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Star-crossed cube: an alternative to star graph

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Abstract: This paper introduces a new interconnection network topology called the star-crossed cube (SCQ). The SCQ is observed to be superior to other contemporary networks in terms of various topological parameters such as diameter, cost, average distance, and message traffic density. The various performance metrics of the proposed network, including cost effectiveness and time-cost effectiveness, are found to be better. The optimal routing and broadcasting algorithms for the new network are also presented. Embedding properties for the proposed network are also studied.

Key words: Interconnection networks, topology, traffic density, performance

1. Introduction

The performance of a distributed memory parallel computer heavily depends on the effectiveness of its interconnection network (IN) [1,2]. There exist numerous INs, such as crossbar, mesh, tree, and cube [3]. Among all of these networks, the hypercube (HC) has received much attention due to its attractive properties, such as regularity, symmetry, small diameter, low degree, and link complexity [4,5]. However, in a HC the link complexity increases with the increase in the network size, which highly affects the performance of the network.

There are several cube-based derivatives. The prominent and most recent networks are the crossed cube (CQ) [6], star graph [7,8], starcube [9], dualcube [10], metacube [11], folded metacube [12], and extended CQ [13]. The diameter of the CQ is almost half that of the HC and also has a reduced mean distance. The star graph is a permutation graph whose node degree is (n – 1) with $n!$ nodes. The star graph, being regular and vertex (edge) symmetric, is asymptotically superior to the HC. In spite of these improved features, the n-star has a major disadvantage, which is that it grows to its next highest level by a very large value. For example, a 4-dimensional star (4-star) has 4! = 24 nodes and a 5-star has 5! = 120 nodes. The significant gap between 2 consecutive dimensions of an n-star graph is considered as a major drawback. The incomplete star (IS), an alternative star, has been introduced to eliminate this problem [14,15]. However, the IS is nonsymmetric and irregular. Thus, it is not suitable for many practical systems. Another variation of the n-star, called the hierarchical star network (HS(n,n)), is a 2-level IN [16]. The HS(n,n) consists of $n!$ modules interconnected with additional edges and each module is an n-star. Thus, the HS(n,n) contains $(n!)^2$ nodes with node degree n. The growth rate of the HS is very high. When n is 3 and 4, the network contains 36 and 576 nodes. This significant gap in the 2 consecutive sizes of the HS becomes a major disadvantage. Another disadvantage of the HS is that the dimension cannot take any values of n like the metacube. It only takes values like (3,3), (4,4),

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(5,5), etc. The hyperstar(n,k) is another alternative to the n-star, in which there are $\frac{n!}{k!(n-k)!}$ nodes at node degree '$n--k'$ [17]. The 2 parameters n and k decide the nature of the network. If $n = 2k$, then the network is regular and symmetric and has a diameter of $n-1$. In the case that the network is irregular, the construction and addressing of the nodes becomes difficult.

Recently, the starcube, denoted as SC(m,n), has been proposed as a new network to eliminate the above said problem. However, the CQ has been proven to be a better network when compared to the HC. The current study proposes a new class of IN topology called the star-CQ (SCQ(m,n)). It is a product graph on the n-star and m-dimensional CQ (CQ(m)). The proposed graph possesses all of the interesting properties that are common to both the star and CQ. In terms of topological properties, it lies in between the n-star and CQ(m). It is also very suitable for variable node size architectures.

The paper is organized as follows: Section 2 describes the basic terminologies and topological features of the base networks. In Section 3, the topological properties of the proposed architecture are presented. Sections 4 to 7, respectively, discuss the performance analysis, embedding, routing, and results. Section 8 concludes the paper.

2. Background

The basic terminologies of the INs are discussed below.

2.1. Basics

Throughout the paper, the IN is treated as an undirected graph, in which the vertices correspond to the processor and the edges correspond to the bidirectional communication links.

Definition 1 *The IN is a finite graph $G = \{V, E\}$, where V and E are a set of n tuples, $v_1 v_2 .. v_n$ and $e_1, e_2, \ldots e_n$, respectively.*

Definition 2 *The degree of a vertex v in G is equal to the number of edges incident on v.*

Definition 3 *The diameter of a graph G denoted as D_G is defined to be max. $\{d_G(u,v)\colon u,v \in V\}$, where d_G is the distance between 2 nodes.*

Definition 4 *A graph is said to be regular if all of its vertices have the same degree.*

Definition 5 *A graph $G(V, E)$ is vertex symmetric if for every pair of vertices, u and v, there exists an automorphism of the graph that maps u into v, $u, v \in V$.*

Definition 6 *A set of paths is said to be node disjoint if no node except for the source and destination nodes appear in more than one path. The number of such paths provides a measure of the fault tolerance and reliability of the network.*

2.2. Construction

The proposed topology is based on the CQ and the n-star graph. The network structures of both the CQ and star are briefly discussed below.

2.2.1. Crossed cube

The CQ(m) is a regular graph of 2^m nodes. Every node in the CQ(m) is identified by a unique binary string of length m [6].

Definition 1 *Two binary strings,* $X = X_1X_0$ *and* $Y = Y_1Y_0$*, of length 2 are said to be pair related if and only if xy* $\in$ *{(00,00) (10,10), (01,11),(11,01)}.*

Definition 2 *The CQ(m) is recursively defined as follows:*

CQ_1 is a complete graph on 2 vertices with labels 0 and 1. For m >1, the CQ(m) contains CQ^0_{m-1} and $CQ^1_{n-1}CQ^1_{m-1}$, joined according to the following rule: the vertex u = $0u_{m-2}\ldots u_0$ from CQ^0_{m-1} and the vertex v = $1v_{m-1}\ldots\ldots v_0$ from CQ^1_{m-1} CQ^1_{n-1} are adjacent in the CQ(m) if:

1. $u_{m-2} = v_{m-2}$ if m is even, and

2. for 0 <= i < $\lfloor \frac{l-1}{2} \rfloor$ $\lfloor (n-1)/2 \rfloor$, $u_{2i+1}u_{2i} \sim v_{2i+1}v_{2i}$.

Figure 1 shows the CQ(m) topology for m = 3 and m = 4. Every vertex in the CQ(m) with a leading 0 bit has exactly 1 neighbor with a leading 1 bit and vice versa. In the CQ(m), when 2 adjacent vertices u and v have a leftmost differing bit at position d, then v is called the d-neighbor of u and the edge (u,v) is called the edge of dimension d. For any 2 nodes u and v of the CQ(m), it is possible to reach v from u in at most $\lceil \frac{m+1}{2} \rceil$ hops.

