

## Using the finite element method to calculate parameters for a detailed model of transformer winding for partial discharge research

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**Abstract:** Power transformers are considered to be one of the most essential and costly pieces of equipment in a power system. Identifying insulation faults in the shortest possible time prevents the occurrence of irreparable damage. Partial discharge (PD) is one of the most significant insulation faults. The first step in the study of PD is the precise modeling of transformer winding at high frequencies. In this paper, the finite element method is used to calculate the parameters for a detailed model of transformer winding. For this reason, a detailed model of transformer winding and the analytic formulations are first presented for the calculation of the parameters of the model. Using two-dimensional finite element methods, the 20-kV transformer winding is then simulated according to exact technical specifications and designed using Maxwell software. After that, the parameters for the detailed model presented in this paper are derived and calculated. Finally, the validity of the model in the frequency range is determined by applying a similar PD pulse on the real and simulated models.

**Key words:** Partial discharge, power transformers, finite element analysis, detailed model

### 1. Introduction

One of the main reasons for the complete removal of high-voltage transformers from the power system is the errors that occur in the insulating structure. Partial discharge (PD) is the most common reason for these errors, which can cause insulation corrosion over time, so that the insulation can no longer tolerate the transient and extra tension of electrical voltages. Therefore, early detection of PD can prevent complete failure of the transformer insulation and a total breakdown of the transformer. Moreover, the exact detection of where PD occurs in the insulating structure near the transformer windings is of great importance, due to the significant reduction of maintenance time and cost.

So far, quite a large number of studies have been conducted on PD location in power transformers. The main methods of PD detection and localization can be divided into 4 general methods: audio, optical, chemical, and electrical techniques. Given the many constraints that the audio, optical, and chemical methods face, these methods cannot be used as a reliable approach to detecting the exact location of the PD [1,2].

Using electrical methods of PD location, we can find quite useful information hidden in the PD signal waveforms, and in that way the exact location where the PD occurs can be recognized.

In general, the various aforementioned methods of locating PD in the transformer windings can be classified into 2 general types: positioning based on winding modeling and positioning based on pattern recognition [3].

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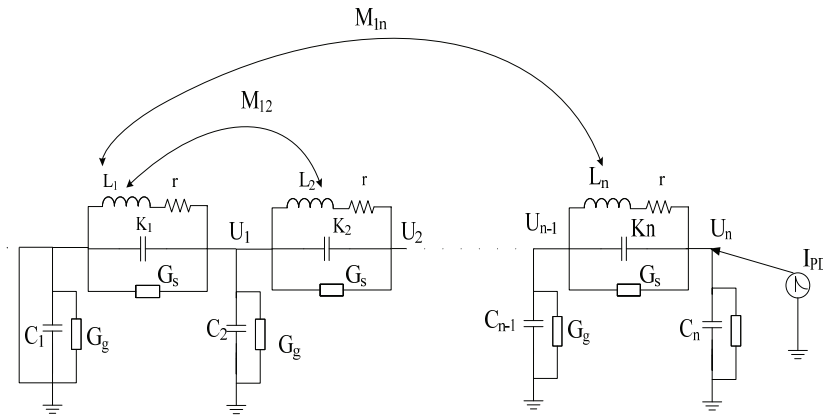
The latter type functions in such a way that the PD pulses occurring in different places have different patterns, so by extracting the appropriate features of these patterns and using pattern recognition methods, we can find the place where the PD occurs. Several studies have been done in the field of winding transformer modeling to evaluate and analyze the way PD signals are distributed for their positioning. According to these studies, there are 4 main methods for modeling the winding: modeling using transfer functions [4], the traveling wave model [5], the multiconductor transmission line model, and the detailed model [6].

The detailed model has thus far been highly regarded by researchers because it is simple to apply in modeling the high-frequency transformer windings [7]; however, in all these studies the parameters were not obtained with sufficient precision because analytical analyses were used for calculating the model parameters. Consequently, locating the PD was not done correctly. Using an analytical equation to obtain the winding parameters, especially in obtaining an inductance matrix (L), is a time-consuming process and has many limitations. This problem is even worse in windings that have more complex structures. For that reason, it is very important to provide a suitable method to find precise parameters of the detailed model in order to achieve the correct location of PD. In this paper, a new method for finding these parameters is presented.

