

## Abrupt and incipient fault detection and compensation for a 4-tank system benchmark

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**Abstract:** Fault detection and compensation play a key role in enhancing the high demand for performance and security in technological systems. This paper proposes an active fault-tolerant control scheme that detects and compensates for actuator faults in a 4-tank system benchmark. The faults are modeled as a drastic gain loss in actuators (pumps), which could lead to a large loss in the nominal performance. The model-based approach uses a recursive least squares parameter estimation algorithm to form a fault detection and diagnosis subsystem and utilizes a parametric eigenstructure assignment method to reconfigure the state feedback controller. The designed controller is simulated for the nonlinear system, and the results demonstrate promising performance increases in faulty cases in comparison with the nonfault-tolerant controller.

**Key words:** Active fault-tolerant control systems, fault detection and diagnosis, online parameter estimation, parametric eigenstructure assignment

### 1. Introduction

Today's technological systems rely on sophisticated control systems to achieve high levels of performance and reliability. A fault or malfunction in sensors, actuators, or other system components in a complex system with ordinary feedback controllers can lead to extreme performance degradation or even instability [1].

To deal with such weaknesses in these systems, new control strategies have been developed that can tolerate faults in system components and maintain the desirable performance and stability characteristics. This is particularly vital in safety-critical systems such as aircrafts, spacecrafts, nuclear power plants, and chemical plants containing dangerous materials [1,2].

In such systems, minor faults can result in catastrophic consequences. The control system must be designed in such a way that it can tolerate potential faults in system components and compensate for their effects while increasing the overall reliability of the system and maintaining the desirable performance and stability. These types of control systems are known as fault-tolerant control systems (FTCSs) [1].

In general, FTCSs are classified into 2 types: passive (PFTCS) and active (AFTCS). In a PFTCS, the controller is designed to tolerate some presumed faults using robust control techniques [3]. The controller in these systems is unvarying. In contrast, an AFTCS uses a reconfigurable controller that reacts to system component failure actively so that the stability and acceptable performance of the entire system can be maintained [1].

AFTCSs depend on a fault detection and diagnosis (FDD) subsystem to provide the most up-to-date

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information about the fault and its magnitude. The FDD subsystem must be insensitive to noise, uncertainties, and input changes, and it must be sensitive to faults in the system components [4].

One group of FDD methods is model-based; it uses the mathematical model of the fault-free system and compares it with the actual system response. In the ideal case, the behavior of the system and its model is exactly the same; when there are differences, the faults can be detected [1,5,6].

Online parameter estimation is one of the frequently used methods of model-based FDD. With estimation of some or all system parameters, system faults can be detected and isolated. Furthermore, it can be used directly in order to evaluate the fault magnitude and reconfigure the controller [7].

The other important subsystem of the AFTCS is the reconfigurable controller. This controller should be designed so that it can be changed easily, and it must be able to maintain the appropriate performance and stability of the system not only when the system works normally but also when there are faults in the system [8].

The aim of this paper is to design a fault-tolerant controller for a 4-tank system benchmark. The controller must tolerate partial actuator failures modeled as a sudden decrease in actuator gains. For the FDD subsystem, the online parameter estimation method is used, and for designing a reconfigurable controller, an adaptive eigenstructure assignment algorithm is selected [8].

The system under study is a MIMO nonlinear 4-tank system benchmark. This system is a benchmark experimental facility developed for research purposes for the processing and aerospace industries [9,10].

Fault-tolerant methods have been applied to multitank system benchmarks in a few recent research works. Some examples are mentioned here. In [11], fault-tolerant methods were implemented in a 4-tank system using a command governor controller. In [12], high-order sliding-mode observers were used for a 3-tank system. The authors of [5] used predictive control and fuzzy logic to design a fault-tolerant control for a 3-tank benchmark. In [13], using feedback linearization, an approach was proposed for fault-tolerant control in a 3-tank benchmark. Other similar works are [8,14,15].

This paper is organized as follows. In Section 2, the model of the 4-tank system benchmark is described. The linearized model has been derived using perturbation theory. Section 3 is devoted to controller design methods, and Section 4 portrays the online parameter estimation method used for FDD. Simulation results of implementation of the controller on the nonlinear model are shown in Section 5 by comparing the performance of a nonreconfigurable controller with a reconfigurable controller by some indexes. Section 6 presents the conclusion and future works.