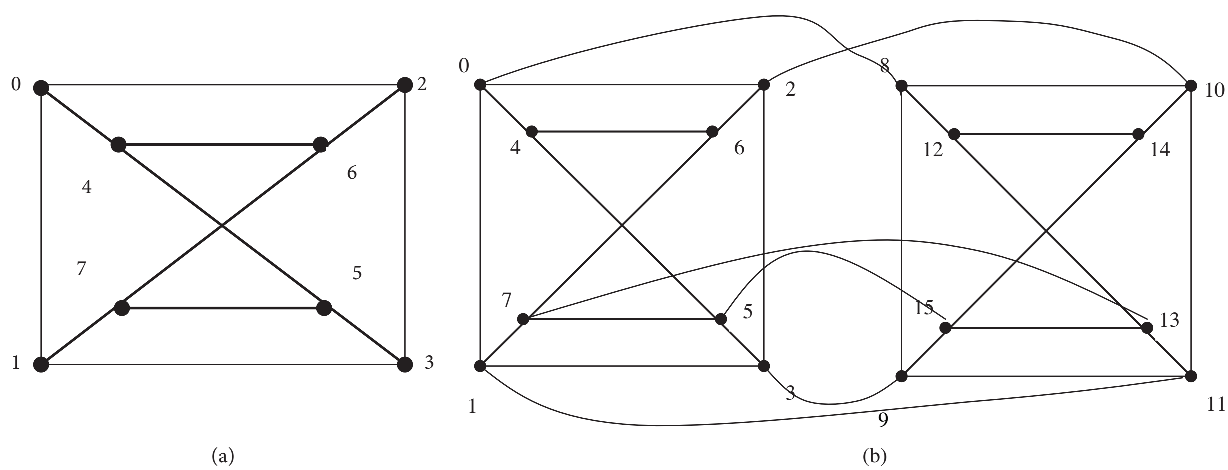

Figure 1. CQ of dimension 3 and 4: a) CQ(3) and b) CQ(4).

2.3. Star graph

The n-dimensional star graph S(n) consists of a set of nodes $\{x_0, x_1, x_2, .x_{n-1}\}$, where $i =$ *1,2,3...n* and $x_i \neq x_j$ for $0 \leq i$ and $j \leq n-1$, as shown in Figure 2. Hence, the total number of vertices is $n!$. There is an edge between any 2 vertices if their labels differ in the first and in 1 additional position. That is, 2 permutations, $a_1a_2...a_n$ and $b_1b_2\ldots b_n$, are adjacent if and only if there exists an $i \neq 1$, such that $a_1 = b_i$, $a_i = b_1$ and $a_j = b_j$ for $j \notin \{1,i\}$. Such an edge is called an i-edge. For any 2 nodes a and b belonging to S(n), traveling from a to b takes at most $\left\lfloor \frac{3(n-1)}{2} \right\rfloor$ steps [8].

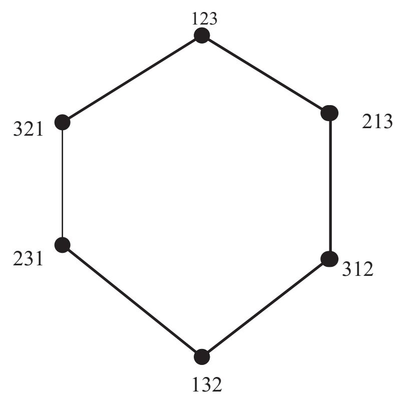

Figure 2. Star graph of dimension 3.

Both of the base networks are regular, vertex, and edge symmetric.

3. Proposed topology

The network details of the proposed topology SCQ(m,n) are described below.

3.1. Star-crossed cube

The SCQ(m,n) is the product graph of the CQ(m) and S(n). That is, in an n-star, each vertex is replaced with a CQ. Next, the node address of each vertex in the resulting graph will have 2 parts $< x_{m-1}, x_{m-2} \ldots x_0, y_0, y_1, \ldots, y_{n-1} >$, where the x_is represent the CQ part and the y_is represent the star part. Each node will have 2 types of neighbors, namely the CQ-part neighbor and star-part neighbor with node addresses $< x_{m-1}, x_{m-2..} \ldots . x_i'.x_0, y_0, y_1, \ldots, y_{n-1} >$ and $< x_{m-1}, x_{m-2} \ldots x_0, y_i, y_1, y_{i-1,} y_0 \ldots, y_{n-1} >$, respectively.

The SCQ(3,3), along with the submodules, is shown in Figures 3a, 3b, and 3c, respectively. In Figure 3a, the first submodule with the star part labeled as 123 and the corresponding labels of the CQ are shown. Similarly, Figure 3b shows the second submodule of the SCQ(3,3) with 213 as their star part level.

3.2. Topological properties

The following are the various topological properties of the proposed network.

Theorem 1 *The total number of nodes in the SCQ(m,n) graph is* $n! \times 2^m$.

Proof For the n-star, the total number of nodes is $n!$and in the CQ(m) it is 2^m. The SCQ is a product graph, where the total number of nodes is:

$$p = n! \times 2^m.$$

□

Theorem 2 *The total number of edges in the SCQ(m,n) is* $n! \times 2^{m-1}(m + n - 1)$.

Proof In the SCQ(m,n) there are n! CQ(m)s connected to and from an n-star. Hence, n!

CQs will have n! × m $\times 2^{m-1}$ edges. Next, each CQ(m) is connected to (n − 1) neighbors. Hence, the total number of edges interconnecting n! CQ(m)s is n! $\times (\frac{n-1}{2}) \times 2^m$.

