Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2014) 22: 1017 – 1031

doi:10.3906/elk-1207-47

# New throughput-based antenna selection scheme

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**Received:** 15.07.2012 • **Accepted:** 17.12.2012 • **Published Online:** 17.06.2014 • **Printed:** 16.07.2014

**Abstract:** In this paper, a new throughput-based transmit antenna selection (AS) scheme is evaluated. First, a closed-form solution for the throughput rate of a multiple-input multiple-output (MIMO) system, with a truncated selective repeat automatic repeat request (TSR-ARQ) or selective repeat plus go-back-N automatic repeat request (SR+GBN-ARQ) protocol at the data link layer, is proposed. In addition, a throughput-based AS technique with adaptive modulation is also investigated. For this purpose, the throughput and packet error rates of a MIMO system with a transmit AS and TSR-ARQ protocol at the data link layer are analyzed. This study is conducted in 2 different cases. In the single-transmit AS case, the probability density functions (PDFs) of the throughput and packet error rates at the data link layer are derived. Moreover, in the case with multiple-transmit AS, a lower bound for the throughput rate and an upper bound for the packet error rate at the data link layer are determined. Consequently, the PDFs of these lower and upper bounds are extracted.

**Key words:** Adaptive modulation, antenna selection, automatic repeat request, cross-layer design, MIMO channel, packet error rate, throughput rate

## 1. Introduction

With the increasing demand for high data-rate services in emerging communication systems, such as third-generation (3G) and 4G wireless systems, high-speed and high-quality data communications have become extremely necessary. The major challenge in providing high-speed and high-quality data communications is the wireless channel, which restricts the overall throughput of the system. Multiple-input multiple-output (MIMO) technology is an attractive solution to overcome the limitations of the wireless channel. On the other hand, an alternative way to combat channel fading is the automatic repeat request (ARQ) protocol [1,2]. Since the ARQ retransmission is only activated when it is necessary, this scheme is very efficient for the overall throughput rate of the system.

MIMO systems can increase the data rate with the spatial multiplexing configuration [3]. However, the multiple-antenna front-end architecture design traditionally results in greater complexity and higher hardware costs in the radio frequency (RF) section [4,5]. One simplifying and cost-reducing solution may be the utilization of a single RF front-end, where a single RF path is used instead of multiple parallel RF chains [4,5]. In addition, the antenna selection (AS) technique is a common method for dealing with this issue [6–10]. AS can be implemented at the transmitter and/or receiver, but only transmit AS is our concern here. The conventional criteria for the AS technique are channel capacity maximization [6] or error rate minimization [7–10]. In [6],

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analytical bounds for the channel capacity of a MIMO system with AS were derived. In [7], a selection algorithm for the minimization of the error probability in the spatial multiplexing configuration with a linear receiver was presented. AS techniques in MIMO systems with orthogonal space time block codes and space time trellis codes were discussed in [8,9], respectively.

Cross-layer design is a new approach in wireless research that tries to overcome the deficiencies of the old multilayer configuration of networks [11,12]. In these papers, the AS scheme was not considered and a cross-layer–based analysis was performed in a MIMO system. In the same way, cross-layer based AS was considered recently in [13–15]. Instead of capacity or error rate optimization, the purpose of cross-layer–based AS is throughput maximization. Hence, the authors in [13] investigated the performance of a cross-layer design with AS when employing ARQ and transmission control protocol over Rayleigh fading channels. In [14,15], a throughput-based AS algorithm was presented. The authors of [14] assumed the stop and wait ARQ (SAW-ARQ) protocol, while the authors of [15] adopted the go-back-N ARQ (GBN-ARQ) protocol at the data link layer. In both [14] and [15], the throughput rate in the MIMO system with 2 simple ARQ protocols at the data link layer was extracted.

The SAW-ARQ and GBN-ARQ protocols [2] are very simple. However, when throughput rate maximization is the goal, selective repeat ARQ (SR-ARQ) performs much better [2,16]. In the SR-ARQ protocol, packets are transmitted continuously and the transmitter only resends those negatively acknowledged packets [2]. The SR-ARQ protocol theoretically requires an infinite buffer length. Consequently, similar protocols, such as truncated SR ARQ (TSR-ARQ) and SR + go-back-N ARQ (SR+GBN-ARQ), with limited buffer lengths, are more interesting [16]. In the TSR-ARQ technique, just $\nu$ retransmissions are allowed, and after $\nu + 1$ attempts with no successful packet transmission, an error is declared. In mixed mode strategies such as SR+GBN-ARQ [16], $\nu$ retransmissions in the SR mode are allowed. After that, the transmitter switches to the GBN mode and previous $N$ packets are resent, regardless of whether they were received error-free or not. Here, $N$ is the number of transmit packets during the packet trip time.

In this paper, the throughput-based transmit AS technique is investigated. First, the throughput rate of a MIMO system with TSR-ARQ or SR+GBN-ARQ at the data link layer is studied. The adaptive modulation scheme is used at the physical layer here. Moreover, a MIMO multiplexing configuration is assumed. Hence, the throughput rate of the system with 2 types of detectors, zero forcing (ZF) and successive interference canceler (SIC), are derived. In this case, a simple throughput-based transmit AS algorithm is proposed. In addition, the throughput rate and the packet error rate at the data link layer of the MIMO systems with transmit AS are studied in this paper. We assume a MIMO multiplexing configuration, ZF detector, and adaptive modulation at the physical layer and TSR-ARQ protocol at the data link layer. With the transmit AS technique, the throughput and packet error rates at the data link layer are examined in 2 different cases. When a single antenna is selected at the transmitter, an exact expression is derived for the probability density function (PDF) of the throughput and packet error rates. Here, there is no limit on the number of receive antennas. On the other hand, when multiple antennas are selected at the transmitter, a lower bound for the throughput rate and an upper bound for the packet error rate are derived. Consequently, the PDFs of these lower and upper bounds are extracted. In this case, an equal number of receive antennas and selected transmit antennas are assumed. Based on these derived PDFs, the statistical average of the throughput and packet error rates at the data link layer is examined. The main contributions of this paper can be summarized here as:

1. Presentation of the new throughput-based transmit AS technique with efficient ARQ protocols at the data link layer and adaptive modulation at the physical layer.
2. Statistical analysis of the throughput and packet error rates in the MIMO systems with the transmit AS technique.

The rest of this paper is organized as follows: First, the system model is introduced in Section 2. Next, the throughput rate of the MIMO system with 2 different detectors at the receiver and 2 types of ARQ protocols at the data link layer is described in Section 3. A simple throughput-based transmit AS is also presented in this section. Statistical analysis of the single- and multiple-transmit AS are also presented in Sections 4 and 5, respectively. Finally, the simulation results are shown in Section 6, and Section 7 concludes the paper.

