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Model-free controller with an observer applied in real-time to a 3-DOF helicopter

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Abstract: In this paper, a model-free controller with an observer is presented for a class of uncertain continuous-time multiinput-multioutput nonlinear dynamic systems. The proposed model-free control law consists of 2 parts: the first part is a linear control term used to specify the dynamics of the closed loop system, and the second part is a compensator of the effects of uncertainties and external disturbances. The compensator is synthesized from an estimator of the effects of uncertainties and disturbances based on the Lyapunov approach. In order to estimate the unavailable states of the controlled system, a linear state observer is designed. All of the signals in the closed-loop system are proved to be uniformly ultimately bounded using the Lyapunov stability theory. The effectiveness and feasibility of the proposed control strategy are examined in a real-time application for a helicopter with 3 degrees of freedom.

Key words: 3-DOF helicopter, model-free control, linear state observer, MIMO nonlinear systems

1. Introduction

Model-based control techniques have been studied in several papers in recent years and many algorithms have been proposed. The real-time application of these control schemes requires a good understanding of process dynamics and their operational environment. However, it is difficult to obtain a good mathematical model of the process dynamics and to know the different disturbances acting on the process. Therefore, these techniques cannot provide satisfactory results when applied to poorly modeled processes, which can operate in ill-defined environments. In order to overcome this problem, model-free control techniques that can be directly applied to complex processes have been given much attention in the control community during the last decade [1–9]. In [1], Fliess and Join proposed a model-free control technique synthesized based on a local nonphysical model. Within this scheme, an online numerical differentiator was used to estimate the derivatives of the output signal, and this control law was applied in real time to control DC/DC converters [2]. In [3], Coelho presented a model-free learning adaptive control based on the pseudogradient concept and optimization procedure for a general discrete single-input and single-output nonlinear system. In [4], Kim and Lewis introduced a model-free H_∞ control design algorithm for unknown linear discrete-time systems using Q-learning, which is a reinforcement learning method based on an actor-critic structure. In [5], Coelho et al. also suggested a model-free learning adaptive control based on the pseudogradient concept with compensation using a radial basis function neural network and optimization approach with the differential evolution technique. In [6], Qi et al. proposed an adaptive higher-order differential feedback control approach of affine chaotic systems, which do not rely on the system's model.

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In [7], Han proposed an active disturbance rejection control (ADRC) scheme using the proportional-integral-derivative (PID) control algorithm and nonlinear control feedback. The proposed approach was a model-free controller consisting of a simple differential equation used as a transient trajectory generator, a noise-tolerant tracking differentiator, and an extended state observer to obtain the total disturbance estimation and rejection. The stability analysis of the ADRC was addressed in [8] and an experimental validation of the ADRC can be found in [9]. In [10–13], one can find other model-free control approaches.

In this paper, we introduce a model-free controller with an observer for a class of uncertain continuous-time multiinput-multioutput (MIMO) nonlinear dynamic systems composed of q interconnected subsystems whose states are not all available for measurements. The proposed model-free control law consists of a linear control term used to specify the dynamics of the closed-loop system and a compensator of the effects of uncertainties and external disturbances. In addition, a state observer is designed for estimating the states of the plant. The overall closed-loop system stability is studied using the Lyapunov approach. The proposed controller guarantees the boundedness of all of the variables in the closed-loop system. The ability of the proposed control scheme is examined experimentally in the control of a 3 degrees of freedom (3-DOF) helicopter system. The objective is to drive the helicopter to a desired elevation and travel angles while keeping the stability of the pitch.

Controller design for helicopter systems has been a topic of active research in recent years due to their important potential applications. The 3-DOF helicopter prototype is often the system used in helicopter research and education for the design and implementation of control concepts. In our study, we consider the 3-DOF helicopter laboratory produced by Quanser Consulting Inc. [14]. Because this helicopter has nonlinear and unstable dynamics as well as significant cross-coupling between its control channels, the control of this MIMO system is a challenging task. Many researchers have investigated the control of 3-DOF helicopters [15–27]. In [15], Fradkov et al. presented a PID control law for a 3-DOF helicopter with a scheme for state estimation. In [16], Rios et al. developed a PID controller with a sliding-mode observer used to compensate and identify the disturbance. Rios et al. also proposed in [17] a control structure based on the PID controller combined with a quasicontinuous controller. In [18], Liu et al. proposed an optimal tracking control strategy based on fuzzy logic and a linear quadratic regulator (LQR). In [19], Hao et al. suggested a robust LQR attitude control method consisting of 3 parts: a nominal feedforward controller, a nominal LQR state feedback controller, and a robust compensator. Kiefer et al. [20] proposed a control scheme to ensure the trajectory tracking of a 3-DOF helicopter under input and state constraints. It consisted of an inversion-based feedforward controller for trajectory tracking and a feedback controller for the trajectory error dynamics. In [21], Kutay et al. introduced an adaptive output feedback control method based on model inversion with feedback linearization and linearly parameterized neural networks to cancel modeling errors. Other control approaches can be found in the literature, such as fuzzy logic control [22], fuzzy-sliding mode control [23], predictive control [24], H_∞ control [25], neural networks control [26], and adaptive control [27].

This paper is organized as follows. Section 2 presents the control problem formulation and control objectives. The proposed model-free control scheme and the used state observer are developed in Section 3, with the stability analysis of the overall system. In Section 4, the proposed control scheme is used to control a 3-DOF helicopter in real time. Section 5 concludes the paper.

2. Problem formulation

Consider the MIMO nonlinear dynamic systems Σ composed of q interconnected subsystems $\Sigma_i,\ i = 1,\ldots,q$, represented in the following normal form:

$$\sum\nolimits_i \begin{cases} \dot{x}_{i1} = x_{i2}, \dot{x}_{i2} = x_{i3}, \ldots, \dot{x}_{ir_i-1} = x_{ir_i} \\ \dot{x}_{ir_i} = f_i(\mathbf{x}, u_i) + d_i(t) \\ y_i = x_{i1} \end{cases}, \tag{1}$$

where $\mathbf{x} = \left[x_{11}, x_{12}, \ldots, x_{1r_1}, \ldots, x_{q1}, x_{q2}, \ldots, x_{qr_q}\right]^{\mathrm{T}} \in R^n$, is the overall state vector; r_i, $i = 1, \ldots, q$, is the relative order of the subsystem Σ_i with $n = \sum_{i=1}^{q} r_i$; $\mathbf{u} = [u_1, \ldots, u_q]^{\mathrm{T}} \in R^q$ is the control input vector; $\mathbf{y} = [y_1, \ldots, y_q]^{\mathrm{T}} \in R^q$ is the output vector; $f_i(\mathbf{x}, u_i)$, $i = 1, \ldots, q$, are smooth unknown nonlinear functions; and $\mathbf{d} = [d_1, \ldots, d_q]^{\mathrm{T}} \in R^q$ is the external disturbances vector of the system.

Differentiating each y_i with respect to time for r_i times, i.e. until the input u_i appears, one obtains the input-output form of Eq. (1) as follows:

$$y_i^{(r_i)} = f_i(\mathbf{x}, u_i) + d_i(t), \quad i = 1, \ldots, q. \tag{2}$$

In order to design a control law for the system in Eq. (2), we rewrite this system as follows:

$$y_i^{(r_i)} = \alpha_i\, u_i + \Delta_i(\mathbf{x}, u_i, t), \quad i = 1, \ldots, q, \tag{3}$$

where α_i is a given design parameter, and $\Delta_i(\mathbf{x}, u_i, t)$ includes the effects of uncertainties and external disturbances and it is given by:

$$\Delta_i(\mathbf{x}, u_i, t) = -\alpha_i\, u_i + f_i(\mathbf{x}, u_i) + d_i(t), \quad i = 1, \ldots, q. \tag{4}$$

We rewrite Eq. (3) in the following form:

$$\begin{cases} \dot{\mathbf{x}}_i = A_i\, \mathbf{x}_i + B_i\left(\Delta_i(\mathbf{x}, u_i, t) + \alpha_i\, u_i\right) \\ y_i = C_i^{\mathrm{T}} \mathbf{x}_i \end{cases}, \quad i = 1, \ldots, q, \tag{5}$$

where $\mathbf{x}_i = [x_{i1}, x_{i2}, \ldots, x_{ir_i}]^{\mathrm{T}} \in R^{r_i}$ and

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{r_i \times r_i}, \quad B_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{r_i \times 1}, \quad C_i^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}_{1 \times r_i}.$$

Let us denote:

$$\Delta(\mathbf{x}, \mathbf{u}, t) = \left[\Delta_1(\mathbf{x}, u_1, t), \ldots, \Delta_q(\mathbf{x}, u_q, t)\right]^{\mathrm{T}}, \quad \alpha = diag\left[\alpha_1, \ldots, \alpha_q\right], \quad A = diag\left[A_1, \ldots, A_q\right],$$

$$B = diag\left[B_1, \ldots, B_q\right], \quad C^{\mathrm{T}} = diag\left[C_1^{\mathrm{T}}, \ldots, C_q^{\mathrm{T}}\right],$$

and the overall dynamic system, in state space, can be rewritten as:

$$\begin{cases} \dot{\mathbf{x}} = A\, \mathbf{x} + B\left(\Delta(\mathbf{x}, \mathbf{u}, t) + \alpha\, \mathbf{u}\right) \\ \mathbf{y} = C^{\mathrm{T}} \mathbf{x} \end{cases}. \tag{6}$$