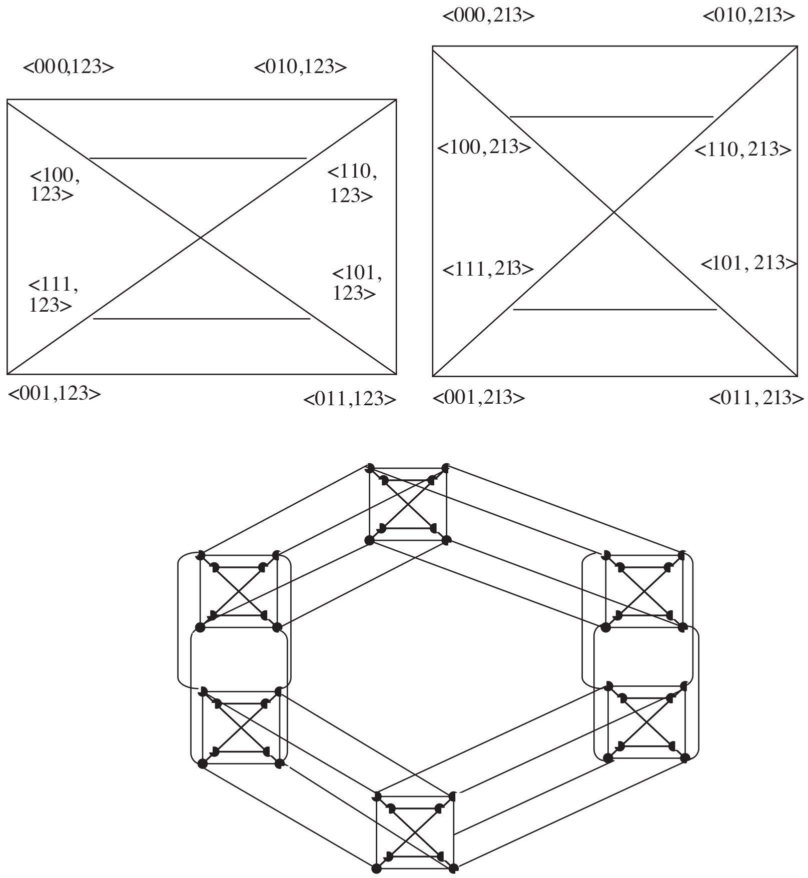

Figure 3. The SCQ and its basic modules: a) SCQ_{123}, b) SCQ_{213}, and c) SCQ(3,3).

Hence, the total number of edges in the SCQ(m,n) is:

$E =$ n! × m $\times 2^{m-1}_{+}$ n! $\times (\frac{n-1}{2}) \times 2^m$,

$$E = n! \times 2^{m-1}(m + n - 1).$$

□

Theorem 3 *The degree of the SCQ(m,n) is $(m + n - 1)$.*

Proof In the CQ(m), the degree of each node is m. Next, there are (n – 1) edges incident on each vertex to form the n-star. Hence, the degree of each vertex in the SCQ(m,n) is $(m + n - 1)$. □

Theorem 4 *The diameter of the SCQ(m,n) is* $\left\lfloor \frac{3(n-1)}{2} \right\rfloor + \left\lceil \frac{m+1}{2} \right\rceil$.

Proof Let (u, v) and (u', v') be 2 nodes in the SCQ(m,n). Traveling from (u, v) to (u', v) then takes at most $\left\lceil \frac{m+1}{2} \right\rceil$ steps, as they belong to the same CQ(m). Next, traveling from

(u', v) to (u', v') takes at most $\left\lfloor \frac{3(n-1)}{2} \right\rfloor$ steps in the S_n, as the nodes with the same CQ part label form an n-star.

Hence, in at most $\left\lfloor \frac{3(n-1)}{2} \right\rfloor + \left\lceil \frac{m+1}{2} \right\rceil$ steps, the node (u', v') can be reached from (u, v). □

Theorem 5 *The cost of the SCQ network is* $\xi = (m+n-1)\left(\left\lfloor \frac{3(n-1)}{2} \right\rfloor + \left\lceil \frac{m+1}{2} \right\rceil\right)$

Proof In symmetric networks, the cost factor ξ is defined as the product of the degree and diameter. From Theorem 3, the degree of the SCQ is $(m+\ n\ –\ 1)$. From Theorem 4, the diameter is $\left\lfloor \frac{3(n-1)}{2} \right\rfloor + \left\lceil \frac{m+1}{2} \right\rceil$.

Hence, the cost is given by $\xi = (m+n-1)\left(\left\lfloor \frac{3(n-1)}{2} \right\rfloor + \left\lceil \frac{m+1}{2} \right\rceil\right)$. □

Theorem 6 *The average distance of the SCQ(m,n) is* $\bar{d}_{scq} = \frac{11x+4y}{8} + n - 4 + \frac{2}{n} + \sum_{i=1}^{n} \frac{1}{i}$ *, where* $m = 3x + y$, $y < 3$*, and x ,y are integer values.*

Proof The average distance of the CQ is $\frac{11x+4y}{8}$ and for the star graph it is

$n - 4 + \frac{2}{n} + \sum_{i=1}^{n} \frac{1}{i}$. Hence, the result follows. □

Theorem 7 *The message density of the SCQ is given by* $\rho = \frac{2\bar{d}_{scq}}{(m+n-1)}$.

Proof This message density is defined as $\rho \equiv \frac{\bar{d}N}{E}$, where N is the total number of nodes, $\bar{d}$ is the average node distance, and E is the total number of links. It is assumed that each node is sending one message to a node at distance d on the average.

From Theorem 6, $\bar{d}_{scq} = \frac{11x+4y}{8} + n - 4 + \frac{2}{n} + \sum_{i=1}^{n} \frac{1}{i}$. Next, using Theorems 1 and 2,

$p = n! \times 2^m$ and $E = n! \times\ 2^{m-1}(m+n-1)$.

$Hence, the\, message\, density\, is\, given\, by$:

$$\frac{11x+4y}{8} + n - 4 + \frac{2}{n} + \sum_{i=1}^{n} \frac{1}{i} n! 2^{m-1}(m+n-1)\rho = \frac{n!2^m(\,)}{} = \frac{11x+4y}{8} + n - 4 + \frac{2}{n} + \sum_{i=1}^{n} \frac{1}{i} \frac{(\,)}{(\frac{m+n-1}{2})}$$

$$or,\ \rho = \frac{2\bar{d}_{scq}}{(m+n-1)}$$

□

Theorem 8 *Partitioning of the SCQ(m,n) can be done into 2 SCQ(m–1,n)s in m ways and into 2 SCQ(m,n–1)s in* $(n-1)$ *ways.*

Proof From the structure of the SCQ, it is evident that the basic modules are CQs, which are connected in star graph fashion. The CQ(m)s can be partitioned into CQ(m–1)s in m ways. The skeleton of the SCQ is an n-star that can be partitioned into $(n-1)$ stars in $(n-1)$ ways. □

Theorem 9 *The SCQ(m,n) has* $(m+n-1)$ *node disjoint paths between any 2 of its nodes.*

Proof The node connectivity of the CQ(m) is m. Similarly for the n-star, it is $(n-1)$. Hence, in the SCQ, the node connectivity becomes $(m+n-1)$. Hence, there are $(m+n-1)$ node disjoint paths between any 2 nodes. □

The different topological parameters of the proposed topology are compared with those of the parent networks in Table 1.

Table 1. Comparison of the topological parameters.

