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Simultaneous identification and correction of measurement and branch parameter errors

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Abstract: The performance of the state estimation (SE) depends on the accuracy of the received measured data, as well as the parameter data of the power system. In this paper, a new algorithm is proposed for the simultaneous identification and correction of measurement and branch parameter errors (series and shunt admittances) in the power system SE problem. The proposed method uses Lagrange multipliers for the identification and correction of branch parameter errors without the need for a-priori specification of suspect parameter vectors. Erroneous measurements and branch parameter values can be corrected using the proposed method to estimate the measurement and branch parameter errors. Finally, IEEE 14-, 30-, and 57-bus test systems are used to show the validity and robustness of the proposed algorithm. Single, multiple, and simultaneous errors in the conventional measurements and branch parameters are considered for different case studies.

Key words: State estimation, measurement errors, branch parameter errors, Lagrange multiplier, normalized residual

1. Introduction

The input data of conventional state estimation (SE) are a redundant collection of measurements and a mathematical model that relates these measurements to the nodal voltage magnitudes (V) and their phase angles (θ), which are taken as state variables of system. This model relies on several assumptions, among which the network configuration and associated parameters are considered to be known without any errors. Unfortunately, these assumptions do not hold true. However, the network parameter values stored in the static database may be incorrect due to inaccurate data supplied by the manufacturers and network changes that are not properly updated in the database. Moreover, these parameters can change due to temperature (especially the series resistance) or environmental conditions (especially the shunt conductance), etc. [1]. As a result, parameter errors, which are usually assumed not to exist normally, can have adverse impacts on SE solutions; hence, the detection, identification, and correction of network parameter errors are very important.

Most state estimators are designed to suspect only the conventional measurement errors and to ignore all other types of errors. Most of the conventional measurement errors can be effectively detected, identified, and corrected using methods such as the largest normalized residual test [1]. In [2], a topological/geometrical based approach was used to define the undetectability index for bad data analysis and measurement error detection. To detect gross errors in the power system SE, a new method based on the definition of innovation index and concepts of [3] was proposed in [4]. Moreover, bad data detection was presented in [5] using a robust method for the solution of the power system SE with equality constraints. A systematic approach for SE based on

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wavelet analysis to detect and eliminate bad data in measurements was developed in [6]. The identification of measurement and topology errors was analyzed in [7]. Furthermore, in [8], a simple multiarea decentralized SE procedure and bad data detection problem were proposed in multiarea systems.

On the other hand, the influence of parameter errors on the SE solution was studied in detail in [9]. There are several published methods for the identification and estimation of branch parameter errors using Supervisory Control and Data Acquisition measurements [10–13] and considering the power management unit's (PMU's) measurements in static SE [14]. Moreover, the joint estimation of state and parameter estimation with PMUs were presented in [15,16] in a dynamic SE. Sensitivities of the SE solution with respect to line reactances and shunt susceptances were analyzed in [17]. Thus, there are 2 types of methods for parameter error identification [9]. The first type is based on residual sensitivity analysis [18,19], where the sensitivities of the measurement residuals to the assumed parameter errors are used for identification. The second type uses a state vector augmented by additional variables, which are the suspected parameters. This approach can be implemented in 2 different ways using the static normal equations [20–22] and the Kalman filter theory [23–25].

Although there are various branch parameter estimation approaches, most of them address only the branch series admittances and assume that the influence of branch shunt admittances is insignificant on the SE solution [13,26], while the influence of shunt admittances is important. As a result, all of the branch parameter error identification and estimation methods have common limitations as follows:

1. A primary set of suspect parameters is required before the estimation and correction of parameter error.
2. The branch parameter estimation with an augmented state vector requires a high computational volume and an extra iteration in the estimation process because the weighted least square (WLS) algorithm should first be solved to estimate the actual values of the branch parameters.
3. When the augmented state vector method is applied, it may yield some unreasonable results, such as negative resistances and unacceptable large branch parameter values.
4. Bad data in the measurement vector have to be removed before the parameter error identification.
5. There is no obvious difference for distinguishing 2 kinds of residuals caused by measurement and branch parameter errors.

However, in order to obtain a reliable SE, simultaneous identification and correction of the measurement and branch parameter errors still represents a challenging task. The method proposed in this paper overcomes these limitations and problems, as described in the next sections.

In this paper, a new algorithm is proposed for the simultaneous identification and correction of measurement and branch parameter errors by eliminating the necessity for the augmented state vector. Using the proposed method, the erroneous branch parameter values can be corrected using a linear approximation for the estimated branch parameter errors and its effectiveness is illustrated by test systems. The main advantage of the proposed method is that the normalized measurement residuals and Lagrange multipliers of the parameter errors can be computed, which allow them to be identified and then corrected, even when appearing simultaneously. Moreover, all of the common limitations listed above can be solved using the proposed method.

Thus, considering the above sentences, the contributions of this paper are 7-fold:

1. The proposed is a new and useful algorithm based on Lagrange multipliers analysis to detect, identify, and correct the simultaneous measurement and branch parameter errors in power systems SE problem.

2. A straightforward correcting procedure of the erroneous branch parameters using a new linear approximation equation.
3. The eliminating of augmented state vector that were used in all related papers and that may obtain some unreasonable results.
4. The elimination of the need for an a-priori specification of the suspect parameter vectors before correction of the parameter errors.
5. The decreasing of the computation volume if multiple errors in the measurement and branch parameters occur.
6. The correcting of simultaneous errors in the measurement and branch parameter errors. In previous papers in related branch parameter estimation, bad data in the measurement vector have to be removed before the parameter error identification.
7. The analysis and reporting of the results of the IEEE case studies.

The paper is organized as follows: section 2 summarizes the Lagrange multipliers approach for the detection and identification of the branch parameter errors. The proposed algorithm for the simultaneous correction and estimation of conventional measurement and branch parameter errors that replaces the augmented state vector is presented in section 3. Section 4 describes the simulation results of the proposed approach on the IEEE 14-, 30-, and 57-bus systems. Finally, section 5 presents the conclusions and final remarks.

