

## A novel efficient model for the power flow analysis of power systems

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**Abstract:** The overall system status calculated by power flow analysis is the most basic information used for all decisions taken by power system operators and planners. While conventional AC power flow solutions are computationally tractable, approximate DC models are employed in many applications, such as optimal power flow studies and unit commitment problems, mainly due to the linear nature of DC models. These models do not provide any information on the reactive power and voltage magnitude quantities and occasionally inaccurate results of the active power values. This paper presents an efficient power flow approach compromising both the conflicting aspects of speed and accuracy. The proposed model adopts bus voltage magnitudes and phase angles as state variables. Given the nonlinear nature of transmission system losses, an iterative method for solving the problem is proposed. Simulation results reveal that the proposed method outperforms conventional methods from an execution time viewpoint, while preserving acceptable accuracy. Different system conditions are also investigated to reveal the robustness and reliability of the proposed model.

**Key words:** Power flow calculation, linear modeling, steady state power system analysis

### 1. Introduction

Power flow is the most underlying requirement in the present energy management systems (EMSs) and planning tools. Mathematically, power flow calculations require the solution of a set of nonlinear algebraic equations, where numerical methods should be used to approximate the solutions. These methods fall into 2 categories: starting methods and higher order methods. The most popular approaches are Gauss–Seidel from the 1st class and Newton–Raphson from the 2nd class. The performances of all power flow models are assessed from 3 aspects: convergence rate, possibility of divergence in abnormal conditions, and complexity of expressions. Gauss–Seidel was the first power flow method used in digital computers. This method suffers from a slow convergence rate along with the inability to cope with unusual conditions such as negative reactance branches [1]. Newton–Raphson, the most common technique in power system analysis packages, assumes small signal linearity to deal with nonlinear algebraic relationships. Although this method has a good convergence rate, it can be diverged when the power system is highly loaded [2]. Moreover, solutions obtained from Newton–Raphson are highly dependent on the initial guess [2]. With the DC power flow model, although it considerably simplifies the solutions, it leads to approximate results. Moreover, the DC power flow model is not applicable in voltage-dependent problems such as reactive power planning.

To overcome the aforementioned difficulties, various tricks have so far been proposed in the literature. To reduce the probability of divergence of the power flow problem, specifically in abnormal operating situations

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and ill-conditioned systems, the following techniques were employed: optimization techniques and mathematical procedures such as Levenberg–Marquardt [3], quadratic programming [4], nonlinear programming (NLP) [5], principal component analysis [6], the optimal multiplier method [7], and heuristic methods such as genetic algorithm [2], particle swarm optimization [8], and biogeography-based optimization [9]. These techniques, despite an acceptable performance in reducing the risk of divergence, are still of very low convergence speed. Moreover, their performance is highly dependent on user expertise.

In addition to the above techniques, various strategies including initial voltage selection [10], rectangular relations accommodation [11], Jacobian matrix modification [12], and node ordering [13] have been investigated in order to improve the convergence speed of the power flow. The authors in [14] reviewed some approaches and compared their effectiveness on convergence speed enhancement and divergence chance reduction. The authors in [15] formulated the power flow problem as a set of autonomous ordinary differential equations to solve ill-conditioned or badly initialized power flow cases.

Parallel processing schemes were exploited in [16–21] to accelerate the computation of the power flow solution. In [22–24], different versions of the generalized minimal residual method equipped with some accelerating schemes were used to speed up the traditional Newton method. The necessity of advanced technology and communication protocols are the main weaknesses of these approaches. An efficient framework for updating the Jacobian matrix, resulting in a power flow solution in a faster manner, was proposed in [25]. In [26], a combination of the Newton–Raphson method and successive substitution technique was used to achieve a rapid solution.

Among the conducted research, [27] and [28] proposed suitable models from the view point of relationship complexity. Empirical knowledge of the system was used in [27] to enhance the accuracy of the DC power flow. However, the proposed methodology is not applicable in cases where adequate information is not available. Moreover, the method is not sufficiently accurate, specifically in voltage magnitudes and reactive powers. A linear power flow model with line flows and square of voltage magnitudes as state variables was introduced in [28]. Similar to the DC model, the developed formulation is linear but incorporates both active and reactive quantities similar to the conventional AC model. This method has a large number of variables/equations, which is undesirable in large-scale networks.

Techniques reviewed above have particular pros and cons. Some suffer from the computational complexity and burden, while others suffer from bad convergence in abnormal conditions. Metaheuristic methods do not guarantee a feasible solution and their performance strongly depends on the user experiences.

Taking into account the above issues, it is very desirable to develop an approach with the advantages of low computational burden as well as high convergence rate. In this paper, this requirement is focused on devising a new mathematical power flow model. The proposed method adopts voltage magnitudes and phase angles as the problem state variables and uses approximate relationships. Given the nonlinear nature of transmission system losses, an iterative method for solving the problem is proposed. Numerical analyses are conducted through 3 case studies, and the obtained results are compared with those of conventional approaches.