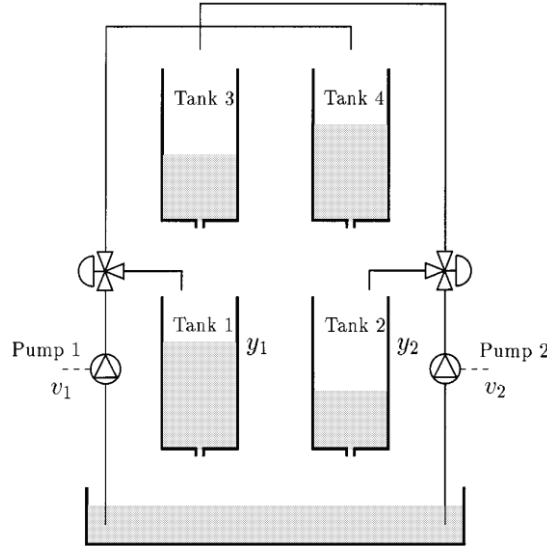
## 2. Process description

The process is called a 4-tank system benchmark and it consists of 4 interconnected water tanks and 2 pumps [16]. The system is shown in Figure 1. The inputs are the voltage to the 2 pumps in the standard range of 0–10 V [17]. The outputs are the water levels in the lower tanks. The height of each tank is 20 cm. The 4-tank process can easily be built by using 2 double-tank processes, which are standard processes in many control laboratories [17,18].

The aim is to control the level in the lower 2 tanks with 2 pumps. The process inputs are  $v_1$  and  $v_2$  (input voltages to the pumps), and the outputs are  $y_1$  and  $y_2$  (voltages from level measurement devices). Mass balance and Bernoulli's law yield are as follows [16]:

$$\begin{aligned}
 \frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\
 \frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\
 \frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\
 \frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1
 \end{aligned}
 \tag{1}$$

where  $A_i$  is the cross-section of tank  $i$ ,  $a_i$  is the cross-section of the outlet hole, and  $h_i$  is the water level.



**Figure 1.** Schematic diagram of the 4-tank process. The water levels in tanks 1 and 2 are controlled by 2 pumps [16].

The voltage applied to pump  $i$  is  $v_i$ , and the corresponding flow is  $k_i v_i$ . The parameters  $\gamma_1 \gamma_2 \in (0, 1)$  are determined according to how the valves are set prior to an experiment. The flow to tank 1 is  $\gamma_1 k_1 v_1$ , and the flow to tank 4 is  $(1 - \gamma_1) k_1 v_1$ , similarly for tank 2 and tank 3. The acceleration of gravity is denoted as  $g$ . The measured level signals are  $k_c h_1$  and  $k_c h_2$ . The parameter values of the process are given in Table 1 [16].

**Table 1.** Process parameter values.

Parameter	Value
$A_1, A_3$	28 cm <sup>2</sup>
$A_2, A_4$	32 cm <sup>2</sup>
$a_1, a_3$	0.071 cm <sup>2</sup>
$a_2, a_4$	0.057 cm <sup>2</sup>
$k_c$	0.50 V/cm
$g$	981 cm/s <sup>2</sup>

Introducing the variables  $x_i h_i - h_i^0$  and  $u_i v_i - v_i^0$ , the linearized state space equation is then given by:

$$\frac{dx}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} x + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} 0 \\ 0 \\ 0 \\ \frac{(1-\gamma_2)k_2}{A_3} \end{bmatrix} u + \begin{bmatrix} \frac{\gamma_2 k_2}{A_2} \\ \frac{(1-\gamma_1)k_1}{A_4} \\ 0 \\ 0 \end{bmatrix} v \quad y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} x, \tag{2}$$

$$T_i = \frac{A_i}{a_i} \sqrt{\frac{2h_i^0}{g}}$$

where the time constants are shown below.

$$i = 1, \dots, 4 \quad (3)$$

The operating point parameters are shown in Table 2 [16].

