Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2014) 22: 12 – 24

doi:10.3906/elk-1206-56

Statistical approach for determining impulse breakdown voltage distribution under DC sweep voltage

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Received: 12.06.2012 • **Accepted:** 22.08.2012 • **Published Online:** 20.12.2013 • **Printed:** 20.01.2014

Abstract: This paper presents a comprehensive approach to the statistical characterization of impulse breakdown voltage under the effect of a DC sweep voltage. Several goodness-of-fit tests are applied to up-and-down test results obtained in the air-insulated rod-plane gap. Three distributions, normal, logistic, and Gumbel, are compared by means of the Kolmogorov–Smirnov goodness-of-fit test, and logistic distribution is also compared to 3-parameter Weibull distributions using the likelihood ratio test. Logistic distribution is found to be a possible alternative to normal and 3-parameter Weibull distributions.

Key words: Sweep voltage, impulse breakdown voltage, swing-motion impulse generator, Kolmogorov–Smirnov test, likelihood ratio test, Monte Carlo optimization

1. Introduction

To estimate the impulse breakdown voltage of gas-insulated test gaps, the up-and-down test method by Dixon and Mood [1] is often used, where the observed impulse breakdown voltage is assumed to follow normal distribution (ND), which is accepted as the standard distribution in IEC 60-1 [2] or IEEE Std 4-1995 [3]. The information on the type of distribution function of the impulse breakdown voltage is of great importance in designing the electrical insulation of high-voltage power equipment.

The sweep voltage resulting from the remanence magnetic flux in the core of power transformers, and possible dielectric polarization existing over the insulating materials of the power equipment undergoing tests, are not usually taken into account. The effect of this voltage on the type of impulse breakdown voltage distribution has not been considered so far. Somerville and Tedford [4] demonstrated that when impulse voltages are applied in the presence of sweep fields, the spatial and temporal variations of the negative ion density in the test gap affect the time-lags in SF_6. They also verified that sweep voltages close to the electrode surface may reduce the insulation strength of electronegative gases [5].

In the present article, we investigate the effect of sweep voltages on the impulse breakdown voltage data obtained from up-and-down tests in the rod-plane gap in air. Different goodness-of-fit test procedures are applied to the test data to decide on the type of distributions: 3 parameter distributions, normal, logistic, and Gumbel, are compared by means of the Kolmogorov–Smirnov (K-S) goodness-of-fit test. The logistic distribution (LD) seems to represent a probabilistic variation of the test data. For generalization, a further attempt is made to determine the best suited type of existing 3-parameter distributions. From among the existing 3-parameter

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distributions, 3-parameter Weibull distribution (3PWD) is selected and compared to LD using the likelihood ratio (LR) test, because of its distinct representative feature in gas-insulated systems [6–9].

2. Statistical characteristics of breakdown phenomena

2.1. Factors affecting impulse breakdown voltage

Two conditions must be simultaneously fulfilled in order for an impulse discharge to occur in gases: there should be at least one suitably located free electron close to the stressed electrode and the electric field stress should be sufficiently high within the critical volume of the stressed electrode. When these 2 conditions are satisfied, the electron produces a sequence of avalanches and streamers that lead to a breakdown. In the absence of an initiatory electron in the critical volume, no single avalanche can lead to a breakdown, even if the electric field exceeds the breakdown field strength of the gas medium [10].

Free electrons are produced naturally in the air as a result of the detachment process caused by external radiation due to cosmic rays or the penetration of ultraviolet radiation from the sun or the presence of local radioactive materials. Once an electron is liberated, it is likely to be removed from the gas medium rapidly [10]. Indeed, the rate of production and the concentration of free electrons are expectedly quite random in areas close to the stressed electrode. When an overvoltage impulse is applied to a spark gap in the air, there is only a small probability that this electron can fall into the critical volume.

Since the distributions of free electrons close to the stressed electrode are statistical in nature, these distribution should be known before any laboratory and field tests are performed on high-voltage power equipment. Since the concentration of free electrons and their probability of occurrence in the critical volume are indeterminate, there is no definitive method that can be implemented for determining $V_{50\%}$ breakdown voltage under lightning and switching impulse stresses.

2.2. Effect of sweep voltage on impulse breakdown voltage

It is thought that the impulse breakdown probability and statistical time-lag in air and other electronegative gases can be correlated directly with the density of the negative ions. There exists a correlation between the statistical time-lag distribution and the initial spatial ion densities [4].

It is assumed that very small electric sweep fields across the test gap can have a drastic effect in reducing the negative ion population [4]. These sweep fields may arise, for example, from induced voltages resulting from charged capacitors of the impulse generator prior to triggering or from remanence magnetization within the ferromagnetic materials of transformers and possible dielectric polarization existing in the power equipment under test or from a difference in the contact potential across the test electrodes. It has been found in a rod-plane gap that sweep fields may cause the ions in the region close to the tip of the rod electrode in the rod-plane gap to sweep either towards or away from the rod, depending on the polarity [5], which causes a variation of the impulse breakdown strength of the test gap.

