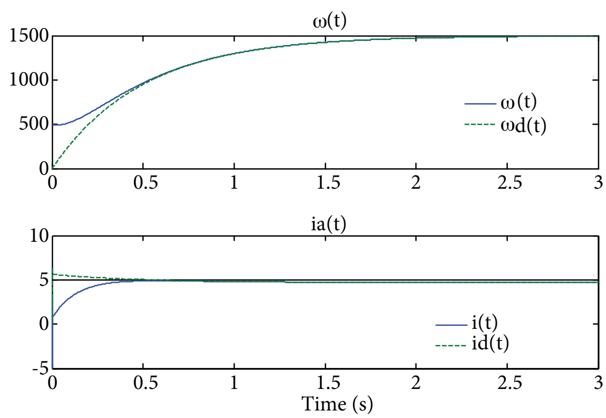

Figure 10. States and desired trajectories.

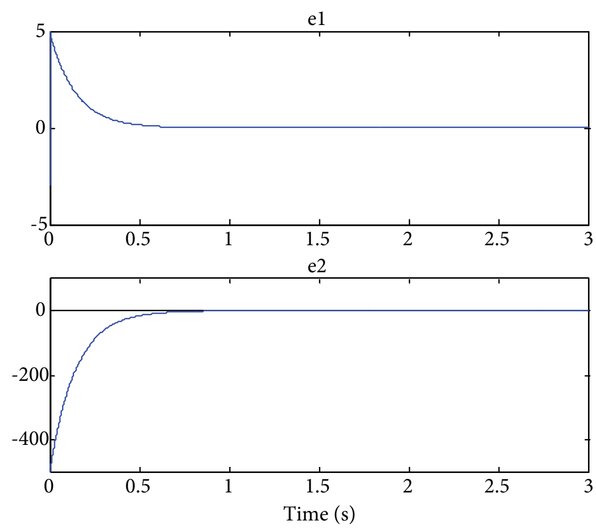

Figure 11. Tracking error.

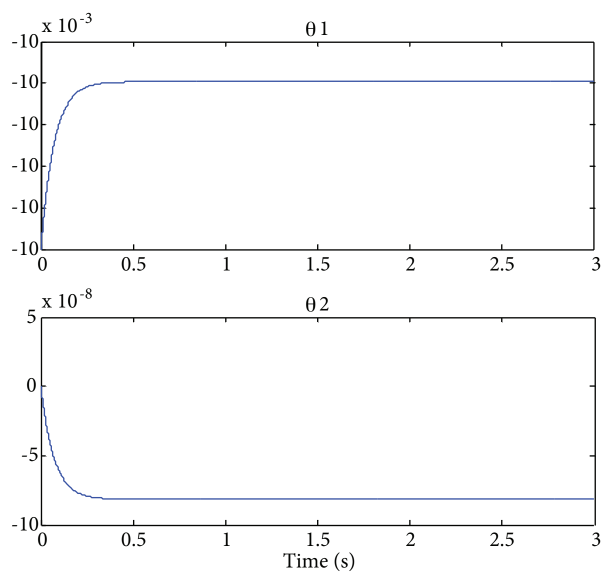

Figure 12. Convergence of the adjustable parameters (vector θ).

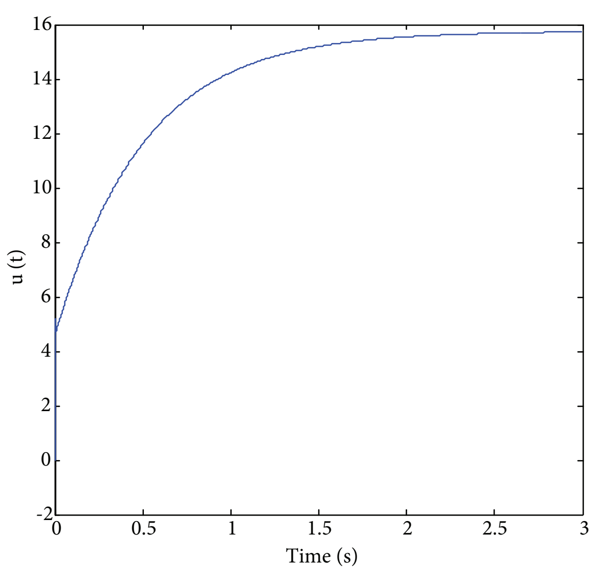

Figure 13. Control input.