**Table 2.** Operating point parameter values of the process.

Parameter	Value
$(h_1^0, h_2^0, h_3^0, h_4^0)$	(12.4, 12.7, 1.8, 1.4) [cm]
$(v_1^0, v_2^0)$	(3.00, 3.00) [V]
$(k_1, k_2)$	(3.33, 3.35) [cm <sup>3</sup> /Vs]
$(\gamma_1, \gamma_2)$	(0.70, 0.60)

Substituting operating point parameters in Eqs. (2) and (3) yields the state space form as follows:

$$\dot{x} = \begin{bmatrix} -0.0159 & 0 & 0.0419 & 0 \\ 0 & -0.0111 & 0 & 0.0333 \\ 0 & 0 & -0.0419 & 0 \\ 0 & 0 & 0 & -0.0333 \end{bmatrix} x + \begin{bmatrix} 0.0833 & 0 \\ 0 & 0.0628 \\ 0 & 0.0479 \\ 0.0312 & 0 \end{bmatrix} u, \quad (4)$$

$$y = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 \end{bmatrix} x.$$

The states can be measured directly through tank levels, so there is no need for designing observers.

### 3. Controller design

The system is observable and controllable. The states of the system are available through direct measurement of tank levels. The process state and output equations are:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}, \quad (5)$$

where  $x_{(t)} \in R^n$  is the state vector,  $u_{(t)} \in R^m$  is the input vector, and  $y_{(t)} \in R^p$  is the output vector. A, B, and C are the system, input, and output matrices, respectively. By defining  $e_{(t)} = r_{(t)} - y_{(t)}$  and augmenting the states  $e$  with system states  $x$ , we can get the integral action in the controller for better tracking. The augmented system equations are as follows [19]:

$$u = - [ k_1 \quad k_2 ] \tilde{x} \quad \begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}\tilde{u} \\ y &= Cx \end{aligned} \quad (6)$$

where

$$\tilde{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \quad \tilde{A} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix} \quad \tilde{x} = \begin{bmatrix} x \\ e \end{bmatrix}. \quad (7)$$

Using Ackerman's method we can calculate the gain of the state feedback controller. For the reconfigurable controller, the gain matrix is calculated online based on the real-time values of the system and input matrices.

#### 4. Parameter estimation algorithm

In this system, a fault in actuators is modeled as a change in actuator gain caused by the clogging of the pumps. For detection and evaluation of faults, the online parameter estimation method with a recursive least squares algorithm is used.

Having the state space equation form of the system as in Eq. (2), faults in actuators are modeled as changes in the input matrix  $B$ . Therefore, system matrix  $A$  can be assumed constant, and the parameter estimation problem is limited to the estimation of matrix  $B$ . The problem is solved separately for each row of the system matrix form equation. Here we study the problem only for the first row of the matrix equation. It is the same for other rows. Consider the following row equation:

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + b_{11}u_1 + b_{12}u_2 \quad (8)$$

Using the Laplace transform and applying a filter to both sides of Eq. 8, and then splitting known and unknown parameters, we will have the following.

$$\frac{sx_1}{s+\lambda} - \frac{a_{11}x_1}{s+\lambda} - \frac{a_{12}x_2}{s+\lambda} - \frac{a_{13}x_3}{s+\lambda} - \frac{a_{14}x_4}{s+\lambda} = \frac{b_{11}u_1}{s+\lambda} + \frac{b_{12}u_2}{s+\lambda} \quad (9)$$

The left side of Eq. (9) is indicated with  $k_1$ . In each row of matrix  $B$ , one element is 0; accordingly, the problem can be simplified to a scalar form. Finally, Eq. (9) arrives at the following:

$$k_1 = b_{11}u_1 \quad (10)$$

The aim is to estimate the  $b_{ij}$  coefficients. The parameter estimation model is shown in Eq. 11 .[20]:

$$\begin{aligned} z &= \theta^T \varphi \\ \dot{\theta} &= P\varepsilon\varphi & \theta_{(0)} &= \theta_0 \\ \dot{P} &= \beta P - P \frac{\varphi\varphi^T}{m_s^2} P & P_{(0)} &= P_0 = Q_0^{-1} \\ m_s^2 &= 1 + \gamma\varphi^T\varphi & \varepsilon &= \frac{z - \theta^T\varphi}{m_s^2} \end{aligned} \quad (11)$$

For Eq. 10, we have:

$$z = k_1 \quad \theta = b_{11} \quad \varphi = u \quad (12)$$

The procedure is the same for other  $b_{ij}$  coefficients.