2. Lagrange multipliers approach for the detection and identification of branch parameter errors

The mathematical model that relates the measurements of the state variables and the branch parameter errors can be formulated as follows:

$$\mathbf{z} = h(\mathbf{x}, \mathbf{p}) + \mathbf{e}, \tag{1}$$

where $\mathbf{z}$ is the vector of measurement $(m \times 1)$ and $\mathbf{x}$ is the system state vector $(n_x \times 1)$. The state vector includes the voltage magnitudes and phase angles, except for the reference bus angle. The nonlinear function $h(\mathbf{x}, \mathbf{p})$ relates the measurement to the system states and power system branch parameter errors. $\mathbf{p}$ is the vector of the power system branch parameter errors and $\mathbf{e}$ is the vector of the measurement errors, which is usually considered to be a random Gaussian variable with a zero mean value and covariance matrix of $\mathbf{R} = diag\left\{\delta_{z1}^2, \delta_{z2}^2, ..., \delta_{zm}^2\right\}$. δ_{zi}^2 is the variance of the ith measurement. If there are no errors in the parameters, the vector of the power system branch parameter error, $\mathbf{p}$, will be zero. Therefore, the conventional WLS SE approach in the presence of the network parameters can be formulated as the following optimization problem:

$$\begin{array}{l} Minimize: \ J(\mathbf{x}) = \mathbf{r}^T \cdot \mathbf{R}^{-1} \cdot \mathbf{r} \\ Subject\ to: \ \mathbf{p} = \ 0 \end{array}, \tag{2}$$

where $\mathbf{r} = \mathbf{z} - h(\mathbf{x}, \mathbf{p})$ and $\mathbf{W} = \mathbf{R}^{-1}$ represent the measurement residual vector and the diagonal matrix, the inverse of which is the measurement error covariance matrix, respectively.

If the analysis of the Lagrange multipliers is applied to solve this problem, the objective function can be written as follows:

$$L(\mathbf{x}, \mathbf{p}, \lambda) = \mathbf{r}^T \cdot \mathbf{R}^{-1} \cdot \mathbf{r} - \lambda^T \mathbf{p}. \tag{3}$$

This function can be solved using the Karush–Kuhn–Tucker first-order optimality conditions:

$$\frac{\partial L}{\partial \mathbf{x}} = \mathbf{H}_x^T \cdot \mathbf{R}^{-1} \cdot \mathbf{r} = 0, \tag{4}$$

$$\frac{\partial L}{\partial \mathbf{p}} = \mathbf{H}_\mathbf{p}^T \cdot \mathbf{R}^{-1} \cdot \mathbf{r} + \lambda = 0, \tag{5}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{p} = 0, \tag{6}$$

where $\mathbf{H}_x = \frac{\partial h(\mathbf{x},\mathbf{p})}{\partial \mathbf{x}}$, $\mathbf{H}_\mathbf{p} = \frac{\partial h(\mathbf{x},\mathbf{p})}{\partial \mathbf{p}}$, and λ are the state Jacobian matrix, parameter Jacobian matrix, and Lagrange multiplier vector for the parameter errors, respectively. Note that λ can now be expressed in terms of $\mathbf{r}$, using Eq. (5) as follows:

$$\lambda = \mathbf{SP} \cdot \mathbf{r} = -\left[\mathbf{H}_\mathbf{p}^T \cdot \mathbf{R}^{-1}\right] \cdot \mathbf{r}. \tag{7}$$

The state vector x can be estimated by an iterative solution of the conventional WLS SE and solving the following normal equations:

$$\Delta\mathbf{x} = \mathbf{G}^{-1} \cdot \mathbf{H}_x^T \cdot \mathbf{R}^{-1} \left(\mathbf{z} - h(\mathbf{x},\, 0\,)\right), \tag{8}$$

where $\mathbf{G} = \mathbf{H}_x^T \cdot \mathbf{R}^{-1} \cdot \mathbf{H}_x$ represents the gain matrix.

In the Lagrange multiplier approach, it is assumed that all Lagrange multipliers are distributed according to a normal distribution with a zero mean value and a nonzero covariance. The covariance matrix can be derived from the relation between the Lagrange multipliers and the measurement residuals as follows:

$$\begin{array}{l} \mathbf{\Lambda} = cov(\lambda) = \mathbf{SP} \cdot cov(\mathbf{r}) \cdot \mathbf{SP}^T = \mathbf{SP} \cdot \mathbf{\Omega} \cdot \mathbf{SP}^T \\ \mathbf{\Omega} = \left\{\mathbf{I} - \mathbf{H}_x(\mathbf{H}_x^T\mathbf{R}^{-1}\mathbf{H}_x)^{-1}\mathbf{H}_x^T\mathbf{R}^{-1}\right\} \end{array}. \tag{9}$$

The Lagrange multipliers for the parameter errors can be normalized using the diagonal elements of the covariance matrix $\mathbf{\Lambda}$, according to the following equation:

$$\lambda_i^N = \frac{\lambda_i}{\sqrt{\Lambda\,(i,i)}}, i = \ 1, ..., n_p, \tag{10}$$

where n_p is the total number of power system branch parameters. The vector λ_i^N is a Gaussian random variable with a zero mean and unit variance. The parameter that has the largest normalized Lagrange multiplier (larger than threshold) is identified as the erroneous parameter and should be corrected using the proposed linear approximation approach, which is described in the following section.

3. The proposed algorithm for the simultaneous correction and estimation of the identified measurement and branch parameter errors

In all of the related literature, if a branch parameter is identified as erroneous, it is corrected by estimating its value using the method described in [9], using the augmented state vector [27]. The estimated branch parameter value is substituted in the database and then the WLS algorithm is repeated. This parameter error correction method needs to solve the WLS algorithm for estimating the correct parameter value with a high computational volume and extra iteration in the estimation process. Following the traditional SE solution, the measurement residuals are used to calculate the Lagrange multipliers associated with the parameter errors. If they are found to be significant, then the associated parameter will be suspected to have an error. The proposed method in this paper overcomes this problem by eliminating the necessity for an augmented state vector. The erroneous branch parameter values could be corrected using a linear approximation for the estimated parameter errors with high accuracy.

