Novel congestion control algorithms for a class of delayed networks

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Abstract: Time delays are often instability sources and give rise to undesired oscillation and performance degradation, so it is necessary to consider delay in general congestion control schemes. In this paper, a new system model for wireless sensor networks (WSNs) is established in discrete time, subsystems including delay are presented, and the overall model is achieved by blending these subsystems. For controller synthesis, common quadratic Lyapunov functions and a new approach based on nonquadratic Lyapunov functions are utilized, and a controller is designed to stabilize each subsystem. The controller synthesis results are expressed in terms of linear matrix inequalities. Moreover, the performance of our proposed scheme is considered and the decay rate is guaranteed. Finally, a set of novel congestion control schemes is derived for WSNs, and the resulting closed-loop systems are globally asymptotically stable in case of queue length changes and the consequent delay changes. The simulation results are also presented to illustrate the effectiveness of our proposed method.

Key words: Congestion control, controller synthesis, nonquadratic Lyapunov stability, linear matrix inequality

1. Introduction

Congestion control is an important issue in communication networks, especially with the growing need of bandwidth, load, size, and connectivity of these networks. The aforementioned fact has necessitated the design and utilization of networks including more efficient congestion control algorithms, especially for wireless sensor networks (WSNs) [1] that play a dominant role in recent technologies.

Congestion of packets results in poor performance and low reliability of networks. Recently, a large and growing number of results have been investigated on congestion control schemes [2–19]. Due to the major role of control theory in solving different problems such as congestion, the main idea in this paper is to use control theory to design and analyze suitable congestion controllers for WSNs as a closed loop system.

Since delays are ubiquitous in networks, it is essential to consider delay in the study of different congestion control schemes. Unlike previous methods and decentralized predictive congestion control (DPCC) [9], in our proposed method, a new system model for WSNs is established in discrete time, subsystems including delay are presented, and the overall model is achieved by blending these subsystems (for further details on DPCC, please refer to Section 2). In this paper, the control signal and buffer occupancy error are considered at different time instances; however, in [9], they are only considered at time instances $n - 1$ and n, respectively.

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Maintaining system functionality despite dynamically changing and uncertain environments is very critical in WSNs. Fading is quite probable in WSNs, which causes bandwidth reduction. In this case, queue size is increased, which causes a delay increase, and satisfactory performance can be achieved if the resultant closed-loop systems are stable in the case of queue length changes and the consequent delay changes. Otherwise, performance degradation occurs and our system tends to be unstable.

In recent years, controller synthesis of different systems has been thoroughly investigated. In this regard, Lyapunov approaches are of paramount importance. Through the Lyapunov-based approaches, systematic stability study and convergence of algorithms in the context of closed-loop control systems can be achieved.

There are several different Lyapunov functions such as the so-called common quadratic Lyapunov functions [20–23], piecewise quadratic Lyapunov functions [24–26], and nonquadratic Lyapunov functions [27–29]. Common quadratic Lyapunov functions tend to be conservative and might not even exist for many complex highly nonlinear systems; however, piecewise and nonquadratic Lyapunov functions are less conservative, but their computation cost is much higher.

In this paper, common quadratic Lyapunov functions are utilized for controller synthesis of our system. Since common quadratic Lyapunov functions tend to be conservative, nonquadratic Lyapunov functions are also used. It is worth mentioning that there have been a few attempts to employ nonquadratic Lyapunov functions for controller synthesis in WSNs with the goal of reducing the conservativeness of common quadratic Lyapunov functions. Therefore, this paper presents a new approach for controller synthesis of our system based on nonquadratic Lyapunov functions. Additionally, the results are expressed in terms of linear matrix inequalities (LMIs), which are numerically feasible with commercially available software. In this paper, a controller is designed for each subsystem. Afterwards, these controllers are blended and system stability is guaranteed. Moreover, unlike [9], performance is considered and the decay rate is guaranteed.

The rest of the paper is structured as follows: the preliminaries are stated in Section 2, while the main results are presented in Section 3. The effectiveness of our approach is demonstrated through simulation results in Section 4. The paper is concluded in Section 5 and the Appendix is given in Section 6.

2. Preliminaries
In this section, DPCC [9] is briefly presented; thereafter, DPCC is compared with our scheme and our proposed model is given.

2.1. Decentralized predictive congestion control
DPCC was introduced in [9]. In this scheme, a hop-by-hop flow control is utilized. DPCC can predict the onset of congestion and reduce the incoming traffic by using a backpressure signal.

2.1.1. Model description in DPCC
Changes of buffer occupancy in terms of incoming and outgoing traffic at a particular node is given as follows:

$$q_a(n+1) = sat_p[q_a(n) + Tu_a(n) - f_a(u_{a+1}(n)) + d(n)] \tag{1}$$

where sat_p is the saturation function expressing finite size queue behavior, $q_a(n)$ is the buffer occupancy of the ath node at time instant n, T is the measurement interval, $u_a(n)$ is a regulated incoming traffic rate, $f_a(\cdot)$ is a dictated outgoing traffic by the next hop node disturbed by channel variations, and $d(n)$ is an unknown

traffic disturbance [9]. It is necessary to calculate and propagate $u_a(n)$ as a feedback to the $(a-1)^{th}$ node to estimate the outgoing traffic for this node.

Considering that q_{ad} is the desired buffer occupancy at the ath node, the buffer occupancy error defined as $e_{ba}(n) = q_a(n) - q_{ad}$ can be written as:

$$e_{ba}(n+1) = sat_p[e_{ba}(n) + Tu_a(n) - f_a(u_{a+1}(n)) + d(n)] \tag{2}$$

In this section, buffer occupancy and buffer occupancy error at a particular node are introduced [9]. In the subsequent section, the adaptive and predictive controller of [9] is briefly presented.