This paper consists of 5 sections. The second section introduces the detailed model and discusses how the parameters of this model should be calculated. Section 3 explains how to obtain the inductance and resistance parameters of the model through the finite element method (FEM). In the fourth section, a comparison is made between model simulation results and experimental results in order to assess the validity of the model, and the fifth section presents the final conclusions.

## 2. Detailed model of transformer winding

A PD that occurs in a transformer has a high frequency range of kHz to MHz. In this situation, the transformer winding model consists of an RLC ladder network. Indeed, as shown in Figure 1, each section of layer transformer winding is modeled by an RLC.



**Figure 1.** A ladder network model for a transformer winding.

In the above figure,  $L_i$  is the self-inductance of section  $i$ ,  $M_{ij}$  is the mutual inductance of  $i$  and  $j$ ,  $K_i$  is the equivalent length capacity for section  $i$ ,  $C_i$  is the capacitance between section  $i$  and the earth (core or tank),  $G_s$  is the ohmic resistance indicating dielectric losses of the  $i$  section,  $G_g$  is the ohmic resistance indicating dielectric losses of section  $i$  and the ground, and  $r$  is the ohmic resistance indicating ohmic losses of section  $i$ .

We can show the occurrences of PD by a current pulse injected into the network. To analyze this network and to obtain the output waveform, we can examine each category separately. Lastly, with the super position theorem, we can obtain the final result. What follows is the calculation of the parameters of the model using the finite element method.

**2.1. Calculation of capacitance**

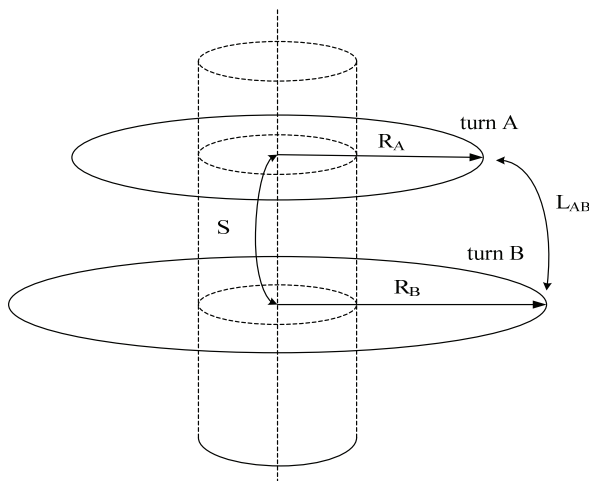
As shown in the equivalent circuit in Figure 1, there is a capacitance between the conductors of each turn, with the next turn, and between each conductor and the ground. In the layer winding, the capacitance between each turn and the next turn is calculated based on Eq. (1):

$$C_T = \frac{\epsilon_0 \epsilon_P \times \pi D_m (w + t_p)}{t_p} \tag{1}$$

In this equation,  $D_m$  is the winding diameter,  $w$  is the width of the conductor axis,  $t_p$  is the paper thickness in both sides of the conductor,  $\epsilon_0$  is the dielectric coefficient of air, and  $\epsilon_p$  is the relative dielectric coefficient of the paper [8,9].

**2.2. Calculation of inductance**

In layer winding, as shown in Figure 2, mutual inductance between 2 circular conductors whose radius is  $R_A$  and  $R_B$  (respectively) and the distance between them is  $S$  can be obtained via Eq. (2).



**Figure 2.** Mutual inductance between 2 circular conductors.

$$L_{AB} = \frac{2\mu_0}{k'} \sqrt{R_A R_B} [K(k') - E(k')] \tag{2}$$

In the above equation:

$$k' = \frac{1 - \sqrt{1 - k^2}}{1 + \sqrt{1 - k^2}} \tag{3}$$

Also:

$$K = \sqrt{\frac{4R_A R_B}{(R_A + R_B)^2 + S^2}} \tag{4}$$

$K(k')$  and  $E(k')$  are full integration of parabolic types 1 and 2. Self-inductance of each winding is also calculated from the following equation:

$$L_{AA} = \mu_0 R_A \left( \ln \frac{16R_A}{D} - 1.75 \right) \quad (5)$$

In this equation,  $\mu_0$  is the correlation coefficient of the permeability of a vacuum;  $R_A$  and  $D$  are the radius and the diameter of the turn. Radius  $R_A$  is calculated as a geometrical mean distance of the turn.