In this section, our goal is to design a control law $\mathbf{u}(\mathbf{t})$ that ensures the boundedness of all of the variables in the closed-loop system and guarantees output tracking of a specified desired trajectory $y_d(t) = [y_{d1}(t), \ldots, y_{dq}(t)]^{\mathrm{T}}$. Let us define $\mathbf{x}_d = \left[y_{d1}(t), y_{d1}^{(1)}(t), \ldots, y_{d1}^{(r_1-1)}(t), \ldots, y_{dq}(t), y_{dq}^{(1)}(t), \ldots, y_{d1}^{(r_q-1)}(t)\right]^{\mathrm{T}} \in R^n$ as the desired overall state vector and $\mathrm{E} = \mathbf{x}_d - \mathbf{x} = [\mathrm{E}_1, \ldots, \mathrm{E}_q]^{\mathrm{T}} \in R^n$ as the overall output tracking error vector, where $\mathrm{E}_i = \left[y_{di}(t) - y_i(t), \ldots, y_{di}^{(r_i-1)}(t) - y_i^{(r_i-1)}(t)\right]^{\mathrm{T}} = \left[e_i(t), \ldots, e_i^{(r_i-1)}(t)\right]^{\mathrm{T}} \in R^{r_i}$ is the output tracking error vector of the subsystem Σ_i, $i = 1, \ldots, q$. Next, from Eq. (3), we get:

$$\dot{\mathbf{E}}_i = A_i\,\mathbf{E}_i + B_i\left(y_{di}^{(r_i)} - \alpha_i\,u_i - \Delta_i(\mathbf{x}, u_i, t)\right), \quad i = 1, \ldots, q. \tag{7}$$

Denoting $y_d^{(r)} = \left[y_{d1}^{(r_1)}, \ldots, y_{dq}^{(r_q)}\right]^{\mathrm{T}}$, the closed-loop error dynamic of the overall system Σ can be written as:

$$\dot{\mathbf{E}} = A\,\mathbf{E} + B\left(y_d^{(r)} - \alpha\,\mathbf{u} - \Delta(\mathbf{x}, \mathbf{u}, t)\right). \tag{8}$$

Regarding the development of the control law, the following assumptions should be made.

Assumption 1 *The partial derivative* $\frac{\partial f_i(\mathbf{x}, u_i)}{\partial u_i}$, $i = 1, \ldots, q$, *is nonzero and the constant* α_i *fulfills the condition* $\left|\frac{1}{\alpha_i}\frac{\partial f_i(\mathbf{x}, u_i)}{\partial u_i} - 1\right| < 1$.

Assumption 2 *The desired trajectory* $y_{di}(t)$, $i = 1, \ldots, q$, *is a known bounded function of time with bounded known derivatives, and* $y_{di}(t)$ *is assumed to be* r_i *times differentiable.*

Remark 1 We point out that the objective is to find u_i to cancel $\Delta_i(\mathbf{x}, u_i, t)$ that includes u_i, which constitutes a fixed-point problem. Assumption 1 guarantees that the map $u_i \mapsto \Delta_i(\mathbf{x}, u_i, t)$ is a contraction and thus guarantees the existence of a solution to the control problem. Moreover, from Assumption 1, α_i and $\frac{\partial f_i(\mathbf{x}, u_i)}{\partial u_i}$ should be of the same sign and $|\alpha_i| > \frac{1}{2}\left|\frac{\partial f_i(\mathbf{x}, u_i)}{\partial u_i}\right|$.

If the function $f_i(\mathbf{x}, u_i)$ and $d_i(t)$, $i = 1, \ldots, q$, are known, i.e. $\Delta(\mathbf{x}, \mathbf{u}, t)$ is known, to achieve the control objectives, one can use the following nonlinear control law $\mathbf{u}(\mathbf{t}) = [u_1, \ldots, u_q]^{\mathrm{T}}$, such that:

$$u_i = \frac{1}{\alpha_i}\left[y_{di}^{(r_i)} + K_{ci}^{\mathrm{T}}\mathbf{E}_i - \Delta_i(\mathbf{x}, u_i, t)\right], \quad i = 1, \ldots, q, \tag{9}$$

where $K_{ci} = \left[k_{ci}^{r_i}, \ldots, k_{ci}^{1}\right]^{\mathrm{T}} \in R^{r_i}$ is the feedback gain vector chosen, such that the matrix $\left(A_i - B_i K_{ci}^{\mathrm{T}}\right)$ is Hurwitz. Substituting Eq. (9) into Eq. (7), we have:

$$\dot{\mathbf{E}}_i = \left(A_i - B_i K_{ci}^{\mathrm{T}}\right)\mathbf{E}_i, \quad i = 1, \ldots, q. \tag{10}$$

Denoting $K_c^{\mathrm{T}} = diag\left[K_{c1}^{\mathrm{T}}, \ldots, K_{cq}^{\mathrm{T}}\right]$, the overall closed-loop error dynamics can be written as:

$$\dot{\mathbf{E}} = \left(A - BK_c^{\mathrm{T}}\right)\mathbf{E}. \tag{11}$$

Next, when we select the control law as in Eq. (9), the closed-loop system will be asymptotically stable, which implies that $\lim_{t\to\infty} \mathbf{E}(t) = 0$. Hence, each subsystem output $y_i(t)$ converges asymptotically to the reference output $y_{di}(t)$, i.e. $\lim_{t\to\infty} e_i(t) = 0$, where $e_i(t) = y_{di}(t) - y_i(t)$.

According to the above analysis, the control law in Eq. (9) is easily obtained if $\Delta_i(\mathbf{x}, u_i, t)$, $i = 1, \ldots, q$, are known. Unfortunately, in this work, the nonlinear functions $f_i(\mathbf{x}, u_i)$, $i = 1, \ldots, q$, and the external disturbances $d_i(t)$ are assumed unknown, i.e. $\Delta_i(\mathbf{x}, u_i, t)$ is unknown, so the controller in Eq. (9) cannot be implemented. In order to overcome this problem, we design in the following section an estimator for the unknown function $\Delta_i(\mathbf{x}, u_i, t)$.

3. Proposed model-free controller

3.1. Control design with measurement states

In the preceding section, we established that there exists a control law, given by Eq. (9), that can achieve control objectives. However, this controller cannot be used since it depends on unknown function $\Delta_i(\mathbf{x}, u_i, t)$. In this section, we propose to use an estimator for approximating the unknown term $\Delta_i(\mathbf{x}, \mathbf{u}, t)$, and the estimated term $\hat{\Delta}_i(t)$ will be used to compute the control law.

When we use the estimated term $\hat{\Delta}_i(t)$ instead of $\Delta_i(\mathbf{x}, u_i, t)$, the control law in Eq. (9) can be rewritten as:

$$u_i = \frac{1}{\alpha_i}\left[y_{di}^{(r_i)} + K_{ci}^{\mathrm{T}}\mathbf{E}_i - \hat{\Delta}_i(t)\right], \quad i = 1, \ldots, q. \tag{12}$$

Substituting Eq. (12) into Eq. (7), we have:

$$\dot{\mathbf{E}}_i = \left(A_i - B_i K_{ci}^{\mathrm{T}}\right)\mathbf{E}_i + B_i\tilde{\Delta}_i(t), \quad i = 1, \ldots, q, \tag{13}$$

where $\tilde{\Delta}_i(t) = \hat{\Delta}_i(t) - \Delta_i(\mathbf{x}, \mathbf{u}, t)$, $i = 1, \ldots, q$. Denoting $\hat{\Delta}(t) = \left[\hat{\Delta}_1(t), \ldots, \hat{\Delta}_q(t)\right]^{\mathrm{T}}$ and $\tilde{\Delta}(t) = \hat{\Delta}(t) - \Delta(\mathbf{x}, \mathbf{u}, t)$, the closed-loop dynamics of the overall system can be written as:

$$\dot{\mathbf{E}} = \left(A - BK^{\mathrm{T}}\right)\mathbf{E} + B\tilde{\Delta}(t). \tag{14}$$

Assumption 3 *For the given positive-definite matrix Q_1, there exists a positive solution P_1 for the following matrix equation:*

$$\left(A - BK_c^{\mathrm{T}}\right)^{\mathrm{T}} P_1 + P_1\left(A - BK_c^{\mathrm{T}}\right) + 4\varepsilon_\Delta P_1^{\mathrm{T}} BB^{\mathrm{T}} P_1 = -Q_1, \tag{15}$$

where ε_Δ is a small positive design parameter.

Assumption 4 *The lumped uncertainty $\Delta(\mathbf{x}, \mathbf{u}, t)$ is a continuous function with a bounded time derivative $\dot{\Delta}(\mathbf{x}, \mathbf{u}, t)$ that satisfies $0 \le \left|\dot{\Delta}(\mathbf{x}, \mathbf{u}, t)\right|^2 \le \psi_0 + \mathbf{E}^{\mathrm{T}} P_1^{\mathrm{T}} BB^{\mathrm{T}} P_1 \mathbf{E}$, where ψ_0 is an unknown positive constant and P_1 is the matrix defined in Eq. (15).*

In order to achieve the control objectives, we develop an estimation law for $\hat{\Delta}(t)$. To this end, let us define the following Lyapunov function:

$$V = \frac{1}{2}\mathbf{E}^{\mathrm{T}} P_1 \mathbf{E} + \frac{1}{2}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}. \tag{16}$$

The time derivative of V is:

$$\dot{V}=\frac{1}{2}\dot{\mathbf{E}}^{\mathrm{T}}P_1\mathbf{E}+\frac{1}{2}\mathbf{E}^{\mathrm{T}}P_1\dot{\mathbf{E}}+\tilde{\Delta}^{\mathrm{T}}\dot{\tilde{\Delta}}. \tag{17}$$