2.3. Determining the distributions of the impulse breakdown voltage test data

In self-restoring insulation systems using air, vacuum, or SF_6, the type of impulse breakdown voltage distribution is generally assumed to follow either normal or Weibull distribution [6–9]. Vibholm and Thyregod assessed 4 different impulse breakdown distributions determined by the up-and-down test method: normal, logistic, 3PW, and Gumbel distributions (GDs). The parameter estimates of these distributions obtained from the method of maximum likelihood for a number of up-and-down test data are compared with the estimates corresponding to ND [11]. The cumulative distribution function (CDF) of the distributions under investigation is given by:

Normal distribution: The CDF of ND is:

$$P(v) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{v} \exp\left(\frac{-(t-\mu)^2}{2\sigma^2}\right) dt, \tag{1}$$

where μ is the mean value and σ is the standard deviation.

Logistic distribution: The CDF of LD is defined by:

$$P(v) = \frac{1}{1+\exp[-(v-\mu)/\sigma]}, \tag{2}$$

where μ is the mean value like in ND and σ is the shape parameter.

Three-parameter Weibull distribution: The CDF of 3PWD is:

$$P(v) = 1 - \exp\left[-\left(\frac{v-\gamma}{\alpha}\right)^{\beta}\right], \tag{3}$$

where α, β, and γ are the scale, shape, and location (threshold) parameters, respectively.

Gumbel distribution: The CDF of GD is given by:

$$P(v) = 1 - \exp\left[-\exp\left(\frac{v-\beta}{\alpha}\right)\right], \tag{4}$$

where α and β are the scale and location parameters, respectively.

After analyzing the works of the researchers in [6–9], in this article, 4 different types of probability distributions are considered to decide on the best-fitted distribution to the present impulse breakdown voltage data: normal, logistic, 3PW, and Gumbel distributions. For the selection of the distribution, parameter estimates are performed by applying the method of maximum likelihood estimation (MLE). The MLE method basically depends on the solution of the likelihood function, which is defined as the product of the probability density functions of the selected distributions (see Appendix 1). The parameter estimates that maximize the likelihood function are obtained by numerical methods [8]. Comparative parameter estimates of the distributions are performed with both the Newton–Raphson (N-R) and Monte Carlo (MC) optimization methods to ensure the correctness of the parameter estimates of the distributions.

2.4. Goodness-of-fit procedure

The most suitable types of statistical distributions that are applied for the evaluation of impulse breakdown data under different experimental test conditions are found [6–9,11] to fit logistic, normal, Gumbel, and 3PW distributions. Hence, in this work, for the statistical evaluation of a 50% impulse breakdown voltage, these 4 types of distributions are selected. Since logistic, normal, and Gumbel distributions are of the 2-parameter type, they are compared by means of K-S and LR tests. The LR test is only applied to compare logistic and 3PW distributions. The compatibility between the curves of the empirical and theoretical distributions is investigated by the K-S test. The largest deviation between these curves is taken as a goodness-of-fit measure [12].

3. Experimental setup

3.1. Two-stage swing-motion impulse generator

As shown in Figure 1a, the impulse generator used in the experiments is 2-stage and arranged to fire with a specially designed swing-motion stage-gap system. The values of the charging capacitor and resistor are 0.26 μF and 600 kΩ, respectively, providing a time constant of RC = 156 ms. The impulse voltage is applied in intervals of 1–2 min; hence, the rate of rise of the applied voltage increase is always lower than this value. The generator stage voltage is 120 kV and it is equipped with 0.26 μF polystyrene capacitors, capable of delivering a maximum of 1.872 kJ. The position of the spheres at each stage resembles a classical impulse generator circuit, with a distinct feature, which is that the column of one group sphere makes a free swing-motion that allows the generator to trigger for any impulse voltage requirement. The standard lightning impulse voltage (1.2/50 μs) is applied to the test gap by means of this generator. The impulse wave shape is recorded by a digital oscilloscope, and the impulse voltage and time-lag are measured with the aid of a computer feed from the digital oscilloscope.

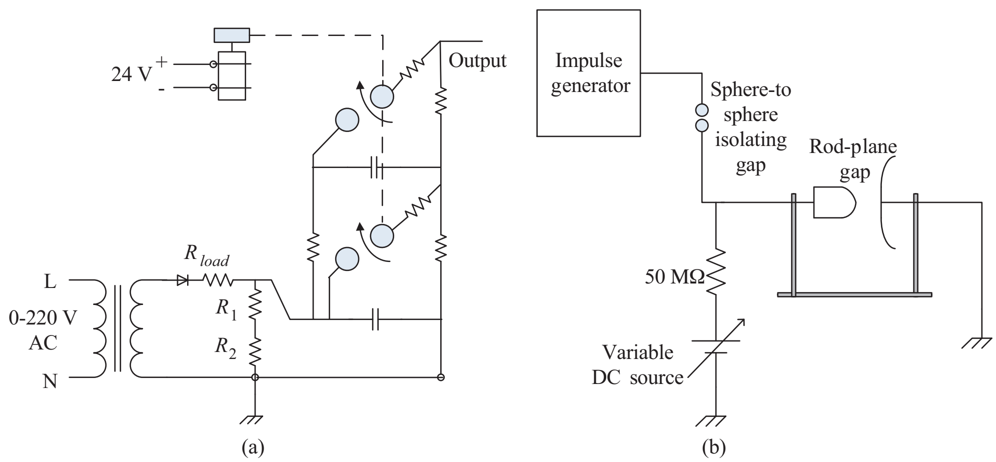

Figure 1. a) Swing-motion 2-stage impulse generator; b) experimental setup and sweep voltage-deterring sphere-sphere gap.

