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Robust stability of linear uncertain discrete-time systems with interval time-varying delay

Mehmet Nur Alpaslan PARLAKÇI[*]
Department of Electrical and Electronics Engineering, İstanbul Bilgi University,
34060 Eyüp, İstanbul, Turkey

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Abstract: This paper presents a robust stability problem for linear uncertain discrete-time systems with interval time-varying delay and norm-bounded uncertainties. First, a necessary and sufficient stability condition is obtained by employing a well-known lifting method and switched system approach for nominal discrete-time delay systems. Both the stability method of checking the characteristic values inside the unit circle and a Lyapunov function-based stability result are taken into consideration. Second, a simple Lyapunov–Krasovskii functional (LKF) is selected, and utilizing a generalized Jensen sum inequality, a sufficient stability condition is presented in the form of linear matrix inequalities. Third, a novel LKF is proposed together with the use of a convexity approach in the LKF. Finally, the proposed method is extended to the case when the system under consideration is subject to norm-bounded uncertainties. Three numerical examples are introduced to illustrate the effectiveness of the proposed approach, along with some numerical comparisons.

Key words: Discrete-time systems, time-varying delay, norm-bounded uncertainties, robust stability, lifting method, Lyapunov–Krasovskii functional, linear matrix inequalities

1. Introduction

The aftereffect, or so-called time-delay, is one of the important issues for physical systems. Most real dynamical systems are often subject to a time delay that leads to instability, a loss in performance, or a degradation in the system's response. The stability and/or stabilization of time-delay systems have been broadly investigated in the literature for several decades; see, for example [1,2] and the references therein. In particular, many results on the stability of discrete-time systems with constant or time-varying time-delay were developed in the existing literature, such as those in [3–13].

Utilizing relations among all of the systems' states, Liu et al. [14] developed some results on the stability and stabilization of uncertain discrete-time systems with time-varying delay. The so-called lifting method was employed in [15] to transform discrete-time systems with constant delay into a delay-free system, and thus some necessary and sufficient stability conditions have been derived. Moreover, for the time-varying delay case, the system was interpreted as a switched system. To avoid enlarging the time-varying delay to the upper bound of the time delay, several conditions were obtained in [16] for the asymptotic stability of discrete-time systems. Introducing an augmented form of the Lyapunov–Krasovskii functional (LKF) with a descriptor-type model transformation and a generalized free-weighting matrix method, Yoneyama and Tsuchiya [17] obtained some stability conditions for discrete-time delay systems. Yue et al. [18] divided the variation interval of the time-

[*]Correspondence: aparlakci@bilgi.edu.tr

delay into subintervals and employed a different LKF in every subinterval to derive some stability criteria for linear discrete-time systems with time-varying delay. Chen and Fong [19] converted the discrete-time system into an augmented one and developed some stability conditions that do not require the assumption of stability of the system when the delay vanishes to 0. The delay-partitioning idea was used in [20] to investigate the stability of linear discrete-time systems with time-varying delay in the state. Huang and Feng [21] avoided employing slack variables or free-weighting matrices when studying the asymptotical stability analysis problem for discrete-time systems with time-varying delay. Concerning the global asymptotic stability of a class of uncertain discrete-time systems with time-varying delay, Kandanvli and Kar [22] utilized various combinations of quantization and overflow nonlinearities to obtain delay-dependent stability conditions in terms of the linear matrix inequalities (LMIs). An approximation model was adopted for the time-varying delay in [23], and using the lifting method and LKF approach led to the deriving of sufficient conditions guaranteeing the robust asymptotic stability of the discrete-time delay system. On the basis of the integral quadratic constraint framework, Kao [24] interpreted the discrete-time systems with time-varying delay as the feedback interconnection of a linear time-invariant stable operator and delay difference operator to get a set of stability criteria. Finally, Ramakrishnan and Ray [25] developed a delay-dependent stability analysis for a class of uncertain discrete-time systems with time-varying delay and nonlinear perturbations using the LKF approach. Inspired by the idea of combining the lifting method and the LKF approach, we propose to convert the discrete-time system with time-varying delay into a switched system and apply the LKF technique to conduct a stability analysis.

In this paper, we consider the robust stability problem for discrete-time systems with interval time-varying delay and norm-bounded uncertainties. First, a lifting method is employed to develop necessary and sufficient stability conditions. Introducing a simple form of LKF, secondly, some sufficient stability results are obtained in the form of LMIs. As a third part of the stability analysis, the discrete-time system is transformed into a switched system by viewing the time-varying delay, such that it can take one of the values from the interval when the switching signal is applied. Therefore, some improved delay-dependent stability criteria are developed to achieve less conservative results for the maximum admissible delay bound. Finally, the proposed stability result is extended to take into account the existence of norm-bounded uncertainties. Several numerical examples are given to exhibit the application of the proposed approach in terms of achievement on the maximum allowable upper bounds of both the delay and the uncertainty.

2. Problem statement and preliminaries

Let us consider a linear uncertain discrete-time system with interval time-varying delay and norm-bounded uncertainties as follows:

$$\begin{aligned} &x(k+1) = [A + DF(k)E_a]\, x(k) + [A_d + DF(k)E_d]\, x(k-d(k)) \\ &x(k) = \phi(k), k = -d_M, -d_M+1, \ldots, 0 \end{aligned}, \tag{1}$$

where $x(k) \in \Re^n$ is the memoryless state vector; $A \in \Re^{n\times n}$, $A_d \in \Re^{n\times n}$, $D \in \Re^{n\times l}$, and $E \in \Re^{p\times n}$ are the known real matrices; $F(k) \in \Re^{l\times p}$ is a real matrix-valued function with Lebesgue measurable elements satisfying $F^T(k)F(k) \leq \mathrm{I}_p$, $\forall(k)$; $d(k)$ is a time-varying function representing the time-delay and satisfying

$$0 < d_m \leq d(k) \leq d_M, \tag{2}$$

with d_m and d_M being positive integers denoting the lower and upper bound of the time-delay; and $\phi(\cdot)$ represents the initial condition, which is a set of points. Now, we first consider the nominal form of Eq. (1) by letting $D = 0_{n\times l}$ to get:

$$\begin{array}{l} x(k+1) = Ax(k) + A_d x(k-d(k)) \\ x(k) = \phi(k), k = -d_M, -d_M+1, \ldots, 0 \end{array} . \tag{3}$$