2.1.2. Adaptive and predictive controller

Based on the scenario in [9], if the outgoing traffic $f_a(.)$ is unknown, the traffic rate input is as follows:

$$u_a(n) = Sat_p\left(\frac{\hat{f}_a(u_{a+1}(n)) + (\kappa_{bv} - 1)e_{ba}(n)}{T}\right) \tag{3}$$

where κ_{bv} is the gain parameter and $\hat{f}_a(u_{a+1}(n))$ is the estimate of $f_a(u_{a+1}(n))$. Buffer occupancy error at time instant $n+1$ is given by:

$$e_{ba}(n+1) = Sat_p[k_{bv}e_{ba}(n) + \tilde{f}_a(u_{a+1}(n)) + d(n)] \tag{4}$$

where $\tilde{f}_a(u_{a+1}(n)) = f_a(u_{a+1}(n)) - \hat{f}_a(u_{a+1}(n))$ is the outgoing traffic estimation error. If there is no estimation error, the traffic estimate is defined as [9]:

$$\hat{f}_a(u_{a+1}(n)) = \hat{\theta}_a(n)f_a(n-1) \tag{5}$$

where $\hat{\theta}_a(n)$ is the actual vector of the traffic parameter and $\hat{f}_a(u_{a+1}(n))$ and $f_a(n-1)$ are the estimation of unknown outgoing traffic and the past value of outgoing traffic, respectively. If the parameter θ_a is updated as

$$\hat{\theta}_a(n+1) = \hat{\theta}_a(n) + \lambda u_a(n)e_{fa}(n+1) \tag{6}$$

provided that $\lambda \|u_a(n)\|^2 < 1$ and $\kappa_{fv\,max} < 1/\sqrt{\delta}$, where λ is the adaptation gain, $\kappa_{fv\,max}$ is the maximum singular value of κ_{fv}, $\delta = 1/[1 - \lambda \|u_a(n)\|^2]$, and $e_{fa}(n) = f_a(n) - \hat{f}_a(n)$, then the mean estimation error of θ_a along with queue utilization mean error converges to zero asymptotically [9].

Remark 1 *It is important to note that fading channels are ignored in Eq. (3) and congestion is detected just by monitoring the buffer occupancy, so in order to mitigate congestion due to fading channels, the rate from Eq. (3) has to be decreased. Back-off interval selection for nodes plays an important role in deciding which node can access the channel. The back-off interval at the ath node is defined as BO_a and it is denoted that $VR_a = 1/BO_a$, where VR_a is the ath node virtual rate. The ath node actual rate is:*

$$R_a(t) = \frac{B(t).VR_a(t)}{\sum_{l \in S_a} VR_l(t)} = \frac{B(t).VR_a(t)}{TVR_a(t)} \tag{7}$$

where $B(t)$ is channel bandwidth, VR_a is the ath node virtual rate, and $TVR_a(t)$ is the sum of all virtual rates for all neighbors S_a [9].

2.2. Comparison of our proposed scheme with DPCC

- Unlike in [2–8] and [10–19], congestion due to fading channels in dynamic environments is studied in both DPCC and our scheme.
- The control problem in this paper and [9] are both solved offline; however, outgoing estimation is accomplished online in both schemes.
- Since time delays are often instability sources and give rise to undesired oscillation and performance degradation, unlike previous schemes and DPCC, in our method, more realistic models including delay are introduced and a new system model for WSNs is established in discrete time. Unlike [9], the control signal and buffer occupancy error are considered at different time instances. However, in DPCC, the control signal and buffer occupancy error are only considered at time instances $n-1$ and n, respectively. Therefore, in this paper, delay is considered and some augmented states are used, which is an important difference between our scheme and [9]. Since our model includes delay, it is more realistic compared to [9], and unlike [9], controller synthesis is presented in case of delayed systems.
- Since WSNs are normally set up in adverse conditions, bandwidth reduction is quite probable (fading causes bandwidth reduction and fading is very probable in WSNs); in this case, the queue size increases, which then renders a delay increase. In our proposed method, the resultant closed-loop systems are stable in the case of queue length changes and the consequent delay changes; however, in [9], performance degradation occurs and the system tends to be unstable.
- There have been a few attempts to employ nonquadratic Lyapunov functions in the context of controller synthesis in WSNs with the goal of reducing the conservativeness of the quadratic framework. Unlike [9] and the available literature, this paper presents a novel approach for controller synthesis of our WSN system based on nonquadratic Lyapunov functions.
- The control signals in DPCC and our scheme are different from each other. In [9], the control signal is given as in Eq. (3); however, our control signals are presented in Eq. (13) and Eq. (27).
- The gain parameter k_{bv} in [9] is an important factor in the design of the controller in Eq. (3) and DPCC is highly dependent on it. However, unlike [9], our controller is not dependent on k_{bv}.

2.3. Model description in our proposed method

Network modeling is a critical issue in network traffic estimation and it is, in general, quite complex for WSNs. In this section, a novel system model for WSNs is established in discrete time.

Since queue length increases cause delay increase in a system and the change of the model, in our proposed scheme, different subsystems are introduced based on different delays and the control signal and buffer occupancy error are considered at different time instances; however, in [9], they are only considered at time instances $n-1$ and n, respectively. In this paper some augmented states are added, which is an important difference between our scheme and [9]. Since our model includes delay, it is more realistic compared to [9], and the controller synthesis is presented in case of delayed systems. The states are as follows:

$$x(n) = \begin{bmatrix} e_{ba}(n) & e_{ba}(n-1) & ... & e_{ba}(n-o) & u(n-1) & ... & u(n-p) & s(n) \end{bmatrix}^T \qquad (8)$$

where $x(n) \in \Re^z$ includes buffer occupancy errors $e_{ba}(n-o)$ and control signals $u(n-p)$ in different time instances and integrator $S(n) \in \Re$. $o.p \in N$ are the number of states (o and p are the number of states considered for buffer occupancy error and control signal, respectively).

$A_j \in \Re^{z \times z}$ and $B_j \in \Re^{z \times z}$ are considered as known constant matrices to describe our system and they show the jth subsystem. $z \in N$ is the number of state variables.