## 2. System model

The throughput rate is defined as the number of correct received bits per unit time. The throughput rate is a cross-layer concept that is jointly affected by the physical layer and data link layer. The physical layer and data link layer model are briefly explained in this section.

### 2.1. Physical layer model

The physical layer is a MIMO system with $L_t$ transmit and $L_r$ receive antennas. The link between each transmit and receive antenna is an uncorrelated Rayleigh fading channel and $\mathbf{H}^{(L_t)}$ indicates the $L_r \times L_t$ channel matrix between the $L_t$ transmit and $L_r$ receive antennas. We also assume transmit AS, where $K_t$ out of $L_t$ antennas are selected for transmission. Here, $\mathbf{H}^{(K_t)}$ indicates the $L_r \times K_t$ channel matrix between the $K_t$ selected transmit and $L_r$ receive antennas. In the MIMO system, a spatial multiplexing configuration is assumed. Therefore, $K_t$ symbols transmit and receive by $L_r > K_t$ antennas. When the ZF detector is employed at the receiver, the instantaneous signal-to-noise ratio (SNR) at the $k$th $(k = 1, ..., K_t)$ path is [17]:

$$\gamma_k = \frac{\gamma}{K_t \left[ \left( \mathbf{H}^{(K_t)\dagger} \mathbf{H}^{(K_t)} \right)^{-1} \right]_{k,k}}, \tag{1}$$

where $\gamma$ is the average SNR, $[.]_{k,k}$ denotes the $k$th diagonal element of the matrix, and $\dagger$ represents the conjugate transpose of the matrix. Moreover, when the SIC detector is used, the instantaneous SNR at the $k$th $(k = 1, ..., K_t)$ path is written as [17]:

$$\gamma_k = \frac{\gamma |r_{k,k}^{(K_t)}|^2}{K_t}, \tag{2}$$

where $r_{k,k}^{(K_t)}$ represents the $k$th diagonal element of $\mathbf{R}^{(K_t)}$. Here, $\mathbf{R}^{(K_t)}$ is the upper triangular matrix in the QR decomposition [18] of the channel matrix as $\mathbf{H}^{(K_t)} = \mathbf{Q}^{(K_t)} \mathbf{R}^{(K_t)}$.

In the MIMO multiplexing configurations, there are $S$ symbols in the frame, which are transmitted from $K_t$ antennas. Therefore, $S/K_t$ symbols are transmitted from each antenna in the frame duration $T_f$. The exact packet error rate of $M_j$-state quadrature amplitude modulation (QAM) at the physical layer of the MIMO system can be written as:

$$P_e = 1 - \prod_{k=1}^{K_t} \left(1 - S_{e,k}(\gamma_k)\right)^{S/K_t}, \tag{3}$$

where

$$S_{e,k}(\gamma_k) = 2pQ\left(a\sqrt{\gamma_k}\right) + 2qQ\left(b\sqrt{\gamma_k}\right) - 4pqQ\left(a\sqrt{\gamma_k}\right)Q\left(b\sqrt{\gamma_k}\right), \tag{4}$$

the symbol error rate, and $p = 1 - 1/M_{j,I}$, $q = 1 - 1/M_{j,Q}$,

$$a = b = \sqrt{6/\left[(M_{j,I}^2 - 1) + (M_{j,Q}^2 - 1)\right]},$$

and the modulation order is $M_j = M_{j,I} \times M_{j,Q}$ [19]. Here, the adaptive modulation policy is to maximize the throughput rate of the systems. There are $J$ modes of transmission. Hence, the SNR range can be divided into $J$ nonoverlapping consecutive intervals, where the boundary points of these $J$ intervals are denoted by $\{\Gamma_j\}_{j=1}^{J+1}$ and $\Gamma_{J+1} = +\infty$. When $\gamma_k \in [\Gamma_j, \Gamma_{j+1})$, the $j$th modulation order $M_j$ is used for the subsequent transmission.

### 2.2. Data link layer model

While the data link layer has different tasks, its error correction capability is our concern here. In the data link layer, the ARQ protocol is applied. Therefore, some corrupted packets at the physical layer are retransmitted and corrected at this layer. When just $\nu$ retransmissions are allowed in the ARQ protocol, the packet error rate at the data link layer can be defined as:

$$P_d = P_e^{\nu+1}, \tag{5}$$

where $P_e$ is the packet error rate at the physical layer, which is defined in Eq. (3).

## 3. Throughput-based transmit AS

For the throughput-based transmit AS scheme, those antennas and modulation orders that maximize the throughput rate are selected for transmission. For this purpose, an exhaustive search can be executed to determine the best subset of antennas and modulation order in the adaptive modulation policy. There are $L_K = \left(\begin{array}{c} L_t \\ K_t \end{array}\right)$ subsets of antennas available for selection and, in each set, there are $J$ modulation orders available for use. Therefore, the complexity of this exhaustive search is in the order of $L_K J$.

A MIMO multiplexing configuration with the ZF or SIC detector at the receiver and TSR-ARQ or SR+GBN-ARQ protocol at the data link layer are assumed. Furthermore, the adaptive modulation scheme helps the transmitter to increase the throughput rate. The throughput rate of the single-input single-output systems [2,16] can be extended to MIMO systems as [20]

$$\begin{aligned} \eta_{TSR} &= K_t\,(1 - P_e)\,S\log_2(M_j) \\ &= K_t\left[\prod_{k=1}^{K_t}\left(1 - S_{e,k}(\gamma_k)\right)\right]^{S/K_t} S\log_2(M_j) \end{aligned} \tag{6}$$

when the TSR-ARQ protocol is applied and [20]

$$\begin{aligned} \eta_{SR+GBN} &= K_t \tfrac{1-P_e}{1+(N-1)P_e^{\nu+1}} S\log_2(M_j) \\ &= K_t \frac{\left[\prod\limits_{k=1}^{K_t}\left(1-S_{e,k}(\gamma_k)\right)\right]^{S/K_t} S\log_2(M_j)}{1+(N-1)\left\{1-\left[\prod\limits_{k=1}^{K_t}\left(1-S_{e,k}(\gamma_k)\right)\right]^{S/K_t}\right\}^{\nu+1}} \end{aligned} \tag{7}$$

when the SR+GBN-ARQ protocol is applied at the data link layer. $N$ is the number of packets that are transmitted during the packet trip time, $\nu$ is the maximum number of retransmissions in the SR ARQ protocols, and $S_{e,k}$ is introduced in Eq. (4). It is worth noting that the throughput rate depends on the instantaneous SNRs $\{\gamma_k\}_{k=1}^{K_t}$ and $\gamma_k$ is a function of $K_t$ selected transmit antennas. Therefore, in the new throughput-based transmit AS scheme, $K_t$ out of $L_t$ antennas with the modulation order $M_j$ that maximize the throughput rate [e.g., Eq. (6) or (7)] have to be selected. Therefore, an exhaustive search over all of the possible combinations of antennas and modulation orders is required. This may be complicated, especially when there are a large