Following the approach introduced in [15], the discrete-time time delay system can be easily lifted to a delay free system. We first let

$$\begin{array}{c} \eta(k) = \left[\begin{array}{ccccc} x^T(k) & x^T(k-1) & \ldots & x^T(k-d_m+1) & x^T(k-d_m) \end{array} \right. \\ \left. \begin{array}{ccccc} \ldots & x^T(k-d_m-i) & \ldots & x^T(k-d_M+1) & x^T(k-d_M) \end{array} \right]^T \end{array} , \tag{4}$$

with

$$\begin{array}{c} \eta(0) = \left[\begin{array}{ccccc} \phi^T(0) & \varphi^T(-1) & \ldots & \phi^T(-d_m+1) & \phi^T(-d_m) \end{array} \right. \\ \left. \begin{array}{ccccc} \ldots & \phi^T(-d_m-i) & \ldots & \phi^T(-d_M+1) & \phi^T(-d_M) \end{array} \right]^T \end{array} ,$$

$i = 0, 1, \ldots, d_{Mm}$, and $d_{Mm} = d_M - d_m$. The discrete-time system in Eq. (3), with an interval time-varying delay satisfying Eq. (2), is then equivalent to the following switched system:

$$\eta(k+1) = A_\sigma \eta(k), \tag{5}$$

where σ is a piecewise constant switching signal taking value from the finite index set $\Im = \{0, 1, \ldots, d_{Mm}\}$, and $A_\sigma = A_{\sigma 0} + A_{\sigma 1}$ with $A_{\sigma 0} = \left[\begin{array}{ll} 0_{n\times d_M n} & 0_n \\ \mathrm{I}_{d_M n} & 0_{d_M n\times n} \end{array} \right]$,

$A_{\sigma 1} = \left[\begin{array}{llll} A & 0_{n\times(d_m+\sigma-1)n} & A_d & 0_{n\times(d_{Mm}-\sigma)n} \\ 0_{d_M n\times n} & 0_{d_M n\times(d_m+\sigma-1)n} & 0_{d_M n\times n} & 0_{d_M n\times(d_{Mm}-\sigma)n} \end{array} \right]$.

Moreover, we now introduce a generalized Jensen sum inequality as follows:

Proposition 1 *Given the integers a, b, c such that $a < b < c$ and a positive definite real symmetric matrix $0 < X^T = X \in \Re^{n\times n}$, then $\forall \upsilon(\cdot) \in \Re^n$, and the following inequality is always satisfied:*

$$-(c-a)\sum_{i=-c}^{-a-1} \upsilon^T(i) X \upsilon(i) \le -\left(\sum_{i=-c}^{-b-1} \alpha\upsilon(i) + \sum_{i=-b}^{-a-1} \beta\upsilon(i)\right)^T X \left(\sum_{i=-c}^{-b-1} \alpha\upsilon(i) + \sum_{i=-b}^{-a-1} \beta\upsilon(i)\right), \tag{6}$$

where α, β take values from the set $\{-1, 1\}$.

Proof Let us define

$$\varphi(i) = \begin{cases} \alpha, & -c \le i < -b \\ \beta, & -b \le i < -a \end{cases} . \tag{7}$$

It is a fact that $\forall i$, and we have $\varphi^2(i) = 1$; then it follows from the classical Jensen sum inequality [1] that we have:

$$
\begin{aligned}
-(c-a)\sum_{i=-c}^{-a-1}\upsilon^T(i)X\upsilon(i) &= -(c-a)\sum_{i=-c}^{-a-1}\varphi^2(i)\upsilon^T(i)X\upsilon(i) \\
&\leq -\left(\sum_{i=-c}^{-a-1}\varphi(i)\upsilon(i)\right)^T X\left(\sum_{i=-c}^{-a-1}\varphi(i)\upsilon(i)\right) \\
&= -\left(\sum_{i=-c}^{-b-1}\alpha\upsilon(i)+\sum_{i=-b}^{-a-1}\beta\upsilon(i)\right)^T X\left(\sum_{i=-c}^{-b-1}\alpha\upsilon(i)+\sum_{i=-b}^{-a-1}\beta\upsilon(i)\right).
\end{aligned}
\tag{8}
$$

This completes the proof. Note that when $\alpha=\beta$, the generalized Jensen sum inequality reduces to the conventional one [1]. The primary objective of this paper is to develop some improved stability criteria for discrete-time systems that ensure a larger upper bound for the time-varying delay. Moreover, a secondary goal is to achieve this aforementioned objective with less computational complexity. □

3. Main results

In this section, we develop a stability analysis in 3 phases: 1) a necessary and sufficient stability result using the lifting method with the switching system, 2) a sufficient stability result based on a simple LKF, and 3) an improved sufficient stability result based on the utilization of a novel LKF and switched system approach. We first consider the nominal discrete-time system defined in Eq. (3).

Lemma 1 [15] *The discrete-time system in Eq. (5) with an interval time-varying delay, $d(k)$, satisfying Eq. (2), is globally and asymptotically stable for any $\sigma\in\{0,1,\ldots,d_{Mm}\}$:*

1. if and only if the polynomial $\det(\lambda\mathrm{I}-A_\sigma)=0$ has all roots lying inside the unit circle, or, equivalently,
2. given any positive definite real symmetric matrix Q, if and only if there exists a positive definite real symmetric matrix P, such that:

$$A_\sigma^T P A_\sigma - P = -Q < 0. \tag{9}$$

Proof The proof is referred to in [15] and [26].