Using Eq. (14), the above equation becomes:

$$\dot{V}=\frac{1}{2}\mathbf{E}^{\mathrm{T}}\left(A-BK_c^{\mathrm{T}}\right)^{\mathrm{T}}P_1\mathbf{E}+\frac{1}{2}\mathbf{E}^{\mathrm{T}}P_1\left(A-BK_c^{\mathrm{T}}\right)\mathbf{E}+\frac{1}{2}\tilde{\Delta}^{\mathrm{T}}B^{\mathrm{T}}P_1\mathbf{E}+\frac{1}{2}\mathbf{E}^{\mathrm{T}}P_1B\tilde{\Delta}+\tilde{\Delta}^{\mathrm{T}}\dot{\tilde{\Delta}} \tag{18}$$

or

$$\dot{V}=\frac{1}{2}\mathbf{E}^{\mathrm{T}}\left(\left(A-BK_c^{\mathrm{T}}\right)^{\mathrm{T}}P_1+P_1\left(A-BK_c^{\mathrm{T}}\right)\right)\mathbf{E}+\frac{1}{2}\tilde{\Delta}^{\mathrm{T}}B^{\mathrm{T}}P_1\mathbf{E}+\frac{1}{2}\mathbf{E}^{\mathrm{T}}P_1B\tilde{\Delta}+\tilde{\Delta}^{\mathrm{T}}\dot{\hat{\Delta}}-\tilde{\Delta}^{\mathrm{T}}\dot{\Delta}. \tag{19}$$

As an estimation law for $\hat{\Delta}(t)$, we use the following first-order differential equation:

$$\dot{\hat{\Delta}}=-\frac{1}{\varepsilon_\Delta}\tilde{\Delta}=-\frac{1}{\varepsilon_\Delta}\hat{\Delta}+\frac{1}{\varepsilon_\Delta}\Delta. \tag{20}$$

With Eq. (20), Eq. (19) becomes:

$$\begin{aligned}\dot{V}=&\tfrac{1}{2}\mathbf{E}^{\mathrm{T}}\left(\left(A-BK_c^{\mathrm{T}}\right)^{\mathrm{T}}P_1+P_1\left(A-BK_c^{\mathrm{T}}\right)\right)\mathbf{E}\\&+\tfrac{1}{2}\tilde{\Delta}^{\mathrm{T}}B^{\mathrm{T}}P_1\mathbf{E}+\tfrac{1}{2}\mathbf{E}^{\mathrm{T}}P_1B\tilde{\Delta}-\tfrac{1}{\varepsilon_\Delta}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}-\tilde{\Delta}^{\mathrm{T}}\dot{\Delta}\end{aligned}. \tag{21}$$

Using the inequality

$$\frac{1}{2}\tilde{\Delta}^{\mathrm{T}}B^{\mathrm{T}}P_1\mathbf{E}+\frac{1}{2}\mathbf{E}^{\mathrm{T}}P_1B\tilde{\Delta}\leq\frac{1}{4\varepsilon_\Delta}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}+\varepsilon_\Delta\mathbf{E}^{\mathrm{T}}P_1^{\mathrm{T}}BB^{\mathrm{T}}P_1\mathbf{E}, \tag{22}$$

Eq. (21) can be bounded as:

$$\dot{V}\leq\frac{1}{2}\mathbf{E}^{\mathrm{T}}\left(\left(A-BK_c^{\mathrm{T}}\right)^{\mathrm{T}}P_1+P_1\left(A-BK_c^{\mathrm{T}}\right)+2\varepsilon_\Delta P_1^{\mathrm{T}}BB^{\mathrm{T}}P_1\right)\mathbf{E}-\frac{3}{4\varepsilon_\Delta}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}-\tilde{\Delta}^{\mathrm{T}}\dot{\Delta}, \tag{23}$$

and with the following inequality:

$$\left|\tilde{\Delta}^{\mathrm{T}}\dot{\Delta}\right|\leq\varepsilon_\Delta\left\|\dot{\Delta}\right\|^2+\frac{1}{4\varepsilon_\Delta}\left\|\tilde{\Delta}\right\|^2\leq\frac{1}{4\varepsilon_\Delta}\left\|\tilde{\Delta}\right\|^2+\varepsilon_\Delta\left(\psi_0+\mathbf{E}^{\mathrm{T}}P_1^{\mathrm{T}}BB^{\mathrm{T}}P_1\mathbf{E}\right), \tag{24}$$

the inequality in Eq. (23) can be rewritten as:

$$\dot{V}\leq\frac{1}{2}\mathbf{E}^{\mathrm{T}}\left(\left(A-BK_c^{\mathrm{T}}\right)^{\mathrm{T}}P_1+P_1\left(A-BK_c^{\mathrm{T}}\right)+4\varepsilon_\Delta P_1^{\mathrm{T}}BB^{\mathrm{T}}P_1\right)\mathbf{E}-\frac{1}{2\varepsilon_\Delta}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}+\varepsilon_\Delta\psi_0. \tag{25}$$

From Eq. (15), we have:

$$\dot{V}\leq-\frac{1}{2}\mathbf{E}^{\mathrm{T}}Q_1\mathbf{E}-\frac{1}{2\varepsilon_\Delta}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}+\varepsilon_\Delta\psi_0. \tag{26}$$

Using Eq. (26) we can obtain the following inequality:

$$\dot{V}\leq-\frac{\lambda_{\min}\left(Q_1\right)}{\lambda_{\max}\left(P_1\right)}\frac{1}{2}\lambda_{\max}\left(P_1\right)\|\mathbf{E}\|^2-\frac{1}{\varepsilon_\Delta}\frac{1}{2}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}+\varepsilon_\Delta\psi_0. \tag{27}$$

Choosing $\gamma = \min\left(\frac{\lambda_{\min}(Q_1)}{\lambda_{\max}(P_1)}, \frac{1}{\varepsilon_\Delta}\right)$, Eq. (27) can be rewritten as:

$$\dot{V} \leq -\frac{\gamma}{2}\lambda_{\max}\left(P_1\right)\left\|\mathbf{E}\right\|^2 - \frac{\gamma}{2}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta} + \varepsilon_\Delta\psi_0, \tag{28}$$

and with Eq. (16), we have:

$$\dot{V} \leq -\gamma V + \varepsilon_\Delta\psi_0. \tag{29}$$

Now we can prove the following theorem, which shows the boundedness of all of the variables in the closed-loop system.

Theorem 1 *Consider system Eq. (1). Suppose that Assumptions 1–4 are satisfied. Next, the control law defined by Eq. (12) guarantees that the closed-loop system is uniformly ultimately bounded (UUB) stable and that the output tracking error converges to a small neighborhood of the origin.*

Proof From Eq. (29) we have:

$$0 \leq V\left(t\right) \leq \frac{\varepsilon_\Delta\psi_0}{\gamma} + \left(V\left(0\right) - \frac{\varepsilon_\Delta\psi_0}{\gamma}\right)e^{-\gamma t}_{.} \tag{30}$$

This implies that signals $\mathbf{E}\left(t\right)$ and $\tilde{\Delta}\left(t\right)$ in the closed-loop system are UUB. Moreover, in order to achieve the tracking error $\mathbf{E}\left(t\right)$ convergence to a small neighborhood around zero, the parameter ε_Δ should be chosen small enough. This completes the proof. □

Let us consider again the estimation law in Eq. (20), which can be rewritten in the form of:

$$\hat{\Delta}_i\left(s\right) = \frac{1}{1+\varepsilon_\Delta s}\Delta_i\left(s\right), \quad i = 1,\ldots,q, \tag{31}$$

where s is the Laplace variable. From Eq. (31), we conclude that $\hat{\Delta}_i$ is obtained by filtering Δ_i using the filter $\frac{1}{1+\varepsilon_\Delta s}$, and we have $\hat{\Delta}_i\left(t\right) \to \Delta_i\left(t\right)$ when $\varepsilon_\Delta \to 0$. Unfortunately, the function Δ is unknown. In order to overcome this difficulty, Eq. (3) is used to compute $\Delta_i\left(t\right)$ as follows:

$$\Delta_i\left(t\right) = \dot{x}_{ir_i} - \alpha_i u_i. \tag{32}$$

Thus, Eq. (31) becomes:

$$\hat{\Delta}_i\left(s\right) = \frac{1}{1+\varepsilon_\Delta s}\left(\dot{x}_{ir_i}\left(s\right) - \alpha_i u_i\left(s\right)\right), \quad i = 1,\ldots,q. \tag{33}$$

Substituting $\hat{\Delta}_i$ into the control law Eq. (12), we have:

$$u_i\left(s\right) = \frac{1}{\alpha_i}\left[y_{di}^{(r_i)}\left(s\right) + K_{ci}^{\mathrm{T}}\mathbf{E}_i\left(s\right) - \frac{1}{1+\varepsilon_\Delta s}\left(\dot{x}_{ir_i}\left(s\right) - \alpha_i u_i\left(s\right)\right)\right], \quad i = 1,\ldots,q \tag{34}$$

or

$$u_i\left(s\right) = \frac{1}{\alpha_i}\left(1 + \frac{1}{\varepsilon_\Delta s}\right)\left(y_{di}^{(r_i)}\left(s\right) + K_{ci}^{\mathrm{T}}\mathbf{E}_i\left(s\right)\right) - \frac{1}{\alpha_i}\frac{\dot{x}_{ir_i}\left(s\right)}{\varepsilon_\Delta s}, \quad i = 1,\ldots,q. \tag{35}$$

Finally, the control law $u_i(s)$, $i = 1, \ldots, q$ becomes:

$$u_i(s) = \frac{1}{\alpha_i}\left(y_{di}^{(r_i)}(t) + K_{ci}^{\mathrm{T}}\mathbf{E}_i(t)\right) + \frac{1}{\alpha_i \varepsilon_\Delta}\int_0^t \left(y_{di}^{(r_i)}(\tau) + K_{ci}^{\mathrm{T}}\mathbf{E}_i(\tau)\right) d\tau - \frac{1}{\alpha_i \varepsilon_\Delta} x_{ir_i}(t). \tag{36}$$

The control law in Eq. (36) is established assuming that the overall state vector is available for measurement. In the next subsection, we develop a model-free control law when the states are not available.