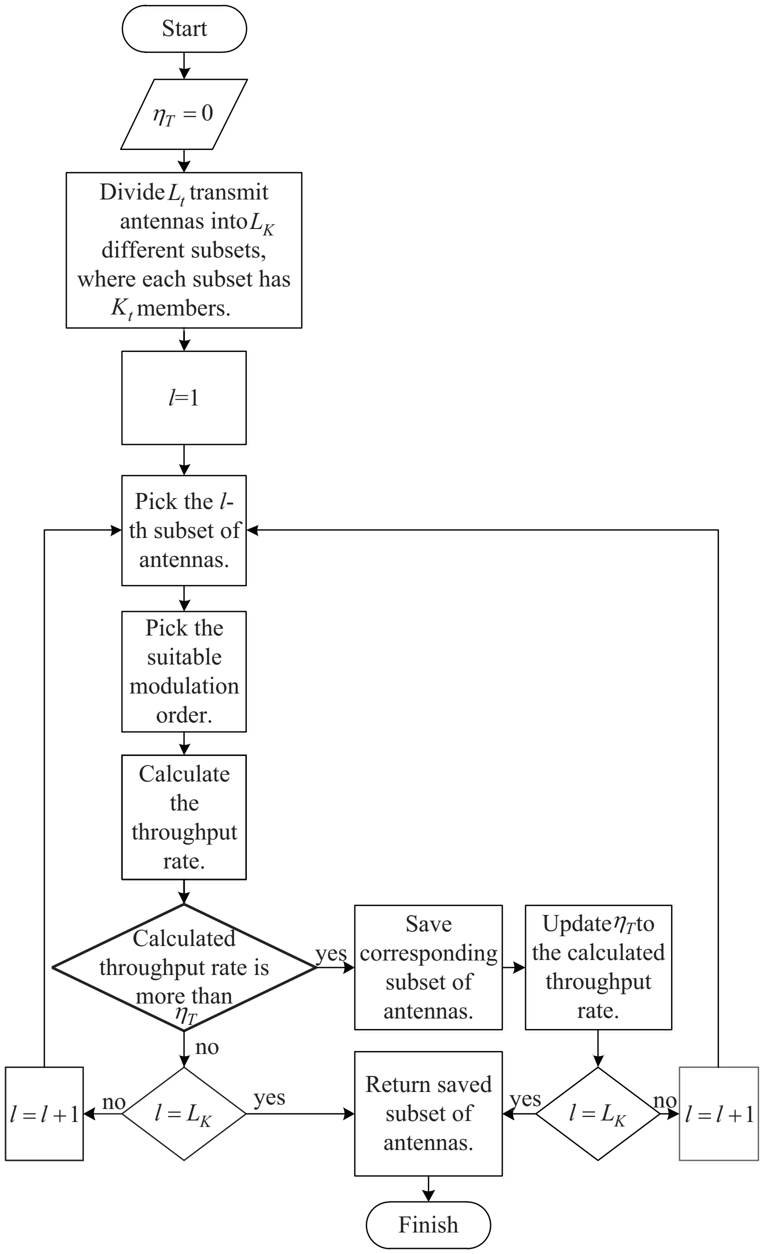


**Figure 1.** The simple throughput-based transmit AS algorithm with the adaptive modulation policy.

number of antennas and modulation orders. However, this exhaustive search, with the complexity of $L_K J$, can be simplified. Indeed, an exhaustive search over all possible modulation orders is not required. It is sufficient to partition the $K_t$-dimensional space of the instantaneous SNR $\{\gamma_k\}_{k=1}^{K_t}$ once and determine the best modulation order in each segment. Therefore, after the selection of $K_t$ antennas, since their instantaneous SNRs are given, the optimal modulation order is determined without any additional search. The optimal modulation orders can be computed off-line and stored in the memory for future usage. In this case, the complexity reduces to $L_K$. The simple throughput-based transmit AS technique with adaptive modulation at the physical layer is summarized in Figure 1. The best modulation orders, when $K_t = 1$ and $K_t = 2$, are also plotted in Figure 2.

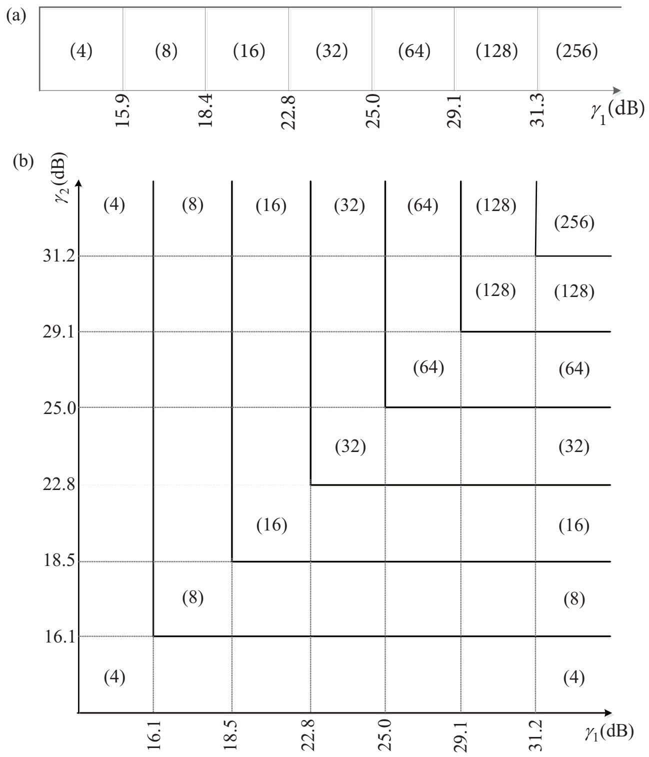


**Figure 2.** The best set of modulation order (a) $K_t = 1$, (b) $K_t = 2$.

## 4. Single-transmit AS

The statistical analysis of the throughput and packet error rates in the MIMO system with single-transmit AS is studied in this section. Adaptive modulation at the transmitter, ZF detection at the receiver, and TSR-ARQ protocol at the data link layer are assumed.