Let the augmented state vector be written as follows:

$$\mathbf{x}_{aug} = [x_1, x_2, ..., x_n | p], \tag{11}$$

where $x_1, x_2, ..., x_n$ are the conventional state variables and p is a parameter that is previously identified as erroneous. Next, execution of the SE solution will provide the optimal estimation of the state variables, as well as the erroneous branch parameters.

As is evident from the above, after identification of the branch parameter errors, an algorithm iteration is required to perform the augmented SE approach for estimating each erroneous branch parameter. If there are multiple branch parameter errors in the power system, they should be estimated one by one, using the augmented SE approach. Hence, this process needs to be run multiple times for correcting all of the branch parameters errors. Such a performance of this method results in the increase of the computational volume and iteration numbers in the estimation process.

A new linear approximation approach is proposed in this paper to overcome this problem by eliminating the necessity for an augmented state vector. The proposed approach deals with the electrical parameters in the classical steady-state π-equivalent model of branches, which consists of series and shunt admittances. If the physical parameters of the line resistances (r_{i-j}) and reactances (x_{i-j}) are required, the chain rule must be used as indicated in Appendix A. The proposed linear approximation approach is described below:

Let Eq. (1) be rewritten as:

$$\mathbf{z} = h(\mathbf{x}, \mathbf{p}_0) + [h(\mathbf{x}, \mathbf{p}) - h(\mathbf{x}, \mathbf{p}_0)] + \mathbf{e}, \tag{12}$$

where $\mathbf{p}$ and $\mathbf{p}_0$ are the actual and erroneous values of the branch parameters, respectively. The term in square brackets in Eq. (12) is equivalent to an additional measurement error. If the parameter errors are large enough, this term may lead to bad data, which should be detected. This term can be linearized as:

$$[h(\mathbf{x}, \mathbf{p}) - h(\mathbf{x}, \mathbf{p}_0)] \simeq \left[\frac{\partial h(\mathbf{x}, \mathbf{p})}{\partial \mathbf{p}}\right] \cdot \mathbf{e_p} = \mathbf{H_p} \cdot \mathbf{e_p}, \tag{13}$$

where $\mathbf{e_p}$ is the vector of the branch parameter errors, considered a random Gaussian variable with a zero mean value and covariance matrix of $\mathbf{R_p}$.

By combining Eqs. (12) and (13), a linear relationship can be established between the vector of the residual measurement $\mathbf{r}$ and the vector of the parameter errors $\mathbf{e_p}$:

$$\mathbf{r} = \mathbf{z} - h(\hat{\mathbf{x}}, \mathbf{p}_0) \simeq \mathbf{H_p} \cdot \mathbf{e_p}, \tag{14}$$

Using Eqs. (14) and (7), the vector of parameter errors $\mathbf{e_p}$ can be written as follows:

$$\mathbf{e_p} = \frac{\lambda}{\mathbf{SP} \cdot \mathbf{H_p}} = \frac{\lambda}{\mathbf{SH_p}}. \tag{15}$$

Suppose that the ith branch parameter is identified as erroneous. Thus, $\lambda_i^{bad} = SH_p(i,i) \cdot e_p(i)$.

Moreover, the covariance matrix of λ can be obtained as follows:

$$\begin{aligned} \mathbf{\Lambda} &= cov(\lambda) = cov\left[(\mathbf{SH_p} \cdot \mathbf{e_p}) \cdot (\mathbf{SH_p} \cdot \mathbf{e_p})^T\right] \\ &= \mathbf{SH_p} \cdot cov\left[\mathbf{e_p} \cdot \mathbf{e_p}^T\right] \cdot \mathbf{SH_p}^T = \mathbf{SH_p} \cdot \mathbf{R_p} \cdot \mathbf{SH_p}^T = \mathbf{SH_p} \cdot \mathbf{R_p} \end{aligned}, \tag{16}$$

Consequently, parameter error $\mathbf{e_p}$ for the ith branch parameter in Eq. (15) can be written as follows:

$$e_p(i) = \frac{\lambda_i^{bad}}{SH_p(i,i)} = \frac{R_p(i,i)}{\Lambda(i,i)} \cdot \lambda_i^{bad}, \tag{17}$$

Thus, the actual branch parameter value can be estimated as:

$$p_i^{correct} = p_i^{bad} - \frac{R_p(i,i)}{\Lambda(i,i)} \cdot \lambda_i^{bad}, \tag{18}$$

where p_i^{bad} and $p_i^{correct}$ are the erroneous values of the identified branch parameter and estimated (corrected) value of the erroneous branch parameter, respectively. Moreover, $R_p(i,i)$ is the ith diagonal element of the branch parameter errors covariance matrix ($\mathbf{R_p}$). As seen in Eq. (18), a linear simple trend is found the between branch parameters, Lagrange multipliers, and their covariance matrices. Applying this proposed method results in faster and more accurate identification and estimation of branch parameter errors in comparison with all other methodologies. Thus, the mentioned limitations in the identification and estimation of branch parameters can be efficiently developed by the proposed method.

