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Active and reactive power transmission loss allocation to bilateral contracts through game theory techniques

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Abstract: Transmission loss has a considerable effect in overall power generation. For fairly distributing the charge of losses to generators and consumers in a deregulated power system, the allocation of this loss is very important. Game-theoretic methods seem fairer for share determination of each participant of a coalition with no discrimination. In this paper, the active and reactive power transmission losses are allocated to bilateral transactions simultaneously through load flow calculations and cooperative game theory solutions. The loss allocation problem and each bilateral transaction are treated as a game and a player of the game, correspondingly. Two game theory-based approaches, the Shapley value and the τ-value, are surveyed. The former is the most relevant game theory allocation method, while the latter is a novel approach. The influences of all loss allocation game players and bilateral bargains on transmission loss are considered. These two proposed methods are applied to a simple 6-bus network and the modified IEEE 57-bus test system. In the 6-bus network positive MVA loss allocations and in the IEEE 57-bus system negative MVA loss allocations are studied. Finally, the results of allocation procedures are compared to each other.

Key words: Bilateral transaction, cooperative game theory, loss allocation, Shapley value method, τ-value method

1. Introduction

With the introduction of energy market concepts and the privatization of electric power systems, which occurred in the early 1980s, most power systems in the world started to move from traditional, monopoly, and vertically integrated structures to competitive markets. For trading electric power in open electrical energy markets, bilateral and pool-based methods are considered. Bilateral transactions are usually long-term agreements determined through individual negotiations between a buyer and a seller [1].

The electric power industry is undergoing a series of challenging changes due to deregulation and competition. One of the most important issues is the allocation of transmission losses among market participants [2]. Different proposals for the allocation of the cost of losses in electricity networks have appeared in the last years for transmission and distribution systems. As a result of the increasing range of agents with open access to transmission networks and the massive quantity of losses concerned, efforts are concentrated on transmission systems [3]. Generally, the transmission loss that is produced by all transactions in the system accounts for 3%–5% of total generation [4]. Thus, the process of loss allocation is important. It determines whether the extra charge can be fairly allocated to each bilateral transaction [5].

Several methods for transmission loss allocation have been recently proposed. One of the earliest methods

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was the pro-rata approach. This method was used for loss allocation in [6]. Losses are allocated to loads and generators with a proportional allocation rule where the system configuration has not been considered. On the other hand, this technique does not have the capability to allocate negative losses. In [7], a proportional sharing method was proposed. This method uses the results of power flow in addition to the linear proportional sharing principle. Another power flow-based method is the loss incremental method, which accurately assesses the impact of a transaction on an area or utility. This approach was proposed in [8]. Some methods use circuit theories for loss allocation. In [9], a loss allocation method based on the circuit theories and the concept of orthogonal projection in deregulated power systems was presented. Meanwhile, [10] introduced a new method for allocating losses in a power system using a loop-based representation of system behavior. Using this method, network behavior is formulated as a series of presumed power transfers directly between market participants. Abdelkader presented a complex power flow tracing transmission loss allocation method that determines the share of each load or generator. Therefore, two algorithms for upstream tracing and downstream tracing of the complex power were proposed [11]. Some other methods, such as bus-wise method based on path-integrals [4], modified Z-bus [12], usage-based [13], and artificial intelligence [14] solutions for loss allocation, were also surveyed lately.

Furthermore, several proposals for transmission loss allocation using cooperative game theory were presented in the literature. It was claimed that these methods allocate losses in a fair manner. In [2], fair schemes for transmission loss allocation under a pool-based electricity market based on equivalent current injection and the Shapley value were proposed. The power generation or loads associated with the market were modeled as individual current injection based on a real-time AC power flow solution. In [3], an analysis and discussion based on cooperative game theory for the allocation of the cost of losses to generators and demands in transmission systems were presented. Other solution concepts such as the Shapley value, the bilateral Shapley value, and the kernel were also explored. Du et al. proposed a nucleolus theory-based method for power loss allocation under bilateral transactions, where the model was put forward and compared with a Shapley value-based allocation method [5]. Complex loss allocation to generators and loads based on circuit theory and Aumann–Shapley method was investigated in [15]. The allocation was calculated for each transmission branch, identifying and quantifying the buses' responsibilities in losses.

The most widespread methods for pay-off allocation in cooperative game theory are the Shapley value and the τ-value [16]. In this paper, transmission active and reactive loss allocation of bilateral transactions simultaneously using cooperative game theory concepts and load flow studies is presented. The Shapley value methodology is one of the most commonly used and fairest techniques for allocation problems. The Shapley value method was compared to other methods in a myriad of papers and has been used for illustration of other methods' efficiency [2,3,5,15,17]. Although [3] illustrated that it is not possible to find an optimal solution for loss allocation for consumers of a network, the Shapley value can always be found. To such an extent, the results of the proposed method are compared to the most pertinent game theory allocation method, the Shapley value.

The contribution of this paper is threefold. First, it offers a novel game theoretic-based method that considers the power system configuration in allocating loss to generators and loads. This is because of the compilation of load flow study and game theoretic tools. In other words, if the power network topology changes, the load flow results change too, and the loss portion for each bilateral contract varies accordingly. Second, it provides active and reactive loss allocation simultaneously without any additional calculations. This advantage is derived from the characteristics of the load flow entity from which MVA loss is calculated; hence, the time for loss allocation calculations is reduced. Third, the allocated loss in this paper does not depend on the amount involved in the bilateral contract. This advantage is derived from the characteristics of game theory features. Hence, the proposed method is better than other methods from previous works.

