Turkish Journal of Electrical Engineering & Computer Sciences

http://journals.tubitak.gov.tr/elektrik/

Research Article

Turk J Elec Eng & Comp Sci
(2016) 24: 262 – 275

doi:10.3906/elk-1308-150

# Modeling based on 3D finite element analysis and experimental study of a 24-slot 8-pole axial-flux permanent-magnet synchronous motor for no cogging torque and sinusoidal back-EMF

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**Received:** 25.08.2013 • **Accepted/Published Online:** 04.11.2013 • **Final Version:** 01.01.2016

**Abstract:** This paper discusses cogging torque minimization techniques in surface-mounted, axial-flux, multirotor permanent magnet synchronous disc motors in order to eliminate cogging components and obtain sinusoidal back-EMF for low-speed applications. Cogging torque minimization techniques are examined in detail with a focus on magnet skewing and pole and rotor shifting in order to obtain zero cogging. Some magnet shape and rotor design alternatives with conventional skew planar magnets were investigated based on 3D finite element analysis (FEA). Improvements compared to unskewed reference axial-flux permanent magnet (AFPM) motor design are presented. A prototype AFPM motor was built and tested based on the analyses. Results obtained from FEA and experimental study are well matched and it was shown that zero cogging and sinusoidal back-EMF can be obtained even for integer slot axial-flux PM motors with the proposed low-cost cogging torque minimization methods.

**Key words:** Axial-flux permanent magnet motor, cogging torque reduction, magnet skewing, pole shifting, magnet shifting, rotor shifting

## 1. Introduction

Permanent magnet (PM) motors are used increasingly in numerous applications today due to their high efficiency, high torque density, small size, weight, and reliability. Although magnet cost looks like an important drawback as opposed to conventional AC and DC motors, the system cost including motor and drive is comparable to standard motors [1,2].

PM motors can be classified into 2 main categories, axial-flux and radial flux PM motors. Radial-flux PM (RFPM) motors are very common and used in various industrial applications. However, axial-flux PM (AFPM) motors are generally used in special applications where large diameter and short stack are required [3–5]. In addition, such disc motors are suitable for applications where a very high number of poles is needed [6].

Although such disc motors have various advantages over conventional PM or AC induction motors if they are designed correctly, special attention is required to torque quality since it becomes a major factor, especially in low-speed applications. Therefore, torque quality issues, including cogging and ripple torque components, must be examined in detail during the design process of AFPM synchronous motors [5–10].

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Torque pulsations in PM motors have 3 major components: cogging torque, torque ripple, and reluctance torque. Cogging torque is generated by the permeance variation between magnets of the rotor and stator slots, while torque ripple occurs from the interaction between the stator magneto-motor force and current excitation. If there is no reluctance variation on the rotor as in surface-mounted PM motors, then no reluctance torque component exists on the torque output. Among these components, cogging torque requires more attention, especially for integer slot motors where slots per pole per phase is an integer.

Some techniques to reduce the cogging torque component exist in the literature and these methods are grouped into two main categories, rotor- and stator-side modifications [11–15]. Stator-side modification methods that can be applied to AFPM motors are slot opening, dummy slots, change slot-pole number ratio configurations, and displaced slots. All these changes on AFPM stators are possible but the manufacturing cost becomes significant. Therefore, stator-side modifications in disc motors are not preferred. However, rotor-side modification methods are easy to implement and cheaper. Some of these techniques include planar magnet skewing, changing the magnet pole arc ratio, magnet or pole shifting, and rotor shifting. Among these methods, magnet skewing is probably the most effective but underrated technique on the rotor side and it has not received much interest in AFPM motors in the literature. Some studies exist in the literature about such skewed magnet design [16]. However, no detailed investigation exists about optimized magnet shape and rotor design with planar skewed magnets in multiple-rotor AFPM motors with integer slots-per-pole-per-phase.

In this study, a TORUS type axial-flux PM synchronous machine with distributed winding (also called a double-rotor single-stator AFPM motor), as shown in Figure 1, was used and designed specifically for maximum cogging torque. This design was a reference motor design and was used in comparison with the designed skewed magnet cases to illustrate the design improvements. Some cogging torque reduction techniques were applied to the design motor, with a focus on magnet skew. Optimized magnet skew and rotor and pole shifting with skewed magnets were applied to the rotor in order to obtain zero cogging components and sinusoidal induced voltage. All rotor designs were examined with 3D finite element analysis (FEA) before prototype rotors were built. The setup system for measuring the cogging torque component and back-EMF was built and the prototypes were tested. The results obtained from the experimental data and 3D FEA were in good agreement. It was also demonstrated that zero cogging torque and sinusoidal back-EMF is possible in AFPM disc motors, even with an integer number of slots-per-pole-per-phase, if the correct magnet skewing techniques are used in the disc rotor.