On the other hand, the redundancy index is an important index in the accuracy of SE results in power systems. All of the parameter error detection and identification approaches need high redundancy. If a system has more measurement errors, removing bad data will reduce redundancy and also decrease the system observability. Therefore, in this paper, the bad data in the measurement set were not deleted and their true value was estimated by a corrective algorithm. A similar corrective equation for parameter errors could be used for measurement errors as follows:

$$\begin{aligned} &Z_i^{correct} = Z_i^{bad} - \tfrac{R(i,i)}{\Omega(i,i)} \cdot r_i^{bad} \\ &r_i^{bad} = Z_i^{bad} - h\left(\hat{x}_i^{bad}\right) \end{aligned}. \tag{19}$$

The above formulation can be used to develop an algorithm in which the detection, identification, and correction of conventional measurement and branch parameter errors are processed simultaneously without losing any measurements. A flowchart of the proposed algorithm is shown in the Figure. It should be noted that the measurement and branch parameter errors are processed simultaneously. This process continues until all of the measurement and branch parameter errors are identified. If the measurement redundancy is low, the parameter errors may be wrong with the measurement errors and ineligible results are detected for the SE.

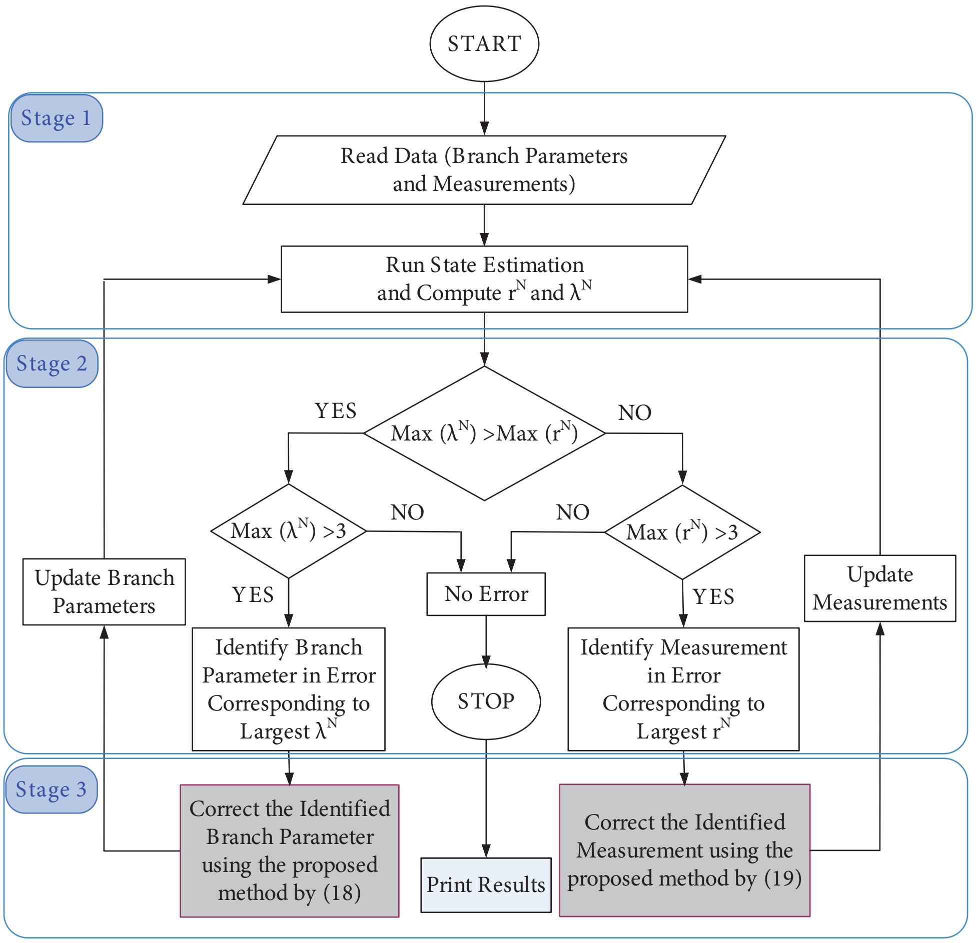

Figure. Flowchart of the identification and correction of the measurement and branch parameter errors.

4. Simulation results

In this section, the validity and performance of the proposed approach are evaluated. In this regard, the proposed approach is implemented and tested on the IEEE 14-, 30-, and 57-bus test systems. The topologies and parameters of these systems can be downloaded from [28]. Different case studies are simulated with errors that are introduced in the branch parameters and conventional measurements. In the simulations, single and multiple errors and simultaneously occurring errors in the conventional measurements and branch parameters are considered. In all of the tests, it is considered that the measurement system is highly redundant. In this regard, it is considered that all of the power injection and power flow measurements are available.

The true value of the measurements is provided by adding Gaussian noise into the calculated values of the load flow solution. On the other hand, to obtain the initial branch parameters (bad parameters), errors are added to the true values of the branch parameters. These errors are considered to be 50% of the true values. Moreover, the initial measurement values (bad measurements) are selected as 50% of the measurement values. In this paper, a typical threshold of 3 is selected for the Lagrange multiplier analysis and normalized residual test.

In the following sections, the parameter and measurement values in the tables are in p.u. Moreover, g_{i-j}, b_{i-j}, and bc_{i-j}^{shunt} are the series conductance, series susceptance, and shunt susceptance of the π-equivalent model of the branch connecting buses i and j, respectively. Moreover, the series and shunt admittances of the line connecting buses i and j are defined as $y_{i-j} = g_{i-j} + jb_{i-j}$ and $y_{i-j}^{shunt} = jbc_{i-j}^{shunt}$, respectively.

4.1. Single error in the branch parameters or conventional measurements

In this section, a single error in the branch parameters or conventional measurements is simulated in the test systems. Table 1 shows tests A and B for the 3 test systems, where tests A and B introduce an error in the branch parameters and the measurements, respectively. These tests are listed in Table 1 for the IEEE 14-, 30-, and 57-bus systems. The true and initial values of the branch parameters and measurements are also given in Table 1.

Table 1. Simulated single errors in the branch parameters and measurements.