3.2. Experimental setup of external DC sweep voltage

Sweep voltages that emerge from some cases, as explained in previous sections, cause voltage to appear at the output terminal of the impulse generator prior to triggering [4]. These voltages are found [4,5] to affect the impulse insulation strength of the air and hence lead to a variation of impulse breakdown voltage being applied across the terminals of the power equipment undergoing tests. In order to investigate the influence of sweep voltages on the impulse breakdown voltage, and on the type of distribution functions, a DC voltage source is connected across a rod-plane test gap and a small sphere-to-sphere gap is inserted to isolate the impulse generator, as shown in Figure 1b. The DC sweep voltage is applied via a 50-MΩ current limiting resistor that is selected in order to prevent damage to the DC source. The impulse-generator leakage current isolator-spheres are placed 1 m away from the test gap and both spheres are 2 cm in diameter.

During the tests, a rod-plane gap is used. The reason for selecting this gap is because of the distinct boundaries of its critical volume around the tip of the rod electrode. The boundaries of critical volumes are not explicitly defined in plane-plane and rod-rod gaps. The rod electrode is 3 cm in diameter and the Rogowski profiled plane electrode is 30 cm in diameter. The gap length is fixed to 3 cm. The impulse breakdown voltage data are obtained by the up-and-down test method and the voltages are recorded and measured with a digital storage oscilloscope (UNI-T) via a generator capacitive voltage divider (a = 1/3500), and the peak voltage and time-lags are recorded with a PC interfaced with the oscilloscope. The laboratory tests are performed without interruption to avoid undesirable environmental conditions on the test results and all of the results are normalized to standard temperature and pressure conditions.

A DC sweep voltage is permanently connected to the test gap during the tests. Positive and negative DC sweep voltages are applied for only the positive impulse tests. A negative impulse voltage is not used since it provides extra electrons into the critical volume in addition to space-born electrons.

The values of the DC sweep voltages are selected similar to those used in [4]. For adjustment of the DC sweep voltage, an electrostatic voltmeter (TREK 520 IT-9265099) is used and the voltage adjustments are done by means of the calibration curve. However, it is found that there is not much difference between the applied and measured DC sweep voltages.

4. Results and discussion

4.1. Up-and-down breakdown test data

The results of normalized up-and-down breakdown test data obtained for different ambient test conditions under positive and negative sweep voltages are illustrated in Figures 2 and 3, where it is seen that 50% impulse breakdown voltage estimates are determined by the LD, which is suggested to be the most likely distribution as a result of this paper.

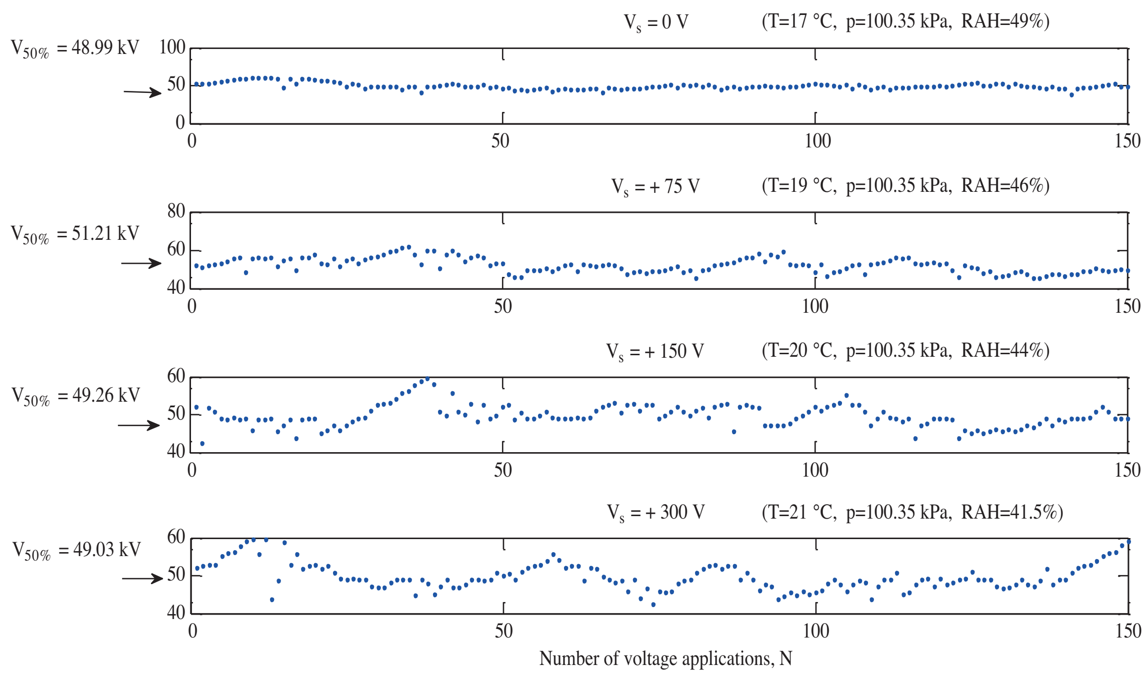

Figure 2. Up-and-down test data for positive sweep voltages. The 50% impulse breakdown voltage estimate was determined by LD, indicated on the vertical breakdown voltage axis. RAH: Relative air humidity.