Single AS is the simplest scheme and has been considered extensively in the literature [21]. In addition, transmission with only one antenna can eliminate practical issues such as mutual coupling, spatial correlation of antennas, and inaccurate time synchronization between antennas [22]. In the single AS scheme, $K_t = 1$ and the ZF detector is similar to the SIC detector. Hence, the instantaneous SNR can be rewritten as:

$$\gamma_1 = \frac{\gamma}{\left[\left(H^{(1)\dagger} H^{(1)}\right)^{-1}\right]_{1,1}} = \gamma |r_{1,1}^{(1)}|^2, \tag{8}$$

where $\gamma$ is the average SNR and $r_{1,1}^{(1)}$ represents the first diagonal element of $\mathbf{R}^{(1)}$. Here, $\mathbf{H}^{(1)} = \mathbf{Q}^{(1)}\mathbf{R}^{(1)}$ and

$\mathbf{R}^{(1)} = r_{1,1}^{(1)}$ is a scalar. $r_{1,1}^{(1)}$ is gamma-distributed [23]; consequently, it can be shown that $\gamma_1$ has a gamma distribution with the shaping parameter $L_r$ and the scaling parameter $\gamma$ [24].

For finding the PDF of $\eta_{TSR}$ and $P_d$, it is necessary to find the PDF of $P_e$ first. The nonlinear function between $P_e$ and the instantaneous SNR is presented in Eqs. (3) and (4). Using Eq. (4) for further developments is not simple. Therefore, the tight approximation

$$P_e \approx \begin{cases} 1, 0 \le \gamma_k < \gamma_{th,j} \\ a_j e^{-b_j \gamma_k}, \gamma_k \ge \gamma_{th,j} \end{cases} \tag{9}$$

can be used [25], where $\gamma_k$ is the instantaneous SNR, and $a_j$, $b_j$, and $\gamma_{th,j}$ are constant parameters that depend on the modulation order. These parameters are listed in the Table for different modulation orders $M_j$, $j = 1, ..., J$. Now, using Eqs. (8) and (9), the PDF of $P_e$ at the fixed modulation order $M_j$ is obtained as:

$$f_{P_e}(x) = \begin{cases} \frac{[\ln(x/a_j)]^{L_r-1}(x/a_j)^{\frac{1}{b_j\gamma}-1}}{-a_j(-b_j)^{L_r}\Gamma(L_r)\gamma^{L_r}}, 0 \le x < 1 \\ P_0\delta(x), x = 1 \end{cases}, \tag{10}$$

where $P_0 = Pr(\gamma_1 < \gamma_{th,j})$, $\Gamma(.)$ is the Gamma function, and $\delta(.)$ represents the delta function. For the following, the integral [26, Eq. 2.722] is used several times.

**Table.** Parameters for the packet error rate approximation.

| Modulation orders, $M_j$ | 4 | 8 | 16 | 32 | 64 | 128 | 256 |
|---|---|---|---|---|---|---|---|
| $a_j$ | 213.9 | 276.1 | 332.9 | 340.2 | 347.8 | 349.7 | 350.1 |
| $b_j$ | 0.5054 | 0.1693 | 0.1024 | 0.0390 | 0.0239 | 0.0094 | 0.0058 |
| $\gamma_{th,j}$(dB) | 10.1 | 15.3 | 17.6 | 21.8 | 23.9 | 28.0 | 30.0 |

### 4.1. Throughput rate

According to the relation between $P_e$ and $\eta_{TSR}$, using Eq. (10), the cumulative distribution function (CDF) of the throughput rate in the MIMO channel with one transmit antenna and with the fixed modulation order can be written as Eq. (11):

$$\begin{aligned} F_{\eta_{TSR}}(x) \quad = \quad & \frac{1}{(-b_j)^{L_r}\Gamma(L_r)\gamma^{L_r}} \left\{ \left(\frac{1-(x/\eta_{\max})}{a_j}\right)^{\frac{1}{b_j\gamma}} \sum_{l=0}^{L_r-1} (-1)^l \frac{(L_r-1)!}{(L_r-1-l)!} \frac{\left[\ln\left(\frac{1-(x/\eta_{\max})}{a_j}\right)\right]^{L_r-1-l}}{[1/(b_j\gamma)]^{l+1}} \right. \\ & \left. -\ (1/a_j)^{\frac{1}{b_j\gamma}} \sum_{l=0}^{L_r-1} (-1)^l \frac{(L_r-1)!}{(L_r-1-l)!} \frac{[\ln(1/a_j)]^{L_r-1-l}}{[1/(b_j\gamma)]^{l+1}} \right\} + P_0, 0 \le x \le \eta_{\max} \end{aligned} \tag{11}$$

where $\eta_{\max} = S\log_2(M_j)$.

There are $L_t$ antennas at the transmitter, but one of them, which maximizes the throughput rate, is selected for the transmission. When $\left\{\eta_{TSR}^1, \eta_{TSR}^2, ..., \eta_{TSR}^{L_t}\right\}$ denotes $L_t$ throughput rates of $L_t$ antennas, then $\left\{\eta_{TSR}^{(1)}, \eta_{TSR}^{(2)}, ..., \eta_{TSR}^{(L_t)}\right\}$ represents the ordered rates, where $\left\{\eta_{TSR}^{(1)} \le \eta_{TSR}^{(2)} \le ... \le \eta_{TSR}^{(L_t)}\right\}$. Using order statistics theories [27], the CDF of $\eta_{TSR}^{(L_t)}$ can be expressed as:

$$F_{\eta_{TSR}}^{(L_t)}(x) = \left(F_{\eta_{TSR}}(x)\right)^{L_t}, 0 \le x \le \eta_{\max}. \tag{12}$$

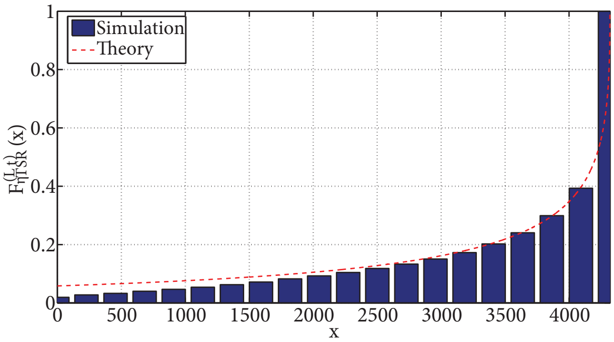


**Figure 3.** Derived and simulated CDF of the throughput with 16-QAM at $\gamma = 12$ dB.

This derived CDF is compared with the simulation result in Figure 3 for 16-QAM and $\gamma = 12$dB. There is a tight match; however, a small gap at the low values of $x$, which is due to the approximation in Eq. (9), can be seen. If the derived CDF in Eq. (11) is divided into 2 parts as $F_{\eta_{TSR}}(x) = \tilde{F}_{\eta_{TSR}}(x) + P_0$, and the derivative of the first part is called $\tilde{f}_{\eta_{TSR}}(x)$, then the PDF of $\eta^{(L_t)}_{TSR}$ can be found by taking the derivative of Eq. (12) with respect to $x$ as:

$$f^{(L_t)}_{\eta_{TSR}}(x) = L_t \tilde{f}_{\eta_{TSR}}(x) \left(\tilde{F}_{\eta_{TSR}}(x) + P_0\right)^{L_t - 1} + P_0^{L_t}\delta(x), 0 \le x \le \eta_{\max}. \tag{13}$$