11. It can be seen that good tracking performance is obtained. Figure 12 shows the convergence of the adaptive parameters of vector θ. Similar to other simulations, θ tends to fixed values. The final value of the adjustable vector θ is $[-0.01 - 8.1 \times 10^{-8}]$ and satisfies (7). The control input is presented in Figure 13, which is smooth and limited. All of the simulation results demonstrate the effectiveness of the proposed control method.

Table. DC motor parameters.

R	K_t	K_w	J	b
0.8Ω	8×10^{-3} v/rpm	8×10^{-3} v/rpm	1.5×10^{-5} NMS	2.5×10^{-5} NM/rpm

5. Conclusion

An adaptive state feedback controller for linear descriptor systems was investigated in this paper and good tracking performance was achieved. The proposed control method ensures the admissibility of the closed-loop system based on the Lyapunov theorem. The effectiveness of the controller was confirmed through simulation results. The control law is simple, bounded, and feasible. It should be pointed out that the proposed approach in this paper can be extended for multiinput systems with some changes in the Lyapunov function, and we can also challenge the method by applying it to more real models in experimental environments.

References

[1] L. Dai, Singular Control Systems, Lecture Notes in Control and Information Sciences Vol. 118, Berlin, Springer-Verlag, 1989.

[2] F.L. Lewis, "A survey of linear singular systems", Circuit Systems and Signal Processing, Vol. 5, pp. 3–36, 1986.

[3] M. Shafiee, P. Karimaghaiee, "Optimal control for singular systems (rectangular case)", Proceedings of the International Conference on Engineering Education, pp. 152–160, 1997.

[4] M. Shafiee, "LQR problem for continues time descriptor systems", Amirkabir Journal of Science and Technology, pp. 5–9, 1993.

[5] E. Fridman, U. Shaked, "H∞-control of linear state-delay descriptor systems: an LMI approach", Linear Algebra and its Applications, Vol. 351, pp. 271–302, 2002.

[6] Y. Xia, P. Shi, G. Liu, D. Rees, "Robust mixed H2/H1 state-feedback control for continuous-time descriptor systems with parameter uncertainties", Circuits Systems Signal Processing, Vol. 24, pp. 431–443, 2005.

[7] E.K. Boukas, "Static output feedback control for linear descriptor systems: LMI approach", Proceedings of the IEEE International Conference on Mechatronics & Automation, pp. 1230–1234, 2005.

[8] M. Dodig, M. Stoši, "Singular systems, state feedback problem", Linear Algebra and its Applications, Vol. 431, pp. 1267–1292, 2009.

[9] A. Bunse-Gerstner, R. Byers, V. Mehrmann, N.K. Nichols, "Feedback design for regularizing descriptor systems", Linear Algebra and its Applications, Vol. 299, pp. 119–151, 1999.

[10] Y. Wang, Z.Q. Sun, F.C. Sun, "Robust fuzzy control of a class of nonlinear descriptor systems with time-varying delay", International Journal of Control, Automation, and Systems, Vol. 2, pp. 76–82, 2004.

[11] S.L. Tung, Y.T. Juang, W.H. Lee, H.C. Chiu, "An improved particle swarm optimization for exponential stabilization of a singular linear time-varying system", Expert Systems with Applications, Vol. 38, pp. 13425–13431, 2011.

[12] K. Takaba, N. Morihira, Y.T. Katayama, "A generalized Lyapunov theorem for descriptor systems", Systems Control Letters, Vol. 24, pp. 49–51, 1995.

[13] I.Y. Masubuchi, Y. Kamitane, A. Ohara, N. Suda, "H∞ control for descriptor systems: a matrix inequalities approach", Automatica, Vol. 33, pp. 669–673, 1997.

[14] Q.L. Zhang, J. Lam, L. Zhang, "Generalized Lyapunov equations for analysing the stability of descriptor systems", Proceedings of the 14th International Federation of Automatic Control World Congress, pp. 19–25, 1999.

[15] S. Xu, J. Lam, "Robust control and filtering of singular systems", Lecture Notes in Control and Information Sciences, Vol. 332, pp. 1–9, 2006.

[16] Q.L. Zhang, L.Q. Zhang, G.Z. Dai, Y.Y. Nie, "Analysis and synthesis of robust stability for descriptor systems", Control Theory and Applications, Vol. 16, pp. 525–528, 1999.

[17] B.C. Kuo, Automatic Control Systems, 6th ed., Upper Saddle River, NJ, USA, Prentice Hall, 1991.