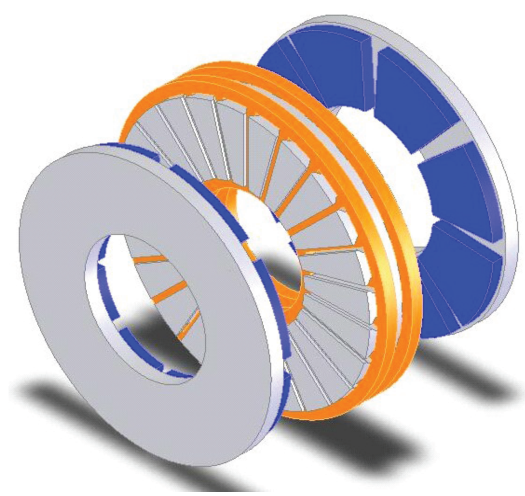


**Figure 1.** TORUS type AFPM motor with distributed windings.

## 2. Axial flux PM motor prototype

The prototype motor used in this study had a double-rotor and one-stator configuration with 24 slots and 8 poles. This is also called an NN type TORUS machine in the literature. The motor was designed so that the cogging torque was maximum with integer slot/pole/phase. The actual picture of the reference AFPM and the standard TORUS motor with fan-shaped magnets with a pole/arc ratio of 0.778 is given in Figure 2. The machine had a peak cogging torque of 8 Nm, which was about 21.2% of the average torque. Variations of cogging torque obtained from FEA and the experimental system are both shown in Figure 3. As seen from this figure, FEA results and experimental data are in excellent agreement. As the reference AFPM motor has 24 slots with 8 poles, the periodicity of cogging torque is 15 mechanical degrees and this is also displayed in Figure 3.

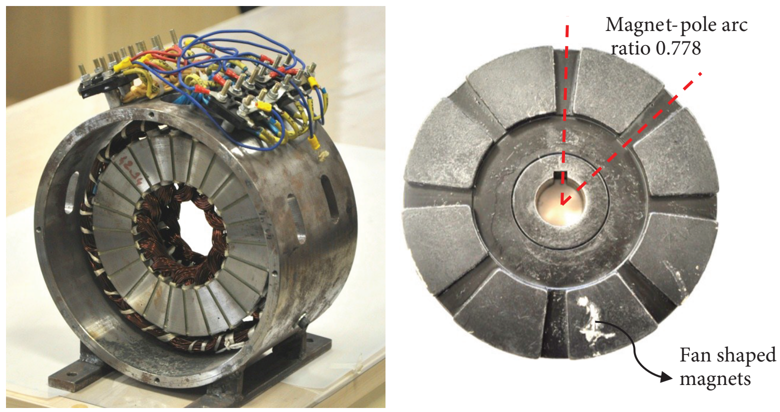



**Figure 2.** Reference double-rotor single-stator axial-flux PM motor.

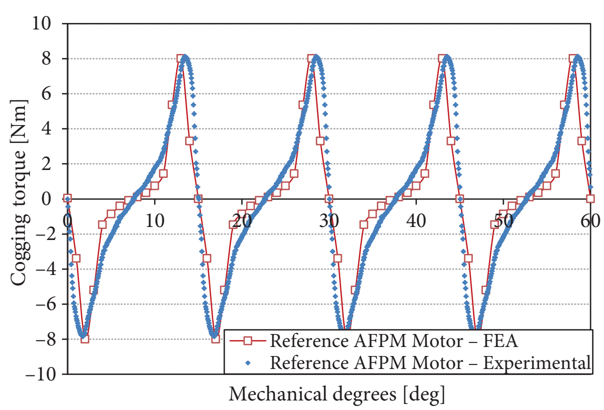



**Figure 3.** Cogging torque waveform of reference AFPM motor (FEA and experimental data).

The reference AFPM motor parameters with fan-shaped magnets and the design details are all shown in the Table. The winding structure of the reference AFPM motor was a standard distributed winding, star-connected with a turn number of 9.

## 3. 3D FEA of prototype AFPM motor

As is well known, FEA can correctly examine permanent magnet motors with nonlinear materials such as silicon steel and NdFeB magnets. The motor designers do not have to go through difficult analytical methods since important parameters of torque, flux, efficiency, and inductance can easily and accurately be extracted from FEA. In addition, critical torque components such as cogging and torque ripple can be precisely calculated using available modern FEA software.