Now the average throughput rate can be obtained numerically. When the adaptive modulation is implemented at the physical layer, then the average throughput rate is evaluated with the efficient Monte Carlo integration method [28] by:

$$\bar{\eta}^{(L_t)}_{TSR} = \sum_{j=1}^{J} \int_{\eta_{TSR}(\Gamma_j)}^{\eta_{TSR}(\Gamma_{j+1})} x f^{(L_t)}_{\eta_{TSR}}(x) dx, \tag{14}$$

where $\eta_{TSR}(\Gamma_j)$ represents the equivalent throughput rate when the instantaneous SNR is $\Gamma_j$.

### 4.2. Packet error rate at the data link layer

According to the relation between $P_e$ and $P_d$, the CDF of the packet error rate at the data link layer with one transmit antenna and with the fixed modulation order is:

$$F_{P_d}(x) = \begin{cases} \frac{1}{-(-b_j)^{L_r}\Gamma(L_r)\gamma^{L_r}} \left(\frac{x^{\frac{1}{\nu+1}}}{a_j}\right)^{\frac{1}{b_j\gamma}} \sum\limits_{l=1}^{L_r-1} (-1)^l \frac{(L_r-1)!}{(L_r-1-l)!} \frac{\left[\ln\left(\frac{x^{\frac{1}{\nu+1}}}{a_j}\right)\right]^{L_r-1-l}}{[1/(b_j\gamma)]^{l+1}}, 0 \le x < 1 \\ \frac{1}{-(-b_j)^{L_r}\Gamma(L_r)\gamma^{L_r}} \left(\frac{x^{\frac{1}{\nu+1}}}{a_j}\right)^{\frac{1}{b_j\gamma}} \sum\limits_{l=1}^{L_r-1} (-1)^l \frac{(L_r-1)!}{(L_r-1-l)!} \frac{\left[\ln\left(\frac{x^{\frac{1}{\nu+1}}}{a_j}\right)\right]^{L_r-1-l}}{[1/(b_j\gamma)]^{l+1}} + P_0, x = 1. \end{cases} \tag{15}$$

There are $L_t$ antennas at the transmitter, but the one with the minimum packet error rate is selected. When $\left\{P_d^1, P_d^2, ..., P_d^{L_t}\right\}$ denotes $L_t$ packet error rates of $L_t$ antennas, then $\left\{P_d^{(1)}, P_d^{(2)}, ..., P_d^{(L_t)}\right\}$ represents the ordered variables, where $\left\{P_d^{(1)} \le P_d^{(2)} \le ... \le P_d^{(L_t)}\right\}$. Using order statistics theories [27], the CDF of $P_d^{(1)}$ can be written as:

$$F^{(1)}_{P_d}(x) = 1 - (1 - F_{P_d}(x))^{L_t}, 0 \le x \le 1. \tag{16}$$

In a similar way, if Eq. (15) is divided into 2 parts as $F_{P_d}(x) = \tilde{F}_{P_d}(x) + P_0 u(x-1)$, where $u(.)$ represents the unit step function and the derivative of the first part is called $\tilde{f}_{P_d}(x)$, then the PDF of $P_d^{(1)}$ can be found by taking the derivative of Eq. (16) with respect to $x$ as:

$$f_{P_d}^{(1)}(x) = L_t \tilde{f}_{P_d}(x) \left(1 - \tilde{F}_{P_d}(x)\right)^{L_t - 1} + P_0^{L_t} \delta(x-1), 0 \ \leq x \leq \ 1. \tag{17}$$

Now the average packet error rate at the data link layer is obtained numerically. With the adaptive modulation at the physical layer, the average packet error rate at the data link layer is evaluated with the efficient Monte Carlo integration method [28] as:

$$\bar{P}_d^{(1)} = \sum_{j=1}^{J} \int_{P_d(\Gamma_j)}^{P_d(\Gamma_{j+1})} x f_{P_d}^{(1)}(x) dx, \tag{18}$$

where $P_d(\Gamma_j)$ represents the equivalent packet error rate at the data link layer when the instantaneous SNR is $\Gamma_j$.

## 5. Multiple-transmit AS

The statistical analysis of multiple-transmit AS is discussed here. A MIMO multiplexing configuration with the adaptive modulation and ZF detector at the physical layer and TSR-ARQ protocol at the data link layer are assumed. For further simplicity, we assume $L_r = K_t$. In this section, a lower bound for the throughput rate and an upper bound for the packet error rate at the data link layer are derived, and the statistical analysis is executed based on these lower and upper bounds.

The throughput rate of MIMO systems is introduced in Eq. (6). First, $\gamma_{\min}$ is defined as $\gamma_{\min} = \min\{\gamma_1, \gamma_2, ..., \gamma_{K_t}\}$. Since $\gamma_k \geq \gamma_{\min}, k = 1, 2, ..., K_t$, then $\eta_{TSR} \geq e_{TSR}$, where

$$\begin{aligned} e_{TSR} &= K_t \left[\prod_{k=1}^{K_t} (1 - S_{e,k}(\gamma_{\min}))\right]^{S/K_t} S \log_2(M_j) \\ &= \eta_{\max}(1 - S_{e,1}(\gamma_{\min}))^S \end{aligned} \tag{19}$$

is the lower bound on the throughput rate and $\eta_{\max} = K_t S \log_2(M_j)$. In addition, since $\gamma_k \geq \gamma_{\min}, k = 1, 2, ..., K_t$, then $P_e \leq Q_e$, where

$$Q_e = \ 1 - \prod_{k=1}^{K_t} (1 - S_{e,k}(\gamma_{\min}))^{S/K_t} = 1 - (1 - S_{e,1}(\gamma_{\min}))^S \tag{20}$$

represents the upper bound on the packet error rate at the physical layer. For the minimum SNR $\gamma_{\min}$, we have [7]:

$$\gamma_{\min} \geq \frac{\gamma}{K_t} \lambda_{\min} \left(\mathbf{H}^{(K_t)\dagger} \mathbf{H}^{(K_t)}\right), \tag{21}$$

where $\lambda_{\min}\left(\mathbf{H}^{(K_t)\dagger} \mathbf{H}^{(K_t)}\right)$ stands for the minimum eigenvalue of $\mathbf{H}^{(K_t)\dagger} \mathbf{H}^{(K_t)}$. Since $L_r = K_t$, the minimum eigenvalue has an exponential distribution with a parameter $K_t$ [17]. Hence, the PDF of $Q_e$ is simply found as:

$$f_{Q_e}(x) = \frac{1}{\alpha_j \beta_j} x^{\frac{1}{\beta_j} - 1} + P_0 \delta(x - \ 1), 0 \leq x \leq 1, \tag{22}$$

where $\alpha_j = a_j^{K_t^2/\gamma}$, $\beta_j = \frac{b_j}{(K_t^2/\gamma)}$, and $P_0 = Pr(\gamma_{\min} < \gamma_{th,j})$. For the following, the integral in [26, Eq. 2.110-8] is used several times.