Parameters	HC	CQ	Star	Star cube	SCQ
Nodes	2^m	2^m	n!	$n!2^m$	$n!2^m$
Links	$m2^{m-1}$	$m2^{m-1}$	$n!\left(\frac{n-1}{2}\right)$	$n!2^{m-1}(m+n-1)$	$n!2^{m-1}(m+n-1)$
Degree	m	m	n-1	$m+n-1$	$m+n-1$
Diameter	m	$\left\lceil\frac{m+1}{2}\right\rceil$	$\left\lfloor\frac{3}{2}(n-1)\right\rfloor$	m+ $\left\lfloor\frac{3}{2}(n-1)\right\rfloor$	$\left\lceil\frac{m+1}{2}\right\rceil+\left\lfloor\frac{3}{2}(n-1)\right\rfloor$
Avg. dist.	$\frac{m}{2}$	$\frac{(11x+4y)}{8}$	n- $4+\frac{2}{n}+\sum_{i=1}^{n}1/i$	$\frac{m}{2}+n-4+\frac{2}{n}+\sum_{i=1}^{n}1/i$	$\frac{11x+4y}{8}+n-4+\frac{2}{n}+\sum\limits_{i=1}^{n}\frac{1}{i}$
Cost	m^2	m$\left\lceil\frac{m+1}{2}\right\rceil$	(n-1)$\left\lfloor\frac{3}{2}(n-1)\right\rfloor$	$(m+n-1)(m+\left\lfloor\frac{3}{2}(n-1)\right\rfloor)$	$(m+n-1)\left(\left\lceil\frac{m+1}{2}\right\rceil+\left\lfloor\frac{3}{2}(n-1)\right\rfloor\right)$
Msg. density	1	$\frac{2\bar{d}_{cq}}{n}$	$\frac{2\bar{d}_s}{n-1}$	$\frac{2\bar{d}_{sc}}{(m+n-1)}$	$\frac{2\bar{d}_{scq}}{(m+n-1)}$

4. Performance analysis

In this section, 2 important measures of performance of the proposed network, namely the cost effectiveness factor (CEF) and the time cost effectiveness factor (TCEF), are analyzed.

The CEF is defined as the ratio of the cost effectiveness and the efficiency. It takes into account the cost of the system (that is, the processor cost and link cost), as well as the processor utilization [18].

Theorem 10 *The cost effectiveness factor of the SCQ(m,n) is* $\frac{1}{1+\rho\frac{m+n-1}{2}}$ *, where* ρ *is the ratio of the link cost to the processor cost.*

Proof In general, the number of links is a function of the number of nodes in the system. In the proposed network, the number of nodes is given by $p=2^m n!$. The total number of links is given by $E=2^{m-1}n!(m+n-1)$ $=n!\,2^m\left(\frac{m+n-1}{2}\right)=\;p(\frac{m+n-1}{2})=f(p)$.

$$g\left(p\right)=\frac{f\left(p\right)}{p}=\frac{m+n-1}{2}$$

$$\text{CEF(p)}=\frac{1}{1+\rho g(p)}=\frac{1}{1+\rho\left(\frac{m+n-1}{2}\right)}$$

□

Theorem 11 *The TCEF of the SCQ(m,n) network is given by TCEF* $=\frac{1+\sigma}{1+\rho\left(\frac{m+n-1}{2}\right)\left(\frac{1}{2^m n!}\right)}$.

Proof The TCEF is given by TCEF(p,T$_p$) $=\frac{1+\sigma T_1^{\alpha-1}}{1+\rho g(p)+\frac{\sigma}{p}}$, where T$_1$ is the time required to solve the problem by a single processor using the fastest sequential algorithm, T$_p$ is the time required to solve the problem by a parallel algorithm using a multiprocessor system having p processors, and ρ is the cost of the penalty / cost of the processors [18]. As is seen in Theorem 10 above, $g\left(p\right)=\frac{m+n-1}{2}$ and $\sigma\, and\, \alpha$ are assumed to be 1.

Hence, TCEF $=\frac{1+\sigma}{1+\rho g(p)+\left(\frac{\sigma}{p}\right)}=\frac{1+\sigma}{1+\rho\left(\frac{m+n-1}{2}\right)+\left(\frac{\sigma}{2^m n!}\right)}$.

□

5. Embedding

The embedding is regarded as an important property of any topology. The embedding of rings, meshes, and binary trees into star graphs and CQs was previously studied [19,20]. This section investigates the embedding of rings and meshes into the SCQ.

The embedding of a guest graph $G(V, E)$ into another host graph S is a mapping of the set of vertices $V(G)$ into that of S, that is, $V(S)$ and $E(G)$ into E(S). The mapping is denoted by R. Thus, any vertex x of G is mapped through R(x) of S uniquely. That means R(x) $\neq$ R(y) for x$\neq$y and x, y $\in V(G)$. However, for the embedding to exist, $|S| \geq |G|$ and S has to be connected. The ratio $|S|/|G|$ is called the *expansion.*

The *dilation* is defined as follows: *dilation (d)* = *max.* {*length of the shortest path from R(x) to R(y)*} . The *congestion* of any edge e of S is the number of paths (each path representing an edge of G mapped to S) of G, which contains e. The maximum of the congestions of all of the edges of S is the congestion of the embedding.

5.1. Embedding of the ring in the SCQ(m,n)

A ring can be embedded into the SCQ(m,n) using Gray codes. A Gray code is a well-known sequence of binary bits, where 2 consecutive codes differ by only 1 bit, G_1 = (0,1). From G_1, G_2 can be derived as (00,01,11,10). For n > 2, $G_n = (0G_{n-1}, 1G^r_{n-1}))$, where G^r_n is the reverse string of G_n. Hence, $G_3 = (0G_2,\ 1G^r_2)$ = (000,001,011,010,110,111,101,100).

Next, $G_4 = (0G_3, 1, G^r_3)$ and $G_5 = (00G_3, 01G^r_3, 11G^r_3, 10G_3)$. The first and last labels differ by 1 bit only.

A ring can be successfully embedded in the SCQ. In Figure 4, the arrow heads show the sequence of embedding. A ring with 8 nodes bearing the codes as in G_3 = (0,1,3,2,6,7,5,4) can be easily embedded in SCQ_{123} as defined above.

R(0) = (0,123), R(1) = (1,123), R(3) = (3,123) .... , and R(4) = (4,123).

For a 16-node ring with codes as in G_4 = (0,1,3,2,6,7,5,4,12,13,15,14,10,11,9,8), the embedding extended to SCQ_1 and SCQ_2 is as follows:

R(0) = (0,123), R(1) = (1,123), R(3) = (3,123) .... , and R(4) = (4,123).

R(12) = (4,213), R(13) = (5,213), R(15) = (7,213), R(14) = (6,213) ..., and R(8) = (0,213).

Obviously, the dilation and expansion for this embedding are both 1.

5.2. Mesh embedding in the SCQ(m,n)

Generally, in a CQ(m), a family of disjoint 3-dimensional (3D) meshes measuring $2 \times 2 \times 2^{m-3}$ can be embedded for m $\geq$ 4 [20]. The current work illustrates the embedding of 3D meshes into the SCQ topology.