The organization of this paper is as follows. Section 2 and Section 3 introduce the basis of the Shapley value and τ-value theories, respectively. While in Section 2 the presentation of the Shapley value method is brief and restricted to its application to the loss allocation problem, in Section 3 a more detailed presentation of the τ-value method as a solution in a generic cooperative game is given, as well as the method's properties. The novel contributions of this work with regards to previously proposed methods is that we use the τ-value method as a novel approach for loss allocation problems and active and reactive MVA loss allocated simultaneously to bilateral contracts. In the next section, two case studies, the 6-bus and IEEE 57-bus power systems, are considered. It is to be noted that in the former case study, the 6-bus network, because of simplicity in the topology of the power system and the existence of few transactions, all calculations can be exhibited. The second case study, the IEEE 57-bus network, yields negative loss allocation and only brief results are given. Active and reactive power losses for bilateral contracts in the aforesaid systems are allocated and the results of the two methods are compared to each other.

2. Shapley value method

The Shapley value method is one of the basic methods to solve cooperative game models. Because of the uniqueness of the results of this method, it is considered as a basic method to solve cooperative game models. The Shapley value is utilized in the literature; therefore, this method will be explained briefly. For a detailed description about this method, see [16]. The loss allocation in an electrical network with bilateral contracts can be formulated by the Shapley value method. The loss allocated to each contract is calculated using Eq. (1):

$$\phi_i\left(v\right) = \sum_{S:i\notin S} \frac{|S|!\left(n-1-|S|\right)!}{n!}\left[v\left(S\cup\{i\}\right)-v\left(S\right)\right], \tag{1}$$

where n is the number of bilateral transactions. The above summation that allocates the loss to transaction i contains $2^{(n-1)}$ addends. These addends are related to all of the coalitions of transactions in the electric network, except the ones that contain transaction i. Each coalition is a set of bilateral contracts that have been used in the electrical market. S is a group of bilateral transactions not containing transaction i, and $|S|$ is the number of bilateral transactions in set S. $v(S)$ shows the transmission loss related to all of the bilateral transactions of set S while $v(S\{i\})$ shows the transmission loss related to the transactions of set S and transaction i.

$[v(S)-v(S\{i\})]$ represents the incremental loss when transaction i is added to the bilateral transmissions of set S. In other words, $[v(S)-v(S\{i\})]$ is the marginal loss related to the entrance of transaction i into the electrical market.

$\phi_i\left(v\right)$ is the loss allocated to transaction i. The Shapley value considers all of the coalitions for which transaction i is available and assesses them. The flow diagram of the Shapley value method is given in Figure 1.

The same flow diagram has been used in order to calculate the reactive power loss allocated to all of the transactions. The difference is the calculation of reactive power loss of the network in each coalition, except the calculation of active power loss.

The losses allocated to each bilateral transaction are related to its marginal loss. The weighted sum of the marginal losses determines the loss allocated to the transaction. These lead to more fair and acceptable cost allocations to each transaction.

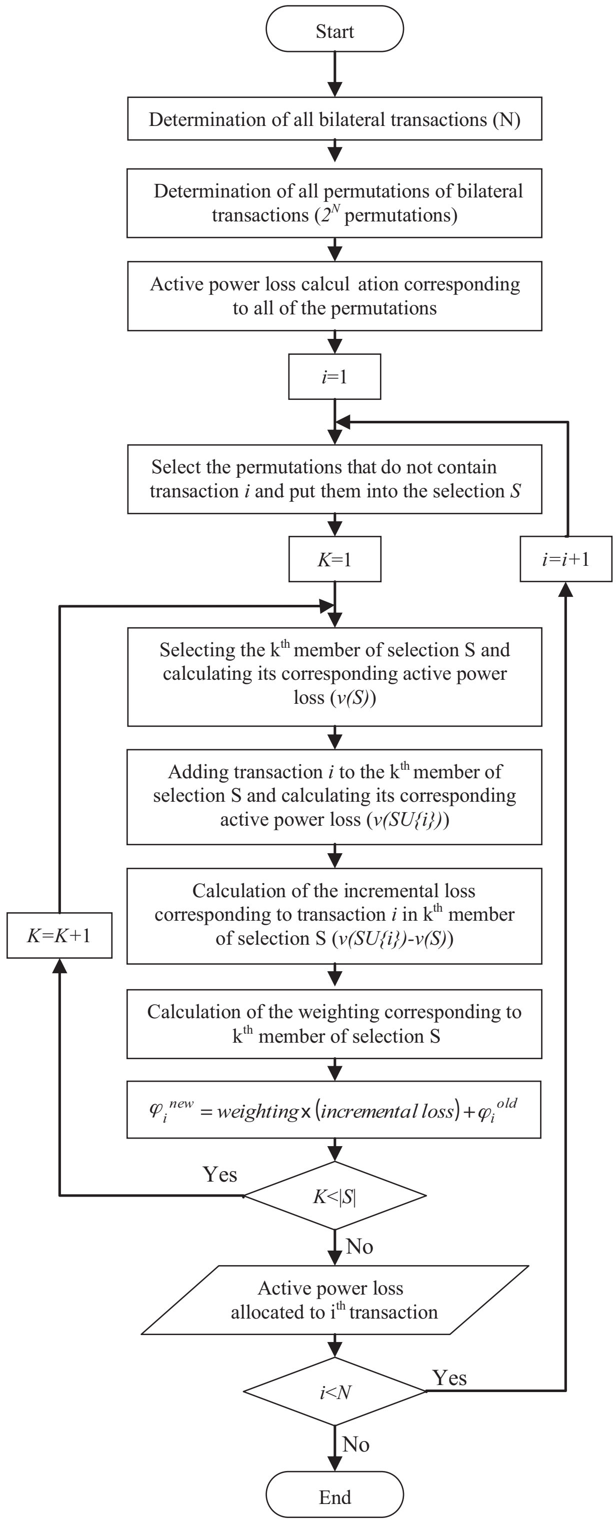

Figure 1. Flow diagram of the Shapley value active power loss allocation method.