3.2. Model-free control design with an observer

It is well known, unfortunately, that the state vector $\mathbf{x}_i$ of the controlled system is not always available. In order to overcome this problem, a linear state observer is used to estimate the unavailable state variables.

Let us define an observer of system Eq. (5), as follows:

$$\begin{cases} \dot{\hat{\mathbf{x}}}_i = A_i\,\hat{\mathbf{x}}_i + B_i\left(\hat{\Delta}_i + \alpha_i\,u_i\right) + K_{0i}\left(y_i - \hat{y}_i\right) \\ \hat{y}_i = C_i^{\mathrm{T}}\hat{\mathbf{x}}_i \end{cases}. \tag{37}$$

where $\hat{\mathbf{x}}_i$ is the estimate of $\mathbf{x}_i$, and K_{0i} is the observer gain matrix chosen such that the matrix $A_i - K_{0i}C_i^{\mathrm{T}}$ is Hurwitz. Denoting $\hat{e}_i = y_{di}(t) - \hat{y}_i(t)$ and $\hat{\mathrm{E}}_i = \left[\hat{e}_i(t), \ldots, \hat{e}_i^{(r_i-1)}(t)\right]^{\mathrm{T}} \in R^{r_i}$, using Eq. (37), one obtains:

$$\dot{\hat{\mathbf{E}}}_i = A_i\,\hat{\mathbf{E}}_i + B_i\left(y_{di}^{(r_i)} - \alpha_i\,u_i - \hat{\Delta}_i\right) + K_{0i}C_i^{\mathrm{T}}\left(\mathbf{E}_i - \hat{\mathbf{E}}_i\right). \tag{38}$$

In this case, we design the control law as follows:

$$u_i = \frac{1}{\alpha_i}\left[y_{di}^{(r_i)} + K_{ci}^{\mathrm{T}}\hat{\mathbf{E}}_i - \hat{\Delta}_i\right], \quad i = 1, \ldots, q. \tag{39}$$

Substituting Eq. (39) into Eq. (38), we get:

$$\begin{cases} \dot{\hat{\mathbf{E}}}_i = \left(A_i - B_iK_{ci}^{\mathrm{T}}\right)\hat{\mathbf{E}}_i + K_{0i}\left(e_i - \hat{e}_i\right), \quad i = 1, \ldots, q \\ \hat{e}_i = C_i^{\mathrm{T}}\hat{\mathbf{E}}_i \end{cases}. \tag{40}$$

Substituting Eq. (39) into Eq. (7), the closed-loop dynamic system can be written as:

$$\begin{cases} \dot{\mathbf{E}}_i = A_i\mathbf{E}_i - B_iK_{ci}^{\mathrm{T}}\hat{\mathbf{E}}_i + B_i\tilde{\Delta}_i, \quad i = 1, \ldots, q \\ e_i = C_i^{\mathrm{T}}\mathbf{E}_i \end{cases}. \tag{41}$$

Define the observation error $\tilde{e}_i = e_i - \hat{e}_i$, $\tilde{\mathrm{E}}_i = \mathrm{E}_i - \hat{\mathrm{E}}_i = \left[\tilde{e}_i(t), \ldots, \tilde{e}_i^{(r_i-1)}(t)\right]^{\mathrm{T}} \in R^{r_i}$. Subtracting Eq. (40) from Eq. (41) yields:

$$\begin{cases} \dot{\tilde{\mathbf{E}}}_i = \left(A_i - K_{0i}C_i^{\mathrm{T}}\right)\tilde{\mathbf{E}}_i + B_i\tilde{\Delta}_i, \quad i = 1, \ldots, q \\ \tilde{e}_i = C_i^{\mathrm{T}}\tilde{\mathbf{E}}_i \end{cases}. \tag{42}$$

Denote $\hat{\mathrm{E}} = \left[\hat{\mathrm{E}}_1, \ldots, \hat{\mathrm{E}}_p\right]^{\mathrm{T}} \in R^n$, $\tilde{\mathrm{E}} = \left[\tilde{\mathrm{E}}_1, \ldots, \tilde{\mathrm{E}}_p\right]^{\mathrm{T}} \in R^n$, $\hat{\mathbf{e}} = [\hat{e}_1, \ldots, \hat{e}_p]^{\mathrm{T}} \in R^p$, $\tilde{\mathbf{e}} = [\tilde{e}_1, \ldots, \tilde{e}_p]^{\mathrm{T}} \in R^p$, and $K_0 = diag\,[K_{01}, \ldots, K_{0p}]$. Next, the overall error observer becomes:

$$\begin{array}{l} \dot{\hat{\mathbf{E}}} = \left(A - BK_c^{\mathrm{T}}\right)\hat{\mathbf{E}} + K_0\left(\mathbf{e} - \hat{\mathbf{e}}\right) \\ \hat{\mathbf{e}} = C^{\mathrm{T}}\hat{\mathbf{E}} \end{array}, \tag{43}$$

and the overall observation error dynamics is given by:

$$\begin{array}{l} \dot{\tilde{\mathbf{E}}} = \left(A - K_0C^{\mathrm{T}}\right)\tilde{\mathbf{E}} + B\tilde{\Delta} \\ \tilde{\mathbf{e}} = C^{\mathrm{T}}\tilde{\mathbf{E}} \end{array}. \tag{44}$$

Assumption 5 *For the given positive-definite matrices $\bar{Q}_1$ and $\bar{Q}_2$, there exist positive solutions $\bar{P}_1$ and $\bar{P}_2$ for the following matrix equations:*

$$\left(A - K_0C^{\mathrm{T}}\right)^{\mathrm{T}}\bar{P}_1 + \bar{P}_1\left(A - K_0C^{\mathrm{T}}\right) + 2\varepsilon_{\Delta}\bar{P}_1^{\mathrm{T}}BB^{\mathrm{T}}\bar{P}_1 + \varepsilon_{\Delta}^2 I_{n\times n} = -\bar{Q}_1, \tag{45}$$

$$\left(A - BK_c^{\mathrm{T}}\right)^{\mathrm{T}}\bar{P}_2 + \bar{P}_2\left(A - BK_c^{\mathrm{T}}\right) + \frac{1}{\varepsilon_{\Delta}^2}\bar{P}_2K_0K_0^{\mathrm{T}}\bar{P}_2 = -\bar{Q}_2. \tag{46}$$

Assumption 6 *The time derivative $\dot{\Delta}(\mathbf{x}, \mathbf{u}, t)$ of the function $\Delta(\mathbf{x}, \mathbf{u}, t)$ fulfills $0 \le \left|\dot{\Delta}(\mathbf{x}, \mathbf{u}, t)\right|^2 \le \bar{\psi}_0 + \tilde{\mathbf{E}}^{\mathrm{T}}\bar{P}_1^{\mathrm{T}}BB^{\mathrm{T}}\bar{P}_1\tilde{\mathbf{E}}$, where $\bar{\psi}_0$ is an unknown positive constant and $\bar{P}_1$ is the matrix defined by Eq. (45).*

Consider the Lyapunov function candidate:

$$V = \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}}\bar{P}_2\hat{\mathbf{E}} + \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\bar{P}_1\tilde{\mathbf{E}} + \frac{1}{2}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta}. \tag{47}$$

The time derivative of V is:

$$\dot{V} = \frac{1}{2}\dot{\hat{\mathbf{E}}}^{\mathrm{T}}\bar{P}_2\hat{\mathbf{E}} + \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}}\bar{P}_2\dot{\hat{\mathbf{E}}} + \frac{1}{2}\dot{\tilde{\mathbf{E}}}^{\mathrm{T}}\bar{P}_1\tilde{\mathbf{E}} + \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\bar{P}_1\dot{\tilde{\mathbf{E}}} + \tilde{\Delta}^{\mathrm{T}}\dot{\tilde{\Delta}}. \tag{48}$$

From Eqs. (43) and (44), Eq. (48) becomes:

$$\begin{array}{l} \dot{V} = \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\left(\left(A - K_0C^{\mathrm{T}}\right)^{\mathrm{T}}\bar{P}_1 + \bar{P}_1\left(A - K_0C^{\mathrm{T}}\right)\right)\tilde{\mathbf{E}} + \frac{1}{2}\tilde{\Delta}B^{\mathrm{T}}\bar{P}_1\tilde{\mathbf{E}} + \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\bar{P}_1B\tilde{\Delta} + \tilde{\Delta}^{\mathrm{T}}\dot{\hat{\Delta}} - \tilde{\Delta}^{\mathrm{T}}\dot{\Delta} \\ \quad + \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}}\left(\left(A - BK_c^{\mathrm{T}}\right)^{\mathrm{T}}\bar{P}_2 + \bar{P}_2\left(A - BK_c^{\mathrm{T}}\right)\right)\hat{\mathbf{E}} + \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}CK_0^{\mathrm{T}}\bar{P}_2\hat{\mathbf{E}} + \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}}\bar{P}_2K_0C^{\mathrm{T}}\tilde{\mathbf{E}} \end{array}. \tag{49}$$