Since delay causes different subsystems to be introduced and $r \in N$ is the number of existing subsystems due to delay, we have:

$$\begin{aligned} A_j &\in \{ \ A_1 \quad A_2 \quad ... \quad A_r \ \} \\ B_j &\in \{ \ B_1 \quad B_2 \quad ... \quad B_r \ \} \end{aligned} \tag{9}$$

For further detail, please see the Appendix.

$u(n) \in \Re$ is the control signal vector and $S(n)$ is an integrator defined as:

$$\begin{aligned} S(n+1) &= S(n) + e_{ba}(n) \\ S(n) &= \sum_{0}^{n-1} e_{ba}(n) + e_{ba}(n) \end{aligned} \tag{10}$$

Since queue size increase causes a delay increase in the system and delay causes different subsystems to be introduced, in order to choose different subsystems in $A_j \in \{ \ A_1 \quad A_2 \quad ... \quad A_r \ \}$ and $B_j \in \{ \ B_1 \quad B_2 \quad ... \quad B_r \ \}$, a new parameter $\beta_j(n)$ is introduced and our proposed system is written as:

$$\begin{aligned} x(n+1) &= \sum_{j=1}^{r} \beta_j(n)(A_j x(n) + B_j u(n)) \\ \sum_{j=1}^{r} \beta_j(n) &= 1, \beta_j(n) \in \{0,1\} \end{aligned} \tag{11}$$

where $\beta_j(n)$ indicates which subsystem is chosen based on delay. $\sum_{j=1}^{r} \beta_j(n) = 1$ and $\beta_j(n)$ can be either 0 or 1.

In the subsequent section, the controller, stability, and simulation results will be addressed in detail.

3. Main results

Since time delays can lead to undesired oscillation and performance degradation, in this section, unlike [9], queue length changes and the consequent delay changes are considered and controller synthesis for our proposed system is accomplished. Conditions in which the resultant closed-loop systems are globally asymptotically stable are achieved.

In this paper, common quadratic Lyapunov functions are utilized for controller synthesis of our system. Since common quadratic Lyapunov functions tend to be conservative, nonquadratic Lyapunov functions are also used. Finally, unlike [9], performance is considered and decay rate is guaranteed.

Lemma 1 (30) : *If matrices C_m and S_m are of appropriate dimensions and S_m is positive definite, then:*

$$C_m^T S_m^{-1} C_m \geq C_m^T + C_m - S_m \tag{12}$$

By defining the control signal as

$$u(n) = [\sum_{j=1}^{r} \beta_j N_j G_j^{-1}]x(n) \tag{13}$$

one has the following result:

Theorem 1 *The system (Eq. (11)) with the control signal (Eq. (13)) is globally asymptotically stable if:*

i) *There is a symmetric positive definite matrix $P \in \Re^{z \times z}$ and matrices $G_j \in \Re^{z \times z}$ and $N_j \in \Re^{1 \times z}$ for every $j \in L$ and $L = \left\{ \begin{matrix} 1 & 2 & ... & r \end{matrix} \right\}$, such that the following LMIs (Eq. (14)) are satisfied:*

$$(A_j G_j + B_j N_j)^T P (A_j G_j + B_j N_j) - G_j^T P G_j < 0 \tag{14}$$

Or equivalently:

ii) *There is a symmetric positive definite matrix $X \in \Re^{z \times z}$ where $X = P^{-1}$ and matrices $G_j \in \Re^{z \times z}$ and $N_j \in \Re^{1 \times z}$ for every $j \in L$ such that the following LMIs (Eq. (15)) are satisfied:*

$$\begin{bmatrix} G_j + G_j^T - X & (A_j G_j + B_j N_j)^T \\ (A_j G_j + B_j N_j) & X \end{bmatrix} > 0 \, j \in L \tag{15}$$

Moreover, the controller gains are given by:

$$F_j = N_j G_j^{-1} \tag{16}$$

Proof Consider the following Lyapunov function candidate:

$$V(x(n)) = x^T(n) P x(n) \tag{17}$$

It is necessary to first check the existence of G_j^{-1}. Note that if condition (ii) of Theorem 1 (Eq. (15)) holds true, we have:

$$\begin{bmatrix} I & 0 \end{bmatrix} \begin{bmatrix} G_j + G_j^T - X & (A_j G_j + B_j N_j)^T \\ (A_j G_j + B_j N_j) & X \end{bmatrix} \begin{bmatrix} I \\ 0 \end{bmatrix} > 0 \, j \in L$$

Thus, $G_j^T + G_j - X > 0$, which can be written as $G_j^T + G_j > X$, since $X \in \Re^{z \times z}$ is considered a symmetric positive definite matrix in (Part ii, Theorem 1), and it can be concluded that $G_j^T + G_j > 0$.

If G_j is not invertible, it means that there exists a nonzero Π_j where $G_j \Pi_j = 0$ (Π_j is in the null space of G_j). Since $G_j^T + G_j > 0$, it is possible to write it as $\Pi_j^T (G_j^T + G_j) \Pi_j > 0$, which can be written as $\Pi_j^T G_j^T \Pi_j + \Pi_j^T G_j \Pi_j > 0$, or equivalently $(G_j \Pi_j)^T \Pi_j + \Pi_j^T (G_j \Pi_j) > 0$. Since $G_j \Pi_j = 0$, we have $0 + 0 > 0$, which is incorrect, so there is no Π_j where $G_j \Pi_j = 0$ and thus G_j is invertible.