4.2. Results of parameter estimates

The parameter estimates are determined by the MLE algorithm for 4 different distributions (see Section 2.3), which are expected to fit the impulse breakdown voltage data. For optimizing the MLE equations, the MC

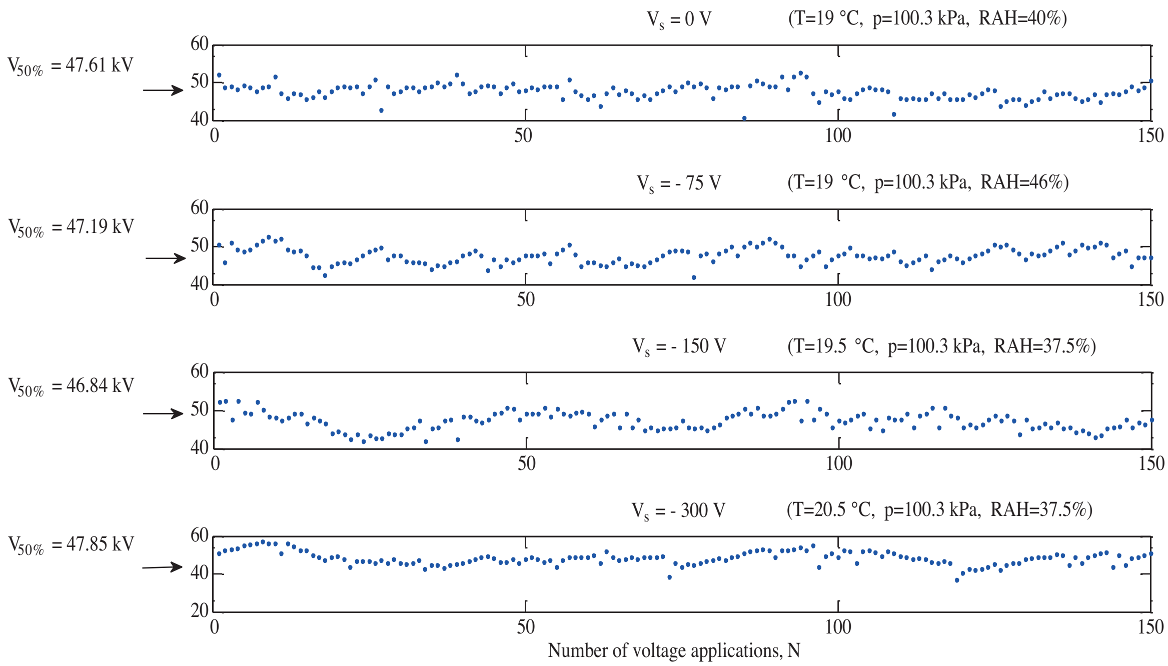

Figure 3. Up-and-down test data for negative sweep voltages. The 50% impulse breakdown voltage estimate was determined by LD, indicated on the vertical breakdown voltage axis. RAH: Relative air humidity.

Table 1. Maximum likelihood estimates for positive sweep voltage data.

Sweep voltage	Dist. type	MC opt	N-R opt
0 V	ND	$\hat{\theta}_1 = (49.1949,\ 4.3738)$	$\hat{\theta}_1 = (49.1959,\ 4.3685)$
	LD	$\hat{\theta}_2 = (48.9953,\ 2.3993)$	$\hat{\theta}_2 = (48.9952,\ 2.3986)$
	3PWD	$\hat{\theta}_3 = (14.0028,\ 3.0510,\ 36.7128)$	$\hat{\theta}_3 = (13.8305,\ 3.0086,\ 36.8138)$
	GD	$\hat{\theta}_4 = (4.7628,\ 51.4372)$	$\hat{\theta}_4 = (4.7614,\ 51.4373)$
75 V	ND	$\hat{\theta}_1 = (51.3760,\ 3.7909)$	$\hat{\theta}_1 = (51.3757,\ 3.7891)$
	LD	$\hat{\theta}_2 = (51.2159,\ 2.2155)$	$\hat{\theta}_2 = (51.2156,\ 2.2160)$
	3PWD	$\hat{\theta}_3 = (7.8761,\ 1.9218,\ 44.4437)$	$\hat{\theta}_3 = (7.6477{,}1.8408,\ 44.6049)$
	GD	$\hat{\theta}_4 = (3.9077,\ 53.2685)$	$\hat{\theta}_4 = (3.9055,\ 53.2834)$
150 V	ND	$\hat{\theta}_1 = (49.2607,\ 2.8397)$	$\hat{\theta}_1 = (49.2561,\ 2.8474)$
	LD	$\hat{\theta}_2 = (49.2636,\ 1.5728)$	$\hat{\theta}_2 = (49.2674,\ 1.5727)$
	3PWD	$\hat{\theta}_3 = (9.3833,\ 3.2282,\ 40.7559)$	$\hat{\theta}_3 = (9.5516,\ 3.2510,\ 40.6727)$
	GD	$\hat{\theta}_4 = (3.1151,\ 50.6441)$	$\hat{\theta}_4 = (3.1115,\ 50.6539)$
300 V	ND	$\hat{\theta}_1 = (49.1023,\ 3.2930)$	$\hat{\theta}_1 = (49.1052,\ 3.2984)$
	LD	$\hat{\theta}_2 = (49.0370,\ 1.8892)$	$\hat{\theta}_2 = (49.0322,\ 1.8910)$
	3PWD	$\hat{\theta}_3 = (8.8184,\ 2.5912,\ 41.1790)$	$\hat{\theta}_3 = (8.7913,\ 2.5343,\ 41.3095)$
	GD	$\hat{\theta}_4 = (3.5515,\ 50.7447)$	$\hat{\theta}_4 = (3.5545,\ 50.7460)$