### 5.1. Throughput rate

According to the relation between $Q_e$ and $e_{TSR}$ as $e_{TSR} = \eta_{\max}(1 - Q_e)$, using Eq. (22), the CDF of $e_{TSR}$ with the fixed modulation order can be written as:

$$F_{e_{TSR}}(x) = \frac{1}{\alpha_j}\left[1 - \left(1 - \frac{x}{\eta_{\max}}\right)^{1/\beta_j}\right] + P_0, 0 \leq x \leq \eta_{\max}. \tag{23}$$

In the multiple-transmit AS case, when $K_t$ antennas are selected, there are $L_K = \left(\begin{array}{c} L_t \\ K_t \end{array}\right)$ subsets of antennas available and the one that causes the maximum throughput rate is selected. When $\left\{e_{TSR}^1, e_{TSR}^2, ..., e_{TSR}^{L_K}\right\}$ denotes $L_K$ throughput rates of the available subsets, then $\left\{e_{TSR}^{(1)}, e_{TSR}^{(2)}, ..., e_{TSR}^{(L_K)}\right\}$ represents the ordered throughput rates, where $\left\{e_{TSR}^{(1)} \leq e_{TSR}^{(2)} \leq ... \leq e_{TSR}^{(L_K)}\right\}$. Using order statistics theories [27], the CDF of $e_{TSR}^{(L_K)}$ can be expressed as:

$$F_{e_{TSR}}^{(L_K)}(x) = \left(F_{e_{TSR}}(x)\right)^{L_K}, 0 \leq x \leq \eta_{\max}. \tag{24}$$

By taking the derivative of Eq. (24) with respect to $x$, the PDF of $e_{TSR}^{(L_K)}$ is expressed as:

$$f_{e_{TSR}}^{(L_K)}(x) = \frac{L_K}{\alpha_j\beta_j\eta_{\max}} \times \left\{\frac{1}{\alpha_j}\left[1 - \left(1 - \frac{x}{\eta_{\max}}\right)^{1/\beta_j}\right] + P_0\right\}^{L_K - 1}\left(1 - \frac{x}{\eta_{\max}}\right)^{\frac{1}{\beta_j} - 1} + P_0^{L_K}\delta(x), 0 \leq x \leq \eta_{\max}. \tag{25}$$

After some manipulations, it can be shown that Eq. (25) is equal to Eq. (13) when $K_t = L_r = 1$.

With the fixed modulation order, a closed-form solution can be expressed for the average throughput rate as:

$$\bar{e}_{TSR}^{(L_K)} = L_K\eta_{\max}\sum_{l=0}^{L_K-1}\left(\begin{array}{c} L_K - 1 \\ l \end{array}\right)\frac{P_0^{L_K-1-l}}{\alpha_j^{l+1}} \times \left[\frac{1}{l+1} - \frac{l!}{(1+\beta_j)(2+\beta_j)...(l+\beta_j)(l+1+‘\beta_j)}\right]. \tag{26}$$

However, with the adaptive modulation we have:

$$\bar{e}_{TSR}^{(L_K)} = \sum_{j=1}^{J}\int_{e_{TSR}(\Gamma_j)}^{e_{TSR}(\Gamma_{j+1})} x f_{e_{TSR}}^{(L_K)}(x)dx, \tag{27}$$

where $e_{TSR}(\Gamma_j)$ represents the equivalent throughput rate when the instantaneous SNR is $\Gamma_j$ and the integral in Eq. (27) has a closed-form solution as in Eq. (28):

$$\int_{e_{TSR}(\Gamma_j)}^{e_{TSR}(\Gamma_{j+1})} x f_{e_{TSR}}^{(L_K)}(x)dx = L_K \eta_{\max} \sum_{l=0}^{L_K-1} \begin{pmatrix} L_K - 1 \\ l \end{pmatrix} \frac{P_0^{L_K-1-l}}{\alpha_j^{l+1}} \times$$
$$\left\{ \frac{\left[1-\left(1-\frac{e_{TSR}(\Gamma_{j+1})}{\eta_{\max}}\right)^{1/\beta_j}\right]^{l+1} - \left[1-\left(1-\frac{e_{TSR}(\Gamma_j)}{\eta_{\max}}\right)^{1/\beta_j}\right]^{l+1}}{l+1} - \sum_{l'=0}^{l} \frac{l!}{(l-l')!} \frac{1}{(\beta_j+1)...(\beta_j+l')} \times \right. \tag{28}$$
$$\left. \frac{\left(1-\frac{e_{TSR}(\Gamma_j)}{\eta_{\max}}\right)^{(\beta_j+l'+1)/\beta_j}\left[1-\left(1-\frac{e_{TSR}(\Gamma_j)}{\eta_{\max}}\right)^{1/\beta_j}\right]^{l-l'} - \left(1-\frac{e_{TSR}(\Gamma_{j+1})}{\eta_{\max}}\right)^{(\beta_j+l'+1)/\beta_j}\left[1-\left(1-\frac{e_{TSR}(\Gamma_{j+1})}{\eta_{\max}}\right)^{1/\beta_j}\right]^{l-l'}}{\beta_j+l'+1} \right\}.$$

### 5.2. Packet error rate at the data link layer

Similar to Eq. (5), when just $\nu$ retransmissions are allowed in the ARQ protocol, the packet error rate at the data link layer can be expressed as:

$$Q_d = Q_e^{\nu+1}, \tag{29}$$

and the CDF of $Q_d$ is obtained as:

$$F_{Q_d}(x) = \begin{cases} \frac{x^{1/[\beta_j(\nu+1)]}}{\alpha_j}, 0 \le x < 1 \\ \frac{x^{1/[\beta_j(\nu+1)]}}{\alpha_j} + P_0, x = 1. \end{cases} \tag{30}$$