Choosing the estimation law for $\hat{\Delta}(t)$ as in Eq. (20), we have:

$$\begin{array}{l} \dot{V} = \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\left(\left(A - K_0C^{\mathrm{T}}\right)^{\mathrm{T}}\bar{P}_1 + \bar{P}_1\left(A - K_0C^{\mathrm{T}}\right)\right)\tilde{\mathbf{E}} + \frac{1}{2}\tilde{\Delta}B^{\mathrm{T}}\bar{P}_1\tilde{\mathbf{E}} + \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\bar{P}_1B\tilde{\Delta} - \frac{1}{\varepsilon_{\Delta}}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta} \\ \quad - \tilde{\Delta}^{\mathrm{T}}\dot{\Delta} + \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}}\left(\left(A - BK_c^{\mathrm{T}}\right)^{\mathrm{T}}\bar{P}_2 + \bar{P}_2\left(A - BK_c^{\mathrm{T}}\right)\right)\hat{\mathbf{E}} + \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}CK_0^{\mathrm{T}}\bar{P}_2\hat{\mathbf{E}} + \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}}\bar{P}_2K_0C^{\mathrm{T}}\tilde{\mathbf{E}} \end{array}. \tag{50}$$

Using the following inequalities,

$$\frac{1}{2}\tilde{\Delta}B^{\mathrm{T}}\bar{P}_1\tilde{\mathbf{E}} + \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\bar{P}_1B\tilde{\Delta} \le \frac{1}{4\varepsilon_{\Delta}}\tilde{\Delta}^2 + \varepsilon_{\Delta}\tilde{\mathbf{E}}^{\mathrm{T}}\bar{P}_1^{\mathrm{T}}BB^{\mathrm{T}}\bar{P}_1\tilde{\mathbf{E}}, \tag{51}$$

$$\frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}} C K_0^{\mathrm{T}} \bar{P}_2 \hat{\mathbf{E}} + \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}} \bar{P}_2 K_0 C^{\mathrm{T}} \tilde{\mathbf{E}} \le \frac{1}{2\varepsilon_\Delta^2}\hat{\mathbf{E}}^{\mathrm{T}} \bar{P}_2 K_0 K_0^{\mathrm{T}} \bar{P}_2 \hat{\mathbf{E}} + \frac{\varepsilon_\Delta^2}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\tilde{\mathbf{E}}, \tag{52}$$

$$\left|\tilde{\Delta}^{\mathrm{T}}\dot{\Delta}\right| \le \varepsilon_\Delta \left\|\dot{\Delta}\right\|^2 + \frac{1}{4\varepsilon_\Delta}\left\|\tilde{\Delta}\right\|^2 \le \frac{1}{4\varepsilon_\Delta}\left\|\tilde{\Delta}\right\|^2 + \varepsilon_\Delta\left(\bar{\psi}_0 + \tilde{\mathbf{E}}^{\mathrm{T}}\bar{P}_1^{\mathrm{T}} B B^{\mathrm{T}} \bar{P}_1 \tilde{\mathbf{E}}\right), \tag{53}$$

Eq. (50) can be upper bounded as:

$$\begin{array}{l} \dot{V} \le \frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\left(\left(A - K_0 C^{\mathrm{T}}\right)^{\mathrm{T}} \bar{P}_1 + \bar{P}_1\left(A - K_0 C^{\mathrm{T}}\right) + 2\varepsilon_\Delta \bar{P}_1^{\mathrm{T}} B B^{\mathrm{T}} \bar{P}_1 + \varepsilon_\Delta^2 I_{n\times n}\right)\tilde{\mathbf{E}} \\ \quad + \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}}\left(\left(A - B K_c^{\mathrm{T}}\right)^{\mathrm{T}} \bar{P}_2 + \bar{P}_2\left(A - B K_c^{\mathrm{T}}\right) + \frac{1}{\varepsilon_\Delta^2}\bar{P}_2 K_0 K_0^{\mathrm{T}} \bar{P}_2\right)\hat{\mathbf{E}} - \frac{1}{2\varepsilon_\Delta}\tilde{\Delta}^2 + \varepsilon_\Delta\bar{\psi}_0 \end{array}. \tag{54}$$

From Eqs. (45) and (46), Eq. (54) becomes:

$$\dot{V} \le -\frac{1}{2}\tilde{\mathbf{E}}^{\mathrm{T}}\bar{Q}_1\tilde{\mathbf{E}} - \frac{1}{2}\hat{\mathbf{E}}^{\mathrm{T}}\bar{Q}_2\hat{\mathbf{E}} - \frac{1}{2\varepsilon_\Delta}\tilde{\Delta}^2 + \varepsilon_\Delta\bar{\psi}_0. \tag{55}$$

Using Eq. (55), we can obtain the following inequality:

$$\dot{V} \le -\frac{\lambda_{\min}\left(\bar{Q}_1\right)}{\lambda_{\max}\left(\bar{P}_1\right)}\frac{1}{2}\lambda_{\max}\left(\bar{P}_1\right)\left\|\tilde{\mathbf{E}}\right\|^2 - \frac{\lambda_{\min}\left(\bar{Q}_2\right)}{\lambda_{\max}\left(\bar{P}_2\right)}\frac{1}{2}\lambda_{\max}\left(\bar{P}_2\right)\left\|\hat{\mathbf{E}}\right\|^2 - \frac{1}{\varepsilon_\Delta}\frac{1}{2}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta} + \varepsilon_\Delta\bar{\psi}_0. \tag{56}$$

Choosing $\bar{\gamma} = \min\left(\frac{\lambda_{\min}\left(\bar{Q}_1\right)}{\lambda_{\max}\left(\bar{P}_1\right)}, \frac{\lambda_{\min}\left(\bar{Q}_2\right)}{\lambda_{\max}\left(\bar{P}_2\right)}, \frac{1}{\varepsilon_\Delta}\right)$, Eq. (56) can be rewritten as:

$$\dot{V} \le -\frac{\bar{\gamma}}{2}\lambda_{\max}\left(\bar{P}_1\right)\left\|\tilde{\mathbf{E}}\right\|^2 - \frac{\bar{\gamma}}{2}\lambda_{\max}\left(\bar{P}_2\right)\left\|\hat{\mathbf{E}}\right\|^2 - \frac{\bar{\gamma}}{2}\tilde{\Delta}^{\mathrm{T}}\tilde{\Delta} + \varepsilon_\Delta\bar{\psi}_0, \tag{57}$$

and with Eq. (47), we have:

$$\dot{V} \le -\bar{\gamma} V + \varepsilon_\Delta\bar{\psi}_0. \tag{58}$$

Now we can prove Theorem 2, which shows the boundedness of all of the variables in the closed-loop system.

Theorem 2 *Consider system Eq. (1). Suppose that Assumptions 1–6 are satisfied. Next, the control law defined by Eq. (39) with the observer in Eq. (40) guarantees that the closed-loop system is UUB stable and that the signals* $\tilde{\mathbf{E}}(t)$ *and* $\hat{\mathbf{E}}(t)$ *converge to a small neighborhood of the origin.*

Proof From Eq. (58), we can have:

$$0 \le V(t) \le \frac{\varepsilon_\Delta\bar{\psi}_0}{\bar{\gamma}} + \left(V(0) - \frac{\varepsilon_\Delta\bar{\psi}_0}{\bar{\gamma}}\right)e^{-\bar{\gamma}t}. \tag{59}$$

Next, it follows that signals $\tilde{\mathbf{E}}(t)$, $\hat{\mathbf{E}}(t)$, and $\tilde{\Delta}(t)$ in the closed-loop system are UUB. Moreover, in order to ensure that $\tilde{\mathbf{E}}(t)$ and $\hat{\mathbf{E}}(t)$ converge to a small neighborhood around zero, the parameter ε_Δ should be chosen sufficiently small. This completes the proof. □

Consider now the estimation law in Eq. (20), which can be rewritten as:

$$\hat{\Delta}_i(s) = -\frac{1}{\varepsilon_\Delta s}\tilde{\Delta}_i(s), \quad i = 1, \ldots, q. \tag{60}$$

Replacing $\hat{\Delta}_i$ by Eq. (60) in the control law in Eq. (39), we obtain:

$$u_i(s) = \frac{1}{\alpha_i}\left[y_{di}^{(r_i)}(s) + K_{ci}^{\mathrm{T}}\hat{\mathbf{E}}_i(s) + \frac{1}{\varepsilon_\Delta s}\tilde{\Delta}_i(s)\right], \quad i = 1,\ldots,q. \tag{61}$$

From Eq. (42), we have $\tilde{e}_i = H(s)\tilde{\Delta}_i$ with $H(s) = C_i^{\mathrm{T}}\left(sI - \left(A_i - K_{0i}C_i^{\mathrm{T}}\right)\right)^{-1}B_i$, and then Eq. (61) becomes:

$$u_i(s) = \frac{1}{\alpha_i}\left[y_{di}^{(r_i)}(s) + K_{ci}^{\mathrm{T}}\hat{\mathbf{E}}_i(s) + \frac{H^{-1}(s)}{\varepsilon_\Delta s}\tilde{e}_i(s)\right], \quad i = 1,\ldots,q. \tag{62}$$

Remark 2 Since $H^{-1}(s)$ in Eq. (62) is not causal, the control law $u_i(s)$ will be filtered, such as $u_{fi}(s) = \frac{1}{(1+w_i s)^{(r_i-1)}}u_i(s), \quad i = 1,\ldots,q$, where w_i are design parameters.