Normal distribution (ND), $\hat{\theta}_1 = (\hat{\mu}, \hat{\sigma})$ (in kV); logistic distribution (LD), $\hat{\theta}_2 = (\hat{\mu}, \hat{\sigma})$ (in kV); 3-parameter Weibull (3PWD), $\hat{\theta}_3 = \left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right)$ (in kV); and Gumbel distribution (GD), $\hat{\theta}_4 = \left(\hat{\alpha}, \hat{\beta}\right)$ (in kV).

and N-R methods are applied. The results of parameter estimates for positive and negative sweep voltages are shown in Tables 1 and 2, respectively. The reason for illustrating the results of both methods is to ensure that the optimum point during the iterations is reached and, hence, that the correct estimates are attained. It can be seen that the parameter estimates of the 4 distributions obtained with both methods seem to be close to each other. Notably, the effect of sweep voltages on parameter estimates appears to be negligible for the 4 distributions.

Table 2. Maximum likelihood estimates for negative sweep voltage data.

Sweep voltage	Dist. type	MC opt	N-R opt
0 V	ND	$\hat{\theta}_1 = (47.5616,\ 2.4315)$	$\hat{\theta}_1 = (47.5565,\ 2.4288)$
	LD	$\hat{\theta}_2 = (47.6041,\ 1.3791)$	$\hat{\theta}_2 = (47.6108,\ 1.3792)$
	3PWD	$\hat{\theta}_3 = (12.0656,\ 5.1940,\ 36.4312)$	$\hat{\theta}_3 = (12.3427,\ 5.3660,\ 36.1514)$
	GD	$\hat{\theta}_4 = (2.2504,\ 48.6952)$	$\hat{\theta}_4 = (2.2489,\ 48.6958)$
–75 V	ND	$\hat{\theta}_1 = (47.2615,\ 2.3956)$	$\hat{\theta}_1 = (47.2639,\ 2.3989)$
	LD	$\hat{\theta}_2 = (47.2016,\ 1.4149)$	$\hat{\theta}_2 = (47.1949,\ 1.4198)$
	3PWD	$\hat{\theta}_3 = (7.2208,\ 2.9168,\ 40.8867)$	$\hat{\theta}_3 = (7.2310,\ 2.9448,\ 40.8223)$
	GD	$\hat{\theta}_4 = (2.3343,\ 48.4341)$	$\hat{\theta}_4 = (2.3406,\ 48.4308)$
–150 V	ND	$\hat{\theta}_1 = (46.8637,\ 2.2760)$	$\hat{\theta}_1 = (46.8743,\ 2.2706)$
	LD	$\hat{\theta}_2 = (46.8474,\ 1.3253)$	$\hat{\theta}_2 = (46.8435,\ 1.3273)$
	3PWD	$\hat{\theta}_3 = (6.8974,\ 2.9175,\ 40.7225)$	$\hat{\theta}_3 = (6.7806,\ 2.9089,\ 40.8367)$
	GD	$\hat{\theta}_4 = (2.2936,\ 47.9866)$	$\hat{\theta}_4 = (2.2979,\ 47.9829)$
–300 V	ND	$\hat{\theta}_1 = (47.8055,\ 3.7486)$	$\hat{\theta}_1 = (47.8077,\ 3.7555)$
	LD	$\hat{\theta}_2 = (47.8476,\ 2.0669)$	$\hat{\theta}_2 = (47.8507,\ 2.0680)$
	3PWD	$\hat{\theta}_3 = (17.3321,\ 4.6303,\ 31.8752)$	$\hat{\theta}_3 = (17.3131,\ 4.6931,\ 31.9167)$
	GD	$\hat{\theta}_4 = (3.6964,\ 49.6185)$	$\hat{\theta}_4 = (3.7015,\ 49.6169)$

ND: $\hat{\theta}_1 = (\hat{\mu}, \hat{\sigma})$ (in kV); LD: $\hat{\theta}_2 = (\hat{\mu}, \hat{\sigma})$ (in kV); 3PWD: $\hat{\theta}_3 = \left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right)$ (in kV); and GD: $\hat{\theta}_4 = \left(\hat{\alpha}, \hat{\beta}\right)$ (in kV).

Table 3. K-S test results for positive sweep voltage data.

Sweep voltage	Dist. type	MLE	K-S
0 V	ND	$\hat{\theta}_1 = (49.1959,\ 4.3685)$	0.1896
	LD	$\hat{\theta}_2 = (48.9952,\ 2.3986)$	0.1807
	GD	$\hat{\theta}_3 = (4.7614,\ 51.4373)$	0.2079
75 V	ND	$\hat{\theta}_1 = (51.3757,\ 3.7891)$	0.1524
	LD	$\hat{\theta}_2 = (51.2156,\ 2.2160)$	0.1329
	GD	$\hat{\theta}_3 = (3.9055,\ 53.2834)$	0.2010
150 V	ND	$\hat{\theta}_1 = (49.2561,\ 2.8474)$	0.1279
	LD	$\hat{\theta}_2 = (49.2674,\ 1.5727)$	0.1263
	GD	$\hat{\theta}_3 = (3.1115,\ 50.6539)$	0.1617
300 V	ND	$\hat{\theta}_1 = (49.1052,\ 3.2984)$	0.1033
	LD	$\hat{\theta}_2 = (49.0322,\ 1.8910)$	0.0999
	GD	$\hat{\theta}_3 = (3.5545,\ 50.7460)$	0.1506

ND: $\hat{\theta}_1 = (\hat{\mu}, \hat{\sigma})$(in kV); LD: $\hat{\theta}_2 = (\hat{\mu}, \hat{\sigma})$(in kV); and
GD: $\hat{\theta}_3 = \left(\hat{\alpha}, \hat{\beta}\right)$(in kV).