$L_K$ subsets of antennas are available at the transmitter, but the one with the minimum packet error rate is selected. When $\left\{Q_d^1, Q_d^2, ..., Q_d^{L_K}\right\}$ denotes $L_K$ packet error rates of these $L_K$ subsets, then $\left\{Q_d^{(1)}, Q_d^{(2)}, ..., Q_d^{(L_K)}\right\}$ represents the ordered variables, where $\left\{Q_d^{(1)} \le Q_d^{(2)} \le ... \le Q_d^{(L_K)}\right\}$. Using order statistics theories [27], the CDF of $Q_d^{(1)}$ can be written as:

$$F_{Q_d}^{(1)}(x) = 1 - (1 - F_{Q_d}(x))^{L_K}, 0 \le x \le 1, \tag{31}$$

and by the derivative of Eq. (31), the PDF is obtained as:

$$f_{Q_d}^{(1)}(x) = L_K \left(1 - \frac{x^{1/[\beta_j(\nu+1)]}}{\alpha_j}\right)^{L_K-1} \frac{x^{\left[\frac{1}{\beta_j(\nu+1)}\right]-1}}{\alpha_j\beta_j(\nu+1)} + P_0^{L_K}\delta(x-1), 0 \le x \le 1. \tag{32}$$

After some manipulations, it can be shown that Eq. (32) is equal to Eq. (17) when $K_t = L_r = 1$.

With the fixed modulation order, a closed-form solution can be expressed for the average packet error rate as:

$$\bar{Q}_d^{(1)} = L_K \sum_{l=0}^{L_K-1} \begin{pmatrix} L_K - 1 \\ l \end{pmatrix} \frac{(-1)^l}{\alpha_j^{l+1}\beta_j(\nu+1)\left(1 + \frac{l+1}{\beta_j(\nu+1)}\right)} + P_0^{L_K}, \tag{33}$$

and when the adaptive modulation is applied, $\bar{Q}_d^{(1)}$ is written as:

$$\bar{Q}_d^{(1)} = \sum_{j=1}^{J} \int_{Q_d(\Gamma_j)}^{Q_d(\Gamma_{j+1})} x f_{Q_d}^{(1)}(x)dx, \tag{34}$$

where $Q_d(\Gamma_j)$ represents the equivalent packet error rate at the data link layer when the instantaneous SNR is $\Gamma_j$. For further simplification, the integral of Eq. (34) can be expressed as:

$$\int_{Q_d(\Gamma_j)}^{Q_d(\Gamma_{j+1})} x f_{Q_d}^{(1)}(x)dx =$$

$$\begin{cases} L_K \sum_{l=0}^{L_K-1} \frac{(-1)^l}{\alpha_j^{l+1}\beta_j(\nu+1)\left(1+\frac{l+1}{\beta_j(\nu+1)}\right)} \left( [Q_d(\Gamma_{j+1})]^{1+\frac{l+1}{\beta_j(\nu+1)}} - [Q_d(\Gamma_j)]^{1+\frac{l+1}{\beta_j(\nu+1)}} \right), Q_d(\Gamma_{j+1}) \neq 1 \\ L_K \sum_{l=0}^{L_K-1} \frac{(-1)^l}{\alpha_j^{l+1}\beta_j(\nu+1)\left(1+\frac{l+1}{\beta_j(\nu+1)}\right)} \left( [Q_d(\Gamma_{j+1})]^{1+\frac{l+1}{\beta_j(\nu+1)}} - [Q_d(\Gamma_j)]^{1+\frac{l+1}{\beta_j(\nu+1)}} \right) + P_0^{L_k+1}, Q_d(\Gamma_{j+1}) = 1. \end{cases} \tag{35}$$

## 6. Simulation results

For all of the simulations, we assume MIMO quasi-static Rayleigh fading channels, $L_t = 4$, $L_r = 4$, and $S = 1080$. For the Monte Carlo simulation, at each average SNR, 10,000 symbols are generated and simulated.

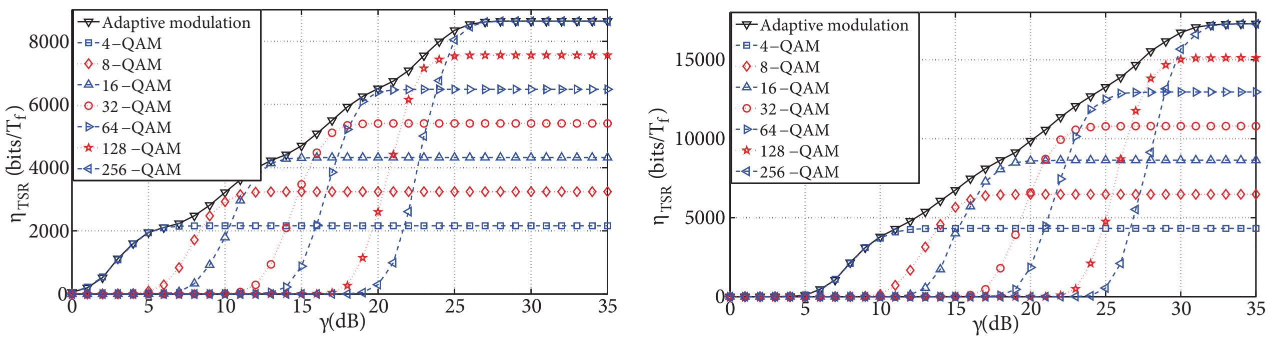


**Figure 4.** The average throughput in a MIMO system with throughput-based AS, with and without adaptive modulation: a) $K_t = 1$ and b) $K_t = 2$.

The average throughput with and without adaptive modulation and throughput-based AS are plotted when $K_t = 1$ and $K_t = 2$ in Figures 4a and 4b, respectively. The TSR-ARQ protocol is assumed at the data link layer and the modulation orders are $M_j = 4, 8, 16, 32, 64, 128$ and $\sim$256. The high advantages of the adaptive modulation and AS are clear in Figure 4. At a low SNR, selecting 1 antenna is much better than using 2 antennas for transmission. In contrast, at a high SNR, the throughput rate with 2 antennas is more than the throughput rate with just 1 antenna. Therefore, choosing a suitable number of antennas at the given SNR is essential. At moderate SNRs, it seems that using both 1 and 2 antennas alternatively can improve the performance. To show this, the average throughput in the MIMO system with transmit AS when $K_t = 1$ or 2 is plotted in Figure 5. In this case, at moderate SNRs, the performance is better than when just 1 or just 2 antennas is selected for transmission.