Test system	Bad parameter or measurement	True value	Initial value	
14-bus	Test A	g_{2-5}	1.702	2.553
	Test B	Q_{7-8}^{flow}	–0.229	–0.343
30-bus	Test A	b_{12-15}	–6.097	–9.146
	Test B	P_{17}^{inj}	–0.090	–0.135
57-bus	Test A	bc_{12-13}^{shunt}	0.060	0.090
	Test B	P_{9-11}^{flow}	0.132	0.198

A cycle of successive SE with the proposed algorithm is run and the erroneous branch parameters are identified and estimated until $\left|r_{i.\max}^{N}\right| < 3$ and $\left|\lambda_{i.\max}^{N}\right| < 3$. After the successful identification and correction of bad data and parameters, all normalized residuals and Lagrange multipliers are lower than 3. Notice that the convergence tolerance of the SE algorithm is equal to 10^{-6}.

The results of the error identification for single errors are shown in Table 2. The estimated (corrected) values obtained by the proposed method can also be seen in Table 2. A comparison of these estimated values with the true values of the measurements and parameters is listed in Table 1, which reveals that the proposed method very accurately estimates and corrects the erroneous branch parameters and measurements. This table also demonstrates the 3 largest normalized residuals (r^N) for the measurements and Lagrange multipliers (λ^N) for the parameters. In Table 2, the percentage of correction is defined as follows:

$$\text{percentage of correction} = \left|\frac{\text{true value} - \text{estimated value}}{\text{true value}}\right| \times 100 \tag{20}$$

4.2. Multiple errors in the branch parameters

In this section, multiple branch parameter errors are simultaneously added to the parameters of the selected branches in the IEEE 14-, 30-, and 57-bus systems, as listed in Table 3. Table 3 also presents the actual values, initial values, and estimated values of the series conductances, series susceptances, and shunt susceptances of the selected branches in the test systems using the proposed method. The results of the proposed method are also compared with those of the existing methods in [11] (for the 14-bus system) and [12] (for the 30- and 57-bus system) by the percentage of correction index. It should be noted that to obtain the initial branch parameter values, errors are added to the actual values of the parameters. These errors are considered as 30% of the actual values.

Table 2. Total results of the error identification for single errors in the branch parameters or measurements.

a) 14-bus system.

	Step	$J(x)$ in last iteration	Identified bad parameter	Estimated parameter	Percentage of correction	r^N or λ^N	Measurement or parameter
Test A	1	$5.52 \times 10^{+3}$	g_{2-5}	1.699	0.176	54.069	g_{2-5}
						22.250	g_{4-5}
						8.907	Q_{2-5}^{flow}
	2	15.485	-	-	-	$r^N \, and \, \lambda^N < 3$	-
Test B	Step	$J(x)$ in last iteration	Identified bad measurement	Estimated measurement	Percentage of correction	$r^N or \lambda^N$	Measurement or parameter
	1	$9.52 \times 10^{+3}$	Q_{7-8}^{flow}	–0.230	0.436	47.499	Q_{7-8}^{flow}
						30.559	b_{7-8}
						11.432	bc_{7-8}^{shunt}
	2	14.351	-	-	-	$r^N \, and \, \lambda^N < 3$	-

b) 30-bus system.

	Step	$J(x)$ in last iteration	Identified bad parameter	Estimated parameter	Percentage of correction	$r^N or \lambda^N$	Measurement or parameter
Test A	1	$1.965 \times 10^{+4}$	b_{12-15}	–6.086	0.18	10.015	b_{12-15}
						5.933	b_{12-14}
						4.454	b_{14-15}
	2	8.539	-	-	-	$r^N \, and \, \lambda^N < 3$	-
Test B	Step	$J(x)$ in last iteration	Identified bad measurement	Estimated measurement	Percentage of correction	$r^N or \lambda^N$	Measurement or parameter
	1	$1.165 \times 10^{+3}$	P_{17}^{inj}	–0.09	0	34.013	P_{17}^{inj}
						13.663	P_{10-17}^{flow}
						11.584	P_{17-10}^{flow}
	2	17.636	-	-	-	$r^N \, and \, \lambda^N < 3$	-

c) 57-bus system.

	Step	$J(x)$ in last iteration	Identified bad parameter	Estimated parameter	Percentage of correction	$r^N or \lambda^N$	Measurement or parameter
Test A	1	$2.267 \times 10^{+3}$	bc_{12-13}^{shunt}	0.06	0	47.561	bc_{12-13}^{shunt}
						33.792	Q_{13-12}^{flow}
						19.725	Q_{12-13}^{flow}
	2	5.208	-	-	-	$r^N \, and \, \lambda^N < 3$	-
Test B	Step	$J(x)$ in last iteration	Identified bad measurement	Estimated measurement	Percentage of correction	$r^N or \lambda^N$	Measurement or parameter
	1	$3.65 \times 10^{+3}$	P_{9-11}^{flow}	0.132	0	60.381	P_{9-11}^{flow}
						23.351	b_{9-11}
						19.392	b_{11-13}
	2	4.944	-	-	-	$r^N \, and \, \lambda^N < 3$	-

In a comparison of the results of the proposed method with those of [11,12], it is evident that the proposed method estimated and corrected the erroneous branch parameters with higher precision.

4.3. Simultaneous errors in the conventional measurements and branch parameters

The main goal of this paper is to detect, identify, and correct conventional measurement and branch parameter errors with high accuracy. This section shows the identification and correction of multiple errors in conventional measurements and branch parameters in the IEEE 14-, 30-, and 57-bus system tests. The simulated errors are

Table 3. Total results of the multiple error identification in the branch parameters.
a) 14-bus system.

Parameter	Initial value	Actual value	Estimated value (proposed method)	Estimated value by [11]	Percentage of correction (proposed method)	Percentage of correction by [11]
g_{1-5}	0.71806	1.0258	1.02651	1.0231	0.0692	0.2632
b_{1-5}	–2.96443	–4.2350	–4.23245	–4.2161	0.0602	0.4439
bc_{1-5}^{shunt}	0.03444	0.0492	0.04904	0.049	0.3252	0.4065
g_{2-3}	0.8731	1.1350	1.13393	1.1297	0.0942	0.4669
b_{2-3}	–3.34726	–4.7818	–4.77401	–4.7602	0.1631	0.4517
bc_{2-3}^{shunt}	0.03066	0.0438	0.044106	0.0442	0.6937	0.9132
g_{2-4}	1.1802	1.6860	1.68748	1.6793	0.0877	0.3974
b_{2-4}	–3.58106	–5.1158	–5.11648	–5.0768	0.0133	0.7623
bc_{2-4}^{shunt}	0.0238	0.034	0.034137	0.0348	0.4029	2.3529

b) 30-bus system.