Theorem 12 *For m $\geq$ 3, a family of $2^{m-2}(n)(n-1)(n-2)\ldots 3$ disjoint 3D meshes measuring $2 \times 2 \times 2^{m-3}$ can be embedded into the SCQ(m,n) with unit dilation and unit expansion.*

The Theorem is proven by the induction method.

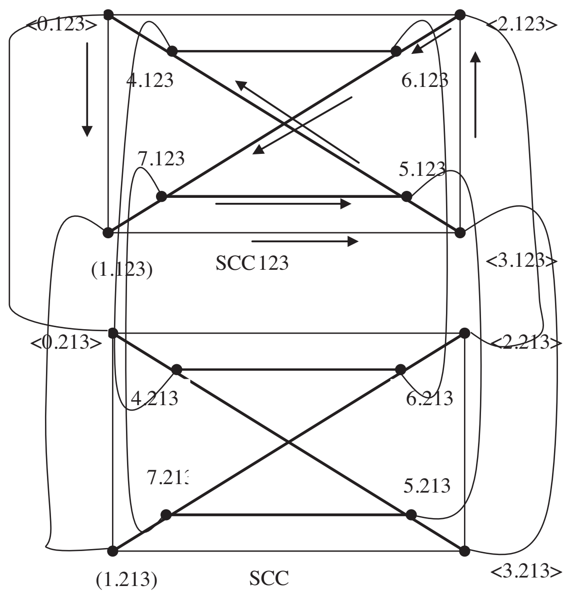

Figure 4. Ring embedding in SCQ_1 and SCQ_2.

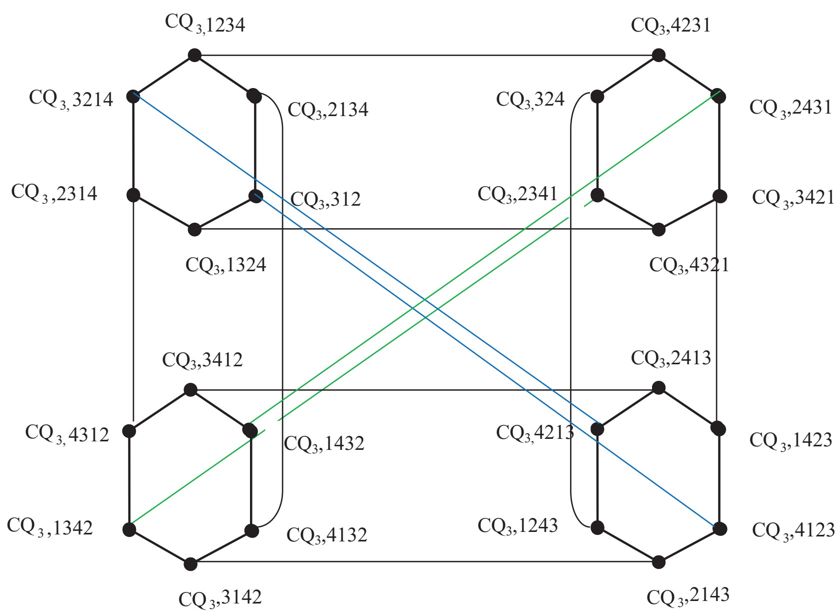

Figure 5. SCQ(3,4).

Lemma 1 *In SCQ(3,3), a family of* $2^{3-2} \times 3 = 6$ *disjoint 3D meshes can be embedded.*

Proof In SCQ(3,3), the subgraph SCQ(CQ_3,123) will combine with SCQ(CQ_3, 321). Next, SCQ(CQ_3,213) will combine with SCQ(CQ_3,312) and SCQ(CQ_3,231) with SCQ(CQ_3,132). Thus, there are 3 sets of interlinks and each interlink forms 2 disjoint meshes. Hence, the number of meshes embedded = 2 × 3= 6. The meshes are as follows:

$$m_{11} = \left(\begin{pmatrix} 0,123 & 2,123 \\ 0,321 & 2,321 \end{pmatrix} \begin{pmatrix} 1,123 & 3,123 \\ 1,321 & 3,321 \end{pmatrix} \right) m_{12} = \left(\begin{pmatrix} 4,123 & 6,123 \\ 4,321 & 6,321 \end{pmatrix} \begin{pmatrix} 5,123 & 7,123 \\ 5,321 & 7,321 \end{pmatrix} \right)$$

$$m_{21} = \left(\begin{pmatrix} 0,231 & 2,231 \\ 0,132 & 2,132 \end{pmatrix} \begin{pmatrix} 1,231 & 3,231 \\ 1,132 & 3,132 \end{pmatrix} \right) m_{22} = \left(\begin{pmatrix} 4,231 & 6,231 \\ 4,132 & 6,132 \end{pmatrix} \begin{pmatrix} 5,231 & 7,231 \\ 5,132 & 7,132 \end{pmatrix} \right)$$

$$m_{31} = \left(\begin{pmatrix} 0,213 & 2,213 \\ 0,312 & 2,312 \end{pmatrix} \begin{pmatrix} 1,213 & 3,213 \\ 1,312 & 3,312 \end{pmatrix} \right) m_{32} = \left(\begin{pmatrix} 4,213 & 6,213 \\ 4,312 & 6,312 \end{pmatrix} \begin{pmatrix} 5,213 & 7,213 \\ 5,312 & 7,312 \end{pmatrix} \right) \quad \square$$

The cube part node addresses are represented by decimal numbers; that is, 0, 1, 2, ... 7.

This embedding has unit dilation and unit expansion.

Lemma 2 *In SCQ(4,3), a family of* $(2^{4-2} \times 3) =$ *12 disjoint 3-D meshes can be embedded.*

Proof In SCQ(4,3), the CQ_4 is the basic module in the 3-star. Hence, there are 6 CQ_4s. Each CQ can embed disjoint 3D meshes. Again, 3 sets of interlinks can be used in the 3-star. Each interlink can form 4 disjoint meshes. Hence, the number of meshes embedded in SCQ(4,3) $= (2^{4-2} \times 3) = 12$. The meshes are as follows:

$M_1 = \begin{pmatrix} 0(m_{11}) & 0(m_{12}) & 1(m_{12}) & 1(m_{11}) \end{pmatrix}$ (where 0 and 1 are appended in front of the cube part of the node address).