The paper is outlined as follows. A brief description of the present power flow models is given in Section 2. The proposed framework is drawn in Section 3. Simulation results are presented in Section 4. The importance of the proposed model is discussed in Section 5 and Section 6 outlines the concluding remarks.

## 2. Review of traditional power flow methods

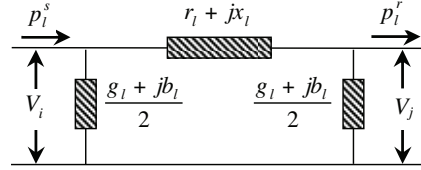
The power flow model consists of a set of nonlinear equations. These equations formulate active and reactive line flows based on bus voltage magnitudes and phase angles. In most power system studies, transmission lines are represented by a pi-equivalent circuit, shown in Figure 1. Using this representation, the equations of the active and reactive flows at the sending and receiving ends of line  $l$ , connecting bus  $i$  to bus  $j$ , are written as:

$$p_l^s = g_l V_i^2 - g_l V_i V_j \cos(\delta_i - \delta_j) - b_l V_i V_j \sin(\delta_i - \delta_j), \quad (1)$$

$$p_l^r = -g_l V_j^2 + g_l V_j V_i \cos(\delta_j - \delta_i) + b_l V_j V_i \sin(\delta_j - \delta_i), \quad (2)$$

$$q_l^s = -b_l V_i^2 - g_l V_i V_j \sin(\delta_i - \delta_j) + b_l V_i V_j \cos(\delta_i - \delta_j), \quad (3)$$

$$q_l^r = b_l V_j^2 + g_l V_j V_i \sin(\delta_j - \delta_i) - b_l V_j V_i \cos(\delta_j - \delta_i). \quad (4)$$



**Figure 1.** The pi-equivalent circuit of a transmission line.

In the literature, many different techniques have been proposed for solving the above equations and deriving system state variables, i.e. bus voltage magnitudes and phase angles. In the following subsections, the most popular methods for the power flow problem are briefly reviewed. Their comprehensive presentations are accessible in most power system analysis textbooks.

### 2.1. The Newton–Raphson power flow model

In this method, an error function using the Taylor expansion of the bus power injections is formed. Next, a set of starting values for the system variables, usually 1 per unit for the voltage magnitudes and 0 radians for the phase angles, is picked. At the end, the mismatch of known quantities with the calculated values should be 0 by adjusting the independent variables. Setting the mismatches to 0 necessitates an iterative procedure. In each iteration, a set of linear equations should be solved. The coefficients of these equations form the well-known Jacobian matrix. In addition to the probable drawbacks discussed in the introduction, calculating all entries of the Jacobian matrix in each step is computationally expensive [1].

### 2.2. The fast decoupled power flow model

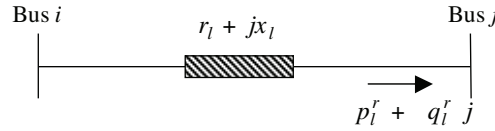
A fast decoupled power flow model is a way to speed up the previous model. The method neglects interactions between the active power and voltage magnitude, as well as between the reactive power and voltage phase angle. Since the differences between the bus voltage phase angles of a line end are usually small, it assumes that  $\cos(\delta_i - \delta_j) = 1$ . Further assumptions of the fast decoupled method are available in [1]. Using these simplifications, the Jacobian matrix will be constant and it is therefore calculated just once. While the fast decoupled method needs less arithmetic to solve the power flow problem, it may fail to converge, especially when some of the underlying assumptions are not correct [1].

### 2.3. The DC power flow model

Further simplification over the previous model is performed by assuming that the voltage magnitude at all of the buses is equal to 1 per unit. This assumption leads to dropping equations associated with the reactive power and voltage magnitude criteria. Considering that the resulting equations are linear, they are providing a noniterative power flow algorithm. The DC power flow is attractive from the computational burden aspect; however, it gives no indication of the reactive powers and voltage magnitudes.

### 2.4. Line flow-based power flow model

In this subsection, the line flow-based power flow model is discussed in brief and interested readers are referred to [28] for a detailed explanation. Contrary to conventional models, the line flow-based power flow adopts active line flows, reactive line flows, and the square of the voltage magnitudes as state variables. In the following, the relationships between these variables are provided to declare the model.



**Figure 2.** Branch  $l$  connecting bus  $i$  to bus  $j$ .

Referring to Figure 2, the branch voltage drop yields to:

$$V_i \angle \delta_i = V_j \angle \delta_j + \frac{p_l^s - j \cdot q_l^s}{V_j \angle -\delta_j} \cdot (r_l + j \cdot x_l). \quad (5)$$

Eq. (5) can be rewritten as follows:

$$2(r_l \cdot p_l^s + x_l \cdot q_l^s) + V_j^2 - V_i^2 = -(r_l \cdot p_l^{loss} + x_l \cdot q_l^{loss}). \quad (6)$$

In Eq. (6), the number of equations is equal to the number of branches. Bus phase angles across a branch are related to the problem variables as:

$$\sin(\delta_i - \delta_j) = \frac{x_l \cdot p_l^s - r_l \cdot q_l^s}{V_i \cdot V_j}. \quad (7)$$

Assuming  $V_i = V_j = 1$  and  $\sin(\delta_i - \delta_j) = \delta_i - \delta_j$ , Eq. (7) can be rewritten as:

$$\delta_i - \delta_j = x_l \cdot p_l^s - r_l \cdot q_l^s. \quad (8)$$

As the sum of the phase angle differences in the branches of a loop is equal to 0, Eq. (8) leads to the following statement:

$$\sum_{l \in L} C_{ml} x_l \cdot p_l^s - \sum_{l \in L} C_{ml} r_l \cdot q_l^s = 0. \quad (9)$$

In Eq. (9), the number of equations is equal to the number of links in the network representative graph.