3. $\tau-$Value method

The τ-value is one-point solution concept in cooperative game theory. This method is defined for each quasibalanced game and was introduced in [16]. This value is based on the upper vector $M(N,v)$ as the marginal contribution and the lower vector $m(v)$ as the minimum right payoff of game $v \in G^N$.

In [16], game $v \in G^N$ is called quasibalanced game if:

$$m\left(v\right) \leq M\left(N,v\right), \tag{2}$$

$$\sum_{i=1}^{n} m_i\left(v\right) \leq v\left(N\right) \leq \sum_{i=1}^{n} M_i\left(N,v\right). \tag{3}$$

The set of $|N|$-person quasibalanced games will be denoted by $v \in G^N$.

The upper vector $M(N,v)$ is calculated using Eq. (4) [18]:

$$M_i\left(N,v\right) = v\left(S\right) - v\left(S-\{i\}\right), \tag{4}$$

where S is a subset of N, which is called a coalition, and $v(S)$ is the worth of the coalition S in the game. The vector $M(N,v)$ is called the upper vector of game v because it is an upper bound for the core of game v.

For calculation of the lower vector $m(v)$, Eq. (5) is used:

$$m_i\left(v\right) = M_i\left(N,v\right) - \lambda_i\left(v\right), \tag{5}$$

where $M_i\left(N,v\right)$ is the upper vector calculated in Eq. (4) and $\lambda_i\left(v\right)$ is a concession vector for which $\lambda_i\left(v\right) = \min\limits_{S:i\in S} g\left(v,S\right)$ for all $i \in N$, where $g\left(v\right)$ is called the gap function and $g\left(v,S\right)$ is the gap of the coalition S in v and can be calculated as:

$$g\left(v,S\right) = \sum_{i\in S} M_i\left(N,v\right) - v\left(S\right) \text{ for all } S \subset N. \tag{6}$$

For game $v \in G^N$ the τ-value, $\tau(v)$, is defined by:

$$\tau_i\left(v\right) = \alpha m_i\left(v\right) + \left(1-\alpha\right) M_i\left(N,v\right), \tag{7}$$

where $\alpha \in [0,1]$ is uniquely determined by $\sum\limits_{i\in N} \tau_i\left(v\right) = v\left(N\right)$ or the τ-value is given by the following equation:

$$\tau_i\left(v\right) = M_i\left(N,v\right) - \frac{g\left(v,N\right)}{\sum\limits_{K=1}^{N} \lambda_K\left(v\right)} \lambda_i\left(v\right). \tag{8}$$

Since the τ-value is a game theory method that allocates a unique value to each player, this method has been used to allocate transmission loss to bilateral contracts. In active and reactive loss allocations, the value of upper vector $M(N,v)$ for each bilateral contract is the transmission loss in the last contribution condition. The lower vector $m(v)$ is the minimum right pay-off corresponding to each transaction.

The τ-value is the allocated transmission loss to each transaction. In Figure 2, the flow diagram of the τ-value approach is shown. For calculation of the reactive power loss allocated to each bilateral contract the similar flow diagram depicted in Figure 2 can be used.