4.3. Kolmogorov–Smirnov test results

For selecting the appropriate distributions among normal, logistic, and Gumbel distributions, which are fitted to the impulse breakdown voltage test data, the K-S goodness-of-fit test is applied. The results of the K-S test between the fitted and the empirical distributions are shown in Tables 3 and 4. Because of its unacceptably large K-S values, Gumbel is the most improper among these distributions under both positive and negative sweep voltages.

LD seems to be more appropriate than the other 2 distributions under positive sweep voltages, as illustrated in Table 3. However, under negative sweep voltage, LD performs better than the others for only the negative sweep voltages of –75 V and –300 V, but for the sweep voltages of 0 V and –150 V, ND outperforms the other distributions [13–15], as shown in Table 4.

Table 4. K-S test results for negative sweep voltage data.

Sweep voltage	Dist. type	MLE	K-S
0 V	ND	$\hat{\theta}_1 = (47.5565, 2.4288)$	0.1199
	LD	$\hat{\theta}_2 = (47.6108, 1.3792)$	0.1362
	GD	$\hat{\theta}_3 = (2.2489, 48.6958)$	0.1580
–75 V	ND	$\hat{\theta}_1 = (47.2639, 2.3989)$	0.1182
	LD	$\hat{\theta}_2 = (47.1949, 1.4198)$	0.1032
	GD	$\hat{\theta}_3 = (2.3406, 48.4308)$	0.1696
–150 V	ND	$\hat{\theta}_1 = (46.8743, 2.2706)$	0.1223
	LD	$\hat{\theta}_2 = (46.8435, 1.3273)$	0.1279
	GD	$\hat{\theta}_3 = (2.2979, 47.9829)$	0.1397
–300 V	ND	$\hat{\theta}_1 = (47.8077, 3.7555)$	0.1299
	LD	$\hat{\theta}_2 = (47.8507, 2.0680)$	0.1169
	GD	$\hat{\theta}_3 = (3.7015, 49.6169)$	0.1833

ND: $\hat{\theta}_1 = (\hat{\mu}, \hat{\sigma})$ (in kV); LD: $\hat{\theta}_2 = (\hat{\mu}, \hat{\sigma})$ (in kV); and GD: $\hat{\theta}_3 = \left(\hat{\alpha}, \hat{\beta}\right)$ (in kV).

4.4. Likelihood ratio test results

In Section 4.3, the results of the K-S test indicate that LD serves better than the normal and Gumbel distributions for all of the values of the sweep voltages, except for 0 V and –150 V. Because of the successful performance of the 3PWD in self-restoring gas-insulated systems [6–9], LD is also compared with the 3PWD by applying the LR test at a 5% significance level for both sweep voltage data samples. The results for both distributions are shown in Tables 5 and 6.

The χ^2 limit, which is 3.841 at a 5% significance level, and the corresponding P-value (0.05) are critical values to test whether the data come from the LD or the 3PWD (see Appendix 3) for our LR test statistic. R values smaller than 3.841 indicate that the data are most likely to come from LD; otherwise, 3WP is the candidate. Hence, for positive sweep voltages (Table 5) of 150 V and 300 V and for negative sweep voltages (Table 6) of 0 V and –300 V, LD serves better than 3PWD. Moreover, P-values greater than 0.05 demonstrate that the data fit the LD better than 3PWD. In Tables 5 and 6, the log-likelihood (LL) values leading to computation of R values are also introduced.

A comparative study for the best-fitted distribution to the impulse breakdown voltage data under positive and negative sweep voltages is also carried out with Q-Q plots. The results of the Q-Q plots confirm the LR test results given in Tables 5 and 6, such that LD has outstanding features among the others.

Table 5. Likelihood ratio test results for positive sweep voltage data.

Sweep voltage	Dist. type	MLE	LL value	R	P-value
0 V	LD	$\hat{\theta}_1 = (48.9952, 2.3986)$	–216.3519	4.0218	0.0449
	3PWD	$\hat{\theta}_2 = (13.8305, 3.0086, 36.8138)$	–218.3628		
75 V	LD	$\hat{\theta}_1 = (51.2156, 2.2160)$	–206.4928	10.1708	0.0014
	3PWD	$\hat{\theta}_2 = (7.6477, 1.8408, 44.6049)$	–201.4074		
150 V	LD	$\hat{\theta}_1 = (49.2674, 1.5727)$	–181.4788	3.7128	0.0540
	3PWD	$\hat{\theta}_2 = (9.5516, 3.2510, 40.6727)$	–183.3352		
300 V	LD	$\hat{\theta}_1 = (49.0322, 1.8910)$	–178.3459	2.4346	0.1187
	3PWD	$\hat{\theta}_2 = (8.7913, 2.5343, 41.3095)$	–177.1286		

LD: $\hat{\theta}_1 = (\hat{\mu}, \hat{\sigma})$(in kV) and 3PWD: $\hat{\theta}_2 = \left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right)$(in kV).