Active and reactive power balance equations at bus  $i$  are expressed in Eqs. (10) and (11), respectively. Note that the last term in the reactive power balance equation is considered to model shunt compensators and line charging susceptances.

$$p_i^g - p_i^d - \sum_{l \in L} A_{il} \cdot p_l^s - \sum_{l \in L} A'_{il} \cdot p_l^{loss} = 0 \quad (10)$$

$$q_i^g - q_i^d - \sum_{l \in L} A_{il} \cdot q_l^s - \sum_{l \in L} A'_{il} \cdot q_l^{loss} + q_i^{shunt} = 0 \quad (11)$$

The active and reactive power balance equations are written in all of the buses, except the slack bus. Therefore, the total number of power balance equations is equal to twice the number of buses minus 2.

Eqs. (6) and (8)–(10) provide linear relationships between the active and reactive line flows, square of bus voltage magnitudes, and active and reactive line losses. The total number of variables and equations in these expressions are  $n + 4n_b - 1$  and  $n + 2n_b - 1$ , respectively. Given the greater number of variables, the system of equations can still not be solved. Active and reactive power losses corresponding to line  $l$ , from bus  $i$  to bus  $j$ , are determined as:

$$p_l^{loss} = \frac{p_i^{s^2} + q_i^{s^2}}{V_i^2} r_l, \quad (12)$$

$$q_l^{loss} = \frac{p_i^{s^2} + q_i^{s^2}}{V_i^2} x_l. \quad (13)$$

The number of above equations is  $2n_b$ ; hence, a system of formulations with an equal number of variables and equations is now developed. However, given the nonlinear nature of Eqs. (12) and (13), an iterative method for solving the problem should be developed. To this end, the loss values are treated as known parameters and updated after each iteration. This assumption makes the model linear, and it can be solved easily. Note that the initial values of the losses are set to 0. The iterations should continue until the settlement of the losses.

Coefficients of the line flow-based model are constant and calculated just once. The results obtained by this technique are relatively accurate; however, the number of variables and equations are greater than those in the previous approaches.

### 3. Proposed formulation

The importance of providing a computationally efficient power flow model that simultaneously addresses voltage and reactive power criteria was stressed in the preceding sections. The purpose of this section is to present a new power flow model in which neither voltage nor reactive power quantities are sacrificed, such as what is done in the DC model, nor does the number of equations increase like the line flow-based model.

In the first step, let:

$$V_i V_j \sin(\delta_i - \delta_j) = \delta_i - \delta_j, \quad (14)$$

which is a good approximation since  $(\delta_i - \delta_j)$  is usually small and voltage magnitudes are near 1 per unit. Note that the above approximation is used in both DC- and line flow-based power flow models [28]. Substituting Eq. (14) into Eqs. (1)–(4):

$$p_l^s = g_l V_i^2 - g_l V_i V_j \cos(\delta_i - \delta_j) - b_l \delta_i + b_l \delta_j, \quad (15)$$

$$p_l^r = -g_l V_j^2 + g_l V_j V_i \cos(\delta_j - \delta_i) + b_l \delta_j - b_l \delta_i, \quad (16)$$

$$q_l^s = -b_l V_i^2 - g_l \delta_i + g_l \delta_j + b_l V_i V_j \cos(\delta_i - \delta_j), \quad (17)$$

$$q_l^r = b_l V_j^2 + g_l \delta_j - g_l \delta_i - b_l V_j V_i \cos(\delta_j - \delta_i), \quad (18)$$

The power exchange in a transmission line in both active and reactive forms is shown in Figure 3. The direction of the arrow indicates the direction of the flow. According to Figure 3, one can derive active and reactive power

losses in a transmission line using the following equations:

$$p_l^{loss} = p_l^s - p_l^r, \quad (19)$$

$$q_l^{loss} = q_l^s - q_l^r. \quad (20)$$

By replacing power flows from Eqs. (15)–(18) into Eqs. (19) and (20), the active and reactive losses could be written as:

$$p_l^{loss} = g_l V_i^2 - g_l V_i V_j \cos(\delta_i - \delta_j) - b_l \delta_i + b_l \delta_j + g_l V_j^2 - g_l V_j V_i \cos(\delta_j - \delta_i) - b_l \delta_j + b_l \delta_i, \quad (21)$$

$$q_l^{loss} = -b_l V_i^2 - g_l \delta_i + g_l \delta_j + b_l V_i V_j \cos(\delta_i - \delta_j) - b_l V_j^2 - g_l \delta_j + g_l \delta_i + b_l V_j V_i \cos(\delta_j - \delta_i), \quad (22)$$

from which:

$$p_l^{loss} = g_l V_i^2 + g_l V_j^2 - 2g_l V_i V_j \cos(\delta_i - \delta_j), \quad (23)$$

$$q_l^{loss} = -b_l V_i^2 - b_l V_j^2 + 2b_l V_i V_j \cos(\delta_i - \delta_j). \quad (24)$$