$$= \left(\begin{pmatrix} 0,123 & 2,123 \\ 0,321 & 2,321 \end{pmatrix} \begin{pmatrix} 1,123 & 3,123 \\ 1,321 & 3,321 \end{pmatrix} \begin{pmatrix} 4,123 & 6,123 \\ 4,321 & 6,321 \end{pmatrix} \begin{pmatrix} 5,123 & 7,123 \\ 5,321 & 7,321 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 8,123 & 10,123 \\ 8,321 & 10,321 \end{pmatrix} \begin{pmatrix} 15,123 & 13,123 \\ 15,321 & 13,321 \end{pmatrix} \begin{pmatrix} 9,123 & 11,123 \\ 9,321 & 11,321 \end{pmatrix} \begin{pmatrix} 12,123 & 14,123 \\ 12,321 & 14,321 \end{pmatrix} \right)$$

$M_2 = (\ 0(m_{21}) \ \ 0(m_{22}) \ \ 1(m_{22}) \ \ 1(m_{21}) \ ))$

$M_3 = (\ 0(m_{31}) \ \ 0(m_{32}) \ \ 1(m_{32}) \ \ 1(m_{31}) \ ))$. $\square$

Lemma 3 *In SCQ(3,4), a family of* $(2^{3-2} \times 4 \times 3) = 24$ *disjoint 3D meshes can be embedded.*

Proof Here, CQ_3 is kept on a 4-star, as shown in Figure 5. Hence, there can be 4! = 24 CQs in total. Next, the number of interlinks is n(n-1) = 4 × 3. In SCQ(3,4), each set of interlink forms 2^{3-2} meshes. In total, there can be $(2^{3-2} \times 4 \times 3) = 24$ disjoint meshes. $\square$

Lemma 4 *In SCQ(4,4) a family of* $(2^{4-2} \times 4 \times 3) = 48$ *disjoint 3D meshes can be embedded.*

Proof In SCQ(4,4), CQ_4 becomes the building block on a 4-star. Hence, there are 24 CQs. These can be connected using n(n–1) = 4 × 3 = 12 interlinks. Now each set of interlinks can form $2^{4-2} = 4$ meshes. Hence, in total, there can be 48 meshes embedded. $\square$

Lemma 5 *In SCQ(3,5) , a family of* $(2^{3-2} \times 5 \times 4 \times 3) = 120$ *disjoint 3D meshes can be embedded.*

Proof In SCQ(3,5), the basic building blocks are 3D CQs on a 5-star. Hence, there can be 5! = 120 CQs. In SCQ(3,5), there are 5 clusters, as shown in Figure 6. Hence, the number of interlinks is 5 × 4 × 3 = 60. Now, each set of interlinks can form a total of $2^{3-2} = 2$ meshes. Hence, in total, there can be 60 × 2 = 120 disjoint meshes embedded. $\square$

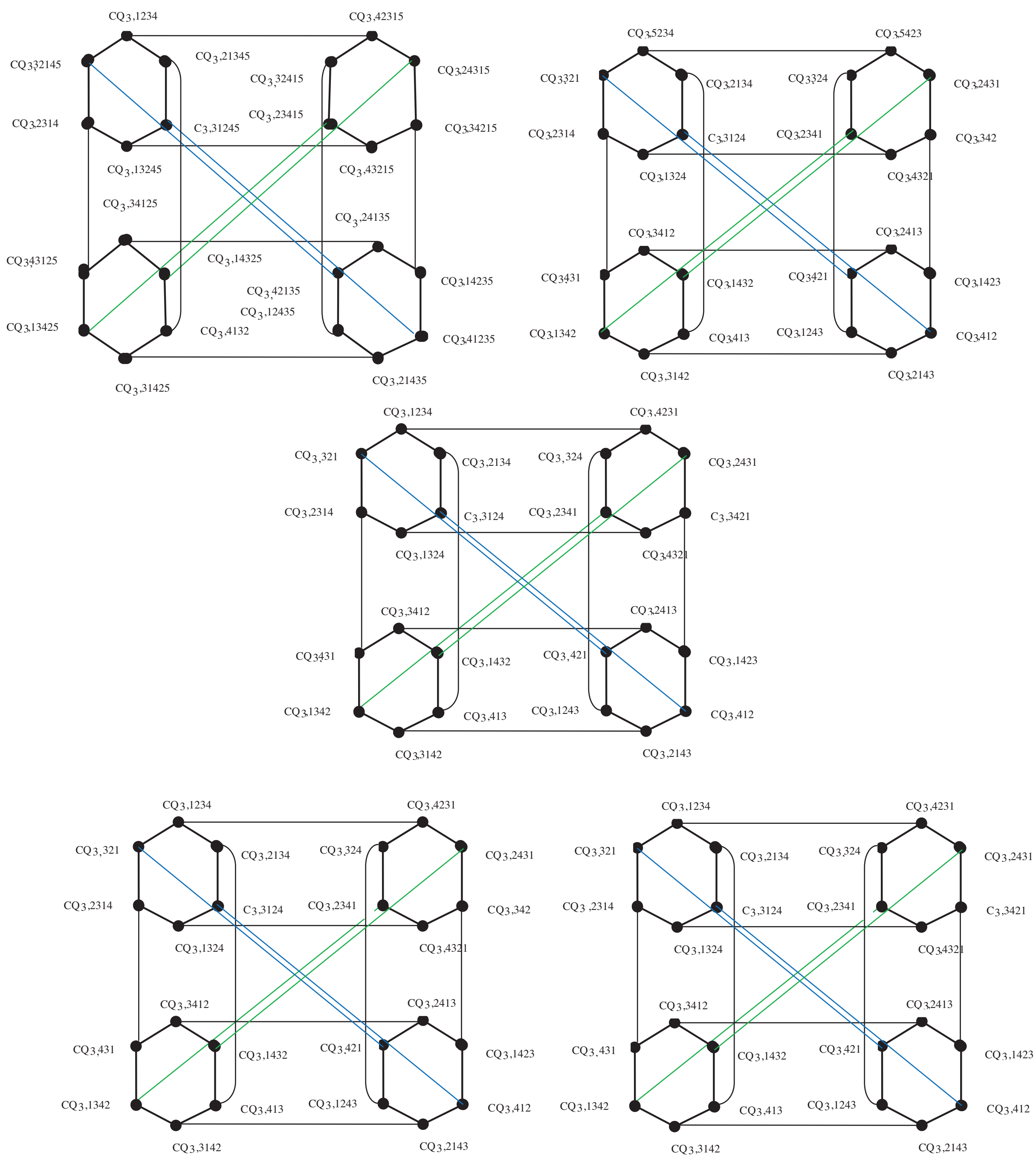

Figure 6. SCQ(3,5) with all 5 clusters.

Proof of Theorem 12

In the SCQ(m,n), the CQ(m) becomes the basic building block on an n-star graph. By induction on m and n, according to Lemma 1 to 5, the CQ(m) can admit 2^{m-2} meshes. Next, in the n-star, there are n clusters and each cluster is an (n-1)-star. Hence, the number of interlinks is n(n-1)(n-2)....3. Thus, the total number of disjoint 3D meshes in the SCQ(m,n) is $2^{m-2} \times n(n-1)(n-2)\ldots.3$. Hence, the result.