Table 6. Likelihood ratio test results for negative sweep voltage data.

Sweep voltage	Dist. type	MLE	LL value	R	P-value
0 V	LD	$\hat{\theta}_1 = (47.6108, 1.3792)$	–170.4623	0.0388	0.8438
	3PWD	$\hat{\theta}_2 = (12.3427, 5.3660, 36.1514)$	–170.4817		
–75 V	LD	$\hat{\theta}_1 = (47.1949, 1.4198)$	–172.5591	4.5044	0.0338
	3PWD	$\hat{\theta}_2 = (7.231036, 2.9448, 40.8223)$	–170.3069		
–150 V	LD	$\hat{\theta}_1 = (46.8435, 1.3273)$	–169.5264	3.9018	0.0482
	3PWD	$\hat{\theta}_2 = (6.7806, 2.9089, 40.8367)$	–167.5755		
–300 V	LD	$\hat{\theta}_1 = (47.8507, 2.0680)$	–191.6012	3.6278	0.0568
	3PWD	$\hat{\theta}_2 = (17.3131, 4.6931, 31.9167)$	–193.4151		

LD: $\hat{\theta}_1 = (\hat{\mu}, \hat{\sigma})$(in kV) and 3PWD: $\hat{\theta}_2 = \left(\hat{\alpha}, \hat{\beta}, \hat{\gamma}\right)$(in kV).

It is observed that LD works best in some sweep voltage data sets according to the consequences of the K-S test. LD is also compared with 3PWD by means of the LR test, according to which LD operates better than 3PWD for 150 V, 300 V, 0 V, and –300 V sweep voltage data samples. Therefore, it is not guaranteed that LD will always behave better than the normal, 3PW, or Gumbel distributions, but at least it can be said under some circumstances that LD might work better than the other 3 distributions.

5. Conclusions

The impulse breakdown voltage data obtained from up-and-down tests to determine the $V_{50\%}$ impulse breakdown voltage are generally observed to follow normal or 3PW probability distributions. However, in the present study, it is shown that LD can sometimes perform better than the other 2 distributions under the influence of externally applied DC sweep voltages. LD may be suggested to be an alternative distribution to normal and 3PW distributions.

Appendixes

Appendix 1. Maximum likelihood estimates

Normal distribution: The probability density function (PDF) of ND is given by:

$$p(v) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(v-\mu)^2}{2\sigma^2}}. \tag{5}$$

The likelihood function L corresponding to n breakdowns occurring at each voltage level v_i is given in Eq. (6):

$$L(v_1, v_2, ..., v_n|\mu, \sigma) = \prod_{i=1}^{n} \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(v_i-\mu)^2}{2\sigma^2}} \right]. \tag{6}$$

Since v_i and n are known, L is a function of μ and σ only. For complete impulse breakdown voltage samples, the natural logarithm of the likelihood function

$$\ln L = -\frac{n}{2}\ln(2\pi) - n\ln(\sigma) - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{v_i-\mu}{\sigma}\right)^2 \tag{7}$$

yields the LL functions:

$$\frac{\partial \ln L}{\partial \mu} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(v_i-\mu) = 0, \tag{8}$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(v_i-\mu)^2 = 0. \tag{9}$$

Logistic distribution: The PDF of LD is given by:

$$p(v) = \frac{e^{-(v-\mu)/\sigma}}{\sigma(1+e^{-(v-\mu)/\sigma})^2}. \tag{10}$$

The likelihood function for LD is:

$$L(v_1, v_2, ..., v_n|\mu, \sigma) = \prod_{i=1}^{n}\left[\frac{e^{-(v_i-\mu)/\sigma}}{\sigma(1+e^{-(v_i-\mu)/\sigma})^2}\right], \tag{11}$$

where $v_1, v_2, ..., v_n$ are the impulse breakdown voltages. The logarithm of the likelihood function is defined by:

$$\ln L = -\sum_{i=1}^{n}\left(\frac{v_i-\mu}{\sigma}\right) - \sum_{i=1}^{n}\ln(\sigma) - 2\sum_{i=1}^{n}\ln\left(1+e^{-(v_i-\mu)/\sigma}\right). \tag{12}$$

To find the parameters that maximize the LL function, the following MLE equations need to be solved simultaneously:

$$\frac{\partial \ln L}{\partial \mu} = \frac{n}{\sigma} - 2\sum_{i=1}^{n}\left(\frac{\frac{1}{\sigma}e^{-(v_i-\mu)/\sigma}}{1+e^{-(v_i-\mu)/\sigma}}\right) = 0, \tag{13}$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\sigma} + \sum_{i=1}^{n}\frac{(v_i-\mu)}{\sigma^2} - 2\sum_{i=1}^{n}\left(\frac{\frac{(v_i-\mu)}{\sigma^2}e^{-(v_i-\mu)/\sigma}}{1+e^{-(v_i-\mu)/\sigma}}\right) = 0. \tag{14}$$