The above equations can be rearranged to form the following equivalents for  $V_i V_j \cos(\delta_i - \delta_j)$ :

$$V_i V_j \cos(\delta_i - \delta_j) = \frac{1}{2} (V_i^2 + V_j^2) - \frac{1}{2g_l} p_l^{loss}, \quad (25)$$

$$V_i V_j \cos(\delta_i - \delta_j) = \frac{1}{2} (V_i^2 + V_j^2) + \frac{1}{2b_l} q_l^{loss}. \quad (26)$$

By substituting Eq. (25) into Eqs. (15) and (16) and Eq. (26) into Eqs. (17) and (18), the equations of the active and reactive flows are derived as:

$$p_l^s = \frac{1}{2} (g_l V_i^2 - g_l V_j^2) + \frac{1}{2} p_l^{loss} - b_l \delta_i + b_l \delta_j, \quad (27)$$

$$p_l^r = -\frac{1}{2} (g_l V_j^2 - g_l V_i^2) - \frac{1}{2} p_l^{loss} + b_l \delta_j - b_l \delta_i, \quad (28)$$

$$q_l^s = -\frac{1}{2} (b_l V_i^2 - b_l V_j^2) - g_l \delta_i + g_l \delta_j + \frac{1}{2} q_l^{loss}, \quad (29)$$

$$q_l^r = \frac{1}{2} (b_l V_j^2 - b_l V_i^2) + g_l \delta_j - g_l \delta_i - \frac{1}{2} q_l^{loss}. \quad (30)$$

So far, the power flow equations are linearized in terms of the bus voltage phase angles, square of the bus voltage magnitudes, and active and reactive line losses.

It is worth noting that the only approximation used in the proposed formulation is to replace  $V_i V_j \sin(\delta_i - \delta_j)$  with  $\delta_i - \delta_j$ , which does not threaten the accuracy of the results. This can clearly be seen in the results presented in the subsequent section.

The derived formulation is in terms of the square bus voltage magnitudes. This parameter, although adopted as a state variable in the line flow-based power flow [28], does not reflect the ultimately required

practical knowledge of the network condition. Therefore, since the voltage magnitudes at all of the buses are always around unity, any function in terms of  $V_i$ , e.g.,  $f(V_i) = V_i^2$ , could be approximated by the Taylor series:

$$f(V_i) \cong f(1) + \left. \frac{df}{dV_i} \right|_{(1)} \cdot (V_i - 1). \quad (31)$$

Accordingly,

$$V_i^2 \cong 2V_i - 1. \quad (32)$$

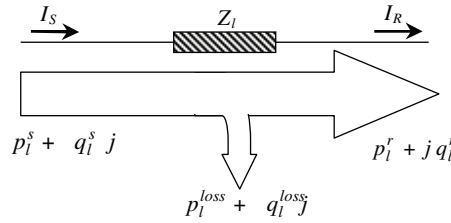
Next, Eqs. (27)–(30) can be expressed as follows:

$$p_i^s = g_l V_i - g_l V_j + \frac{1}{2} p_l^{loss} - b_l \delta_i + b_l \delta_j, \quad (33)$$

$$p_i^r = -g_l V_j + g_l V_i - \frac{1}{2} p_l^{loss} + b_l \delta_j - b_l \delta_i, \quad (34)$$

$$q_i^s = -b_l V_i + b_l V_j - g_l \delta_i + g_l \delta_j + \frac{1}{2} q_l^{loss}, \quad (35)$$

$$q_i^r = b_l V_j - b_l V_i + g_l \delta_j - g_l \delta_i - \frac{1}{2} q_l^{loss}. \quad (36)$$



**Figure 3.** Active and reactive power flow in a transmission line.

So far, the active and reactive line flows in both the sending and receiving ends are linearly expressed in terms of the bus voltage magnitudes, phase angles, and line losses.