Parameter	Initial value	Actual value	Estimated value (proposed method)	Estimated value by [12]	Percentage of correction (proposed method)	Percentage of correction by [12]
g_{1-2}	6.7919	5.2246	5.2385	5.2530	0.2653	0.5436
b_{1-2}	–20.3407	–15.647	–15.5963	–15.5596	0.3231	0.5566
bc_{1-2}^{shunt}	0.0343	0.0264	0.02635	0.0263	0.1897	0.3787
g_{2-4}	2.21715	1.7055	1.7025	1.7001	0.1762	0.3166
b_{2-4}	–6.75649	–5.1973	–5.19218	–5.2177	0.0985	0.3925
bc_{2-4}^{shunt}	0.02392	0.0184	0.01836	0.0183	0.2178	0.5434
g_{8-28}	1.8770	1.4439	1.4513	1.4590	0.5098	1.0457
b_{8-28}	–5.9030	–4.5408	–4.53602	–4.5570	0.1053	0.2923
bc_{8-28}^{shunt}	0.0278	0.0214	0.021331	0.0216	0.3234	0.9345

c) 57-bus system.

Parameter	Initial value	Actual value	Estimated value (proposed method)	Estimated value by [12]	Percentage of correction (proposed method)	Percentage of correction by [12]
g_{7-8}	3.4337	2.6413	2.6397	2.6350	0.0605	0.2385
b_{7-8}	–17.5881	–13.5293	–13.5516	–13.4267	0.1648	0.7583
bc_{7-8}^{shunt}	0.0126	0.0097	0.0099	0.0093	2.0618	4.1237
g_{1-15}	2.6914	2.0703	2.06523	2.0765	0.2449	0.2994
b_{1-15}	–13.7593	–10.5841	–10.5785	–10.6052	0.0529	0.1993
bc_{1-15}^{shunt}	0.0642	0.0494	0.049074	0.0489	0.6599	1.0121
g_{3-15}	6.8567	5.2744	5.32459	5.1101	0.95157	3.1150
b_{3-15}	–22.4715	–17.2481	–17.2083	–17.4715	0.23075	1.2952
bc_{3-15}^{shunt}	0.0354	0.0272	0.02697	0.0276	0.84558	1.4705

shown in Table 4, which include 3 measurement errors and 3 branch parameter errors, simultaneously. Moreover, the true and initial values of these variables are given in Table 4.

Simulation results are demonstrated in Table 5. These results include the objective function in the last iteration, identified bad parameter or measurement, and estimated values of these measurements or parameters. Note that when there are multiple errors in the network parameters as well as conventional measurements, repeated application of the proposed method can identify and correct errors one by one. Table 4 also shows the

3 largest normalized residuals (r^N) for the measurements and Lagrange multipliers (λ^N) for the parameters, as well as the percentage of correction.

Table 4. Simulated simultaneous errors in the branch parameters and measurements.

Test system	Bad parameter or measurement	True value	Initial value
14-bus	g_{3-4}	1.986	2.979
	b_{5-6}	–3.968	–5.952
	bc_{2-5}^{shunt}	0.0346	0.052
	Q_{10}^{inj}	–0.058	–0.087
	P_{4-5}^{flow}	–0.618	–0.927
	Q_{12-13}^{flow}	0.0115	0.0172
30-bus	g_{1-3}	1.540	2.310
	b_{2-6}	–5.116	–7.674
	bc_{9-11}^{shunt}	0	0.050
	P_{7}^{inj}	–0.228	–0.342
	P_{12-14}^{flow}	0.076	0.114
	Q_{23-24}^{flow}	0.008	0.012
57-bus	g_{3-4}	7.645	11.467
	b_{9-10}	–5.681	–8.521
	bc_{12-16}^{shunt}	0.0216	0.0324
	Q_{15}^{inj}	–0.05	–0.075
	P_{38-48}^{flow}	–0.136	–0.204
	Q_{54-55}^{flow}	–0.067	–0.100

Table 5. Total results of the error identification for the simultaneous error in the parameters and measurements.

a) 14-bus system.

Step	$J(x)$ in last iteration	Identified bad measurement or parameter	Estimated measurement or parameter	Percentage of correction	$r^N or \lambda^N$	Measurement or parameter
1	2.849 × 10^{+3}	g_{3-4}	1.975	0.554	38.716	g_{3-4}
					31.541	g_{2-3}
					27.156	P_{4-5}^{flow}
2	1.805 × 10^{+3}	bc_{2-5}^{shunt}	0.035	1.156	25.423	bc_{2-5}^{shunt}
					23.481	Q_{2-5}^{flow}
					21.012	g_{3-4}
3	910.553	Q_{10}^{inj}	–0.058	0	21.703	Q_{10}^{inj}
					20.123	Q_{12-13}^{flow}
					19.343	bc_{2-5}^{shunt}
4	463.290	b_{5-6}	–3.961	0.176	20.035	b_{5-6}
					17.312	b_{4-7}
					12.143	b_{7-9}
5	60.798	Q_{12-13}^{flow}	0.0115	0	5.342	Q_{12-13}^{flow}
					5.164	bc_{2-5}^{shunt}
					4.132	Q_{2-5}^{flow}
6	34.2598	P_{4-5}^{flow}	–0.618	0	3.967	P_{4-5}^{flow}
					3.824	b_{5-6}
					3.213	P_{5-4}^{flow}

Table 5. Continued.

b) 30-bus system.