6. Routing and broadcasting

The problem of finding a path from a source node s to a destination node t and forwarding the message along the path is called routing. For any parallel processing network, the routing should be simple and fast. The CQ and star graph both have self-routing algorithms. Hence, in the hybrid network, the SCQ also allows self-routing. The current section describes the routing algorithm for the SCQ network.

The source and destination nodes are $u(c, s)$ and $v(c^{'}s^{'})$, respectively.

Routing in the SCQ(m,n)

{

Step 1: Route the message from u to u_1. *The address of* u_1 *will be either (* $c^{'}$*, s) or (c,* $s^{'}$ *). For the first case, CQ routing will be used as discussed in [21]. In the second case, star routing is used.*

Step 2: Next, from ($c^{'}s$*) the destination node (* $c^{'}s^{'}$ *) can be easily reached using star routing. If the intermediate node is (c,* $s^{'}$*), then CQ self-routing will be used to reach the destination node, that is, (* $c^{'}s^{'}$*).*

}

Algorithm one-to-all broadcast

An optimal broadcasting algorithm is proposed for the proposed SCQ network. For this, the star-broadcast algorithm [22] is used as a subroutine. Suppose $u(c, s)$ is an arbitrary node in any cluster of the SCQ. The following algorithm broadcasts a message to all of the other nodes in the SCQ(m,n).

Step 1: Suppose the cluster (c,) contains the source node u(c,s). Next, using the CQ-broadcast algorithm, a message can be transmitted to all other nodes within the cluster. Para do*

{

Step 2: Next, for intercluster communication, the star links are used, in which each node (x,v) of cluster (c,) transmits the message to node (x,*$v^{'}$ *) using star-broadcast. Next, in the other clusters, at least 1 node will be loaded with the message.*

Step 3: Now, the loaded nodes in each cluster will do a broadcast within the cluster using the CQ-broadcast algorithm [21].

}

Theorem 13 *The one-to-all broadcast algorithm for the SCQ(m,n) takes* $O(m + nlogn)$ *time.*

Proof The 3 steps in the above algorithm take $O(m)$, $O(nlogn)$, and $O(m)$ time, respectively. Thus, the entire algorithm takes $O(m + nlogn)$ time. □

Algorithm all-to-all broadcast

In all-to-all broadcasting, every node in the network sends a message to all other nodes in the network. Hence, the one-to-all algorithm can be executed for each node for complete broadcasting of the message in the network.

Theorem 14 *In the SCQ(m,n) the time complexity for all-to-all broadcasting is* $O(M + nlogn)$.

Proof For a cube-type network of dimension m, consisting of M nodes, the time complexity for all-to-all broadcasting is $O(M)$ [22]. In an n-star, any sequence of nodes that constitutes a one-to-all broadcasting algorithm is also an all-to-all broadcasting algorithm.

Hence, in the SCQ(m,n), the time complexity of the all-to-all broadcasting algorithm is $O(M + nlogn)$. □

7. Results and discussions

This section presents the different results obtained. Moreover, a comparative study is made to establish the superiority of the proposed topology.

While comparing with star and HS, and hyperstar, the growth of the SCQ is slower, as shown in Figure 7. However, it has a better packing density than the CQ and the HC with same node degree. The node degree of the starcube and the SCQ(m,n) is the same but it is higher than that of the star graph and the traditional HC, as well as CQ due to the hybrid structure. The HS has a degree equal to that of HC and CQ. For even dimensions, the degree of the hyperstar is the same as that of the n-star. The comparison of the node degree against the dimensions is shown in Figure 8.

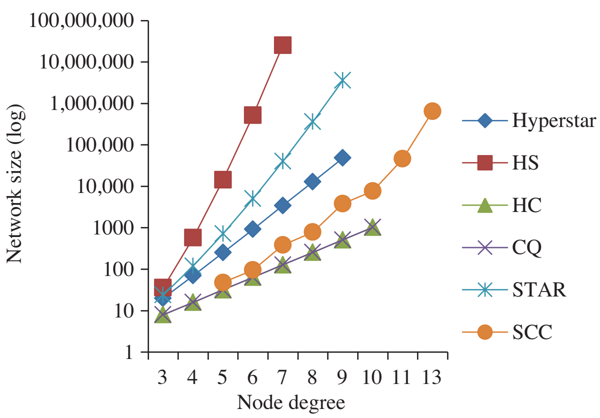

Figure 7. Comparison of the packing densities.

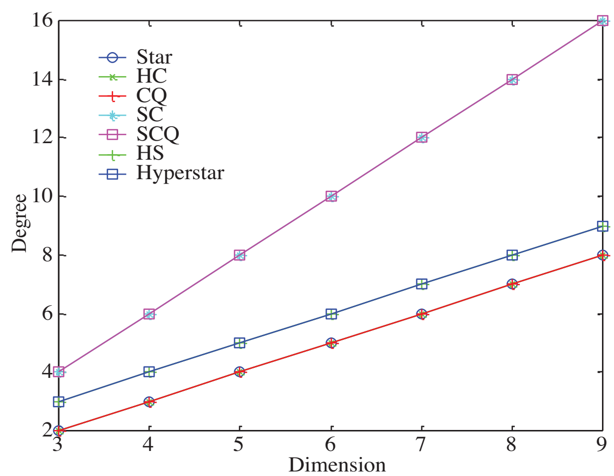

Figure 8. Comparison of the node degrees.

While comparing the diameter, it is observed that the HS possesses the highest value as its size grows at a faster rate. The HC and the CQ both possess smaller values, as they are smaller networks. The SCQ stands lowest among the hybrid networks, as shown in Figure 9. At even dimensions, the SCQ bears lower values than the hyperstar. For odd dimensions, both have the same values.

The SCQ, being a hybrid network, has a cost that is higher than that of its parent networks, namely the HC, CQ, and n-star, as shown in Table 2. However, the cost is found to be less than that of the starcube, the HS, and hyperstar, as shown in Figure 10. In Figure 10, the cost is compared against the node degree. With node degree 5 and beyond, the SCQ bears the lowest value.

Table 2. Comparison of the cost of SCQ with the parent networks.