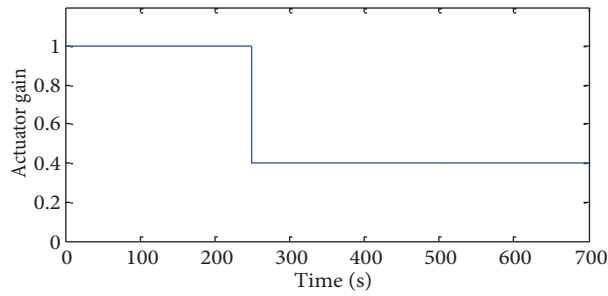
## 5. Simulation results

### 5.1. Abrupt faults

The system with the designed fault-tolerant controller was simulated in a MATLAB Simulink environment. It was controlled once using a state feedback eigenstructure assignment controller that is not fault-tolerant. The poles of the closed loop system were placed at:

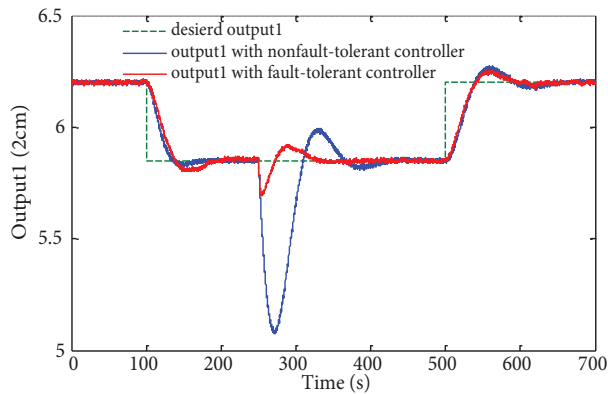
$$P = [ -0.0678 \pm 0.0683i \quad -0.0617 \pm 0.0591i \quad -0.0172 \quad -0.0562 ]$$

There are 2 step changes in the reference signal at 100 s and 500 s. Both actuator gains fell abruptly by 60% at 250 s, as shown in Figure 2.

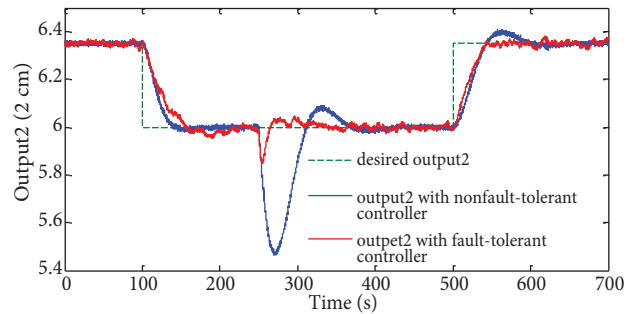


**Figure 2.** Fault in actuators modeled as a fall in gain by 60%.

Next, this scenario was repeated using the fault-tolerant controller. In Figures 3 and 4 the outputs and desired responses of the system under the 2 controllers are shown and compared in a single diagram. It is obvious that while the controller is not fault-tolerant, the performance of the system is extremely degraded, and the occurring fault has a great effect on system output. The fault-tolerant controller partially compensated for the fault effect and improved system performance under the faulty situation. The chosen parameters of the parameter estimation algorithm in this stage are shown in Table 3.



**Figure 3.** Output 1 of the system with the 2 controllers and abrupt fault.



**Figure 4.** Output 2 of the system with the 2 controllers and abrupt fault.

**Table 3.** Parameter values of the estimation algorithm.

Parameter	Value
$\beta$	3
$\gamma$	1
$\lambda$	0.2
$P_0$	1

The control command signals for both controllers can be seen in Figures 5 and 6.

In Table 4, the performances of the 2 controllers are compared using the integral absolute error (IAE) index.

Figure 7 illustrates the estimated parameters and their known initial values. It is obvious that the FDD subsystem has correctly estimated the decreased actuator gains at 250 s, and the fault is easily detected in this case.

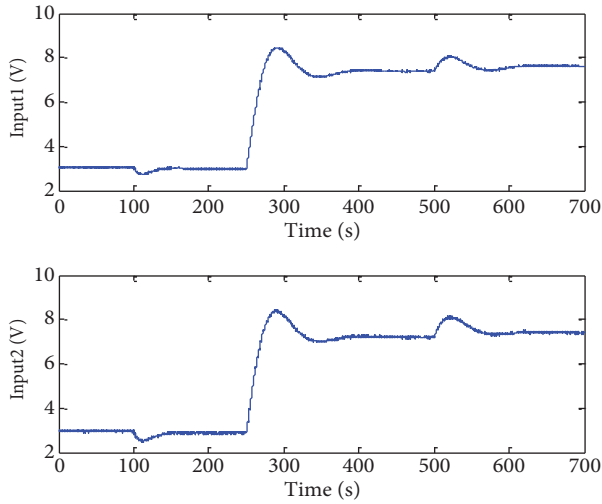


Figure 5. Control command signals of nonfault-tolerant controller.