Substituting Eqs. (33)–(36) into Eqs. (10) and (11) and arranging that in a matrix representation, the active and reactive power injections at the load buses and active power injection in the voltage-controlled buses are as follows:

$$\begin{bmatrix} p_1 \\ p_2 \\ \vdots \end{bmatrix} = [\mathbf{G}] \begin{bmatrix} V_1 \\ V_2 \\ \vdots \end{bmatrix} - [\mathbf{B}] \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \end{bmatrix} + \mathbf{A}' \mathbf{P}^{loss}, \quad (37)$$

$$\begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = -1 \cdot [\mathbf{B}] \begin{bmatrix} V_1 \\ V_2 \\ \vdots \end{bmatrix} - [\mathbf{G}] \begin{bmatrix} \delta_1 \\ \delta_2 \\ \vdots \end{bmatrix} + \mathbf{A}' \mathbf{Q}^{loss}, \quad (38)$$

where

$$\mathbf{G}_{ij} = -g_l \Big|_{i \rightarrow j}, \quad (39)$$

$$\mathbf{G}_{ii} = -\sum_j \mathbf{G}_{ij}, \quad (40)$$

$$\mathbf{B}_{ij} = -b_l |_{i \rightarrow j}, \quad (41)$$

$$\mathbf{B}_{ii} = -\sum_j \mathbf{B}_{ij}. \quad (42)$$

At a load bus, the active and reactive power injections are known and the voltage magnitude and phase angle are unknown. Moreover, at a voltage-controlled bus, the unknown quantities are the reactive power injection and voltage phase angle, while the active power injection and voltage magnitude are known. Removing equations associated with the slack bus and those associated with the reactive power injection at the voltage-controlled buses, the number of unknown variables and equations is  $2n_b + 2n_{pq} + n_{pv}$  and  $2n_{pq} + n_{pv}$ , respectively.

Since the active and reactive power losses, expressed in Eqs. (12) and (13), are nonlinear, the proposed power flow model is nonlinear and cannot be solved using the direct matrix inversion technique. Three different alternatives exist to overcome this difficulty.

- a) Neglecting losses: In this approach,  $p_l^{loss}$  and  $q_l^{loss}$  at all lines are assumed to be 0 and the problem is solved just by a single matrix inversion. Based on numerical evidences, the error level of this method is noticeably less than that of the DC model.
- b) Iterative approach: Loss values are initially set to 0 and updated after each iteration using Eqs. (12) and (13). Hence, they are treated as known parameters in each iteration. This assumption makes the model linear and solvable by means of the matrix inversion technique. The process is terminated when the losses do not change in successive iterations.
- c) Linearized loss equations: Eqs. (12) and (13) are linearized around the point obtained in the last iteration; then the loss terms are considered in the linear model as the problem variable.

The 1st and 3rd solutions result in a linear format and thus are appropriate for applications in which the nonlinearity of the conventional AC model leads to an excessive complexity. The unit commitment problem is an example for such applications.

In this paper, the second alternative is adopted since it incorporates the line losses, while not imposing further calculations. Accordingly, the active and reactive power losses at all lines are neglected in the first iteration. Next, the remained set of linear equations is solved, and losses are accordingly updated. The iterative process is followed until a settlement in the transmission line losses is achieved.

It is worth noting that the main reason behind the divergence occurrence of the Newton–Raphson or Gauss–Seidel methods relates to the feature of starting the procedure from an initial guess. These approaches assume a starting point for unknown voltage magnitudes and phase angles, usually 1 per unit with a 0 radian. In highly loaded or ill-conditioned systems, these assumptions may be inappropriate, causing divergence of the solution. In contrast, 0 losses are assumed as the initial values in the line flow-based and the proposed power flow models, which is a more realistic assumption. Therefore, the convergence of these methodologies is very likely.<sup>1</sup>

#### 4. Simulation results

In this section, 3 case studies, consisting of the Wood Wollenberg 6-bus system, the IEEE 118-bus network, and the Polish power grid are conducted. For the sake of the comparison and verification of the proposed model, the

<sup>1</sup> Indeed, no diverged case was observed during our various simulations.

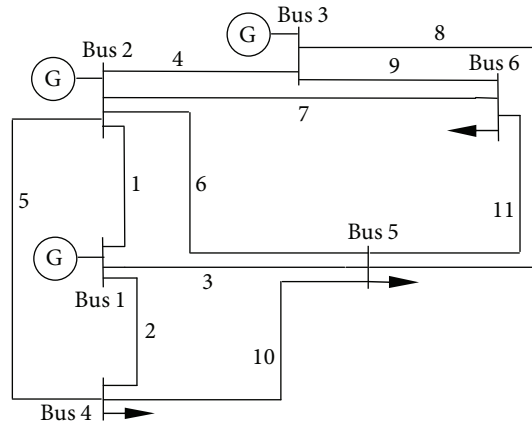


results obtained from the Newton–Raphson, fast decoupled, line flow-based, and DC power flow solutions are presented as well. All of the simulations are conducted in MATLAB on an Intel Core 2 Duo 2.20-GHz processor with 2-GB RAM.

The mean absolute error of the parameters are calculated and compared in various models. The final results of the Newton–Raphson technique are taken as the benchmark for comparison. As only active line flows are calculated in the DC model, bus voltage magnitudes and reactive line flows are assumed to be 1 per unit and 0 MVAR, respectively. The robustness of the proposed formulation is investigated at different loading conditions on the Wood Wollenberg system.

### 5. The Wood Wollenberg 6-bus system

The 6-bus test system, depicted in Figure 4, is a small 230-kV power system including 11 transmission lines, 3 generating buses, and 3 load buses. The technical data of the components and the operating conditions were taken from [1].