It suffices to show that the following inequality is satisfied to prove that our proposed system is globally asymptotically stable. Thus:

$$\Delta V = V(x(n+1)) - V(x(n)) = x^T(n+1) P x(n+1) - x^T(n) P x(n) < 0 \tag{18}$$

Based on the system equation (Eq. (11)) and the control signal (Eq. (13)), we can easily obtain

$$x(n+1) = \sum_{j=1}^{r} \beta_j(n)(A_j + B_j N_j G_j^{-1}) x(n) \tag{19}$$

since

$$\sum_{j=1}^{r} \beta_j(n) = 1 \text{ and } \beta_j \in \{0, 1\}$$

We have

$$x(n + 1) = (A_j + B_j N_j G_j^{-1})x(n) \tag{20}$$

Substituting Eq. (20) into Eq. (18) yields:

$$\begin{aligned}
\Delta V &= x^T(n)(A_j + B_j N_j G_j^{-1})^T P(A_j + B_j N_j G_j^{-1})x(n) - x^T(n)Px(n) < 0 \\
&= x^T(n)[(A_j + B_j N_j G_j^{-1})^T P(A_j + B_j N_j G_j^{-1}) - P]x(n) < 0
\end{aligned} \tag{21}$$

which in turn implies that:

$$(A_j + B_j N_j G_j^{-1})^T P(A_j + B_j N_j G_j^{-1}) - P < 0 \tag{22}$$

Via the Schur complement lemma [31], Eq. (22) can be rewritten as:

$$\begin{bmatrix} P & (A_j + B_j N_j G_j^{-1})^T \\ (A_j + B_j N_j G_j^{-1}) & P^{-1} \end{bmatrix} > 0 \, j \in L \tag{23}$$

Pre- and postmultiplying Eq. (23) by $\begin{bmatrix} G_j^T & 0 \\ 0 & I \end{bmatrix}$ and $\begin{bmatrix} G_j & 0 \\ 0 & I \end{bmatrix}$, respectively, leads to:

$$\begin{bmatrix} G_j^T P G_j & (A_j G_j + B_j N_j)^T \\ (A_j G_j + B_j N_j) & P^{-1} \end{bmatrix} > 0 \tag{24}$$

Using the Schur complement lemma, the claimed global exponential stability result of condition (i) is established.

It follows from Lemma 1 that:

$$G_j^T P G_j \geq G_j + G_j^T - P^{-1} \tag{25}$$

Therefore, Eq. (24) can be expressed as:

$$\begin{bmatrix} G_j + G_j^T - P^{-1} & (A_j G_j + B_j N_j)^T \\ (A_j G_j + B_j N_j) & P^{-1} \end{bmatrix} > 0 \tag{26}$$

The equivalent condition (ii) follows directly by considering $P^{-1} = X$ as follows:

$$\begin{bmatrix} G_j + G_j^T - X & (A_j G_j + B_j N_j)^T \\ (A_j G_j + B_j N_j) & X \end{bmatrix} > 0 \tag{27}$$

Hence, the claim of Theorem 1 is established and the proof is completed. $\qquad\square$

Remark 2 *It is also possible to obtain some sufficient conditions for asymptotic stability by considering the same Lyapunov function and the state feedback* $u(n) = \sum_{j=1}^{r} \beta_j F_j x(n)$ *where* $F_j = N_j P$ *for some matrix* N_j. *This is accomplished if all slack variables* (G_j^{-1}) *are considered* P *in Theorem 1.*

In the subsequent theorem, the controller synthesis results are presented based on nonquadratic Lyapunov functions. By defining the control signal as

$$u(n) = \sum_{j=1}^{r} \beta_j N_j (\sum_{i=1}^{r} \beta_i G_i)^{-1} x(n) \tag{28}$$

one gets the following result:

Theorem 2 *The system (Eq. (11)) with the control signal (Eq. (27)) is globally asymptotically stable if:*

i) T*here exists a set of symmetric positive definite matrices $P_j \in \Re^{z \times z}$ and matrices $G_i \in \Re^{z \times z}$ and $N_j \in \Re^{1 \times z}$ for every $i, j \in L$ and $L = \{ \; 1 \quad 2 \quad ... \quad r \; \}$ such that the following LMIs are satisfied:*

$$(A_j G_i + B_j N_j)^T G_m^{-T} P_c G_m^{-1} (A_j G_i + B_j N_j) - P_j < 0 i, j, m, c \in L \tag{29}$$

Or equivalently:

ii) *There is a set of symmetric positive definite matrices $P_j \in \Re^{z \times z}$ and matrices $G_i \in \Re^{z \times z}$ and $N_j \in \Re^{1 \times z}$ for every $i, j \in L$ such that the following LMIs are satisfied:*

$$\begin{bmatrix} P_j & (A_j G_i + B_j N_j)^T \\ (A_j G_i + B_j N_j) & G_m + G_m^T - P_c \end{bmatrix} > 0 i, j, m, c \in L \tag{30}$$

Moreover, the controller gains are given as:

$$F_{ji} = N_j G_i^{*-1} \tag{31}$$

with the following notation, which is adopted for simplicity:

$$G_i^* = \sum_{i=1}^{r} \beta_i G_i \tag{32}$$

Proof Consider the following Lyapunov function candidate:

$$V(x(n)) = x^T(n)[G_i^{*-T}(\sum_{j=1}^{r} \beta_j P_j) G_i^{*-1}] x(n) \tag{33}$$

Following the same procedure as in Theorem 1 to prove that G_j is invertible, we can conclude that G_m and G_i are also invertible.

The difference function is given by:

$$\begin{aligned} \Delta V &= V(x(n+1)) - V(x(n)) \\ &= x^T(n+1) G_m^{*-T} \left[\sum_{c=1}^{r} \beta_c P_c \right] G_m^{*-1} x(n+1) - x^T(n) G_i^{*-T} \left[\sum_{j=1}^{r} \beta_j P_j \right] G_i^{*-1} x(n) \end{aligned} \tag{34}$$

where indices i and j are used for time instant n and indices m and c are used for time instant $n+1$.