Next, we investigate a stability analysis via employing the classical LKF method. □

Lemma 2 *Given the positive integers d_M and d_m, the linear discrete-time system in Eq. (3) with an interval time-varying delay, $d(k)$, satisfying Eq. (2), is globally and asymptotically stable if there exist positive definite real symmetric matrices P, Q, R, S, T, and U, all with appropriate dimensions satisfying*

$$\Omega<0, \tag{10}$$

where $\Omega=\Gamma_1^TP\Gamma_1+\Gamma_2^T(-P+T+U)\Gamma_2+\Gamma_3^T\left(d_m^2Q+d_{Mm}^2R+d_M^2S\right)\Gamma_3-\Gamma_4^TQ\Gamma_4-\Gamma_5^TR\Gamma_5-\Gamma_6^TS\Gamma_6-\Gamma_7^TT\Gamma_7-\Gamma_8^TU\Gamma_8$ with $\Gamma_1=[A\ \ A_d\ \ 0_{n\times 2n}]$, $\Gamma_2=[\mathrm{I}_n\ \ 0_{n\times 3n}]$, $\Gamma_3=\left[\begin{array}{ccc} A-\mathrm{I}_n & A_d & 0_{n\times 2n}\end{array}\right]$, $\Gamma_4=\left[\begin{array}{cccc}\mathrm{I}_n & 0_n & -\mathrm{I}_n & 0_n\end{array}\right]$, $\Gamma_5=\left[\begin{array}{cccc}0_n & 2\mathrm{I}_n & -\mathrm{I}_n & -\mathrm{I}_n\end{array}\right]$, $\Gamma_6=\left[\begin{array}{ccc}\mathrm{I}_n & 0_{n\times 2n} & -\mathrm{I}_n\end{array}\right]$, $\Gamma_7=\left[\begin{array}{ccc}0_{n\times 2n} & \mathrm{I}_n & 0_n\end{array}\right]$, $\Gamma_8=\left[\begin{array}{cc}0_{n\times 3n} & \mathrm{I}_n\end{array}\right]$.

Proof Let us choose a candidate of a simple form of LKF, as follows:

$$\begin{aligned} V(k) &= x^T(k)Px(k) + d_m \sum_{i=-d_m}^{-1} \sum_{j=k+i}^{k-1} \eta^T(j)Q\eta(j) + d_{Mm} \sum_{i=-d_M}^{-d_m-1} \sum_{j=k+i}^{k-1} \eta^T(j)R\eta(j) \\ &\quad + d_M \sum_{i=-d_M}^{-1} \sum_{j=k+i}^{k-1} \eta^T(j)S\eta(j) + \sum_{i=k-d_m}^{k-1} x^T(i)Tx(i) + \sum_{i=k-d_M}^{k-1} x^T(i)Ux(i) \end{aligned}, \tag{11}$$

where $\eta(j) = x(j+1) - x(j)$. We compute the forward difference on $V(k)$ in Eq. (11) as:

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) = x^T(k+1)Px(k+1) - x^T(k)Px(k) \\ &\quad + d_m \sum_{i=-d_m}^{-1} \left[\sum_{j=k+1+i}^{k} \eta^T(j)Q\eta(j) - \sum_{j=k+i}^{k-1} \eta^T(j)Q\eta(j) \right] \\ &\quad + d_{Mm} \sum_{i=-d_M}^{-d_m-1} \left[\sum_{j=k+1+i}^{k} \eta^T(j)R\eta(j) - \sum_{j=k+i}^{k-1} \eta^T(j)R\eta(j) \right] \\ &\quad + d_M \sum_{i=-d_M}^{-1} \left[\sum_{j=k+1+i}^{k} \eta^T(j)S\eta(j) - \sum_{j=k+i}^{k-1} \eta^T(j)S\eta(j) \right] \\ &\quad + \sum_{i=k+1-d_m}^{k} x^T(i)Tx(i) - \sum_{i=k-d_m}^{k-1} x^T(i)Tx(i) + \sum_{i=k+1-d_M}^{k} x^T(i)Ux(i) - \sum_{i=k-d_M}^{k-1} x^T(i)Ux(i) \\ &= x^T(k+1)Px(k+1) - x^T(k)Px(k) + d_m \sum_{i=-d_m}^{-1} \left[\eta^T(k)Q\eta(k) - \eta^T(k+i)Q\eta(k+i) \right] \\ &\quad + d_{Mm} \sum_{i=-d_M}^{-d_m-1} \left[\eta^T(k)R\eta(k) - \eta^T(k+i)R\eta(k+i) \right] \\ &\quad + d_M \sum_{i=-d_M}^{-1} \left[\eta^T(k)S\eta(k) - \eta^T(k+i)S\eta(k+i) \right] \\ &\quad + x^T(k)Tx(k) - x^T(k-d_m)Tx(k-d_m) + x^T(k)Ux(k) - x^T(k-d_M)Ux(k-d_M) \end{aligned}$$

$$\begin{aligned} &= x^T(k+1)Px(k+1) + x^T(k)\left(-P+T+U\right)x(k) + \eta^T(k)\left(d_m^2 Q + d_{Mm}^2 R + d_M^2 S\right)\eta(k) \\ &\quad - d_m \sum_{i=k-d_m}^{k-1} \eta^T(i)Q\eta(i) - d_{Mm} \sum_{i=k-d_M}^{k-d_m-1} \varphi^2(i)\eta^T(i)R\eta(i) \\ &\quad - d_M \sum_{i=k-d_M}^{k-1} \eta^T(i)S\eta(i) - x^T(k-d_m)Tx(k-d_m) - x^T(k-d_M)Ux(k-d_M) \end{aligned}. \tag{12}$$