Three-parameter Weibull distribution: The PDF of 3PWD is given by:

$$p(v) = \frac{\beta}{\alpha}\left(\frac{v-\gamma}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{v-\gamma}{\alpha}\right)^{\beta}\right]. \tag{15}$$

The likelihood function is given by:

$$L(v_1, v_2, ..., v_n|\alpha, \beta, \gamma) = \prod_{i=1}^{n} \left[\frac{\beta}{\alpha} \left(\frac{v_i - \gamma}{\alpha} \right)^{\beta - 1} \exp \left[- \left(\frac{v_i - \gamma}{\alpha} \right)^{\beta} \right] \right]. \quad (16)$$

Here, $v_1, v_2, ..., v_n$ are the lightning impulse breakdown voltages. The logarithm of the likelihood function is denoted by:

$$\ln L = n \ln \left(\frac{\beta}{\alpha} \right) + (\beta - 1) \sum_{i=1}^{n} \ln (v_i - \gamma) - \frac{1}{\alpha} \sum_{i=1}^{n} (v_i - \gamma)^{\beta}. \quad (17)$$

LL equations are given by:

$$\frac{\partial \ln L}{\partial \alpha} = -\frac{n}{\alpha} + \frac{1}{\alpha^2} \sum_{i=1}^{n} (v_i - \gamma)^{\beta} = 0, \quad (18)$$

$$\frac{\partial \ln L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \ln (v_i - \gamma) - \frac{1}{\alpha} \sum_{i=1}^{n} \left[(v_i - \gamma)^{\beta} \ln (v_i - \gamma) \right] = 0, \quad (19)$$

$$\frac{\partial \ln L}{\partial \gamma} = \frac{\beta}{\alpha} \sum_{i=1}^{n} \ln (v_i - \gamma)^{\beta - 1} - (\beta - 1) \sum_{i=1}^{n} (v_i - \gamma)^{-1} = 0. \quad (20)$$

Gumbel distribution: The PDF of the GD has the form:

$$p(v) = \left(\frac{1}{\alpha} \right) \exp \left(\frac{v - \beta}{\alpha} \right) \exp \left[- \exp \left(\frac{v - \beta}{\alpha} \right) \right]. \quad (21)$$

The likelihood function is given by:

$$L(v_1, v_2, ..., v_n|\alpha, \beta) = \prod_{i=1}^{n} \left[\left(\frac{1}{\alpha} \right) \exp \left(\frac{v_i - \beta}{\alpha} \right) \exp \left[- \exp \left(\frac{v_i - \beta}{\alpha} \right) \right] \right], \quad (22)$$

where $v_1, v_2, ..., v_n$ are the impulse breakdown voltages obtained from the up-and-down test. The logarithm of likelihood function is denoted by:

$$\ln L = n \ln \left(\frac{1}{\alpha} \right) + \sum_{i=1}^{n} \left(\frac{v_i - \beta}{\alpha} \right) - \sum_{i=1}^{n} \exp \left(\frac{v_i - \beta}{\alpha} \right). \quad (23)$$

The first derivatives of the LL function with respect to distribution parameters α and β yield the following 2 equations:

$$\frac{\partial \ln L}{\partial \alpha} = n\alpha - \sum_{i=1}^{n} \left(\frac{v_i - \beta}{\alpha^2} \right) + \sum_{i=1}^{n} \left(\frac{v_i - \beta}{\alpha} \right) \exp \left(\frac{v_i - \beta}{\alpha} \right) = 0, \quad (24)$$

$$\frac{\partial \ln L}{\partial \beta} = -\frac{n}{\alpha} + \frac{1}{\alpha} \sum_{i=1}^{n} \exp \left(\frac{v_i - \beta}{\alpha} \right) = 0. \quad (25)$$

Appendix 2. Monte Carlo optimization

Alternatively, we can use the MC optimization method to determine the parameters of the probability distribution functions. Maximizing the LL function in this method is realized by means of a random search. Numbers are produced randomly in the specified interval for the parameters and inserted into the LL function. This method repeatedly evaluates the LL function at randomly selected values of the parameters [16] and requires no derivatives of the LL function. It needs no iterative initial values for finding parameters. Generated random values that maximize the LL function are selected as the estimated parameters of the distribution.

Appendix 3. Likelihood ratio test

Distributions that are compared by means of the LR test should have a different number of parameters. Logistic and 3PW distributions have a different number of parameters: LD has 2 parameters and 3PWD has 3 parameters. For this reason, comparing logistic and 3PW using the LR test is appropriate. This test method benefits from the LL values of the distributions.

$\mathbf{H}_0$: Data come from a LD model,

$\mathbf{H}_1$: Data come from a 3PWD model,

$$R = -2 * \ln\left(\frac{L_L}{L_{3PW}}\right) = -2 * \ln L_L + 2 * \ln L_{3PW}, \tag{26}$$

where $\ln L_L$ and $\ln L_{3PW}$ are the LL function values of the logistic and 3PW distributions, respectively.

If R is greater than $\chi^2_{k,0.05}$, then H_0 is rejected and H_1 is accepted. Otherwise, H_1 is rejected and H_0 is accepted. All of the data are considered at a 5% significance level for this work.

$$k = (\text{parameter number of } 3PW \text{distribution}) - (\text{parameter number of logistic distribution}) \tag{27}$$

$$p = 1 - \chi^2(R, k) \tag{28}$$

Here, R is the value of the LR test and k is the degrees of freedom of the χ^2 distribution.

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