Based on the system equation (Eq. (11)) and the control signal (Eq. (27)), we can easily obtain:

$$x(n+1) = \sum_{j=1}^{r} \beta_j (A_j + B_j N_j G_i^{*-1}) x(n) \tag{35}$$

Since $\sum_{j=1}^{r} \beta_j = 1$ and $\beta_j \in \{0,1\}$, we have:

$$x(n+1) = (A_j + B_j N_j G_i^{-1}) x(n) \tag{36}$$

Substituting Eq. (35) into Eq. (33) and considering $\sum_{c=1}^{r} \beta_c = 1$ and $\beta_c \in \{0,1\}$ yields:

$$\begin{aligned} \Delta V &= x^T(n)(A_j + B_j N_j G_i^{-1})^T (G_m^{-T} P_c G_m^{-1})(A_j + B_j N_j G_i^{-1})x(n) - x^T(n) G_i^{-T} P_j G_i^{-1} x(n) \\ &= x^T(n)[(A_j + B_j N_j G_i^{-1})^T (G_m^{-T} P_c G_m^{-1})(A_j + B_j N_j G_i^{-1}) - G_i^{-T} P_j G_i^{-1}]x(n) \end{aligned} \tag{37}$$

which can be written as:

$$\Delta V = x^T(n) G_i^{-T} [(A_j G_i + B_j N_j)^T (G_m^{-T} P_c G_m^{-1})(A_j G_i + B_j N_j) - P_j] G_i^{-1} x(n) \tag{38}$$

where the G_i^{-T} parentheses on the left and G_i^{-1} parentheses on the right of Eq. (37) are taken.

It suffices to show that the following inequality is satisfied to prove that the proposed system (Eq. (35)) is globally asymptotically stable. Therefore:

$$[(A_j G_i + B_j N_j)^T (G_m^{-T} P_c G_m^{-1})(A_j G_i + B_j N_j) - P_j] < 0 \tag{39}$$

Thus, the desired result in condition (i) is satisfied. The equivalent condition (ii) follows directly from the Schur complement as:

$$\begin{bmatrix} P_j & (A_j G_i + B_j N_j)^T \\ (A_j G_i + B_j N_j) & (G_m^{-T} P_c G_m^{-1})^{-1} \end{bmatrix} > 0 \, i,j,m,c \in L \tag{40}$$

Via the matrix inversion lemma, we have:

$$(G_m^{-T} P_c G_m^{-1})^{-1} = G_m P_c^{-1} G_m^T \tag{41}$$

Then it follows from Lemma 1 that:

$$G_m P_c^{-1} G_m^T \geq (G_m + G_m^T - P_c) \tag{42}$$

Therefore, Eq. (39) can be expressed as:

$$\begin{bmatrix} P_j & (A_j G_i + B_j N_j)^T \\ (A_j G_i + B_j N_j) & G_m + G_m^T - P_c \end{bmatrix} > 0 \, i,j,m,c \in L \tag{43}$$

Thus, the closed-loop control system (Eq. (35)) is globally exponentially stable. Moreover, the controller gains can be easily determined by Eq. (30) and the proof is completed. $\square$

In the following theorem, performance is considered and the decay rate is guaranteed.

Theorem 3 *The closed-loop control system of Eq. (35) is globally asymptotically stable with decay rate $\Phi \in \Re^{z \times z}$ if:*

i) *There is a set of symmetric positive definite matrices $P_j \in \Re^{z \times z}$ and $Q_j \in \Re^{z \times z}$ and a positive definite full rank decay rate matrix $\Phi \in \Re^{z \times z}$ and matrices $G_i \in \Re^{z \times z}$ and $N_j \in \Re^{1 \times z}$ for every $i, j \in L$ and $L = \{ \ 1 \quad 2 \quad ... \quad r \ \}$ such that the following LMIs are satisfied:*

$$(A_j G_i + B_j N_j)^T G_m^{-T} P_c G_m^{-1}(A_j G_i + B_j N_j) - (P_j - Q_j) < 0 i,j,m,c \in L \tag{44}$$

$$G_i^T \Phi^{-1} G_i - Q_j \leq 0 i, j \in L \tag{45}$$

Or equivalently:

ii) *There is a set of symmetric positive definite matrices $P_j \in \Re^{z \times z}$ and matrices $Q_j \in \Re^{z \times z}$ and a positive definite full rank decay rate matrix $\Phi \in \Re^{z \times z}$ and matrices $G_i \in \Re^{z \times z}$ and $N_j \in \Re^{1 \times z}$ for every $i, j \in L$ and $L = \{ \ 1 \quad 2 \quad ... \quad r \ \}$ such that the following LMIs are satisfied:*

$$\begin{bmatrix} P_j - Q_j & (A_j G_i + B_j N_j)^T \\ (A_j G_i + B_j N_j) & G_m + G_m^T - P_c \end{bmatrix} > 0 i,j,m,c \in L \tag{46}$$

$$\begin{bmatrix} Q_j & G_i^T \\ G_i & \Phi \end{bmatrix} \geq 0 i,j \in L \tag{47}$$

Moreover, the controller gains are as in Eq. (30) and the control signal can be rewritten as in Eq. (27).

Proof Following the same procedure as in Theorem 1 to prove that G_j is invertible, we can conclude that G_m and G_i are also invertible.

Considering the Lyapunov function candidate defined in Eq. (32), it is sufficient to show that the following inequality is satisfied to prove that the proposed system in Eq. (35) is globally asymptotically stable. In the proof of Theorem 3, it is not assumed that Eq. (46) is satisfied; however, in this theorem, it is sufficient to show that Eq. (46) is satisfied to prove that the proposed system of Eq. (35) is globally asymptotically stable.