### 2.3. Calculation of resistance

To calculate the series resistance per unit length, the following equation can be used:

$$R = \frac{1}{2(h+w)} \sqrt{\frac{\pi f \mu}{\sigma}} \quad (6)$$

Here,  $w$  and  $h$  are the dimensions of a rectangular cross-section conductor,  $\mu$  is the conductor permeability coefficient,  $\sigma$  is the conductivity of the conductor, and  $f$  is the frequency. Note that the above equation is obtained by considering the skin effect.

## 3. Calculation of parameters using the FEM

The FEM is a numerical method that is used for the approximate calculation of partial differential equations and integrals. The basic function of this method is simplifying the equations into ordinary equations, which can then be calculated by numerical methods such as Euler's. The important point in calculating the partial equations is obtaining a simple equation that is numerically constant. In other words, errors converge to zero in several steps of analysis. There are several methods with different advantages and disadvantages for doing this. The FEM is one of the best methods.

Accuracy in calculating self-inductance and mutual inductance has a major effect on modeling transformer winding. Due to the large number of calculations, in many cases short circuit inductance is used in modeling instead of self-inductance and mutual inductance, which causes errors in simulation studies and in locating PD in transformer windings. As mentioned before, the software used in this study in order to obtain the parameters of a detailed model is Maxwell 2D, which solves these problems based on finite element analysis.

### 3.1. Calculation of the capacitance matrix

In order to calculate the capacitance matrix of transformer winding, a set of electrostatic field simulations will be carried out by the software. In each simulation, the electrostatic field applies a voltage to one of the conductors and the voltage applied to other conductors will be considered zero. Eq. (7) shows the energy stored in the electrical field, which is related to the capacitance between 2 conductors [10,11].

$$W_{ij} = \frac{1}{2} \int_{\Omega} D_i E_j d\Omega \quad (7)$$

In this equation,  $W_{ij}$  is the energy of the electrical field and is related to the flux lines connecting conductor  $i$  to conductor  $j$ .  $D_i$  is the electrical flux density where voltage is applied to conductor  $i$ , and  $E_j$  is the electrical field in a state where voltage is applied to conductor  $j$ . The capacitance between conductors  $i$  and  $j$  is thus

obtained via Eq. (8).

$$C = \frac{2W_{ij}}{\nu^2} = \int_{\Omega} D_i E_j d\Omega \tag{8}$$

### 3.2. Calculation of the of the impedance matrix

In Maxwell 2D software, impedance matrix calculations are done in two parts, in such a way that first the calculations of the inductance matrix model are done and then those of the resistance matrix will be done. Finally, after the calculation of the aforementioned matrixes, the result is presented in the form of an impedance matrix using the  $Z = R + j\omega L$  equation. In order to calculate the resistance and inductance matrixes, the eddy current field simulator is used so that in each stage, the circuit of each of the conductors is considered equal to 1 A and the circuit of the other conductors is considered equal to zero.

Eq. (9) represents the energy stored in the magnetic field which couples the two conductors [11].

$$W_{ij} = \frac{1}{2} LI^2 = \frac{1}{2} \int_{\Omega} B_i H_j d\Omega \tag{9}$$

In this equation,  $W_{ij}$  is the energy stored in the magnetic field that couples conductors i and j, I is the current of conductor i, and  $B_i$  is the density of the magnetic field of 1 A flowing through conductor j.

$$L_{ij} = \frac{2W_{ij}}{I^2} = \int_{\Omega} B_i H_j d\Omega \tag{10}$$

In order to calculate the resistance using the software, the ohmic losses (p) are calculated according to Eq. (11).