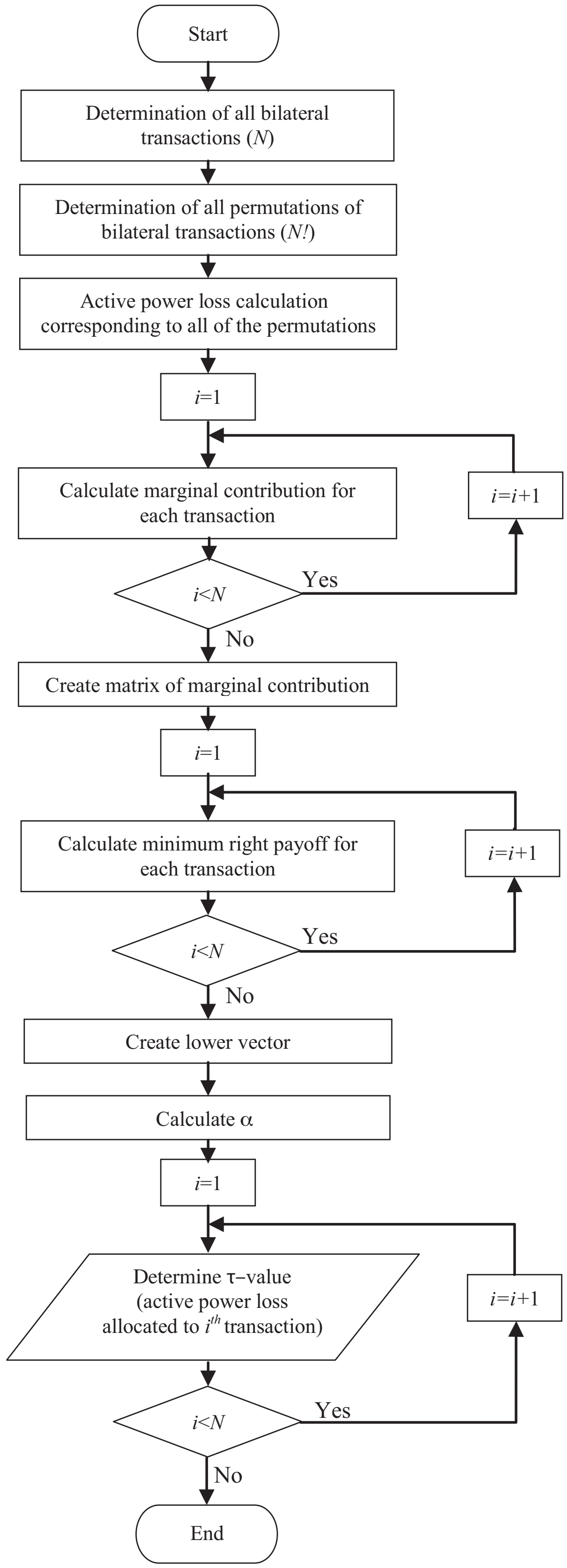

Figure 2. Flow diagram of the τ-value approach for active power loss allocation.

The τ-value method has the following properties [18]:

1. Individual rationality is $\tau_i(v) \geq v(\{i\})$ for all $i \in N$ and $v \in G^N$;
2. Coalition rationality is $\tau_i(v) + \tau_j(v) \geq v(\{i,j\})$ for all $i,j \in N$ and $v \in G^N$;
3. Efficiency is $\sum_{i \in N} \tau_i(v) = v(N)$ for all $v \in G^N$;
4. Symmetry is $\theta_v \in Q^N$ and $\tau_{\theta(i)}(v) = \tau_i(v)$ for all $i \in N$ and any permutation $\theta : N \to N$;
5. Dummy player is if $\tau_i(v) = v(\{i\})$ for all $v \in G^N$ and for all dummy players i in v, i.e. players $i \in N$ such that $v(S \cup \{i\}) = v(\{i\}) + v(S)$ for all $S \in 2^{N/\{i\}}_{;}$
6. S-equivalence property is $w \in Q^N$ and $\tau(w) = k\tau(v) + d$ whenever $w = kv + d$ with $k > 0$ and $d \in R^n$.

4. Test cases

4.1. Six-bus network

In order to apply the proposed method, a simple 6-bus network is used. The single-line diagram of the 6-bus sample system is shown in Figure 3.

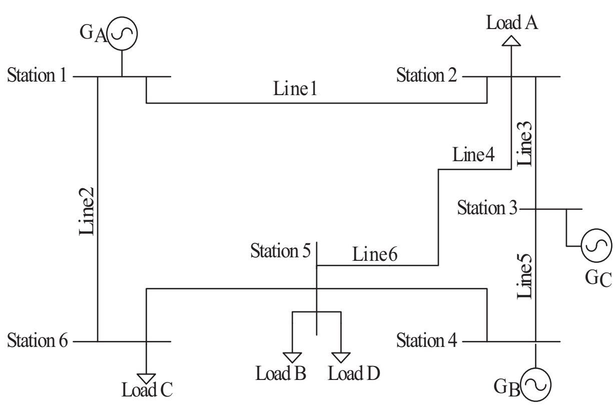

Figure 3. Six-bus single-line diagram.

The sample system contains 3 generators, 4 loads, and 7 transmission lines, and the parameters of the transmission lines are listed in Table 1. Three bilateral contracts are considered in the power network. The rest loads are under pool operation.

Table 1. Parameters of branch of 6-bus system.

Line name	From node	To node	R (Ω)	X (Ω)	B (Ω)
Line 1	Station1	Station2	2	10	0
Line 2	Station1	Station6	3	11	0
Line 3	Station2	Station3	0.5	4	0
Line 4	Station2	Station5	2	8	0
Line 5	Station3	Station4	0.5	3	0
Line 6	Station4	Station5	2	8	0
Line 7	Station5	Station6	1.5	6	0

The bilateral contracts are transacted between 2 generators, GA and GB, and 3 loads, Load A, Load B, and Load C. The transactions are as follows: generator GA contracts bilateral transaction bargains with Load A and Load B, and Generator GB contracts bilateral transaction bargains with Load B. Detailed specifications of bilateral transactions are given in Table 2.

Table 2. Specification of transactions.

Type	Transaction number	User	Supplier	Load quantity	
				P (MW)	Q (MVAR)
Pool operation	-	Load A	GA, GB, GC	110	50
	-	Load D	GA, GB, GC	100	40
Bilateral contracts	T1	Load A	GA	20	20
	T2	Load B	GB	80	35
	T3	Load C	GA	100	50

Supposing that the system is without any bilateral contracts, the transmission loss of the system is equal to 0.385 MW and 1.957 MVAR. If all of the bilateral contracts are considered, 1.969 MW and 8.502 MVAR loss occurs in the power system. Based on the load flow results, the transmission losses due to bilateral contracts are equal to 1.584 MW and 6.546 MVAR and should be allocated to the bilateral contracts. The game theory methods, the Shapley value and the τ-value, have been used to allocate the losses fairly to bilateral transactions. In this paper the transmission loss related to pool base contracts have not been considered.