**Figure 4.** Single-line diagram of the Wood Wollenberg 6-bus system.

The results obtained by different methods are given in Tables 1–3, where bolded columns show the worst result from the accuracy aspect.

**Table 1.** Voltage magnitudes obtained by different methodologies for the Wood Wollenberg 6-bus system (kV).

Bus #	Power flow technique				
	Newton-Raphson	Fast decoupled	Line flow-based	DC model	Proposed model
1	241.5	241.5	241.5	230	241.5
2	241.5	241.5	241.5	230	241.5
3	246.1	246.1	246.1	230	246.1
4	227.6	<b>227.4</b>	227.6	230	227.2
5	226.7	<b>226.5</b>	<b>226.6</b>	230	<b>226.2</b>
6	231.0	230.9	231.0	230	<b>230.5</b>

Compared to the results associated with the Newton–Raphson method, all of the approximate iterative approaches, i.e. fast decoupled, line flow-based, and the proposed model, lead to acceptable solutions from an accuracy point of view. However, the DC method has more error in the active powers and provides no implication of the voltage and reactive power values.

**Table 2.** Active line flows obtained by different methodologies for the Wood Wollenberg 6-bus system (MW).

Line #	Power flow technique				
	Newton–Raphson	Fast decoupled	Line flow-based	DC model	Proposed model
1	28.7	28.8	<b>27.6</b>	<b>25.3</b>	<b>27.5</b>
2	43.6	<b>43.4</b>	44.0	41.6	44.0
3	35.6	35.5	36.0	33.1	35.9
4	2.9	2.9	2.4	1.9	2.4
5	33.1	<b>32.9</b>	32.8	32.5	32.8
6	15.5	15.4	15.4	16.2	15.4
7	26.2	26.2	26.1	24.8	26.1
8	19.1	19.1	18.9	16.9	18.9
9	43.8	<b>43.6</b>	43.5	44.9	43.5
10	4.1	4.1	4.3	4.0	4.3
11	1.6	1.7	2.1	0.3	2.0

**Table 3.** Reactive line flows obtained by different methodologies for the Wood Wollenberg 6-bus system (MVAR).

Line #	Power flow technique				
	Newton–Raphson	Fast decoupled	Line flow-based	DC model	Proposed model
1	-15.4	-15.5	-14.9	0	-14.9
2	20.1	20.7	19.9	0	19.9
3	11.3	11.7	11.0	0	11.1
4	-12.3	-12.3	-12.1	0	-11.6
5	46.1	<b>47.9</b>	46.0	0	46.0
6	15.4	15.8	15.2	0	15.4
7	12.8	12.4	12.3	0	12.7
8	23.7	23.2	23.0	0	22.7
9	61.5	60.7	<b>60.6</b>	0	<b>60.1</b>
10	-4.9	-4.9	-5.0	0	-4.9
11	-9.9	-9.7	-9.7	0	-9.6

Based on Table 1, the voltage magnitudes obtained by the proposed method are almost accurate. The most inaccurate voltage magnitudes using the proposed methodology are associated with buses 5 and 6 with a 0.5-kV error. While the maximum error indices for the fast decoupled and line flow-based methods are 0.2 and 0.1 kV, respectively, it should be noted that all of these errors are negligible; hence, all of the approximate methods, except for the DC model, are acceptable from a voltage magnitude accuracy aspect.

As shown in Table 2, the accuracy of the results obtained by the proposed and conventional methods is almost identical and higher than that of the DC model. The maximum error in the active line flows obtained by the fast decoupled method is related to lines 2, 5, and 9, which is equal to 0.2 MW. The less accurate active line flow associated with the line flow-based model, the DC model, and the proposed methodology occurs in line 1, where the errors are 1.1, 3.4, and 1.2 MW, respectively.

Referring to Table 3, the reactive power flows obtained by all of the methods, except for the DC model, are relatively accurate. The most inaccurate result of the fast decoupled method relates to line 5 with a 1.8-MVAR error. The worst performance of the developed technique and line flow-based model relates to the line 9 reactive flow with errors of 1.4 and 0.9 MVAR, respectively.

Table 4 provides the mean absolute error level of the results obtained by the proposed methodology and its comparison with the conventional methods.

**Table 4.** Error comparison of the methods for the Wood Wollenberg 6-bus system.

Parameter	Mean absolute error			
	Fast decoupled	Line flow-based	DC model	Proposed model
Voltage magnitude (kV)	0.083	0.017	7.633	0.165
Active line flow (MW)	0.091	0.373	1.482	0.337
Reactive line flow (MVAR)	0.473	0.182	21.045	0.287

According to Table 4, the DC model experiences bigger error values in comparison with the others. In the case of voltage magnitudes, the fast decoupled, line flow-based, and proposed methods have almost similar accuracy. However, in the case of active and reactive line flows, the accuracy of the fast decoupled method is much better, while the line flow-based and proposed formulations lead to results with relatively the same accuracy. Consequently, the approximation of the proposed model is trivial and is not a trouble maker.

Table 5 outlines other measures to compare the proposed methodology with the existing ones.