The average throughput in the MIMO system with single-transmit AS is presented in Figure 6. For comparison, the theoretical and simulation results with adaptive modulation when $L_r$ varies from 2 up to 4 are plotted together. TSR-ARQ protocol at the data link layer is assumed. As expected, the average throughput increases when the number of receive antennas is increased. Very tight agreement between theory and simulation is clear in Figure 6 and, therefore, this verifies the accuracy of the theoretical expressions derived in Section 4.1. In addition, the average packet error rate at the data link layer in the MIMO system with single-transmit AS is plotted in Figure 7. The theoretical and simulation results with adaptive modulation and different values of $\nu$ at the TSR-ARQ protocol are plotted. A very tight match between the theory and simulation is also clear in Figure 7.

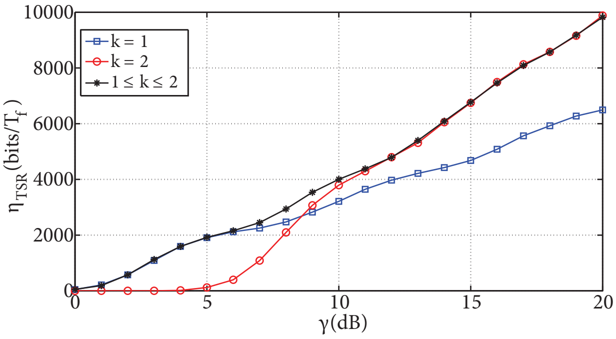


**Figure 5.** The average throughput in the MIMO system with throughput-based AS.

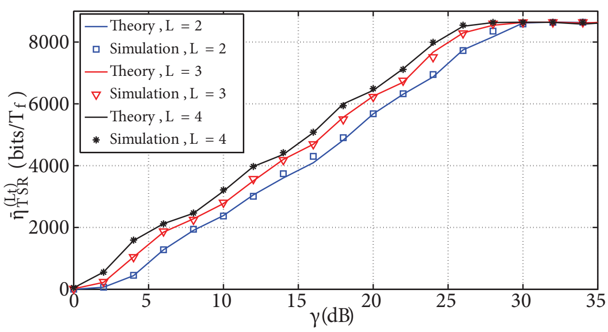


**Figure 6.** The average throughput rate, $\bar{\eta}_{TSR}^{(L_t)}$, in the MIMO system with single-transmit AS and adaptive modulation when $L_r = 2$,3, and 4.

Finally, the lower bound on the average throughput $\bar{e}_{TSR}^{(L_K)}$ and multiple-transmit AS is plotted in Figure 8, where the theoretical and simulation results for the average throughput with adaptive modulation and TSR-

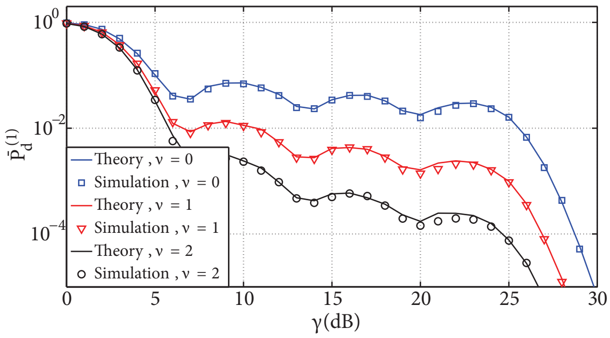


**Figure 7.** The average packet error rate at the data link layer, $\bar{P}_d^{(1)}$, in the MIMO system with single-transmit AS and adaptive modulation for different values of $\nu$.

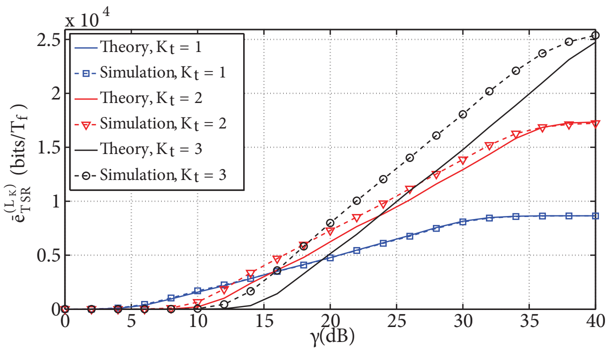


**Figure 8.** The lower bound on the average throughput, $\bar{e}_{TSR}^{(L_K)}$, in the MIMO system with multiple-transmit AS and adaptive modulation.

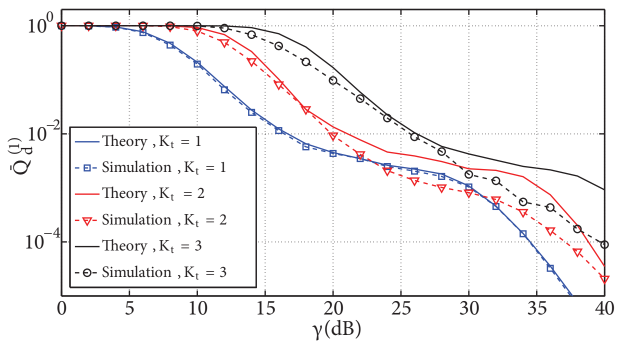


**Figure 9.** The upper bound on the average packet error rate at the data link layer, $\overline{Q}_d^{(1)}$, in the MIMO system with multiple-transmit AS and adaptive modulation when $\nu = 1$.

ARQ protocol at the data link layer are presented. The upper bound on the average packet error rate at the data link layer $\bar{Q}_d^{(1)}$ is also plotted in Figure 9. When just one antenna is selected, the lower and upper bounds become an exact expression for the average throughput and average packet error rate. However, for multiple-transmit AS, the gap between the lower and upper bounds and simulation results increases when the number of selected antennas is increased.

## 7. Conclusion

This paper presents an efficient method of throughput-based transmit AS with adaptive modulation. A closed-form solution is derived for the throughput rate of the MIMO system with TSR-ARQ or SR+GBN-ARQ at the data link layer, and adaptive modulation at the physical layer, when ZF or SIC detector is employed at the receiver. We propose a simple selection algorithm that does not require an exhaustive search over the available modulation order at the adaptive modulation scheme.

Next, the throughput and packet error rates at the data link layer are statistically analyzed. Here, in 2 cases, we derive analytical expressions for the PDF of the throughput and the packet error rate at the data link layer. In single-transmit AS with an arbitrary number of receive antennas, the PDFs of the throughput and the packet error rates are derived and their average value is calculated numerically. Very tight agreement between the theoretical and simulated results indicates that the derived expressions for the throughput and the packet error rates are accurate. In the multiple-transmit AS case, where more than one antenna is selected at the transmitter, we obtain a lower bound for the throughput rate and an upper bound for the packet error rate at the data link layer. There is also an exact PDF for these lower and uppoer bounds. Finally, in this case, a closed-form solution for the averages of the throughput and packet error rates at the data link layer is obtained. The analytically derived expressions are verified by the simulation results in this case, too.

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