The transmission loss allocation of the power system and each bilateral transaction can be regarded as a game theory problem and players, respectively. The set of all players in the game is represented as $N = \{T1, T2, T3\}$, which has 8 subsets as below:

$$\{\varphi\}, \{T1\}, \{T2\}, \{T3\}, \{T1, T2\}, \{T1, T3\}, \{T2, T3\}, \{T1, T2, T3\}.$$

For each subset load flow is performed by DIgSILENT software and total transmission loss is calculated. The results are summarized in Table 3.

Table 3. Transmission losses of sample system for each coalition.

Transaction	Total loss P (MW)	Total loss Q (MVAR)	Subtracted loss P (MW)	Subtracted loss Q (MVAR)
No. transaction	0.385	1.957	0	0
T1	0.479	2.415	0.095	0.458
T2	0.944	4.507	0.559	2.550
T3	1.130	4.822	0.745	2.865
T1 & T2	1.021	4.891	0.636	2.934
T1 & T3	1.306	5.630	0.922	3.674
T2 & T3	1.810	7.766	1.426	5.810
T1 & T2 & T3	1.969	8.503	1.584	6.546

In Table 3, subtracted values for overall active and reactive losses in each coalition are equal to corresponding active and reactive losses minus losses when no bilateral contract is in service.

4.1.1. Shapley value approach for transmission loss allocation

For the calculation of active and reactive power loss we need to determine all coalitions, which are shown in Table 3. According to Figure 1 and Eq. (1), the active power loss allocated to the first bilateral contract can be calculated as shown below.

$$\begin{aligned} APL_{T1} &= \sum_{S:i\notin S} \frac{|S|!\,(n-1-|S|)!}{n!}\left[v\left(S\cup\{i\}\right)-v\left(S\right)\right] \\ &= \frac{0!\times 2!}{3!}\left[v\left(\{T1\}\right)-v\left(\{\varphi\}\right)\right]+\frac{1!\times 1!}{3!}\left[v\left(\{T1,T2\}\right)-v\left(\{T2\}\right)\right] \\ &\quad +\frac{1!\times 1!}{3!}\left[v\left(\{T1,T3\}\right)-v\left(\{T3\}\right)\right] \\ &\quad +\frac{2!\times 0!}{3!}\left[v\left(\{T1,T2,T3\}\right)-v\left(\{T2,T3\}\right)\right] \\ &= 0.127 \end{aligned}$$

Allocated transmission active and reactive power loss can be calculated similar to the above calculation. These values are given in Table 4.

Table 4. Loss allocation in 6-bus sample system.

Transaction no.	$\tau-$value method		Shapley value method		Percent deviation	
	Active power loss (MW)	Reactive power loss (MW)	Active power loss (MW)	Active power loss (MW)	Active power loss (MW)	Reactive power loss (MW)
T1	0.125	0.589	0.127	0.127	1.57	1.34
T2	0.608	2.702	0.611	0.611	0.49	0.33
T3	0.851	3.255	0.847	0.847	–0.47	–0.53
Sum	1.584	6.546	1.584	6.546	0.00	0.00

4.1.2. τ-Value Approach for Transmission Loss Allocation

In order to calculate the portion of each bilateral contract, by means of the τ-value solution, the upper vector $M(N,v)$ and the lower vector $m(v)$ must be produced. The upper vector $M(N,v)$ in the loss allocation problem is the matrix of marginal contribution. For calculation of the upper vector, Table 5 is used.

In Table 5, the active and reactive loss portion of each contract with consideration of permutations is shown. From Table 5, the upper vector for active losses and reactive losses for each bilateral contract are $M_P\left(N,v\right)=\left(0.159,0.663,0.948\right)$ and $M_Q\left(N,v\right)=\left(0.737,2.872,3.612\right)$, respectively. The upper vector can also be calculated using Eq. (4), namely:

Table 5. Loss of each contract for marginal contribution.

Transactions \ Permutations		T2	T3	T1	T3	T1	T2
		T3	T2	T3	T1	T2	T1
		T1	T1	T2	T2	T3	T3
T1	P (MW)	0.159	0.159	0.095	0.176	0.095	0.559
	Q (MVAR)	0.737	0.737	0.458	0.808	0.458	2.550
T2	P (MW)	0.559	0.680	0.663	0.663	0.541	0.077
	Q (MVAR)	2.550	2.944	2.872	2.872	2.476	0.384
T3	P (MW)	0.867	0.745	0.827	0.745	0.948	0.948
	Q (MVAR)	3.260	2.865	3.216	2.865	3.612	3.612

$$M_{P3}(N,v) = V_P(\{T1,T2,T3\}) - V_P(\{T1,T2\}) = 1.584 - 0.636 = 0.948.$$

From Eq. (5), the lower vector $m(v)$, which indicates the minimum right pay-off of the bilateral contracts, is equal to $m_P(v) = (0.095, 0.559, 0.763)$ and $m_Q(v) = (0.458, 2.550, 2.937)$. Subscripts P and Q are used for active and reactive power losses. The calculation for determining the lower vector is as follows below.