Employing the generalized Jensen sum inequality outlined in Proposition 1 allows one to rewrite Eq. (12) in the form of an inequality, as follows:

$$\begin{aligned} \Delta V(k) &\leq x^T(k+1)Px(k+1) + x^T(k)\left(-P+T+U\right)x(k) + \eta^T(k)\left(d_m^2 Q + d_{Mm}^2 R + d_M^2 S\right)\eta(k) \\ &\quad - \left(\sum_{i=k-d_m}^{k-1} \eta(i) \right)^T Q \sum_{i=k-d_m}^{k-1} \eta(i) - \left(\sum_{i=k-d_M}^{k-d_m-1} \varphi(i)\eta(i) \right)^T R \sum_{i=k-d_M}^{k-d_m-1} \varphi(i)\eta(i) \\ &\quad - \left(\sum_{i=k-d_M}^{k-1} \eta(i) \right)^T S \sum_{i=k-d_M}^{k-1} \eta(i) - x^T(k-d_m)Tx(k-d_m) - x^T(k-d_M)Ux(k-d_M) \end{aligned}. \tag{13}$$

Now we introduce the following set of closed form representations as:

$$\begin{aligned}
x(k+1) &= \Gamma_1\chi(k), x(k) = \Gamma_2\chi(k), \eta(k) = x(k+1) - x(k) = (A - \mathrm{I}_n)x(k) + A_d x(k-d(k)) = \Gamma_3\chi(k), \\
&\sum_{i=k-d_m}^{k-1}\eta(i) = \sum_{i=k-d_m}^{k-1}x(i+1) - \sum_{i=k-d_m}^{k-1}x(i) = \sum_{i=k+1-d_m}^{k}x(i) - \sum_{i=k-d_m}^{k-1}x(i) \\
&= x(k) - x(k-d_m) = \Gamma_4\chi(k), \sum_{i=k-d_M}^{k-d_m-1}\varphi(i)\eta(i) = \sum_{i=k-d_M}^{k-d(k)-1} 1\cdot\eta(i) \\
&+ \sum_{i=k-d(k)}^{k-d_m-1}(-1)\cdot\eta(i) = \sum_{i=k-d_M}^{k-d(k)-1}x(i+1) - \sum_{i=k-d_M}^{k-d(k)-1}x(i) - \left[\sum_{i=k-d(k)}^{k-d_m-1}x(i+1) - \sum_{i=k-d(k)}^{k-d_m-1}x(i)\right], \\
&= \sum_{i=k+1-d_M}^{k-d(k)}x(i) - \sum_{i=k-d_M}^{k-d(k)-1}x(i) - \left[\sum_{i=k+1-d(k)}^{k-d_m}x(i) - \sum_{i=k-d(k)}^{k-d_m-1}x(i)\right] = x(k-d(k)) - x(k-d_M) \\
&- [x(k-d_m) - x(k-d(k))] = 2x(k-d(k)) - x(k-d_m) - x(k-d_M) = \Gamma_5\chi(k), \\
&\sum_{i=k-d_M}^{k-1}\eta(i) = \sum_{i=k-d_M}^{k-1}x(i+1) - \sum_{i=k-d_M}^{k-1}x(i) = \sum_{i=k+1-d_M}^{k}x(i) - \sum_{i=k-d_M}^{k-1}x(i) \\
&= x(k) - x(k-d_M) = \Gamma_6\chi(k), x(k-d_m) = \Gamma_7\chi(k), x(k-d_M) = \Gamma_8\chi(k)
\end{aligned} \tag{14}$$

where $\chi(k) = \begin{bmatrix} x^T(k) & x^T(k-d(k)) & x^T(k-d_m) & x^T(k-d_M) \end{bmatrix}^T$ and Γ_i, $i = 1,\ldots,8$, are defined in Lemma 2. As a result, we rewrite Eq. (13) in view of Eq. (14) as:

$$\Delta V(k) \leq \chi^T(k)\Omega\chi(k), \tag{15}$$

where Ω is defined in Eq. (10). Hence, if the inequality in Eq. (10) is satisfied, then we obtain:

$$\Delta V(k) \leq \chi^T(k)\Omega\chi(k) < 0, \tag{16}$$

implying that the nominal discrete-time system in Eq. (3) is guaranteed to be globally asymptotically stable. This completes the proof. □

Finally, we consider the use of a novel LKF combined with the switching system approach. We first interpret the discrete-time state with the time-varying delay, $x(k-d(k))$, as follows:

$$x(k-d(k)) = x(k-d_m-r), \tag{17}$$

where $r \in \{0, 1, \ldots, d_{Mm}\}$. Therefore, the nominal system in Eq. (3) can be transformed into a switching system such that:

$$\begin{array}{l} x_r(k+1) = Ax_r(k) + A_d x_r(k-d_m-r) \\ x_r(k) = \varphi(k), k = -d_M, -d_M+1, \ldots, 0 \end{array}. \tag{18}$$

The following theorem summarizes the main results on the stability of discrete-time systems.