Remark 3 In the case of a second-order MIMO nonlinear dynamic system, i.e. $r_i = 2$, for an observer gain matrix $K_{0i} = diag\left[K_{0i}^1, K_{0i}^2\right]$, the control law $u_i(s)$ can be rewritten as follows:

$$u_i(s) = \frac{1}{\alpha_i}\left[y_{di}^{(r_i)}(s) + K_{ci}^{\mathrm{T}}\hat{\mathbf{E}}_i(s) + \frac{s^2 + K_{0i}^1 s + K_{0i}^2}{\varepsilon_\Delta s}\tilde{e}_i(s)\right], \quad i = 1,\ldots,q, \tag{63}$$

and the filtered control law by:

$$u_{fi}(s) = \frac{1}{(1+w_i s)}u_i(s), \quad i = 1,\ldots,q. \tag{64}$$

Using Eq. (63), Eq. (64) can be written as:

$$u_{fi}(s) = \frac{1}{\alpha_i(1+w_i s)}\left(y_{di}^{(r_i)}(s) + K_{ci}^{\mathrm{T}}\hat{\mathbf{E}}_i(s)\right) + \frac{1}{\alpha_i}\frac{s^2 + K_{0i}^1 s + K_{0i}^2}{\varepsilon_\Delta s(1+w_i s)}\tilde{e}_i(s), \quad i = 1,\ldots,q \tag{65}$$

or

$$\begin{aligned} u_{fi}(s) = {} & \tfrac{1}{\alpha_i(1+w_i s)}\left(y_{di}^{(r_i)}(s) + K_{ci}^{\mathrm{T}}\hat{\mathbf{E}}_i(s)\right) + \tfrac{1}{\alpha_i w_i \varepsilon_\Delta}\tilde{e}_i(s) \\ & + \tfrac{1}{\alpha_i \varepsilon_\Delta}\tfrac{\left(K_{0i}^1 w_i - 1\right)s + w_i K_{0i}^2}{w_i s^2 + s}\tilde{e}_i(s) \end{aligned}. \tag{66}$$

In the next section, we will verify experimentally the effectiveness of the proposed model-free controller in Eq. (66) with the observer in Eq. (43) in control of the 3-DOF helicopter system. The control objective is to drive the helicopter to a desired elevation and travel angles while keeping the stability of the pitch.

4. Application to the 3-DOF helicopter

4.1. System description

The 3-DOF helicopter setup used in our work, which is manufactured by Quanser Consulting Inc., is presented in Figure 1 [14]. It is a platform technology for research in helicopter flight control systems. This setup is an excellent testbed for advanced control methods and it consists of a base on which a long arm is mounted. The arm carries the helicopter body composed of 2 propellers on one end and a counterweight on the other end. Two DC motors are mounted below the propellers to create the forces that drive propellers. The motors' axes are parallel and their thrust is vertical to the propellers. We have 3 degrees of freedom: elevation (ψ), pitch

(θ), and travel (ϕ). To measure these angles, 3 encoders are installed on the elevation axis, pitch axis, and travel axis. The movement range of the elevation ψ and pitch θ angles is limited to between around –1 and 1 rad due to the hardware restriction.

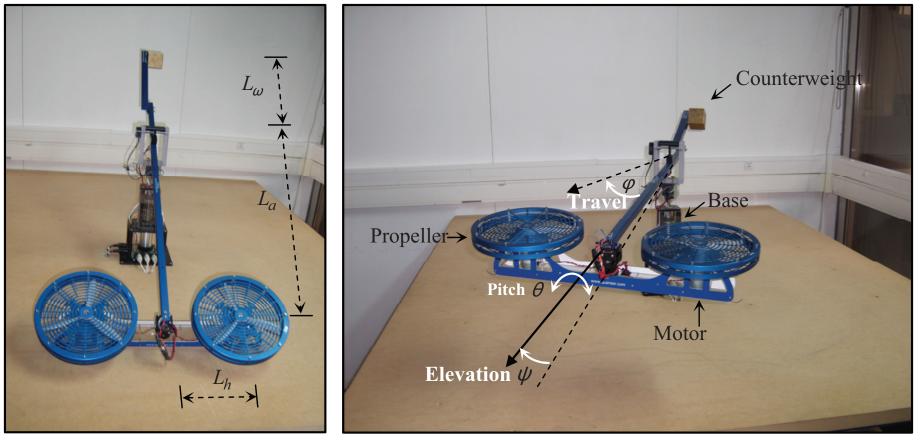

Figure 1. Quanser helicopter with 3-DOF.

In the literature, the dynamic modeling of this system was studied in several papers [22,25,26,28]. The equations of motion about axes ψ, θ, and ϕ are given by Eq. (67).

$$\begin{cases} \ddot{\psi} = (1/J_\psi)\left(-M_h g\cos(\psi)L_a + M_\omega g\cos(\psi)L_\omega + K_f(V_f+V_b)\cos(\theta)L_a - f_\psi\left(\dot{\psi}\right)\right) \\ \quad + d_\psi(t) \\ \ddot{\theta} = (1/J_\theta)\left(K_f(V_f-V_b)L_h - f_\theta\left(\dot{\theta}\right)\right) + d_\theta(t) \\ \ddot{\phi} = (1/J_\phi)\left(K_f(V_f+V_b)\sin(\theta)L_a - f_\phi\left(\dot{\phi}\right)\right) + d_\phi(t) \end{cases} \tag{67}$$

Here, J_ψ, J_θ, and J_ϕ denote the moments of inertia; M_h is the total mass of the helicopter; M_ω is the mass of the counterweight; L_a is the helicopter distance to pivot; L_ω is the counterweight distances to pivot; L_h is the motor distance to the pitch; g is the gravity constant, K_f is the motor volt-to-thrust relationship constant; V_f and V_b are the voltages applied to the front and back motors, respectively; $f_\psi\left(\dot{\psi}\right)$, $f_\theta\left(\dot{\theta}\right)$, and $f_\phi\left(\dot{\phi}\right)$ are the friction terms; and $d_\psi(t)$, $d_\theta(t)$, and $d_\phi(t)$ are the bounded external disturbances. The Table provides the physical parameters of the used helicopter, taken from the Quanser 3-DOF helicopter prototype installed in the IRCCyN laboratory.

By setting $u_1 = (V_f + V_b)$ and $u_2 = (V_f - V_b)$, system Eq. (67) takes the following form.

$$\begin{cases} \ddot{\psi} = (1/J_\psi)\left(-M_h g\cos(\psi)L_a + M_\omega g\cos(\psi)L_\omega - f_\psi\left(\dot{\psi}\right)\right) + (1/J_\psi)(K_f L_a)\cos(\theta)u_1 \\ \quad + d_\psi(t) \\ \ddot{\theta} = (1/J_\theta)\left(-f_\theta\left(\dot{\theta}\right)\right) + (1/J_\theta)(K_f L_h)u_2 + d_\theta(t) \\ \ddot{\phi} = (1/J_\phi)\left(-f_\phi\left(\dot{\phi}\right)\right) + (1/J_\phi)(K_f L_a)\sin(\theta)u_1 + d_\phi(t) \end{cases} \tag{68}$$

In this study, our objective is to ensure the convergence of the elevation and travel angles (ψ,ϕ) to the desired trajectories (ψ_d,ϕ_d) while keeping the stability of the pitch angle (θ). From Eq. (68), it can be seen that the

Table. The 3-DOF helicopter parameter values.

Parameter	Value	Unit
V_f and V_b	$[-24:+24]$	V
K_f	0.1188	N/V
g	9.81	m s^{-2}
M_h	1.426	kg
M_ω	1.87	kg
L_a	0.660	m
L_ω	0.470	m
L_h	0.178	m
J_ψ	1.0348	kg m^2
J_θ	0.0451	kg m^2
J_ϕ	1.0348	kg m^2

travel rotation (ϕ) depends on the control input u_1. Indeed, u_1 is the designed total input vector oriented to obtain the desired elevation and travel angles. Let us define $u_3 = u_1 \sin(\theta)$ as the component of the vector u_1 responsible for the travel rotation. Next, the desired pitch angle θ_d ensuring the suitable travel rotation can be calculated by

$$\theta_d = \arcsin\left(\frac{u_3}{u_1 + \Delta\varepsilon}\right), \tag{69}$$

where $\Delta\varepsilon$ is a small positive constant. The dynamic model in Eq. (68) can be rewritten as follows:

$$\begin{cases} \ddot{\psi} = (1/J_\psi)\left(-M_h g\cos(\psi) L_a + M_\omega g \cos(\psi) L_\omega - f_\psi\left(\dot{\psi}\right)\right) \\ \quad + (1/J_\psi)(K_f \cos(\theta) L_a) u_1 + d_\psi(t) \\ \ddot{\theta} = (1/J_\theta)\left(-f_\theta\left(\dot{\theta}\right)\right) + (1/J_\theta)(K_f L_h) u_2 + d_\theta(t) \\ \ddot{\phi} = (1/J_\phi)\left(-f_\phi\left(\dot{\phi}\right)\right) + (1/J_\phi)(K_f L_a) u_3 + d_\phi(t) \end{cases}. \tag{70}$$

In order to simplify the application of the model-free controller developed in the previous section to the 3-DOF helicopter system, let us define $\mathbf{y} = [\psi, \theta, \phi]$ as the output vector, $\mathbf{u} = [u_1, u_2, u_3]^{\mathrm{T}}$ as the vector of the control inputs and the state space vector by $\mathbf{x} = \left[\psi, \dot{\psi}, \theta, \dot{\theta}, \phi, \dot{\phi}\right]^{\mathrm{T}}$, $\mathbf{d} = [d_\psi, d_\theta, d_\phi]^{\mathrm{T}}$ as the external disturbances vector of the system, and $f_i(\mathbf{x}, \mathbf{u}), \quad i = 1, \ldots, p$ as the nonlinearities of the system given as follows.

$$\begin{aligned} f_1(\mathbf{x},\mathbf{u}) &= (1/J_\psi)\left(-M_h g\cos(\psi) L_a + M_\omega g\cos(\psi) L_\omega - f_\psi\left(\dot{\psi}\right)\right) + (1/J_\psi)(K_f\cos(\theta) L_a) u_1 \\ f_2(\mathbf{x},\mathbf{u}) &= (1/J_\theta)\left(-f_\theta\left(\dot{\theta}\right)\right) + (1/J_\theta)(K_f L_h) u_2 \\ f_3(\mathbf{x},\mathbf{u}) &= (1/J_\phi)\left(-f_\phi\left(\dot{\phi}\right)\right) + (1/J_\phi)(K_f L_a) u_3 \end{aligned}$$

Next, the 3-DOF helicopter system given by Eq. (70) can be expressed as:

$$y_i^{(2)} = f_i(\mathbf{x}, \mathbf{u}) + d_i(t), \quad i = 1, 2, 3, \tag{71}$$

which is in the general input-output form given by Eq. (2), with $r_i = 2$ and $q = 3$ in this case.