$$\Delta V = V(x(n+1)) - V(x(n)) < -x^T(n)[G_i^{-T} Q_j G_i^{-1}]x(n) \tag{48}$$

In the following, it is proven that the right side of Eq. (46) is negative: since $x(n)$ is a nonzero arbitrary vector, in order to show that $x^T(n)[G_i^{-T} Q_j G_i^{-1}]x(n)$ is positive, we show that $G_i^{-T} Q_j G_i^{-1}$ is positive. Since $Q_j \in \Re^{z \times z}$ is symmetric positive definite matrices ($Q_j > 0$) (Theorem 3, conditions 1 and 2) and $G_i \in \Re^{z \times z}$ are invertible, so pre- and postmultiplying ($Q_j > 0$) by G_i^{-T} and its transpose, respectively, leads to $G_i^{-T} Q_j G_i^{-1} > 0$. Otherwise (if $G_i^{-T} Q_j G_i^{-1} > 0$ is not correct), it means that there is at least a nonzero $\alpha(n)$ where $\alpha^T(n) G_i^{-T} Q_j G_i^{-1} \alpha(n) \leq 0$. Considering $G_i^{-1} \alpha(n) = \eta(n)$ and substituting it in $\alpha^T(n) G_i^{-T} Q_j G_i^{-1} \alpha(n) \leq 0$, we have $\eta^T(n) Q_j \eta(n) \leq 0$. Since G_i is invertible and $\alpha(n) \neq 0$, from $G_i^{-1} \alpha(n) = \eta(n)$, we can conclude that $\eta(n) \neq 0$. Thus, if $G_i^{-T} Q_j G_i^{-1} > 0$ is not correct, there is a $\eta(n)$ where $\eta^T(n) Q_j \eta(n) \leq 0$, which is wrong (contrary to our assumption) (Q_j is symmetric positive definite matrices ($Q_j > 0$) and $\eta(n)$ is considered nonzero). Thus, there is no nonzero vector that satisfies $\eta^T(n) Q_j \eta(n) \leq 0$, and it can be concluded that $G_i^{-T} Q_j G_i^{-1} > 0$ and $x^T(n)[G_i^{-T} Q_j G_i^{-1}]x(n) > 0$, and

so $-x^T(n)[G_i^{-T}Q_jG_i^{-1}]x(n) < 0$. $\qquad\qquad\square$

Substituting Eq. (35) into Eq. (46) yields:

$$\Delta V = x^T(n)[(A_j + B_jN_jG_i^{-1})^T(G_m^{-T}P_cG_m^{-1})(A_j + B_jN_jG_i^{-1}) - G_i^{-T}P_jG_i^{-1}]x(n) < \\ -x^T(n)[G_i^{-T}Q_jG_i^{-1}]x(n) \tag{49}$$

which can be written as:

$$x^T(n)G_i^{-T}[(A_jG_i + B_jN_j)^T(G_m^{-T}P_cG_m^{-1})(A_jG_i + B_jN_j) - P_j]G_i^{-1}x(n) < -x^T(n)[G_i^{-T}Q_jG_i^{-1}]x(n) \tag{50}$$

where the G_i^{-T} parentheses on the left and the G_i^{-1} parentheses on the right of Eq. (48) are taken, which in turn implies that:

$$(A_jG_i + B_jN_j)^T(G_m^{-T}P_cG_m^{-1})(A_jG_i + B_jN_j) - (P_j - Q_j) < 0 \tag{51}$$

Therefore, the claimed global exponential stability result of Eq. (42) is established. If Eq. (42) (the condition in Theorem 3) is satisfied, since Eq. (42) is Eq. (49) in the proof of Theorem 3, it means that Eq. (49) is satisfied. If Eq. (49) is satisfied, pre- and postmultiplying Eq. (49) by $x^T(n)G_i^{-T}$ and its transpose renders Eq. (48), which can be written as Eq. (47), and considering Eq. (35) and Eq. (32), Eq. (46) can be achieved (G_i is invertible). Thus, if Eq. (42) or Eq. (49) is satisfied, it can be concluded that Eq. (46) is satisfied. If Eq. (46) is satisfied, since the right side of Eq. (46) is negative, it can be concluded that $\Delta V < 0$, and so the proposed system of Eq. (35) is globally asymptotically stable.

The corresponding equivalent condition in Eq. (44) follows directly from the Schur complement as:

$$\begin{bmatrix} P_j - Q_j & (A_jG_i + B_jN_j)^T \\ (A_jG_i + B_jN_j) & (G_m^{-T}P_cG_m^{-1})^{-1} \end{bmatrix} > 0 \, i, j, m, c \in L \tag{52}$$

Via the matrix inversion lemma, we have:

$$(G_m^{-T}P_cG_m^{-1})^{-1} = G_mP_c^{-1}G_m^T \tag{53}$$

It then follows from Lemma 1 that:

$$G_mP_c^{-1}G_m^T \geq (G_m + G_m^T - P_c)$$

And so we have:

$$\begin{bmatrix} P_j - Q_j & (A_jG_i + B_jN_j)^T \\ (A_jG_i + B_jN_j) & G_m + G_m^T - P_c \end{bmatrix} > 0 \, i, j, m, c \in L \tag{54}$$

It is noted that the following inequality guarantees the performance with the decay rate Φ:

$$G_i^{-T}Q_jG_i^{-1} \geq \Phi^{-1} \tag{55}$$

In fact, pre- and postmultiplying Eq. (52) by G_i^T and its transpose, respectively, leads to:

$$G_i^T\Phi^{-1}G_i - Q_j \leq 0 \tag{56}$$

It follows from the Schur complement that:

$$
\begin{bmatrix} Q_j & G_i^T \\ G_i & \Phi \end{bmatrix} \geq 0 i, j \in L
\tag{57}
$$

Thus, Eq. (43) and Eq. (45) are also satisfied and the claim of Theorem 3 is established, and the proof is completed.