Step	$J(x)$ in last iteration	Identified bad measurement or parameter	Estimated measurement or parameter	Percentage of correction	$r^N or \lambda^N$	Measurement or parameter
					43.762	g_{1-3}
1	$3.913 \times 10^{+4}$	g_{1-3}	1.538	0.308	39.342	g_{1-2}
					37.921	g_{3-4}
					35.521	bc_{9-11}^{shunt}
2	$2.205 \times 10^{+4}$	bc_{9-11}^{shunt}	0	0	33.342	b_{9-11}
					29.561	Q_{9-11}^{flow}
					27.391	P_7^{inj}
3	$1.134 \times 10^{+4}$	P_7^{inj}	–0.227	0.438	24.133	Q_{23-24}^{flow}
					23.012	b_{2-6}
					23.399	b_{2-6}
4	$3.874 \times 10^{+3}$	b_{2-6}	–5.114	0.039	21.753	Q_{23-24}^{flow}
					19.537	g_{1-3}
					17.191	P_{12-14}^{flow}
5	$1.334 \times 10^{+3}$	P_{12-14}^{flow}	0.076	0	14.162	g_{14-15}
					9.534	b_{12-14}
					3.368	Q_{23-24}^{flow}
6	24.486	Q_{23-24}^{flow}	0.008	0	3.254	g_{1-3}
					2.243	g_{1-2}

c) 57-bus system.

Step	$J(x)$ in last iteration	Identified bad measurement or parameter	Estimated measurement or parameter	Percentage of correction	$r^N or \lambda^N$	Measurement or parameter
					58.855	P_{38-48}^{flow}
1	$6.023 \times 10^{+4}$	P_{38-48}^{flow}	–0.136	0	48.723	P_{48}^{inj}
					44.812	P_{48-38}^{flow}
					29.413	Q_{54-55}^{flow}
2	$2.556 \times 10^{+3}$	Q_{54-55}^{flow}	–0.0674	0.597	27.351	b_{9-10}
					24.715	Q_{15}^{inj}
					23.819	b_{9-10}
3	$1.717 \times 10^{+3}$	b_{9-10}	–5.694	0.228	23.713	Q_{15}^{inj}
					22.976	g_{10-12}
					22.476	g_{3-4}
4	$1.241 \times 10^{+3}$	g_{3-4}	7.651	0.078	20.678	g_{8-9}
					20.118	g_{4-6}
					19.987	bc_{12-16}^{shunt}
5	153.926	bc_{12-16}^{shunt}	0.02157	0.138	17.483	Q_{54-55}^{flow}
					13.762	Q_{12-16}^{flow}
					11.033	Q_{15}^{inj}
6	78.537	Q_{15}^{inj}	–0.0499	0.20	7.712	bc_{15-45}^{shunt}
					6.452	bc_{14-15}^{shunt}

A comparison of these estimated values with the true values of the conventional measurements and branch parameters listed in Table 4 shows that the proposed method estimated and corrected the erroneous branch parameters and measurements with high precision. Moreover, it can be observed that the multiple errors in the measurements and parameters were identified and corrected by the proposed approach.

Table 6 lists the number of simulated errors, average number of iterations in any runs, and total central processing unit (CPU) time for a laptop computer with a 2-GHz Pentium 2 CPU and 1-GB RAM using the proposed method based on Lagrangian analysis for validating the measurement and branch parameter errors.

As is evident above, using the proposed method, a CPU time of less than 1 s is required to obtain the correct solution and to identify the suspected measurements and branch parameters with reference to the IEEE 14-, 30-, and 57-bus systems.

5. Conclusions

This paper proposes a new algorithm for the simultaneous identification and correction of conventional measurement and branch parameter errors to enhance the efficiency of power system SE. There is no need for an a-priori specification of suspect parameter vectors. The proposed method uses Lagrange multipliers for the identification of branch parameter errors, which are calculated based on the results of the conventional WLS SE. Erroneous measurement and branch parameter values could be corrected using a new linear approximation approach by eliminating the necessity for augmented state vectors. Finally, the proposed method for branch parameter error correction is implemented and tested on the IEEE 14-, 30-, and 57-bus test systems. Different cases are simulated, in which errors are introduced in the measurements and branch parameters. All single, multiple, and simultaneous errors in the conventional measurements and branch parameters are simulated. The performance of the proposed method is illustrated through these examples and it is shown that it can identify and correct erroneous measurements and branch parameters with high accuracy.

Appendix A: derivatives with respect to the physical line parameter

In order to simplify the mathematical formulation of the SE problem, system parameters are expressed in terms of the network branch admittances, namely series ($y_{i-j} = g_{i-j} + jb_{i-j}$) and shunt ($y_{i-j}^{shunt} = jbc_{i-j}^{shunt}$) admittances, and the proposed error correction method is applied for these parameters. If the physical parameters of the line resistances (r_{i-j}), and reactances (x_{i-j}) are required, the chain rule must be used as indicated below.

Terms of the network branch admittances are related to the line resistances and reactances as:

$$g_{i-j} = \frac{r_{i-j}}{r_{i-j}^2 + x_{i-j}^2},\ b_{i-j} = \frac{-x_{i-j}}{r_{i-j}^2 + x_{i-j}^2},\ \forall i, \forall j \in \Omega_i \tag{A1}$$

and

$$g_{i-i} = \sum_j g_{i-j},\ b_{i-i} = \sum_j b_{i-j} + bc_{i-j}^{shunt},\ \forall i, \forall j \in \Omega_i, \tag{A2}$$

where Ω_i is set of buses adjacent to bus i.