Theorem 1: Given the positive integers d_M and d_m, the nominal discrete-time system in Eq. (3) with an interval time-varying delay, $d(k)$, satisfying Eq. (2), is stable if there exist positive definite real symmetric matrices P, Q, R, S, T, U, and Z, all with appropriate dimensions satisfying for all $r \in \{0, 1, \ldots, d_{Mm}\}$

$$\Sigma_r < 0, \tag{19}$$

where

$$\Sigma_r = \Gamma_{1r}^T P \Gamma_{1r} + \Gamma_{2r}^T \{-P + [1-\delta(r)]\,T + U + [1-\delta(r-d_{Mm})]\,Z\}\,\Gamma_{2r}$$
$$+\Gamma_{3r}^T \left\{[1-\delta(r)]\,d_m^2 Q + (d_m+r)^2 R + [1-\delta(r-d_{Mm})]\,d_M^2 S\right\}\Gamma_{3r} - [1-\delta(r)]\,\Gamma_{4r}^T Q \Gamma_{4r} - \Gamma_{5r}^T R \Gamma_{5r}$$
$$-[1-\delta(r-d_{Mm})]\,\Gamma_{6r}^T S \Gamma_{6r} - [1-\delta(r)]\,\Gamma_{7r}^T T \Gamma_{7r} - \Gamma_{8r}^T U \Gamma_{8r} - [1-\delta(r-d_{Mm})]\,\Gamma_{9r}^T Z \Gamma_{9r} \quad \text{with}$$
$$\Gamma_{1r} = \begin{bmatrix} A & 0_{n\times[1-\delta(r)]n} & A_d & 0_{n\times[1-\delta(r-d_{Mm})]n} \end{bmatrix}, \Gamma_{2r} = \begin{bmatrix} \mathrm{I}_n & 0_{n\times[3-\delta(r)-\delta(r-d_{Mm})]n} \end{bmatrix},$$
$$\Gamma_{3r} = \begin{bmatrix} A-\mathrm{I}_n & 0_{n\times[1-\delta(r)]n} & A_d & 0_{n\times[1-\delta(r-d_{Mm})]n} \end{bmatrix}, \Gamma_{4r} = \begin{bmatrix} \mathrm{I}_n & -\mathrm{I}_n & 0_{n\times[2-\delta(r)-\delta(r-d_{Mm})]n} \end{bmatrix},$$
$$\Gamma_{5r} = \begin{bmatrix} \mathrm{I}_n & 0_{n\times[1-\delta(r)]n} & -\mathrm{I}_n & 0_{n\times[1-\delta(r-d_{Mm})]n} \end{bmatrix}, \Gamma_{6r} = \begin{bmatrix} \mathrm{I}_n & 0_{n\times[2-\delta(r)-\delta(r-d_{Mm})]n} & -\mathrm{I}_n \end{bmatrix},$$
$$\Gamma_{7r} = \begin{bmatrix} 0_n & \mathrm{I}_n & 0_{n\times[2-\delta(r)-\delta(r-d_{Mm})]n} \end{bmatrix}, \Gamma_{8r} = \begin{bmatrix} 0_{n\times[2-\delta(r)]n} & \mathrm{I}_n & 0_{n\times[1-\delta(r-d_{Mm})]n} \end{bmatrix},$$
$$\Gamma_{9r} = \begin{bmatrix} 0_{n\times[3-\delta(r)-\delta(r-d_{Mm})]n} & \mathrm{I}_n \end{bmatrix}, \quad \text{and}$$
$$\delta(r) = \begin{cases} 1, & r=0 \\ 0, & r\neq 0 \end{cases}.$$

Proof We choose a set of candidate LKFs as follows:

$$\begin{aligned}
V_r(k) &= x_r^T(k) P x_r(k) + [1-\delta(r)]\,d_m \sum_{i=-d_m}^{-1}\sum_{j=k+i}^{k-1} \eta_r^T(j) Q \eta_r(j) \\
&+ (d_m+r) \sum_{i=-d_m-r}^{-1}\sum_{j=k+i}^{k-1} \eta_r^T(j) R \eta_r(j) + [1-\delta(r-d_{Mm})]\,d_M \sum_{i=-d_M}^{-1}\sum_{j=k+i}^{k-1} \eta_r^T(j) S \eta_r(j) \\
&+ [1-\delta(r)] \sum_{i=k-d_m}^{k-1} x_r^T(i) T x_r(i) + \sum_{i=k-d_m-r}^{k-1} x_r^T(i) U x_r(i) + [1-\delta(r-d_{Mm})] \sum_{i=k-d_M}^{k-1} x_r^T(i) Z x_r(i)
\end{aligned}, \tag{20}$$

where $\eta_r(j) = x_r(j+1) - x_r(j)$. We calculate the forward difference on $V_r(k)$ in Eq. (20) as:

$$\begin{aligned}
\Delta V_r(k) &= V_r(k+1) - V_r(k) = x_r^T(k+1) P x_r(k+1) - x_r^T(k) P x_r(k) \\
&+ [1-\delta(r)]\,d_m \sum_{i=-d_m}^{-1} \left[\sum_{j=k+1+i}^{k} \eta_r^T(j) Q \eta_r(j) - \sum_{j=k+i}^{k-1} \eta_r^T(j) Q \eta_r(j)\right] \\
&+ (d_m+r) \sum_{i=-d_m-r}^{-1} \left[\sum_{j=k+1+i}^{k} \eta_r^T(j) R \eta_r(j) - \sum_{j=k+i}^{k-1} \eta_r^T(j) R \eta_r(j)\right] \\
&+ [1-\delta(r-d_{Mm})]\,d_M \sum_{i=-d_M}^{-1} \left[\sum_{j=k+1+i}^{k} \eta_r^T(j) S \eta_r(j) - \sum_{j=k+i}^{k-1} \eta_r^T(j) S \eta_r(j)\right] \\
&+ [1-\delta(r)] \sum_{i=k+1-d_m}^{k} x_r^T(i) T x_r(i) - [1-\delta(r)] \sum_{i=k-d_m}^{k-1} x_r^T(i) T x_r(i) + \sum_{i=k+1-d_m-r}^{k} x_r^T(i) U x_r(i) \\
&- \sum_{i=k-d_m-r}^{k-1} x_r^T(i) U x_r(i) + [1-\delta(r-d_{Mm})] \sum_{i=k+1-d_M}^{k} x_r^T(i) Z x_r(i) \\
&- [1-\delta(r-d_{Mm})] \sum_{i=k-d_M}^{k-1} x_r^T(i) Z x_r(i) \\
&= x_r^T(k+1) P x_r(k+1) - x_r^T(k) P x_r(k) + [1-\delta(r)]\,d_m \sum_{i=-d_m}^{-1} \left[\eta_r^T(k) Q \eta_r(k) - \eta_r^T(k+i) Q \eta_r(k+i)\right]
\end{aligned}$$