$$P = \frac{1}{2\sigma} \int_V \vec{J} \cdot \vec{J} \times dV \tag{11}$$

In the above equation,  $\sigma$  is the conductivity of the conductor and J is the current density. Resistance is then calculated based on Eq. (12), which shows the relationship between resistance and ohmic losses [12].

$$P = RI_{RMS}^2 \Rightarrow R = \frac{P}{I_{RMS}^2} = \frac{2P}{I_{Peak}^2} \tag{12}$$

The simulator considers the peak of current flowing in each conductor as equal to 1 A, so the resistance is equal to 2p. It should be noted that due to the consideration of the skin effect on eddy current field, the resistance obtained from this method is higher than the DC resistance.

In order to evaluate the method, we have made use of a 20-kV transformer winding whose characteristics are shown in the Table.

**Table.** The basic characteristics of the 20-kV transformer winding.

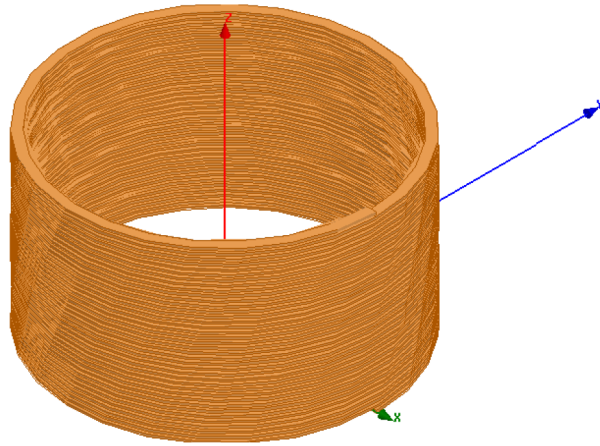
Type of winding	Layer
Number of turns	38 turns
Winding diameter	26 cm
Height of winding	16 cm
Conductor width	8 mm
Conductor height	2 mm

Figure 3 represents the outward view of the 20-kV transformer winding.

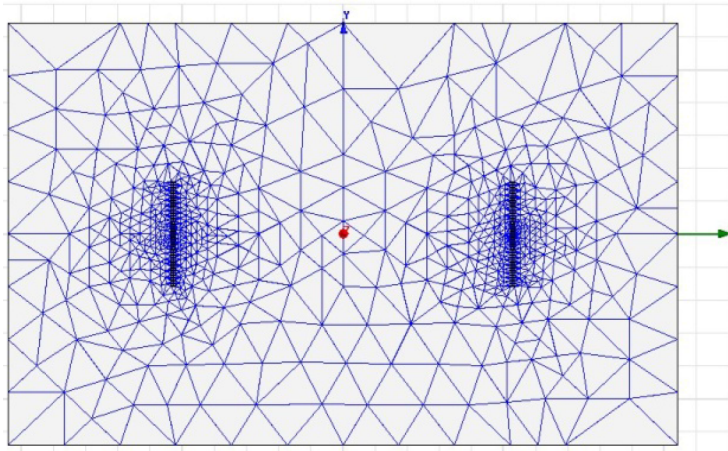
Then 20-kV transformer winding is simulated by Maxwell software. Figure 4 shows a 3D view of the simulated transformer in Maxwell software. Figure 5 also demonstrates the meshing around conductors, which is obtained by 2D analysis.



**Figure 3.** The 20-kV transformer winding.



**Figure 4.** Three-dimensional view of the 20-kV transformer winding simulated via Maxwell software.



**Figure 5.** Classification of 2D mesh of the winding.

Figure 6 is a diagram that shows the saved energy around the different turns of transformer winding. As mentioned before, this energy is used to calculate capacitance. Figure 7 also demonstrates the flux density around the different turns of winding.

All the aforementioned parameters have been calculated for a 20-kV transformer winding. For example, Figure 8 shows self-inductance and mutual inductance per length unit (H/m) in terms of turn for the 10th turn in 3 different frequencies, i.e. 100 kHz, 500 kHz, and 1 MHz.