Degree	Star	SC	SCQ	HC
3	6			9
4	12			16
5	24	20	20	25
6	35	42	36	36
7	54	63	56	49
8	70	80	72	64
9	96	108	99	81

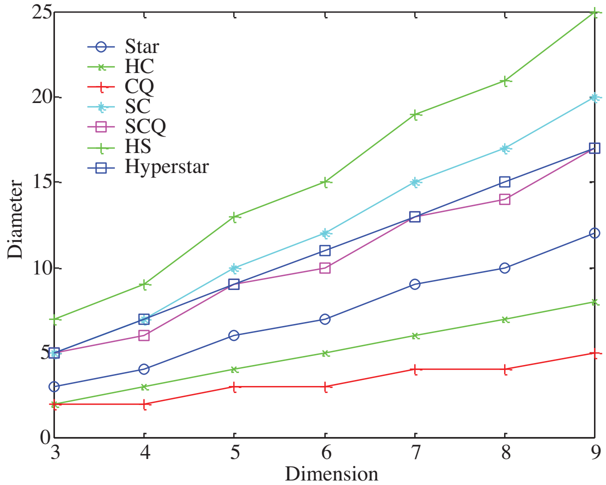

Figure 9. Comparison of the diameter.

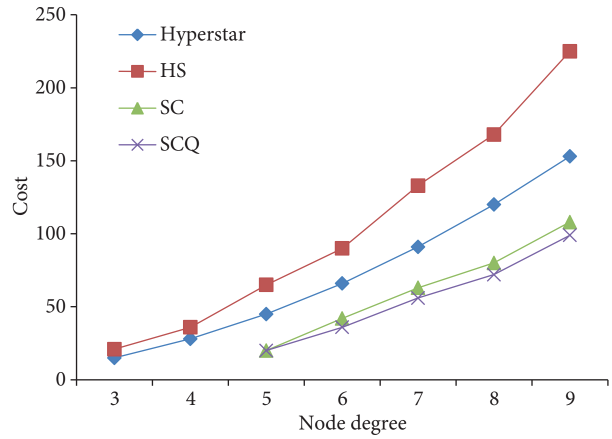

Figure 10. Comparison of the cost.

A comparison of the average node distance of the SCQ shows that it is a better network topology at higher dimensions than the other networks. The comparison is shown in Figure 11.

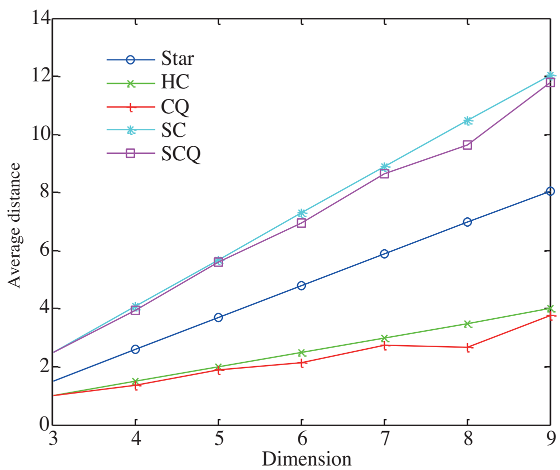

Figure 11. Average node distance comparison.

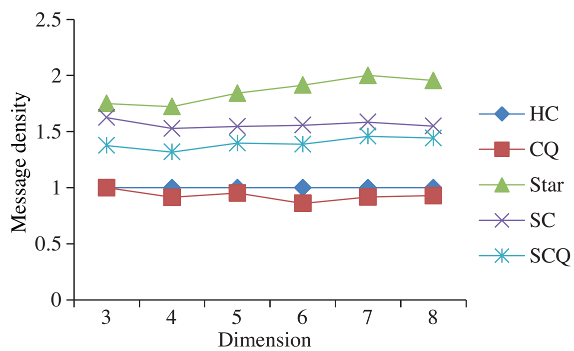

Figure 12. Message density comparison.

Next, the average message density of the SCQ is compared with the parent networks in Figure 12. The message traffic density of the HC is always 1. For the star graph, the value is higher with more nodes. Being a hybrid graph, the SCQ possesses better values with a higher packing density. It is always less than that of the starcube and the star graph.

Figures 13 and 14 show the variation of CEF and TCEF with respect to the dimensions. The proposed network is cost-effective due to the monotonic decreasing nature of the curves, similar to HC. It is more beneficial when the dimensions lie between 4 and 8.

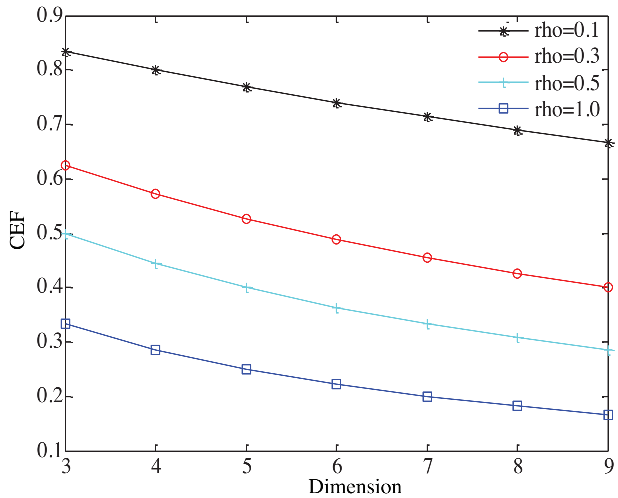

Figure 13. Comparison of the CEF.

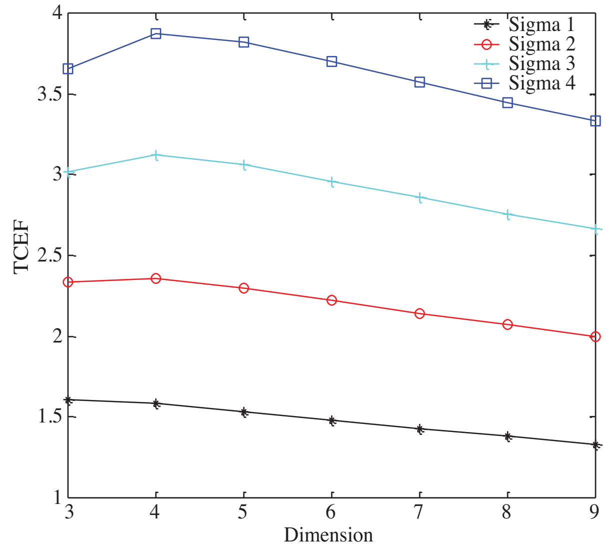

Figure 14. Comparison of the TCEF.

8. Conclusion

This paper proposes a new interconnection topology suitable for large-scale parallel systems. Being a hybrid structure, the SCQ bears the advantages of both of the networks. Compared to the existing hybrid networks, it possesses better characteristics, such as regularity, degree, diameter, cost, average distance, and traffic density. Moreover, mesh and ring embedding into the SCQ is possible with the unit dilation and unit congestion. Different performance measures make the SCQ(m,n) a very promising and suitable candidate for parallel systems.

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