Figure 1 depicts our proposed control scheme. Buffer occupancy and the outgoing rate are used in the controller design, and the resultant closed-loop systems are globally asymptotically stable in case of queue length changes and the consequent delay changes.
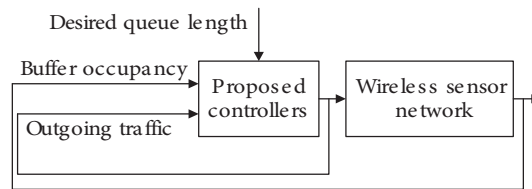
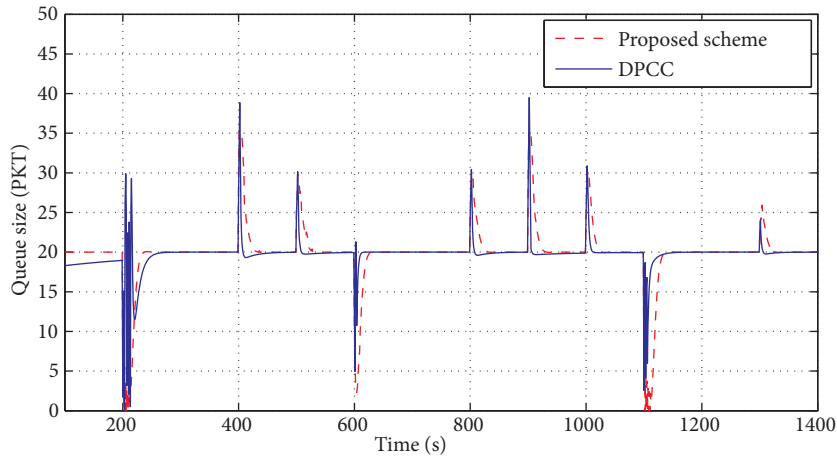**Figure 1.** Our proposed control scheme.

4. Simulation results

In this section, the performance of our proposed scheme (Theorem 2) is compared with DPCC and, in this regard, queue level changes and the control signal are evaluated in the case of outgoing flow rate variations. Thereafter, the mean error of queue size, the sent traffic, and the mean outgoing estimation error in the proposed scheme (all theorems) and DPCC are presented in the Table.

Table. Mean queue size error, sent traffic, and mean outgoing estimation error in the proposed scheme and DPCC.

	Proposed scheme Theorem 1	Proposed scheme Theorem 2	Proposed scheme Theorem 3	DPCC
Mean {Queue size error}^{2}	15.34491	12.98235	14.85314	53.77587
Mean {-Queue size error-}	1.240782	1.156574	1.239565	1.533551
Sent traffic	20035.9	20042.52	20036.81	19920.98
Mean {-Outgoing estimation error-}	0.2229	0.1883	0.2256	3.2074

In this study, o and p are both considered 4, z is 9, and r is 4 (see the Appendix). The maximum and ideal queue sizes are considered as 50 and 20 packets, respectively, and the controller parameters are $k_{bv} = 0.6$ and 0.7 and $\lambda = 0.001$.

Figures 2 and 3 illustrate queue length changes in DPCC with $k_{bv} = 0.6$ and $k_{bv} = 0.7$ and the proposed scheme (Theorem 2), and it is shown that $k_{bv} = 0.7$ leads to poor performance in DPCC. k_{bv} is an important factor in the design of the DPCC controller (Section 2.1.2), which directly affects system performance, and DPCC is highly dependent on it. Figure 4 shows the control signal. Finally, the mean queue size error, the sent traffic, and the mean outgoing estimation error in the proposed scheme (all theorems) and DPCC are presented in the Table.

Figure 2. Queue size of proposed scheme and DPCC with $k_{bv} = 0.6$.
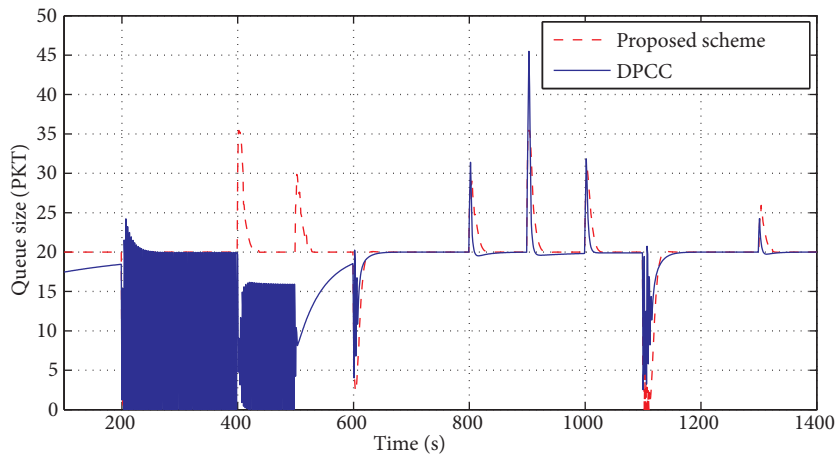
Figure 3. Queue size of proposed scheme and DPCC with $k_{bv} = 0.7$.
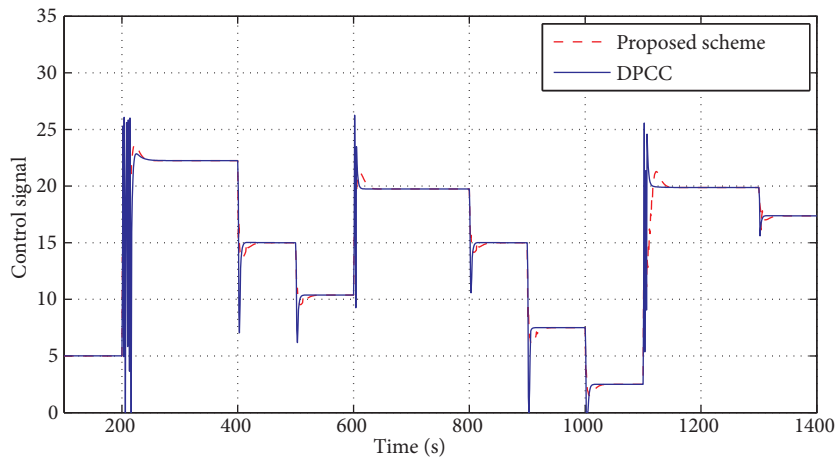
Figure 4. Control signal of proposed scheme and DPCC.