$g_P(v,\{T1,T2,T3\}) = \sum_{i\in S} M_{Pi}(N,v) - v(\{T1,T2,T3\})$

$= 0.159 + 0.663 + 0.948 - 1.584 = 0.185$

$g_P(v,\{T1,T2\}) = \sum_{i\in S} M_{Pi}(N,v) - v(\{T1,T2\})$

$= 0.159 + 0.663 - 0.636 = 0.185$

$g_P(v,\{T1,T3\}) = \sum_{i\in S} M_{Pi}(N,v) - v(\{T1,T3\})$

$= 0.159 + 0.948 - 0.922 = 0.185$

$g_P(v,\{T1\}) = \sum_{i\in S} M_{Pi}(N,v) - v(\{T1\})$

$= 0.159 - 0.095 = 0.064$

$\lambda_{Pi}(v) = \min_{S:i\in S} g_P(v,S) = \min\{0.185, 0.185, 0.185, 0.064\} = 0.064$

$m_{P1}(v) = M_{P1}(N,v) - \lambda_{P1}(v) = 0.159 - 0.064 = 0.095$

Substituting the upper and lower vector into Eq. (7), the value of α is calculated as follows:

$$\alpha_P(0.095 + 0.559 + 0.763) + (1-\alpha_P)(0.159 + 0.663 + 0.948) = 1.584 \rightarrow \alpha_P = 0.526.$$

With determination of α the allocated losses for active and reactive power are shown in Table 4. For instance, the calculation for the first bilateral contract active power loss is as follows:

$$\tau_{P3}(v) = \alpha_P m_{P3}(v) + (1-\alpha_P) M_{P3}(N,v) = 0.526 \times 0.095 + (1 - 0.526) \times 0.159 = 0.125.$$

The contents of Tables 3–5 will be totally different for the same system under the same operating conditions merely by changing the reference bus. In other words, the loss allocated is dependent on the selection of the reference bus.

The allocated loss through the mentioned methods does not depend on the amount involved in the bilateral contract. To prove this claim, the largest bilateral contract, *T3*, is divided into two bilateral contracts. The loss allocations of the divided bilateral contracts are then summed. The calculation shows the same losses for one bilateral contract and one bilateral contract divided as two bilateral contracts.

4.1.3. Results properties

As stated above, for the specifications of the Shapley value and τ-value methods, the results for active and reactive power loss allocation have the following properties:

1. Individual rationality,

2. Coalition rationality,

3. Efficiency.

The individual rationality property means that the allocated losses, i.e. active and reactive power loss, for each transaction are greater than losses when a corresponding transaction exists. For instance, the individual rationality property for transaction 1, *T1*, for the τ-value method is shown below.

$\tau_{P1}(v) \geq v_P(\{T1\}) \rightarrow 0.125\ \text{MW} \geq 0.095\ \text{MW}$

$\tau_{Q1}(v) \geq v_Q(\{T1\}) \rightarrow 0.589\ \text{MVAR} \geq 0.458\ \text{MVAR}$

The coalition rationality property shows that active and reactive allocated losses to each transaction coalition are greater than created losses when corresponding transactions are in service. From Tables 3 and 4, this property for second and third transactions T2 and T3 is shown below.

$\tau_{P2}(v) + \tau_{P3}(v) \geq v_P(\{T2, T3\})$

$\rightarrow 0.611 + 0.847\ \text{MW} \geq 1.426\ \text{MW}$

$\rightarrow 1.458\ \text{MW} \geq 1.426\ \text{MW}$

$\tau_{Q2}(v) + \tau_{Q3}(v) \geq v_Q(\{T2, T3\})$

$\rightarrow 2.711 + 3.238\ \text{MVAR} \geq 5.810\ \text{MVAR}$

$\rightarrow 5.949\ \text{MVAR} \geq 5.810\ \text{MVAR}$

These values are calculated by the Shapley value method. Furthermore, the τ-value method results meet the coalition rationality property.

The final property, efficiency, is shown such that the sum of all allocated losses to each transaction is equal to the coalition with all transactions. With a glance at the last rows of Tables 3 and 4, this property is confirmed.

It is thought that the Shapley value method seems fairer than the other game-theoretic methods as all contributions of all participants are mentioned, and so it is widely used to compute shares of each participant of a coalition with no discrimination [19]. According to Table 4, transmission losses allocated to each transaction through the applied methods in the 6-bus power system are very close to each other. The worst case that has the most deviation for both active and reactive loss allocation is taken for the first transaction. These values are 1.57% and 1.34% for active and reactive power losses, respectively, and percent deviations are shown in Table 4; the differences in results are negligible.

4.2. Negative loss allocation in IEEE 57-bus system

The Shapley value (Section 2) and τ-value method (Section 3) are used for solving the loss allocation problem. These methods have been applied to the IEEE 57-bus test system shown in Figure 4 [20].

In this network, 5 bilateral transactions between generation units and loads have been considered. Table 6 represents the bilateral transactions.

Table 6. Bilateral contracts assigned in electricity market.