**Table 5.** Comparison of other measures for the Wood Wollenberg 6-bus system.

Parameter	Power flow technique				
	Newton–Raphson	Fast decoupled	Line flow-based	DC model	Proposed model
Variable no.	8	8	15	5	8
Equation no.	8	8	15	5	8
Iteration no.	3	5	5	1	3
Run time (s)	0.0059	0.00400	0.00396	0.00014	0.00032

Referring to Table 5, the least computational time and the least number of variables, equations, and iterations belong to the DC model; however, the accuracy of the results is the worst. The proposed technique significantly decreases the computational time of the power flow study. In addition, compared to the line flow-based model, the proposed model has a lower number of variables, equations, and needed iterations. Accordingly, the effectiveness of the proposed formulation is verified.

### 5.1. The IEEE 118-bus test system

This system has 118 buses and 186 transmission lines. The system demand is served through 99 buses and the generation capacity of 9966.2 MW is distributed among 54 buses. The annual peak load of the system is equal to 4242 MW. Moreover, the system has 3 voltage levels of 138, 161, and 345 kV. Because of the system's large dimension, just the mean absolute errors associated with 4 techniques are presented in Table 6.

**Table 6.** Error comparison of methods for the IEEE 118-bus system.

Parameter	Mean absolute error			
	Fast decoupled	Line flow-based	DC model	Proposed model
Voltage magnitude (p.u.)	0.00005	0.00002	0.0231	0.00009
Active line flow (MW)	0.0951	0.4910	3.6107	0.4944
Reactive line flow (MVAR)	0.1007	0.1624	15.117	0.3742

The results presented in Table 6 also verify the satisfactory performance of the proposed methodology. It can be seen that the most and least accurate methods are the fast decoupled method and the DC model, respectively. Dimension measures, iteration numbers, and the execution time associated with various methods, including the proposed technique, are given in Table 7.

**Table 7.** Comparison of other measures for the IEEE 118-bus system.

Parameter	Power flow technique				
	Newton–Raphson	Fast decoupled	Line flow-based	DC model	Proposed model
Variable no.	181	181	436	117	181
Equation no.	181	181	436	117	181
Iteration no.	3	5	5	1	4
Run time (s)	0.0594	0.03738	0.05317	0.00125	0.00213

Likewise, for the Wood Wollenberg 6-bus system, numerical evidence verifies that the proposed technique makes a significant improvement in the computational burden without endangering the accuracy. It is worth noting that the computational burden of the conventional Newton–Raphson method is considerably heavier than that of the new algorithm. The reason goes back to the computational effort required for updating the coefficient matrix at each iteration, in which all elements of the matrix are achieved by nonlinear expressions and in terms of the results associated with the previous iteration. In the case of the fast decoupled method, the large number of iterations for convergence increases the run time. This case study, owing to its dimension, demonstrates the applicability of the proposed method for real-world problems.

## 5.2. The Polish power grid

Here, the proposed method is applied to the Polish power grid, corresponding to the configuration in the winter 2003–2004 evening peak condition. The system comprises 3 voltage levels, including 110, 220, and 400 kV, and has 2746 buses and 3279 transmission lines. The system demand is served through 370 buses and the demanded load of 24,873 MW is distributed among 1993 buses. The technical data for the system are available at <http://www.pserc.cornell.edu/matpower/case2746wp.m>. Given the large dimension of the grid, just the mean absolute errors associated with 4 techniques are provided in Table 8.

**Table 8.** Error comparison of methods for the Polish power grid.

Parameter	Mean absolute error			
	Fast decoupled	Line flow-based	DC model	Proposed model
Voltage magnitude (p.u.)	0.00002	0.00363	0.0614	0.00815
Active line flow (MW)	0.00071	0.1024	1.8131	0.1270
Reactive line flow (MVAR)	0.00605	0.2701	25.607	0.1900

The results presented support the satisfactory performance of the proposed technique. Similar to the preceding case studies, the fast decoupled method and the DC model are the most and least accurate methodologies, respectively. Table 9 provides dimension measures, iteration numbers, and the execution time associated with various techniques, including the proposed one.

**Table 9.** Comparison of other measures for the Polish power grid.

Parameter	Power flow technique				
	Newton–Raphson	Fast decoupled	Line flow-based	DC model	Proposed model
Variable no.	5121	5121	8934	2745	5121
Equation no.	5121	5121	8934	2745	5121
Iteration no.	4	15	6	1	4
Run time (s)	0.72	1.14	0.46	0.08	0.21

Like the Wood Wollenberg 6-bus and IEEE 118-bus systems, the proposed method brings a significant reduction in the run time without jeopardizing the accuracy. Referring to the results, the fast decoupled method has the largest run time, mainly due to the large number of iterations required for convergence.

### 5.3. Different loading conditions

To reveal the robustness and reliability of the proposed method, the Wood Wollenberg 6-bus test system is adopted and repeated at the following loading conditions: 40%, 60%, 80%, 100%, and 120% of the base case. Table 10 outlines the mean absolute error associated with all of the parameters. The results indicate that by changing the system loading condition, the accuracy of the results obtained by the new methodology is not degraded.