$$\begin{aligned}
&+(d_m+r)\sum_{i=-d_m-r}^{-1}\left[\eta_r^T(k)R\eta_r(k)-\eta_r^T(k+i)R\eta_r(k+i)\right]\\
&+[1-\delta(r-d_{Mm})]\,d_M\sum_{i=-d_M}^{-1}\left[\eta_r^T(k)S\eta_r(k)-\eta_r^T(k+i)S\eta_r(k+i)\right]\\
&+[1-\delta(r)]\left[x_r^T(k)Tx_r(k)-x_r^T(k-d_m)Tx_r(k-d_m)\right]\\
&+x_r^T(k)Ux_r(k)-x_r^T(k-d_m-r)Ux_r(k-d_m-r)\\
&+[1-\delta(r-d_{Mm})]\left[x_r^T(k)Zx_r(k)-x_r^T(k-d_M)Zx_r(k-d_M)\right]\\
&=x_r^T(k+1)Px_r(k+1)+x_r^T(k)\left\{-P+[1-\delta(r)]\,T+U+[1-\delta(r-d_{Mm})]\,Z\right\}x_r(k)\\
&+\eta_r^T(k)\left\{[1-\delta(r)]\,d_m^2Q+(d_m+r)^2R+[1-\delta(r-d_{Mm})]\,d_M^2S\right\}\eta_r(k)\\
&-[1-\delta(r)]\,d_m\sum_{i=k-d_m}^{k-1}\eta_r^T(i)Q\eta_r(i)-(d_m+r)\sum_{i=k-d_m-r}^{k-1}\eta_r^T(i)R\eta_r(i)\\
&-[1-\delta(r-d_{Mm})]\,d_M\sum_{i=k-d_M}^{k-1}\eta_r^T(i)S\eta_r(i)-[1-\delta(r)]\,x_r^T(k-d_m)Tx_r(k-d_m)\\
&-x_r^T(k-d_m-r)Ux_r(k-d_m-r)-[1-\delta(r-d_{Mm})]\,x_r^T(k-d_M)Zx_r(k-d_M)
\end{aligned}\quad. \tag{21}$$

Applying the Jensen sum inequality [1] in Eq. (21) yields:

$$\begin{aligned}
&\Delta V_r(k)\le x_r^T(k+1)Px_r(k+1)+x_r^T(k)\left\{-P+[1-\delta(r)]\,T+U+[1-\delta(r-d_{Mm})]\,Z\right\}x_r(k)\\
&+\eta_r^T(k)\left\{[1-\delta(r)]\,d_m^2Q+(d_m+r)^2R+[1-\delta(r-d_{Mm})]\,d_M^2S\right\}\eta_r(k)\\
&-[1-\delta(r)]\left(\sum_{i=k-d_m}^{k-1}\eta_r(i)\right)^T Q\sum_{i=k-d_m}^{k-1}\eta_r(i)-\left(\sum_{i=k-d_m-r}^{k-1}\eta_r(i)\right)^T R\sum_{i=k-d_m-r}^{k-1}\eta_r(i)\\
&-[1-\delta(r-d_{Mm})]\left(\sum_{i=k-d_M}^{k-1}\eta_r(i)\right)^T S\sum_{i=k-d_M}^{k-1}\eta_r(i)-[1-\delta(r)]\,x_r^T(k-d_m)Tx_r(k-d_m)\\
&-x_r^T(k-d_m-r)Ux_r(k-d_m-r)-[1-\delta(r-d_{Mm})]\,x_r^T(k-d_M)Zx_r(k-d_M)
\end{aligned}\quad. \tag{22}$$

In a similar manner as in the proof of Lemma 2, we now introduce the following set of closed form representations as:

$$\begin{aligned}
&x_r(k+1)=\Gamma_{1r}\chi_r(k),x_r(k)=\Gamma_{2r}\chi_r(k),\eta_r(k)=x_r(k+1)-x_r(k)=(A-\mathrm{I}_n)x_r(k)+A_dx_r(k-d(k))\\
&=\Gamma_{3r}\chi_r(k),\sum_{i=k-d_m}^{k-1}\eta_r(i)=\sum_{i=k-d_m}^{k-1}x_r(i+1)-\sum_{i=k-d_m}^{k-1}x_r(i)=\sum_{i=k+1-d_m}^{k}x_r(i)-\sum_{i=k-d_m}^{k-1}x_r(i)\\
&=x_r(k)-x_r(k-d_m)=\Gamma_{4r}\chi_r(k),\sum_{i=k-d_m-r}^{k-1}\eta_r(i)=\sum_{i=k-d_m-r}^{k-1}x_r(i+1)-\sum_{i=k-d_m-r}^{k-1}x_r(i)\\
&=\sum_{i=k+1-d_m-r}^{k}x_r(i)-\sum_{i=k-d_m-r}^{k-1}x_r(i)=x_r(k)-x_r(k-d_m-r)=\Gamma_{5r}\chi_r(k),\\
&\sum_{i=k-d_M}^{k-1}\eta_r(i)=\sum_{i=k-d_M}^{k-1}x_r(i+1)-\sum_{i=k-d_M}^{k-1}x_r(i)=\sum_{i=k+1-d_M}^{k}x_r(i)-\sum_{i=k-d_M}^{k-1}x_r(i)\\
&=x_r(k)-x_r(k-d_M)=\Gamma_{6r}\chi_r(k),x_r(k-d_m)=\Gamma_{7r}\chi_r(k),x_r(k-d_m-r)=\Gamma_{8r}\chi_r(k)\\
&x_r(k-d_M)=\Gamma_{9r}\chi_r(k)
\end{aligned}\quad, \tag{23}$$