Transaction no.	Generator bus	Load bus	Active power (MW)	Reactive power (MVAR)
T1	2	1	30	10
T2	3	7	40	15
T3	6	50	21	10.5
T4	9	47	15	5
T5	12	9	70	20

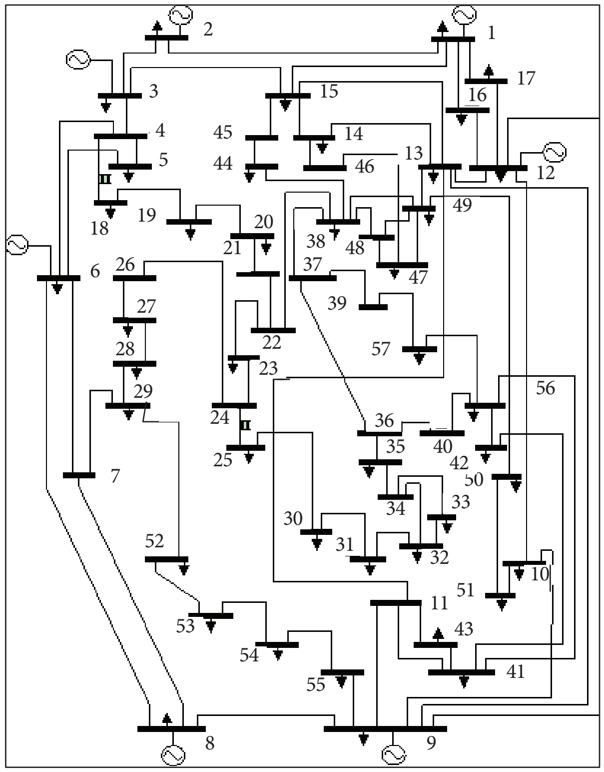

Figure 4. IEEE 57-bus test system.

Data of the test system are shown in Table 7. In this case, for some transmission lines the PI model with shunt capacitance is used. Some values of the IEEE 57-bus parameters have been modified to reach conditions for negative loss allocation purpose.

In order to calculate the share of each bilateral transaction with the τ-value method, it is necessary to calculate the last contribution $(M_i(N,v))$ and the minimum right pay-off $(m_i(v))$ corresponding to that transaction using Eqs. (4) and (5). These values for all of the transactions are listed in Table 8.

The results of the Shapley value and the τ-value method are illustrated in Figures 5 and 6. In this case study, according to Figures 5 and 6, the last bilateral transaction yields negative loss allocation. One of the advantages of bilateral contracts may be transmission loss reduction if both sides of the contract, the generator and load, are electrically far from each other and the power supplier side is near the local loads and the power consumer side is close to another power generation source. Accordingly, the fifth bilateral transaction, which is between the generator and the load connected to bus 12 and bus 9 relatively where it meets the aforementioned conditions, causes a reduction in power flow through transmission lines. This power flow decrement in transmission lines makes for a lower overall transmission loss.

The total transmission loss of the IEEE 57-bus network through these methods is 0.498 + j 3.470 MVA. The first transaction loss allocation results for active and reactive power have the most deviation. These values are 32.67% and 40.44% for active and reactive power losses, correspondingly. The other active and reactive losses allocated to each bilateral transaction have lower deviation than the first one; the average of these values is 7.12%.

Table 7. Line data of 57-bus IEEE sample network.

Line number	Sending bus	Receiving	Branch resistance R (Ω)	Branch resistance R (Ω)	Line charging B (μS)
1	1	2	1.513	5.103	707.819
2	2	3	5.431	15.491	448.834
3	3	4	2.041	6.670	208.505
4	4	5	11.391	24.057	141.564
5	4	6	7.837	26.973	190.947
6	6	7	3.645	18.590	151.440
7	6	8	6.178	31.529	257.888
8	8	9	1.804	9.204	300.686
9	9	10	6.725	30.600	241.427
10	9	11	4.702	15.455	119.616
11	9	12	11.810	53.764	423.594
12	9	13	8.766	28.796	222.771
13	13	14	2.406	7.910	60.357
14	13	15	4.903	15.838	126.200
15	1	15	3.244	16.585	542.112
16	1	16	8.274	37.544	299.588
17	1	17	4.338	19.683	156.927
18	3	15	2.952	9.659	298.491
19	4	18	0.0	101.149	0.0
20	4	18	0.0	78.368	0.0
21	5	6	5.504	11.682	68.038
22	7	8	2.533	12.976	106.447
23	10	12	5.048	23.000	179.973
24	11	13	4.064	13.341	103.155
25	12	13	3.244	10.571	331.413
26	12	16	3.281	14.817	118.519
27	12	17	7.235	32.623	261.180
28	14	15	3.116	9.969	81.207
29	18	19	84.017	124.841	0.0
30	19	20	51.577	79.097	0.0
31	21	20	0.0	141.554	0.0
32	21	22	13.414	21.323	0.0
33	22	23	1.804	2.770	0.0
34	23	24	30.254	46.656	46.091
35	24	25	0.0	215.420	0.0
36	24	25	0.0	224.168	0.0
37	24	26	0.0	8.620	0.0
38	26	27	30.071	46.292	0.0
39	27	28	11.263	17.387	0.0
40	28	29	7.618	10.698	0.0
41	7	29	0.0	11.810	0.0
42	25	30	24.604	36.815	0.0
43	30	31	59.414	90.578	0.0
44	31	32	92.401	137.599	0.0
45	32	33	7.144	6.561	0.0
46	34	32	0.0	173.684	0.0
47	34	35	9.477	14.216	17.558
48	35	36	7.837	9.787	8.779

Table 7. Continued.