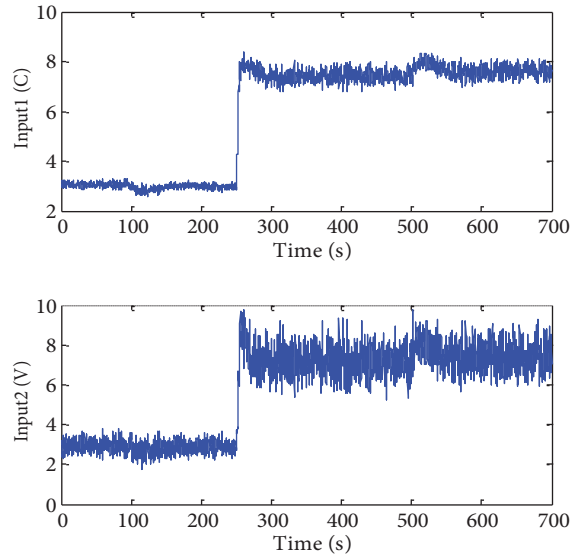


Figure 6. Control command signals of fault-tolerant controller with abrupt fault in the system.

Table 4. Comparison of the 2 controllers using IAE index.

	With nonfault-tolerant controller		With fault-tolerant controller	
	Output 1	Output 2	Output 1	Output 2
IAE factor	50.60	39.63	23.05	21.86

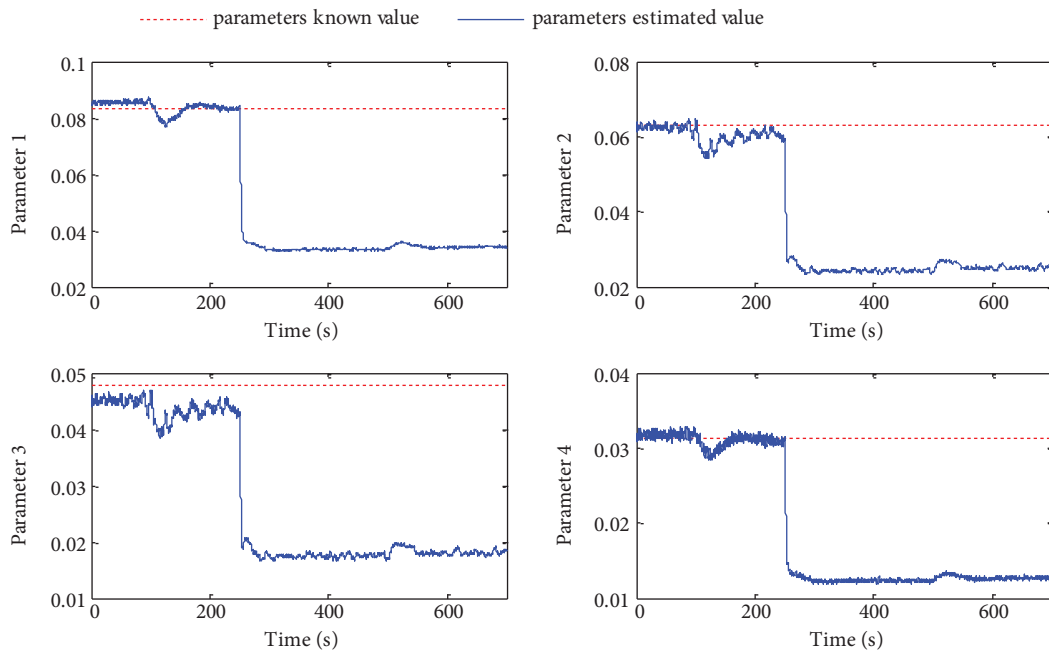


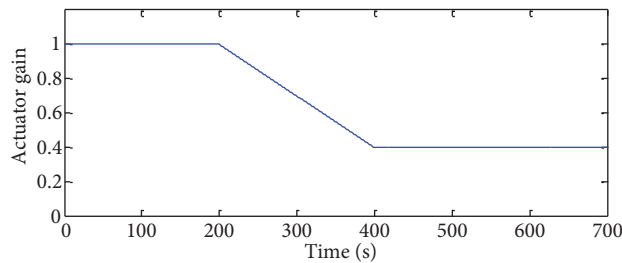
Figure 7. Known and estimated parameter values.