Let F be the variable for which the partial derivatives are looked. These partial derivatives with respect to the resistances (r_{i-j}) can be obtained using the chain rule as follows:

$$\begin{aligned}\frac{\partial F}{\partial r_{i-j}} &= \frac{\partial g_{i-j}}{\partial r_{i-j}}\left[\frac{\partial F}{\partial g_{i-j}} + \frac{\partial F}{\partial g_{i-i}}\cdot\frac{\partial g_{i-i}}{\partial g_{i-j}} + \frac{\partial F}{\partial g_{j-j}}\cdot\frac{\partial g_{j-j}}{\partial g_{i-j}}\right] \\ &+ \frac{\partial b_{i-j}}{\partial r_{i-j}}\left[\frac{\partial F}{\partial b_{i-j}} + \frac{\partial F}{\partial b_{i-i}}\cdot\frac{\partial b_{i-i}}{\partial b_{i-j}} + \frac{\partial F}{\partial b_{j-j}}\cdot\frac{\partial b_{j-j}}{\partial b_{i-j}}\right]\end{aligned} \tag{A3}$$

From Eq. (A1), the derivatives of the network branch admittance terms with respect to the resistances (r_{i-j}) can be obtained as follows:

$$\frac{\partial g_{i-j}}{\partial r_{i-j}} = \frac{x_{i-j}^2 - r_{i-j}^2}{\left(r_{i-j}^2 + x_{i-j}^2\right)^2},\ \frac{\partial b_{i-j}}{\partial r_{i-j}} = \frac{2r_{i-j}x_{i-j}}{\left(r_{i-j}^2 + x_{i-j}^2\right)^2} \tag{A4}$$

whereas from Eq. (A2),

$$\frac{\partial g_{i-i}}{\partial g_{i-j}} = \frac{\partial b_{i-i}}{\partial b_{i-j}} = 1 \tag{A5}$$

Finally, Eq. (A3), using Eqs. (A4) and (A5), could be written as below:

$$\begin{aligned}\frac{\partial F}{\partial r_{i-j}} &= \frac{x_{i-j}^2 - r_{i-j}^2}{\left(r_{i-j}^2 + x_{i-j}^2\right)^2}\left[\frac{\partial F}{\partial g_{i-j}} + \frac{\partial F}{\partial g_{i-i}} + \frac{\partial F}{\partial g_{j-j}}\right] \\ &+ \frac{2r_{i-j}x_{i-j}}{\left(r_{i-j}^2 + x_{i-j}^2\right)^2}\left[\frac{\partial F}{\partial b_{i-j}} + \frac{\partial F}{\partial b_{i-i}} + \frac{\partial F}{\partial b_{j-j}}\right], \forall i, \forall j \in \Omega_i\end{aligned} \tag{A6}$$

Similarly, the partial derivative of the variable F with respect to the reactances (x_{i-j}) can be obtained as:

$$\begin{aligned}\frac{\partial F}{\partial x_{i-j}} &= \frac{\partial g_{i-j}}{\partial x_{i-j}}\left[\frac{\partial F}{\partial g_{i-j}} + \frac{\partial F}{\partial g_{i-i}}\cdot\frac{\partial g_{i-i}}{\partial g_{i-j}} + \frac{\partial F}{\partial g_{j-j}}\cdot\frac{\partial g_{j-j}}{\partial g_{i-j}}\right] \\ &+ \frac{\partial b_{i-j}}{\partial x_{i-j}}\left[\frac{\partial F}{\partial b_{i-j}} + \frac{\partial F}{\partial b_{i-i}}\cdot\frac{\partial b_{i-i}}{\partial b_{i-j}} + \frac{\partial F}{\partial b_{j-j}}\cdot\frac{\partial b_{j-j}}{\partial b_{i-j}}\right]\end{aligned} \tag{A7}$$

From Eq. (A1), the derivatives of the network branch admittance terms with respect to the reactances (x_{i-j}) can be obtained as follows:

$$\frac{\partial g_{i-j}}{\partial x_{i-j}} = \frac{-2r_{i-j}x_{i-j}}{\left(r_{i-j}^2 + x_{i-j}^2\right)^2},\ \frac{\partial b_{i-j}}{\partial x_{i-j}} = \frac{x_{i-j}^2 - r_{i-j}^2}{\left(r_{i-j}^2 + x_{i-j}^2\right)^2} \tag{A8}$$

Finally, Eq. (A7), using Eqs. (A5) and (A8), could be written as below:

$$\begin{aligned}\frac{\partial F}{\partial x_{i-j}} &= \frac{-2r_{i-j}x_{i-j}}{\left(r_{i-j}^2 + x_{i-j}^2\right)^2}\left[\frac{\partial F}{\partial g_{i-j}} + \frac{\partial F}{\partial g_{i-i}} + \frac{\partial F}{\partial g_{j-j}}\right] \\ &+ \frac{x_{i-j}^2 - r_{i-j}^2}{\left(r_{i-j}^2 + x_{i-j}^2\right)^2}\left[\frac{\partial F}{\partial b_{i-j}} + \frac{\partial F}{\partial b_{i-i}} + \frac{\partial F}{\partial b_{j-j}}\right], \forall i, \forall j \in \Omega_i\end{aligned} \tag{A9}$$

If the partial derivative of the variable F with respect to half of the shunt susceptances (bc_{i-j}^{shunt}) are sought, the following expressions should be used:

$$\begin{aligned}\frac{\partial F}{\partial bc_{i-j}^{shunt}} &= \frac{\partial F}{\partial bc_{i-j}^{shunt}} + \frac{\partial F}{\partial b_{i-i}}\cdot\frac{\partial b_{i-i}}{\partial bc_{i-j}^{shunt}} + \frac{\partial F}{\partial b_{j-j}}\cdot\frac{\partial b_{j-j}}{\partial bc_{i-j}^{shunt}} \\ &= \frac{\partial F}{\partial bc_{i-j}^{shunt}} + \frac{\partial F}{\partial b_{i-i}} + \frac{\partial F}{\partial b_{j-j}},\ \forall i, \forall j \in \Omega_i\end{aligned} \tag{A10}$$

Note also that due to Eq. (A2), $\partial b_{i-i}/\partial bc_{i-j}^{shunt} = \partial b_{j-j}/\partial bc_{i-j}^{shunt} = 1$.

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