As shown in Figure 8, the maximum amount of inductance is for the 10th turn, which is self-inductance for this turn. On the other hand, it can be concluded that by increasing the frequency, inductance does not change significantly. Figure 9 demonstrates resistance per length unit (ohm/m) in terms of turn for the 10th turn in those frequencies.

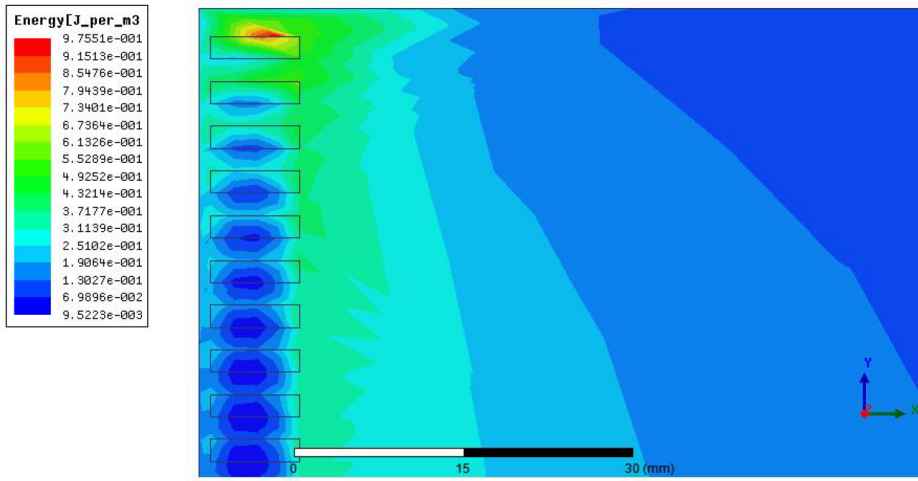


Figure 6. Saved energy around the different turns of transformer winding.

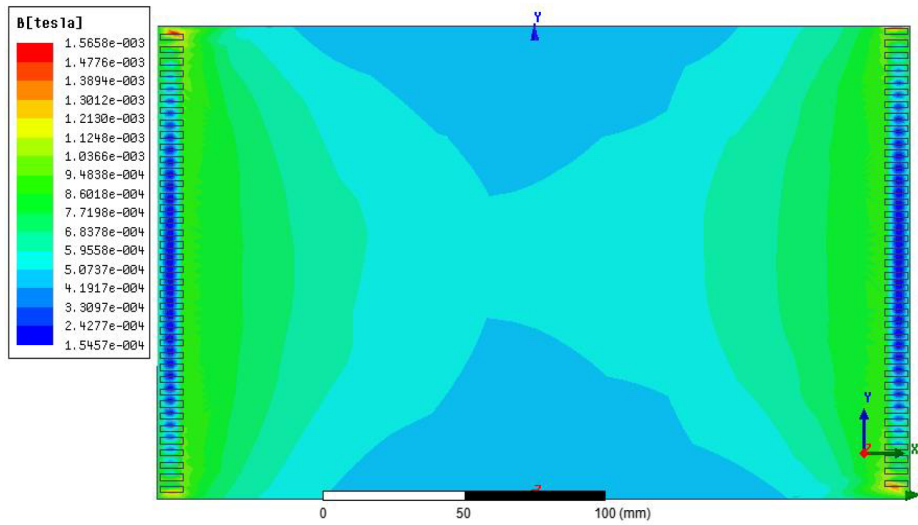


Figure 7. Magnetic flux density around the different turns of winding.

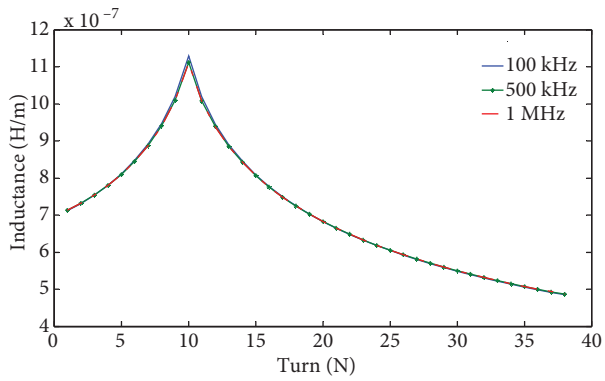


Figure 8. Inductance diagram (H/m) in terms of turn for the 10th turn of winding.