## 5.2. Incipient faults

As is obvious from Figure 6, the control command signal is jittering a lot. That is because the parameter estimation method is vulnerable to process noise. Moreover, the state feedback gain is calculated from these parameters. The effect of noise on the estimation of parameters is passed to the state feedback gains and the control command signal. To deal with this problem, we can filter the high-frequency noise by using filters or changing the estimation algorithm parameters that make the estimation process slower. Although in this case the consequences of noise are reduced, the ability of the fault-tolerant controller to compensate for abrupt faults will deteriorate. Therefore, there is a tradeoff between the speed of reaction to faults and the jitteriness of control command signals.

However, if faults are not abrupt, as is the case in some processes, slower parameter estimation can be used so that the effects of noise on jitteriness of the control command signals might be decreased.

To illustrate this situation, an incipient fault as a gradual gain loss of the actuators was considered. The gains were reduced by 60% at a fixed rate from 200 s to 400 s. Figure 8 shows the actuator gains. In this condition, the estimation parameters are tuned in such a way that they have less effect from the process noise and converge on the actual value at an appropriate pace so that the overall fault-tolerant controller can compensate for the faults. These tuned values of estimation algorithm parameters are given in Table 5.



**Figure 8.** Faults in actuators that have been modeled as a gradual decrease in gain by 60%.

**Table 5.** Parameter values of the estimation algorithm.

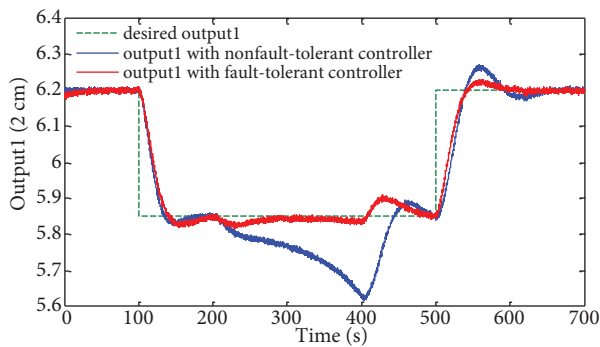
Parameter	Value
$\beta$	10
$\gamma$	1
$\lambda$	0.04
$P_0$	1

Figures 9 and 10 display the system outputs under the fault-tolerant and nonfault-tolerant controllers in this experiment. As is obvious, the fault-tolerant controller has compensated for the faults in the actuators. The performances of the 2 controllers can also be compared quantitatively in Table 6.

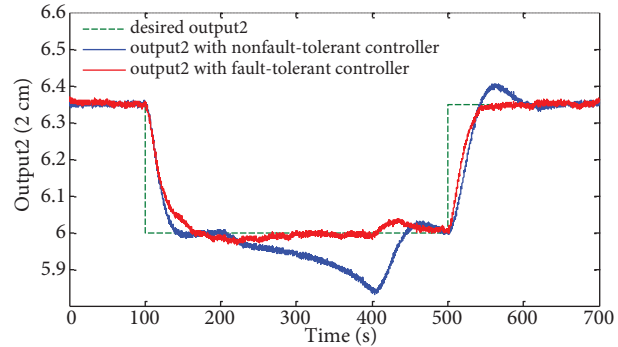
**Table 6.** Comparison of the 2 controllers using IAE index.

	With nonfault-tolerant controller		With fault-tolerant controller	
	Output 1	Output 2	Output 1	Output 2
IAE factor	43.06	34.92	20.17	19.25





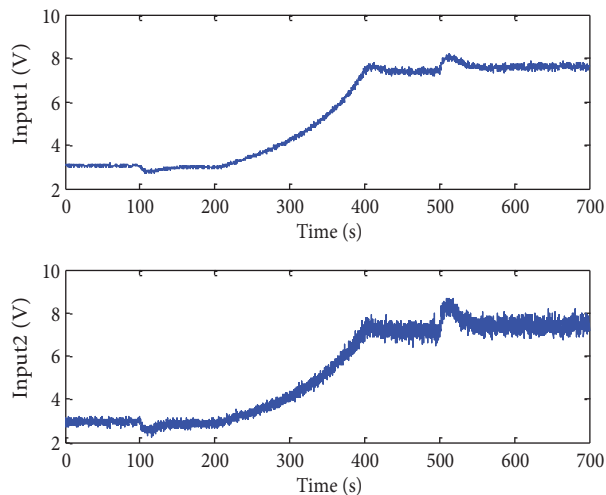
**Figure 9.** Output 1 of the system with the 2 controllers and incipient fault.