where

$$\chi_r(k) = \begin{cases} \begin{bmatrix} x_r^T(k) & x_r^T(k-d_m) & x_r^T(k-d_M) \end{bmatrix}^T, & r \in \{0, d_{Mm}\} \\ \begin{bmatrix} x_r^T(k) & x_r^T(k-d_m) & x_r^T(k-d_m-r) & x_r^T(k-d_M) \end{bmatrix}^T, & \text{else} \end{cases}.$$

Now we substitute Eq. (23) appropriately into Eq. (22) to obtain:

$$\Delta V_r(k) \le \chi_r^T(k)\Sigma_r\chi_r(k), \tag{24}$$

where Σ_r is defined in Eq. (19). Hence, if the inequality in Eq. (19) is satisfied, then we obtain:

$$\Delta V_r(k) \le \chi_r^T(k)\Sigma_r\chi_r(k) < 0, \tag{25}$$

implying that the nominal discrete-time system in Eq. (3) is guaranteed to be globally asymptotically stable. This completes the proof. □

Next we consider the linear uncertain discrete-time system in Eq. (1) and present the following sufficient robust stability result derived using Theorem 1.

Corollary 1 *Given the positive integers d_M and d_m, the linear uncertain discrete-time system in Eq. (1) with interval time-varying delay, $d(k)$, satisfying Eq. (2), is robustly globally asymptotically stable if there exist positive definite real symmetric matrices P, Q, R, S, T, U, and Z, all with appropriate dimensions and a positive scalar, $\varepsilon > 0$, satisfying for all $r \in \{0, 1, \ldots, d_{Mm}\}$*

$$\Sigma_r = \begin{bmatrix} \Sigma_r(1,1) + \varepsilon E^T E & \Gamma_1^T P & \Gamma_{3r}^T \left\{ \begin{array}{l} [1-\delta(r)]\, d_m^2 Q \\ + (d_m + r)^2 R \\ + [1-\delta(r-d_{Mm})]\, d_M^2 S \end{array} \right\} & 0_{[4-\delta(r)-\delta(r-d_{Mm})]n\times n} \\ * & -P & 0_n & PD \\ * & * & -\left\{ \begin{array}{l} [1-\delta(r)]\, d_m^2 Q \\ + (d_m + r)^2 R \\ + [1-\delta(r-d_{Mm})]\, d_M^2 S \end{array} \right\} & \left\{ \begin{array}{l} [1-\delta(r)]\, d_m^2 Q \\ + (d_m + r)^2 R \\ + [1-\delta(r-d_{Mm})]\, d_M^2 S \end{array} \right\} D \\ * & * & * & -\varepsilon \mathrm{I}_l \end{bmatrix} < 0, \tag{26}$$

where

$$\Sigma_r(1,1) = \Gamma_{2r}^T \{-P + [1-\delta(r)]\, T + U + [1-\delta(r-d_{Mm})]\, Z\}\, \Gamma_{2r} - [1-\delta(r)]\, \Gamma_{4r}^T Q \Gamma_{4r} - \Gamma_{5r}^T R\Gamma_{5r} - [1-\delta(r-d_{Mm})]\, \Gamma_{6r}^T S\Gamma_{6r} - [1-\delta(r)]\, \Gamma_{7r}^T T\Gamma_{7r} - \Gamma_{8r}^T U\Gamma_{8r} - [1-\delta(r-d_{Mm})]\, \Gamma_{9r}^T Z\Gamma_{9r},$$

$$E = \begin{bmatrix} E_a & 0_{p\times[1-\delta(r)]n} & E_d & 0_{p\times[1-\delta(r-d_{Mm})]n} \end{bmatrix}.$$

Proof Let us apply the Schur complement in Eq. (19) to obtain:

$$\Psi_r = \begin{bmatrix} \Xi_r(1,1) & \Gamma_1^T P & \Gamma_{3r}^T \left\{ \begin{array}{l} [1-\delta(r)]\, d_m^2 Q + (d_m + r)^2 R \\ + [1-\delta(r-d_{Mm})]\, d_M^2 S \end{array} \right\} \\ * & -P & 0_n \\ * & * & -\left\{ \begin{array}{l} [1-\delta(r)]\, d_m^2 Q + (d_m + r)^2 R \\ + [1-\delta(r-d_{Mm})]\, d_M^2 S \end{array} \right\} \end{bmatrix} < 0. \tag{27}$$

Replacing A and A_d with $A + DF(k)E_a$ and $A_d + DF(k)E_d$, respectively, in Eq. (27) gives:

$$\Psi_r + \Delta\Psi_r^T(k) + \Delta\Psi_r(k) < 0, \tag{28}$$

where

$$\Delta\Psi_r(k) = \begin{bmatrix} 0_{[4-\delta(r)-\delta(r-d_{Mm})]n\times n} & 0_{[4-\delta(r)-\delta(r-d_{Mm})]n\times n} & 0_{[4-\delta(r)-\delta(r-d_{Mm})]n\times n} \\ PDF(k)E & 0_n & 0_n \\ \left\{ \begin{array}{l} [1-\delta(r)]\, d_m^2 Q + (d_m + r)^2 R \\ + [1-\delta(r-d_{Mm})]\, d_M^2 S \end{array} \right\} DF(k)E & 0_n & 0_n \end{bmatrix}.$$