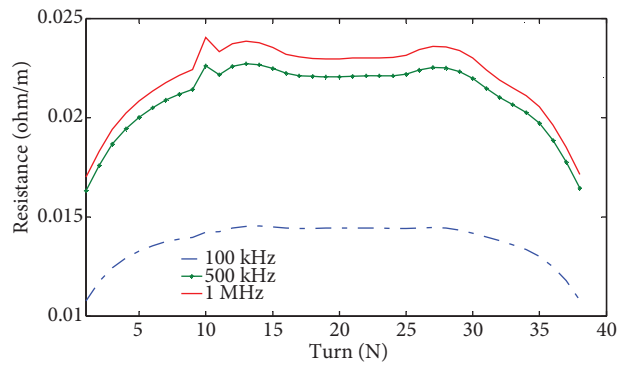


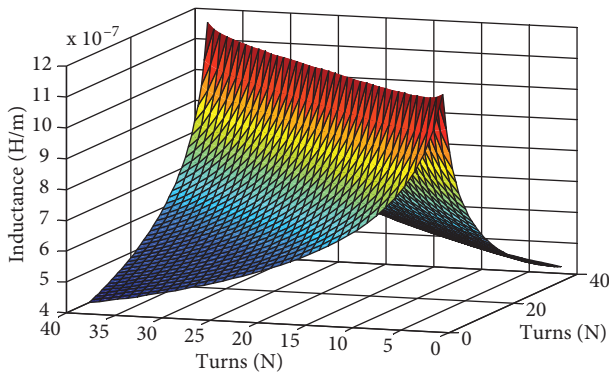
Figure 9. Diagram of resistance (ohm/m) in terms of turn for the 10th turn of winding.



In this figure, as frequency increases, resistance increases too, due to the skin effect in the conductor. As mentioned before, self-inductance and mutual inductance have been calculated for all the turns of a 20-kV transformer winding, which results in a  $38 \times 38$  matrix. The three-dimensional diagram of this matrix in terms of turn is shown in Figure 10.

**4. Comparison of simulation results and experimental**

As was mentioned before, FEM is used in this paper to determine the parameters of the detailed model of transformer winding. After determining the parameters of the model, the accuracy of the model is examined in order to study PD. Figure 11 shows the test done in a lab setting.

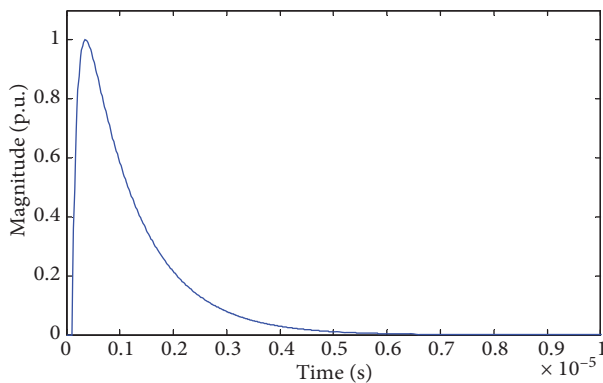


**Figure 10.** Inductance diagram for 38 turns of the winding.

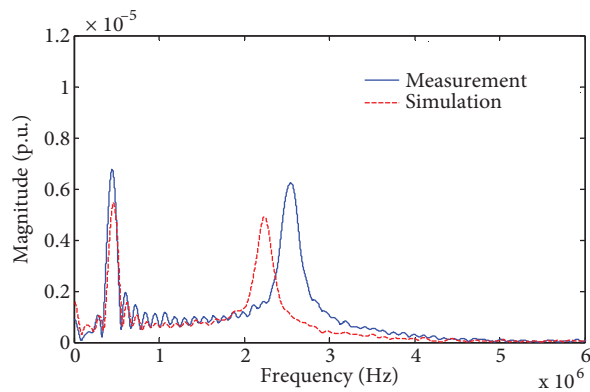


**Figure 11.** A view of the test done on a 20-kV transformer winding in a lab setting.

For this purpose, as shown in Figure 12, a PD pulse with a duration time of  $1.5 \mu s$  and rise time of  $0.4 \mu s$  is injected into both ends of different turns by a PD calibrator. The output voltage signal is then measured from a bushing end. It is worth noting that this has done on a 20-kV transformer winding in a laboratory. Additionally, in MATLAB software, this pulse is applied to the simulated model, which was the result of FEM analysis.