**Figure 10.** Output 2 of the system with the 2 controllers and incipient fault.

In Figure 11, the control command signals are shown. Although the jitters are much lower than in the former experiment with abrupt faults, they do exist. That is due to the fact that the parameter estimation method is inherently susceptible to process noise.

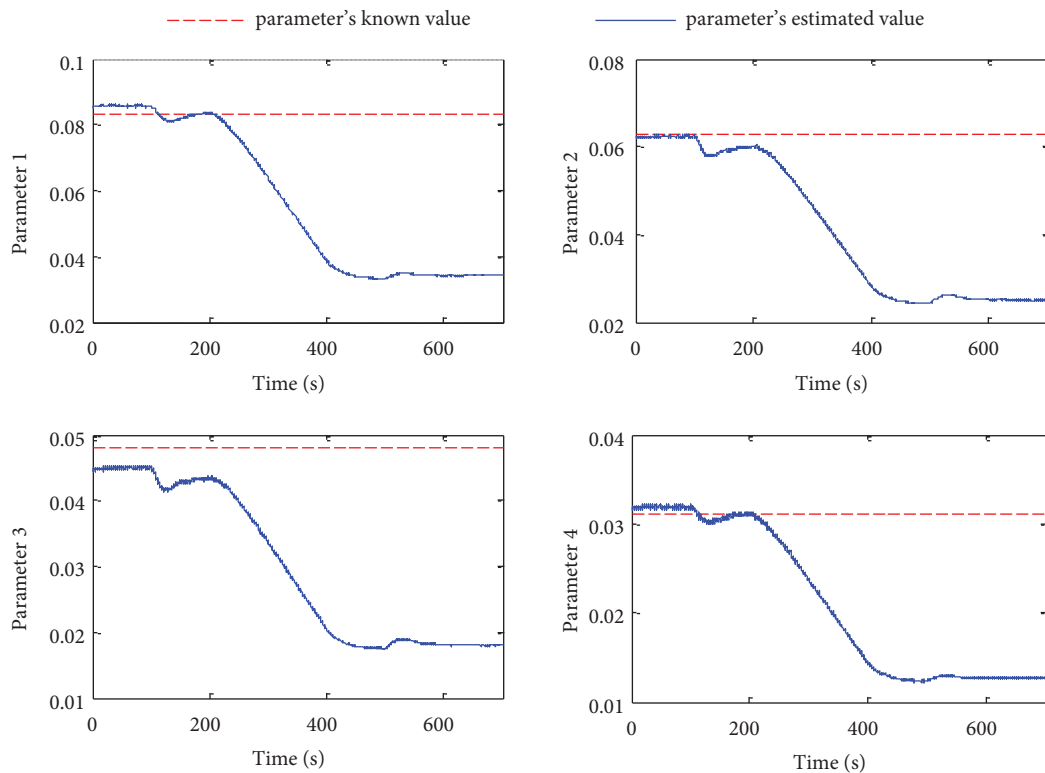
Figure 12 shows the parameter's real and estimated values. This figure shows the competency of the parameter estimation method used for detection and diagnosis of actuator faults in the system.



**Figure 11.** Control command signals of fault-tolerant controller with incipient fault in the system.

## 6. Conclusion

The fault-tolerant controller consists of a FDD subsystem and a reconfigurable controller. For the FDD subsystem, online parameter estimation is used, and an adaptive eigenstructure assignment controller is employed as the reconfigurable controller. The isolation of faults is not always easy using the parameter estimation method, because the reverse relations of physical parameters and faults are usually complicated. This method is more sensitive to noise than other model-based methods such as observer-based methods. On the other hand, it has higher capability to detect multiplicative faults. The effectiveness of the method to tolerate abrupt faults as well as incipient faults in actuators is demonstrated in the application to the simulated 4-tank system benchmark. Although the parameter estimation method is vulnerable to noise, it can improve the performance of the system at the cost of some jitters in the control command signals.



**Figure 12.** Known and estimated parameter values.

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