**Table 10.** Performance of the proposed method at various loading conditions - mean absolute error.

Parameter	120% loading	100% loading	80% loading	60% loading	40% loading
Voltage magnitude (kV)	0.13113	0.16493	0.17584	0.15770	0.11304
Active line flow (MW)	0.59481	0.33746	0.23592	0.12791	0.06531
Reactive line flow (MVAR)	0.33278	0.28670	0.24050	0.22841	0.22176

## 6. Discussion

The numerical evidence in support of the effectiveness of the new proposed power flow model was presented in the preceding section. However, one might ask the following questions. What is the necessity of developing a new linear power flow model? Why do we need to reduce the power flow execution time?

At first glance, one might deduce that the runtime reduction from 0.72 s to 0.21 s, in the case of the Polish power grid, is a minor improvement and not strong enough for involving some level of approximation. In response, we should say that errors introduced in the outcomes of the proposed method are trivial. On the other hand, power flow equations are the basis for many other applications such as unit commitment problems and composite system reliability evaluation studies. In these problems, having a nonlinear conventional AC model leads to an excessive complexity and avoids the utilization of efficient numerical tools. Accordingly, the proposed power flow method could be helpful.

Talking about the unit commitment problem, conventional AC equations cause the entire model to lie within the mixed integer NLP format, which is very rigorous to solve. However, applying the proposed power flow model, the unit commitment problem is converted to the mixed linear programming fashion and can be tackled using powerful commercial solvers.

Composite system reliability assessment encounters a tremendous number of contingencies, which should be analyzed and judged to see whether they result in system success or failure. The judgment is based on the power flow solutions. In such a case, even a marginal improvement in the computational burden and execution time of the power flow problem is extremely desirable. Owing to the considerable acceleration observed by the proposed model, it could hence be an effective tool in enhancing the reliability studies.

## 7. Conclusion

A new formulation for the power flow studies is proposed in this paper. The new approach is more attractive in terms of calculation effort and execution time. The presented technique offers a system of approximate equations wherein the cumbersome system of nonlinear equations related to the conventional AC power flow model is avoided. A comparative study with conventional models of Newton–Raphson, fast decoupled, line

flow-based, and DC model is conducted in the paper. Moreover, simulations on different loading conditions are conducted to demonstrate the robustness of the proposed model. Numerical evidence verifies the acceptable accuracy of the developed formulation. The superiority of the proposed methodology appears more, implying the point that its results are obtained in a faster procedure. While the Newton–Raphson and fast decoupled methods suffer from a computational burden, the line flow-based method suffers from a huge number of variables and equations, and the DC model has inaccurate results, while the developed formulation provides an appropriate compromise between accuracy and speed. The presented method, due to the high speed and low error results, is suitable for studies such as power system reliability assessment, in which thousands of possible outage scenarios must be analyzed in a reasonable time span. Future works will be focused on the application of the model in the reliability evaluation of composite generation and transmission systems.

### Nomenclature

#### Indices and sets

$i, j$  Indices of the bus  
 $l$  Index of the branch  
 $m$  Index of the loop

#### Parameters

$n$  Number of buses  
 $n_{pv}$  Number of voltage-controlled buses  
 $n_{pq}$  Number of load buses  
 $n_b$  Number of branches  
 $A_{il}$  Element of the bus-line incidence matrix, which is equal to 1, if bus  $i$  is the sending bus of line  $l$ ,  $-1$  if bus  $i$  is the receiving bus of line  $l$ , and 0 otherwise  
 $A'_{il}$  Modified  $A_{il}$  with all '+1' set to 0  
 $C_{ml}$  Element of loop-line incidence matrix, which is equal to 1 if loop  $m$  and line  $l$  are collinear,  $-1$  if loop  $m$  and line  $l$  are not collinear, and 0 otherwise  
 $x_l$  Reactance of line  $l$   
 $r_l$  Resistance of line  $l$

$g_l$  Real part of the admittance of line  $l$   
 $b_l$  Imaginary part of the admittance of line  $l$   
 $\mathbf{B}$  Imaginary part of the system admittance matrix  
 $\mathbf{G}$  Real part of the system admittance matrix

#### Variables

$V_i$  Voltage magnitude of bus  $i$   
 $\delta_i$  Voltage phase angle of bus  $i$   
 $q_i^{shunt}$  Shunt compensator and line charging susceptance reactive power generation at bus  $i$   
 $p_i$  Active power injection at bus  $i$   
 $q_i$  Reactive power injection at bus  $i$   
 $p_i^g$  Active power generation at bus  $i$   
 $q_i^g$  Reactive power generation at bus  $i$   
 $p_i^d$  Active power demand at bus  $i$   
 $q_i^d$  Reactive power demand at bus  $i$   
 $p_l^s$  Active power flow of line  $l$  at the sending end  
 $q_l^s$  Reactive power flow of line  $l$  at the sending end  
 $p_l^r$  Active power flow of line  $l$  at receiving end  
 $q_l^r$  Reactive power flow of line  $l$  at the receiving end

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