Line number	Sending bus	Receiving	Branch resistance R (Ω)	Branch resistance R (Ω)	Line charging B (μS)
49	36	37	5.285	6.670	0.0
50	37	38	11.864	18.389	10.974
51	37	39	4.356	6.907	0.0
52	36	40	5.468	8.493	0.0
53	22	38	3.499	5.376	0.0
54	11	41	0.0	136.505	0.0
55	41	42	37.726	64.152	0.0
56	41	43	0.0	75.087	0.0
57	38	44	5.267	10.662	10.974
58	15	45	0.0	18.990	0.0
59	14	46	0.0	13.395	0.0
60	46	47	4.192	12.393	17.558
61	47	48	3.317	4.246	0.0
62	48	49	15.200	23.510	26.337
63	49	50	14.598	23.328	0.0
64	50	51	25.260	40.095	0.0
65	10	51	0.0	12.976	0.0
66	13	49	0.0	34.810	0.0
67	29	52	26.280	34.081	0.0
68	52	53	13.887	17.933	0.0
69	53	54	34.227	42.282	0.0
70	54	55	31.566	41.280	0.0
71	11	43	0.0	27.884	0.0
72	44	45	11.372	22.635	21.948
73	40	56	0.0	217.789	0.0
74	56	41	100.784	100.055	0.0
75	56	42	38.728	64.517	0.0
76	39	57	0.0	246.949	0.0
77	57	56	31.712	47.385	0.0
78	38	49	20.959	32.258	16.461
79	38	48	5.686	8.784	0.0
80	9	55	0.0	21.961	0.0

Table 8. Required values for τ-value method.

Transaction	mi(v)		Mi(N,v)		λi	
T1	1.034	2.933	0.310	0.704	–0.725	–2.229
T2	1.803	6.939	1.432	5.818	–0.371	–1.121
T3	1.423	6.370	0.692	4.129	–0.731	–2.241
T4	1.180	4.826	0.449	2.585	–0.731	–2.241
T5	–2.359	–9.625	–2.742	–10.746	–0.382	–1.121
Sum	3.081	11.444	0.141	2.490	–2.940	–8.954

5. Conclusion

The most important motivation in a power system that causes moves from a conventional and vertically integrated structure to a deregulated one is competition. In deregulated power system transmission loss, the

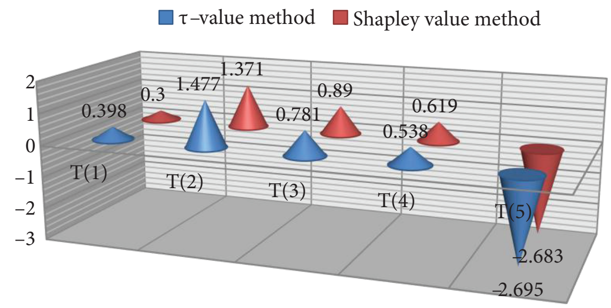

Figure 5. Transmission active loss allocation results of the Shapley value and τ-value method in IEEE 57-bus system.

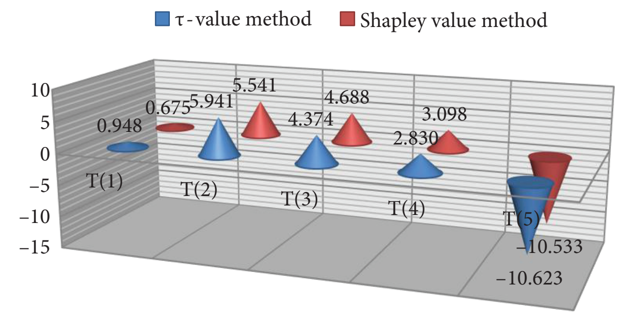

Figure 6. Transmission reactive loss allocation results of the Shapley value and τ-value method in IEEE 57-bus system.

portion of each supplier and consumer should be allocated. In this paper, active and reactive power loss allocation for bilateral transactions based on cooperative game theory, the concepts of the Shapley value and the τ-value, and the load flow analysis have been proposed. The basis and properties of the methods were then discussed. It was shown that these methods try for the best efforts in allocating transmission loss to each participant fairly and firmly so each contract must be responsible only for contribution in transmission loss. The difference between these two approaches is that in the Shapley value method all possible permutations and their relevant load flow studies must be considered, while the τ-value method is based on the minimum right pay-off and the last contribution matrices together with related load flow calculations. These two methods have been applied to a 6-bus test system and a modified IEEE 57-bus test system. Transmission losses allocated by the cooperative game theory techniques were compared to each other and it was discernable that the results were very close to each other. The Shapley value solution is reputed to be the fairest approach in allocation problems. Therefore, the τ-value solution is proper for active power transmission loss allocation to bilateral contracts in conjunction with reactive power loss allocation. Additionally, the allocated loss through the aforementioned methods does not depend on the amount involved in the bilateral contract.

Nomenclature

N	Grand coalition
φ	Empty coalition
$v \in G^N$	Represents a game
n	Number of bilateral transactions or number of players
S	A subset of N that is called a coalition or a group of bilateral transactions/players not containing transaction/player i
$\|S\|$	Number of bilateral transactions/players in set S
$v(S)$	Transmission loss related to all of the bilateral transactions of set S or worth of the coalition S in the game
$v(S\{i\})$	Transmission loss related to the transactions/players of set S and transaction/player i
$\phi_i(v)$	Loss allocated to transaction i
$M(N,v)$	Upper vector or marginal contribution
$m(v)$	Lower vector or minimum right payoff
$\lambda_i(v)$	Concession vector
$g(v)$	Gap function
$g(v,S)$	Gap of the coalition S in v
α	Coefficient
$\tau_i(v)$	$\tau-$ value or loss allocated to transaction i

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