4.2. Experiment results

In order to verify experimentally the effectiveness of the proposed model-free controller with an observer developed in this paper, an application to the 3-DOF helicopter system is conducted. Our objective is to ensure the convergence of the elevation and travel angles (ψ, ϕ) to the desired trajectories (ψ_d, ϕ_d). Since the control of the travel rotation requires the pitch rotation control, another controller is used to also ensure the convergence of the angle (θ) to the desired angle (θ_d). In each case, a model-free controller in the form of Eq. (66) with an observer in the form of Eq. (40) is used. In the first step, let us define the tracking errors: $e_\psi = \psi - \psi_d$, $e_\theta = \theta - \theta_d$, and $e_\phi = \phi - \phi_d$. The inputs (u_1, u_2, u_3) are chosen as the outputs of 3 controllers given by

$$\begin{array}{l} u_1(s) = \frac{1}{\alpha_\psi(1+w_\psi s)}\left(\psi_d^{(2)}(s) + K_{c\psi}^{\mathrm{T}}\hat{\mathbf{E}}_\psi(s)\right) + \frac{1}{\alpha_\psi w_\psi \varepsilon_\Delta}\tilde{e}_\psi(s) \\ \quad + \frac{1}{\alpha_\psi \varepsilon_\Delta}\frac{\left(K_{0\psi}^1 w_\psi - 1\right)s + w_\psi K_{0\psi}^2}{w_\psi s^2 + s}\tilde{e}_\psi(s) \end{array}, \tag{72}$$

$$\begin{array}{l} u_2(s) = \frac{1}{\alpha_i(1+w_i s)}\left(y_{di}^{(r_i)}(s) + K_{ci}^{\mathrm{T}}\hat{\mathbf{E}}_i(s)\right) + \frac{1}{\alpha_\theta w_\theta \varepsilon_\Delta}\tilde{e}_\theta(s) \\ \quad + \frac{1}{\alpha_\theta \varepsilon_\Delta}\frac{\left(K_{0\theta}^1 w_\theta - 1\right)s + w_\theta K_{0\theta}^2}{w_\theta s^2 + s}\tilde{e}_\theta(s) \end{array}, \tag{73}$$

$$\begin{array}{l} u_3(s) = \frac{1}{\alpha_\phi(1+w_\phi s)}\left(\phi_d^{(2)}(s) + K_{c\phi}^{\mathrm{T}}\hat{\mathbf{E}}_\phi(s)\right) + \frac{1}{\alpha_\phi w_\phi \varepsilon_\Delta}\tilde{e}_\phi(s) \\ \quad + \frac{1}{\alpha_\phi \varepsilon_\Delta}\frac{\left(K_{0\phi}^1 w_\phi - 1\right)s + w_\phi K_{0\phi}^2}{w_\phi s^2 + s}\tilde{e}_\phi(s) \end{array}, \tag{74}$$

with $\hat{\mathbf{E}}_\psi$, $\hat{\mathbf{E}}_\theta$, and $\hat{\mathbf{E}}_\phi$ the state estimation errors computed by the following observers:

$$\begin{cases} \dot{\hat{\mathbf{E}}}_i = \left(A_i - B_i K_{ci}^{\mathrm{T}}\right)\hat{\mathbf{E}}_i + K_{0i}\left(e_i - \hat{e}_i\right), \quad i \in \{\psi, \phi, \theta\} \\ \hat{e}_i = C_i^{\mathrm{T}}\hat{\mathbf{E}}_i \end{cases}, \tag{75}$$

where $\tilde{e}_\psi = e_\psi - \hat{e}_\psi$, $\tilde{e}_\theta = e_\theta - \hat{e}_\theta$, $\tilde{e}_\phi = e_\phi - \hat{e}_\phi$, $A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B_i = \begin{bmatrix} 0 & 1 \end{bmatrix}^{\mathrm{T}}$, and $C_i = \begin{bmatrix} 1 & 0 \end{bmatrix}$. The control voltages, V_f and V_b, applied to the front and back motors, are computed from the command signals u_1 and u_2 as follows:

$$\begin{cases} V_f = 0.5\left(u_1 + u_2\right) \\ V_b = 0.5\left(u_1 - u_2\right) \end{cases}. \tag{76}$$

The control structure is illustrated in Figure 2 and the parameters of the used helicopter are given in the Table. The control law is developed and implemented using MATLAB/Simulink and Real-Time Workshop with a fixed step size of $\Delta t = 0.001$ s. The control objective consists of moving the helicopter from the initial position $(\psi = 0°,\ \phi = 0°)$ to the new position $(\psi = 20°,\ \phi = 15°)$. In order to make the desired outputs smooth curves, the reference trajectories chosen for $\psi_d(t)$, $\theta_d(t)$, and $\phi_d(t)$ are filtered, respectively, with a 2nd-order filter, 1st-order filter, and 6th-order filter defined by the transfer functions $H_\psi = 1\big/(s+1)^2$, $H_\theta = 1/(0.5s+1)$, and $H_\phi = 1/(s+1)^6$, where s is the Laplace variable. The controller parameters used in experiment study are $w_\psi = w_\theta = w_\phi = 0.01\,s.$, $\varepsilon_\Delta = 0.01\,s.$, $\alpha_\psi = 0.8$, $\alpha_\theta = \alpha_\phi = 5$, $K_{c\psi} = [75, 20]$, $K_{c\theta} = K_{c\phi} = [10, 1]$, $K_{0\psi} = [200, 3500]$, $K_{0\theta} = [40, 700]$, and $K_{0\phi} = [80, 1400]$. The experiment results are shown in Figures 3–12.

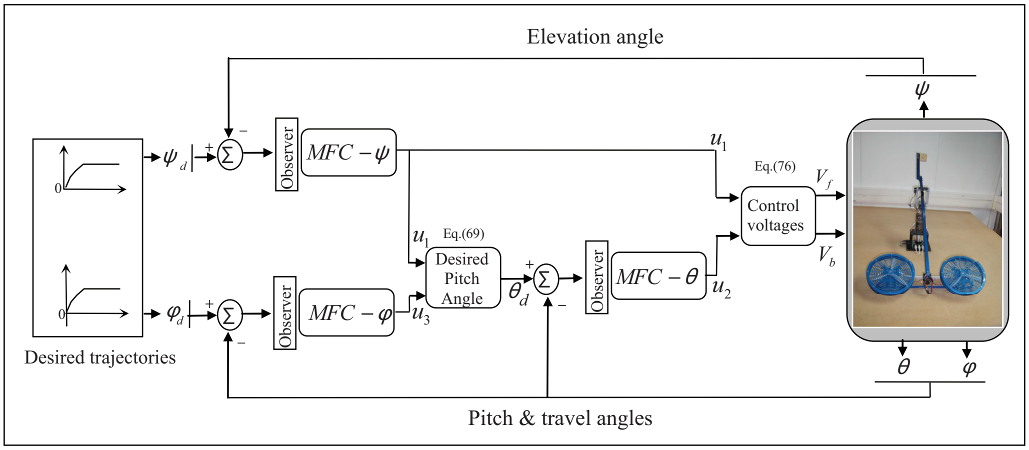

Figure 2. Synoptic scheme of the proposed controller.

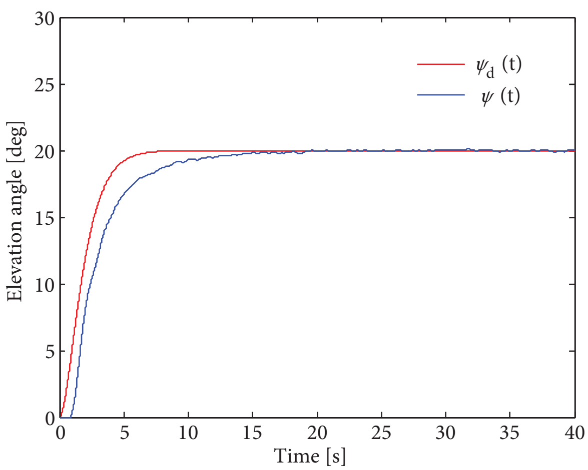

Figure 3. Trajectories $\psi(t)$ and $\psi_d(t)$.

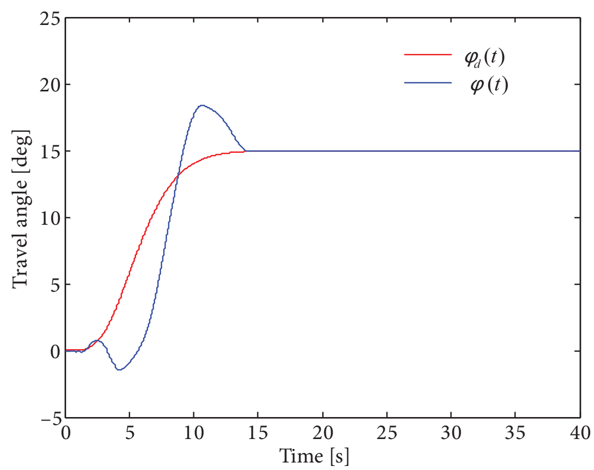

Figure 4. Trajectories $\phi(t)$ and $\phi_d(t)$.