The figures and table demonstrate that our proposed schemes end in better performance in comparison with DPCC, which is due to the use of a better system model for WSNs (unlike DPCC, delay is considered in our

model). It is worth mentioning that, unlike DPCC, our resultant closed-loop systems are globally asymptotically stable in the case of queue length changes and the consequent delay changes.

Furthermore, Theorems 1 and 2 are the weakest and the best in our proposed schemes, respectively, since the latter is less conservative in comparison with the former, and this is due to the fact that nonquadratic Lyapunov functions are used in the latter; however, common quadratic Lyapunov functions are used in the former. In Theorem 3, performance is considered and the decay rate is guaranteed; however, decay rate is not guaranteed in Theorems 1 and 2 or DPCC.

5. Conclusion

In this paper, novel congestion control schemes are presented and the resultant closed-loop systems are globally asymptotically stable in the case of queue length changes and the consequent delay changes. Unlike previous schemes and DPCC, in our method, a novel system model for WSNs is established in discrete time, subsystems including delay are presented, and the overall model is achieved by blending these subsystems. In this paper, common quadratic Lyapunov functions and a new approach based on nonquadratic Lyapunov functions are utilized for controller synthesis. The controller synthesis results are expressed in terms of LMIs, which are numerically feasible with commercially available software. In our method, a controller is designed for each subsystem and, afterwards, these controllers are blended and system stability is guaranteed. Moreover, unlike DPCC, performance is considered and the decay rate is guaranteed. The extended simulation results demonstrate the superior performance of the proposed scheme in comparison with DPCC.

Appendix: Model description, numerical form

As stated in the model description, the system model is expressed as:

$$x(n+1) = \sum_{j=1}^{r} \beta_j(n)(A_j x(n) + B_j u(n))$$

$$\sum_{j=1}^{r} \beta_j(n) = 1, \beta_j(n) \in \{0, 1\}$$

In our simulations, o and p are both considered 4, z is 9, and r is 4. It is straightforward to achieve the subsequent subsystems if the delay is considered as 0, 1, 2, or 3. Based on delay, different subsystems as (A_1, B_1), (A_2, B_2), (A_3, B_3), and (A_4, B_4) are chosen in our simulations.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & T & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & T & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & T & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} T & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad B_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

$$B_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad B_4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Nomenclature

a	Index for node number
n	Time instance
i, j	Index to show subsystem in time instant n
m, c	Index to show subsystem in time instant $n+1$
$r \in N$	Number of existing subsystems due to delay
L	Set of subsystems, $i, j \in L$ and $L = \{1\ 2\ ...\ r\}$
o	Number of states considered for buffer occupancy error
p	Number of states considered for control signal
$z \in N$	Number of state variables
sat_p	Saturation function expressing finite size queue behavior
$\beta_i(n)$	Parameter to indicate which subsystem is chosen based on delay
κ_{bv}	Gain parameter
$\kappa_{fv\,max}$	Maximum singular value of κ_{fv}
λ	Adaptation gain
$\hat{\theta}_a(n)$	Actual vector of traffic parameter
Π_j	Π_j is considered in the null space of G_j
$d(n)$	Unknown traffic disturbance
$e_{ba}(n)$	Buffer occupancy error
$f_a(\cdot)$	Dictated outgoing traffic by the next hop node
$f_a(n-1)$	Past value of outgoing traffic
$\hat{f}_a(u_{a+1}(n))$	Estimate of $f_a(u_{a+1}(n))$
$\tilde{f}_a(u_{a+1}(n))$	Outgoing traffic estimation error
$q_a(n)$	Buffer occupancy of the ath node at time instant n
q_{ad}	Desired buffer occupancy at the ath node
$u_a(n)$	Regulated incoming traffic rate
$x(n) \in R^z$	New states including buffer occupancy errors and control signals in different time instances and the integrator
T	Measurement interval
N	Set of natural numbers
$\Re$	Set of real numbers
$\Re^n$	Set of n component real vectors
$\Re^{n \times m}$	Set of $n \times m$ real matrices
$B(t)$	Channel bandwidth
BO_a	Back-off interval at the ath node
$R_a(t)$	a^{th} node actual rate

VR_a $\qquad$ a^{th} node virtual rate

$TVR_a(t)$ $\qquad$ Sum of all virtual rates of all neighbors S_a

$A_j \in R^{z \times z}$ $\qquad$ Known constant matrices to describe our system

$B_j \in R^z$ $\qquad$ Known constant matrices to describe our system

$S(n) \in R$ $\qquad$ Integrator

C_m $\qquad$ Matrices of appropriate dimensions

S_m $\qquad$ Matrices of appropriate dimensions and positive definite

$F_j \in \Re^{1 \times z}$ $\qquad$ A set of $1 \times z$ real controller gain matrices, $j \in L$ and $L = \{1 \ 2 \ ... \ r \ \}$

$G_j \in \Re^{z \times z}$ $\qquad$ A set of $z \times z$ real matrices

$G_i^* \in \Re^{z \times z}$ $\qquad$ Notation adopted for simplicity $G_i^* = (\sum\limits_{i=1}^{r} \beta_i G_i)$

$P \in \Re^{z \times z}$ $\qquad$ A symmetric positive definite matrix

$P_j \in \Re^{z \times z}$ $\qquad$ A set of symmetric positive definite $z \times z$ real matrices

$N_j \in \Re^{1 \times z}$ $\qquad$ A set of $1 \times z$ real matrices

$Q_j \in \Re^{z \times z}$ $\qquad$ A set of symmetric positive definite $z \times z$ real matrices

$X \in \Re^{z \times z}$ $\qquad$ Symmetric positive definite $z \times z$ real matrices where $X = P^{-1}$

$V(x(n))$ $\qquad$ Lyapunov function candidate

ΔV $\qquad$ Difference function $\Delta V = V(x(n+1)) - V(x(n))$

$\Phi \in \Re^{z \times z}$ $\qquad$ Positive definite full rank $z \times z$ real decay rate matrix

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