We shall now reexpress $\Delta\Psi_r(k)$ in closed form, as follows:

$$\Delta\Psi_r(k) = \Pi_1^T F(k) \Pi_2, \tag{29}$$

where

$$\Pi_1 = \left[\begin{array}{cccc} 0_{[4-\delta(r)-\delta(r-d_{Mm})]n\times n} & D^T P & D^T & \left\{ \begin{array}{l} [1-\delta(r)]\, d_m^2 Q + (d_m + r)^2 R \\ + [1-\delta(r-d_{Mm})]\, d_M^2 S \end{array} \right\} \end{array} \right], \Pi_2 = \left[\begin{array}{ccc} E & 0_{p\times n} & 0_{p\times n} \end{array} \right].$$

Substituting Eq. (29) into Eq. (28) and applying the well-known bounding inequality [1] yields:

$$\Psi_r + \Pi_2^T F^T(k) \Pi_1 + \Pi_1^T F(k) \Pi_2 \le \Psi_r + \varepsilon^{-1} \Pi_1^T \Pi_1 + \varepsilon \Pi_2^T \Pi_2 < 0. \tag{30}$$

Applying the Schur complement to the inequality in Eq. (30) allows one to obtain Eq. (26). This completes the proof. □

Remark 1 *Note that Lemma 1 presents a necessary and sufficient stability condition for the nominal discrete-time system in Eq. (3), while both Lemma 2 and Theorem 1 are capable of providing only sufficient stability criteria. Similarly, when the linear uncertain discrete-time system in Eq. (1) is taken into consideration, Corollary 1 gives only sufficient robust stability results.*

4. Numerical examples

In this section, we introduce several numerical examples to illustrate the application of the stability results presented in the former section.

Example 1 Let us consider the nominal discrete-time time delay system in Eq. (3) with the following parameters:

$A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.97 \end{bmatrix}, A_d = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}$. It is clearly seen that the system is stable for the case where $d(k) = 0$. Using the necessary and sufficient stability criterion given in Lemma 1, we find that the nominal system in Eq. (3) is asymptotically stable for $0 \leq d(k) \leq 18$. Note that the upper bound of the time-varying delay obtained through Lemma 1 is in fact the analytical limit. Second, we consider Lemma 2 and obtain a feasible solution set for Eq. (10) with $\alpha = -\beta$, guaranteeing the asymptotic stability of Eq. (3) for $0 \leq d(k) \leq 15$. Finally, employing Theorem 1 shows that the nominal system in Eq. (3) is guaranteed to be asymptotically stable for $0 \leq d(k) \leq 16$. In order to make a comparison, it was reported in [20] that the asymptotic stability of this system is guaranteed for $0 \leq d(k) \leq 15$. As a result, it is apparently seen among the sufficient stability results given in Lemma 2, [20] and Theorem 1 that Theorem 1 gives a maximum allowable upper bound for $d(k)$, which remains the closest one to the analytical limit obtained by Lemma 1. Moreover, although Theorem 1 is a bit far behind Lemma 1 concerning the admissible delay bound, it requires only 21 decision variables, while Lemma 1 utilizes 136 decision variables to get the analytical limit of the stability bound of the delay.

Example 2 Let us now consider Eq. (1) with the following system parameters:

$$A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.91 \end{bmatrix}, A_d = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, E_a = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}, E_d = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, D = \bar{\rho}\mathrm{I}_n,$$

and $F(k) = \rho(k)/\bar{\rho}$ with $\rho(k) \leq \bar{\rho}$. For $\bar{\rho} = 1$, when $d_m = 8$, the allowable upper bound of d_M was calculated in [18] as 16. We find using Theorem 1 with $\bar{\rho} = 1$ and $d_m = 8$ that Eq. (1) is robustly asymptotically stable for an allowable upper bound of $d_M = 32$. This shows that the proposed method of this paper gives a less conservative result than that given by Yue et al. [18].

Example 3 Let us consider a slightly different version of Example 2 with

$$A = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.9 \end{bmatrix}, A_d = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, E_a = \begin{bmatrix} 1 & 0 \end{bmatrix}, E_d = \begin{bmatrix} 0 & 0 \end{bmatrix}, D = \bar{\rho}\mathrm{I}_n,$$

$F(k) = \rho(k)/\bar{\rho}$ *with* $\rho(k) \leq \bar{\rho}$.

Table. Maximum allowable values of $\bar{\rho}$ for Example 3.

Methods (d_m/d_M)	(2,7)	(5,10)	(8,15)	(20,25)	NoDV
Huang and Feng [21]	0.1920	0.1425	Not reported	0.0886	19
Li and Gao [23]	0.1938	0.1541	0.1032	Not reported	19
Ramakrishnan and Ray [25]	0.1954	0.1541	Not reported	0.0937	44
Corollary 1	0.1957	0.1581	0.1301	0.1119	22

In the Table, NoDV represents the number of decision variables, and the calculated results are listed for the maximum allowable values of $\bar{\rho}$ such that this system is robustly asymptotically stable given the prescribed integers of d_m and d_M in comparison to those reported in the existing literature. The first observation appears in the Table, where Corollary 1 gives better results on the uncertainty bound compared to those reported in [21], [23], and [25]. The number of decision variables required by Corollary 1, however, is much lower than that in [25].

5. Conclusions

This paper presents a further stability result on linear uncertain discrete-time systems with interval time-varying delays. Two different main methods were utilized. The first is the use of classical stability results for linear time-delay systems in a Lyapunov sense based on the modified Jensen-type inequality and the second is the use of a novel LKF together with the use of a convexity approach in the LKF. Three numerical examples were given to illustrate the results. It was shown that the proposed method achieves less conservative results for the numerical examples under consideration. Furthermore, the number of decision variables required in the present paper remains lower than those reported in the literature.

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