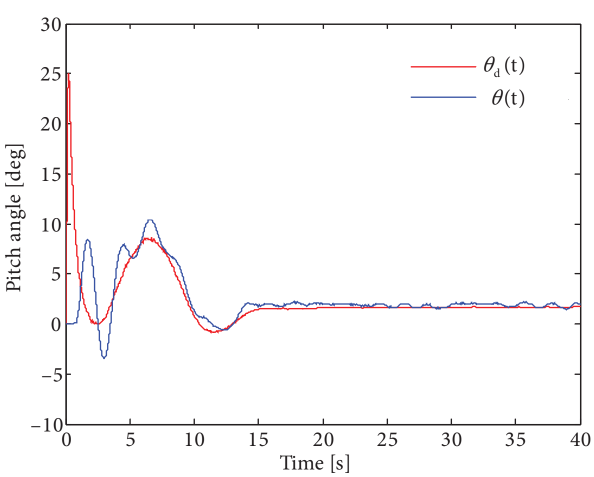

Figure 5. Trajectories $\theta(t)$ and $\theta_d(t)$.

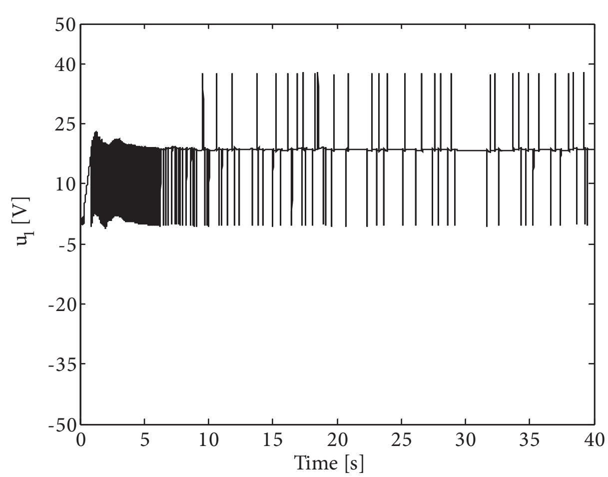

Figure 6. Control input signal u_1.

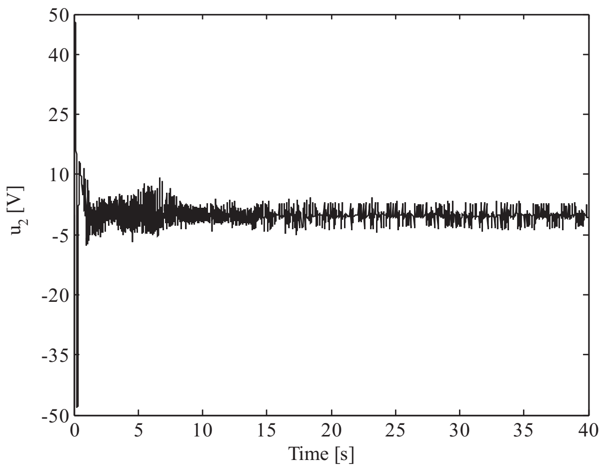

Figure 7. Control input signal u_2.

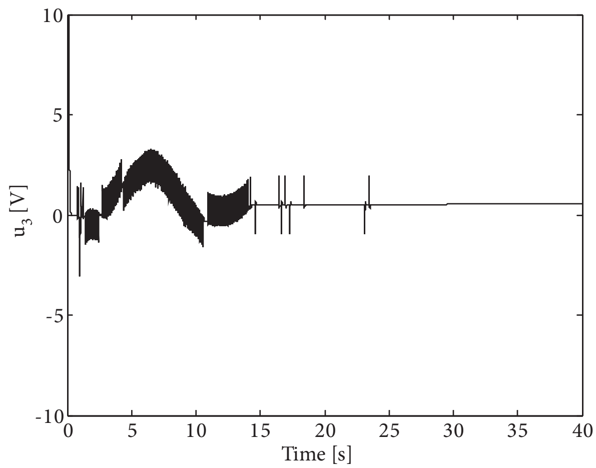

Figure 8. Control input signal u_3.

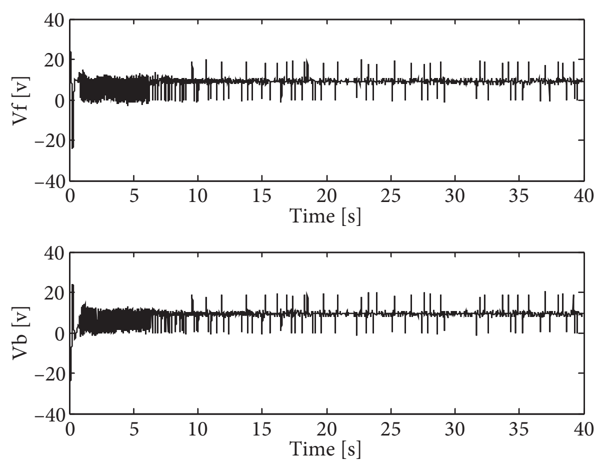

Figure 9. Control voltage for the front and back motors.

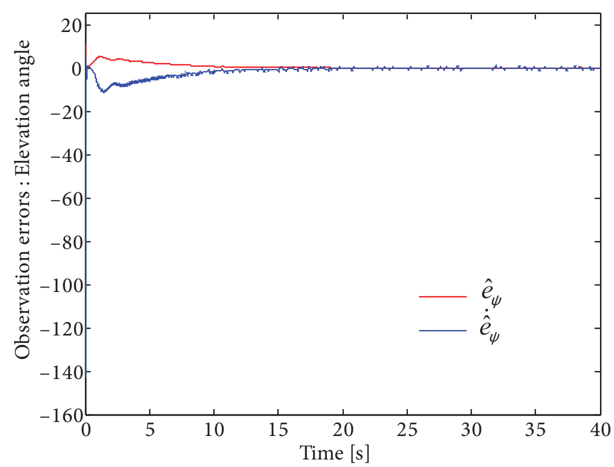

Figure 10. State estimation errors $\hat{e}_\psi$ and $\dot{\hat{e}}_\psi$.

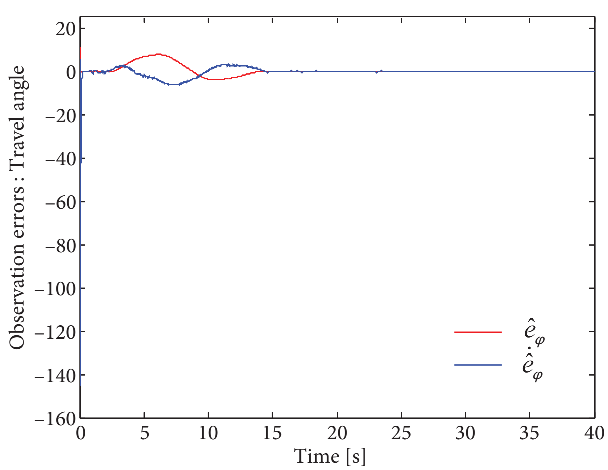

Figure 11. State estimation errors $\hat{e}_\phi$ and $\dot{\hat{e}}_\phi$.

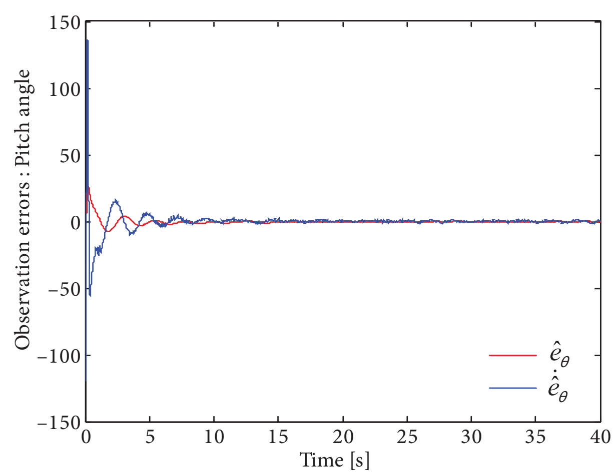

Figure 12. State estimation errors $\hat{e}_\theta$ and $\dot{\hat{e}}_\theta$.

Figure 3 illustrates the time evolution of the desired and actual elevation angles, Figure 4 shows the desired and actual travel angles, and Figure 5 shows the desired and actual pitch angles. It can be seen from Figures 3–5, that the actual trajectories $(\psi(t), \theta(t), \phi(t))$ converge to the desired trajectories $(\psi_d(t), \theta_d(t), \phi_d(t))$. The control input signals (u_1, u_2, u_3) are given in Figures 6, 7, and 8, respectively. The control voltages for the front and back motors are shown in Figure 9. Figure 10 illustrates the time evolution of the state estimation errors $\hat{e}_\psi$ and $\dot{\hat{e}}_\psi$, the state estimation errors $\hat{e}_\phi$ and $\dot{\hat{e}}_\phi$ are given in Figure 11, and the state estimation errors $\hat{e}_\theta$ and $\dot{\hat{e}}_\theta$ are given in Figure 12. From Figures 10, 11, and 12, it can be seen that all of the state estimation errors converge to zero. These experimental results demonstrate the performances of the proposed model-free controller and its effectiveness for the control tracking of dynamic systems.

5. Conclusion

In this paper, a model-free controller with an observer was proposed for a class of uncertain continuous-time MIMO nonlinear dynamic systems with unavailable states. The control scheme consists of 2 parts: the first part is a linear control term used to specify the dynamics of the closed-loop system and the second part is added to compensate the uncertainties of the dynamic system and external disturbances. A state observer is designed to provide an estimate of the state vector. The proposed control scheme does not require the knowledge of the mathematical model of the plant, guarantees the boundedness of all of the signals in the closed-loop system, and ensures the convergence of the tracking errors to a neighborhood of the origin. The proposed controller has been experimentally examined and tested in the control of a helicopter system. Our objective was to drive the helicopter to a desired elevation and travel angles while keeping the stability of the pitch. The experimental results show the good performances of the proposed controller.

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