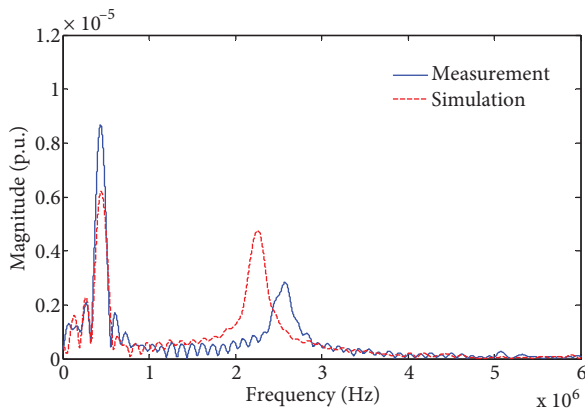
**Figure 12.** PD pulse applied to the winding.



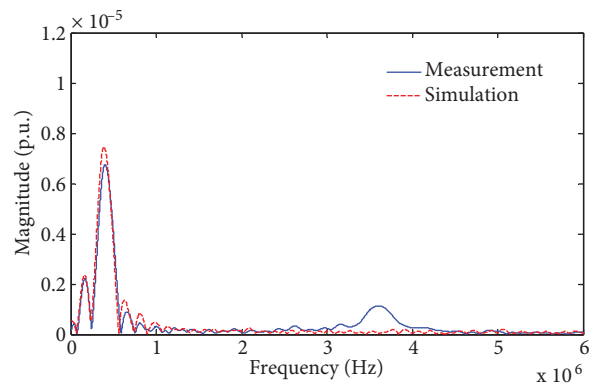
**Figure 13.** The blue diagram shows the output voltage signal of a 20-kV transformer winding when a PD pulse is applied to the 10th turn of winding and the red diagram shows the simulated waveform in the frequency domain.



Figures 13, 14, and 15 show the output voltage signal that results from measurement and simulation after PD pulse injection to the 10th, 18th, and 33rd turns, respectively, in the frequency domain. The resonance frequencies of output voltage signals and their magnitudes in these frequencies are important parameters that show the behavior of winding at high frequencies. As shown in these figures, in measurement and simulation results, the resonance frequencies and the magnitudes of these frequencies are close to each other under 1 MHz. As the frequency increases to more than 1 MHz, the difference between the resonance frequencies and the amplitude in these frequencies increases. This is due to the validity of the detailed model, which is valid for the range below 1 MHz. Finally, comparison between the simulated and measured results shows that the proposed method is an appropriate method for modeling transformer winding at high frequencies. It is also suitable for PD research.



**Figure 14.** The blue diagram shows the output voltage signal of a 20-kV transformer winding when a PD pulse is applied to the 18th turn of winding and the red diagram shows the simulated waveform in the frequency domain.



**Figure 15.** The blue diagram shows the output voltage signal of a 20-kV transformer winding when a PD pulse is applied to the 33rd turn of winding and the red diagram shows the simulated waveform in the frequency domain.

## 5. Conclusion

In this paper, transformer winding is simulated by a detailed model. The function of a detailed model in the studies of transformers' internal phenomena, especially PD, is highly dependent on the accuracy of the parameters. In practical terms, it is usually impossible to calculate with high precision the parameters of a model due to the innate limitations of the model and the inevitable approximations that exist in the calculation formulas for detailed model parameters. This paper has obtained the parameters of a detailed model based on the FEM in Maxwell software. A comparison of the results obtained from a test on a 20-kV winding and the results of simulation clearly suggest the accuracy and validity of this method.

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