**Table.** Parameters of the reference axial flux PM motor.

| | |
|---|---|
| Number of poles | 8 |
| Number of stator slots | 24 |
| Air gap length | 0.8 mm |
| Slots/pole | 3 |
| Diameter ratio | 0.56 |
| Magnet type | NdFeB – N35UH |
| Lamination material | M270-35A |
| Number of turns | 9 |
| Winding | Star-connected |

Since no mathematical model exists for pulsating torque components of AFPM motors, finite element modeling and analysis of such PM motors are critical to predict important motor parameters such as average torque, back-EMF, and cogging torque. In this paper, detailed design, modeling, and analysis of the AFPM motor is all carried out with 3D FEA. The Flux3D FEA package by Cedrat was used in all the simulations of this paper [17]. The key point in 3D FEA modeling is to achieve reasonable mesh density on the motor, especially the motor air gap where energy conversion takes place. In addition, high mesh density results in high accuracy but longer processing time. Therefore, designers should choose adequate mesh density based on motor topology. Figure 4 shows the mesh density of the reference AFPM motor over one pole structure (3 slots and one magnet pole), which consisted of 110,366 volume elements, 2128 line elements, and 153,721 nodes. Each simulation took about 22 min with a Xeon processor workstation.

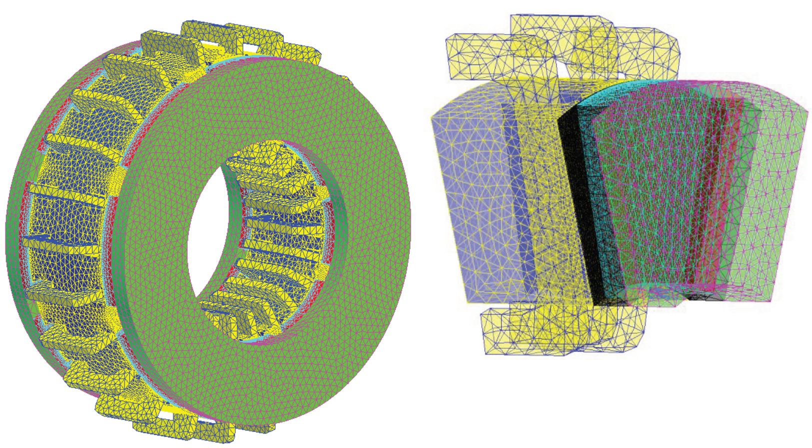


**Figure 4.** Mesh structure of the reference AFPM motor (110,366 volume elements, 2128 line elements, and 153,721 nodes are used in the 1-pole symmetric model).

## 4. Zero cogging torque for AFPM motor

Torque quality is one of the major issues for PM motors, especially in low-speed applications. Motor designers must pay special attention to the torque quality issue before finalizing the motor design [18,19]. Torque quality can be affected by the design criteria and the control techniques. This study focuses on improving torque quality by using the techniques of motor design. The major effect of torque quality on the design side is the magnet shape, which affects both the cogging level and back-EMF harmonics. Cogging torque is expressed as in [13] by using a Fourier series as in Eq. (1).

$$T_{cog} = 2p\, m_1 \sum_{k=1}^{\infty} T_{pk} \sin(N_s k\theta) \tag{1}$$

Here, $2p$ is the number of poles, $m_1$ is the number of rotors, $T_{pk}$ is the Fourier coefficient of cogging per magnet per rotor disc, $\theta$ is the rotor position angle, and $N_S$ is the number of stator slots.

In the literature, there are various techniques for reducing the cogging torque components of PM motors [20,21]. In this study, effective and simple rotor-side cogging torque reduction techniques, which are magnet skewing and shifting and rotor rotating, were implemented in the designed AFPM motor and torque quality of the motor was improved by eliminating the cogging component. In addition, it can be shown that the back-EMF waveform of the AFPM motor can be made perfectly sinusoidal, as in AFPM AC motors, after correctly shaping the rotor magnets.

### 4.1. Cogging torque reduction by magnet skewing

Magnet skewing technique is an effective modification for reducing cogging torque components in PM motors. Due to its fast and easy implementation and effectiveness, magnet skewing is commonly used in the literature for conventional PM motors as well as AFPM motors [22,23]. Magnet skewing has some different alternatives, such as conventional skew, triangular skew, trapezoidal skew, rounded magnets, parallel-sided, and dual skewed options [24].

In this paper, a conventional skewed rotor was applied to the reference AFPM motor and a reduction of cogging torque was observed. The schema of the conventional skewed magnet from a reference fan-shaped magnet is displayed in Figure 5. The fan-shaped magnet was skewed based on the mechanical skew angle, $\theta$. Magnet edges on the rotor ID were slipped based on the skew angle. Magnet edges on the rotor OD were not changed in this modification. As seen from the schema, volume or area of the reference unskewed fan-shaped magnet (ABCD) is the same as the area of the skewed AB$\underline{CD}$ magnet.

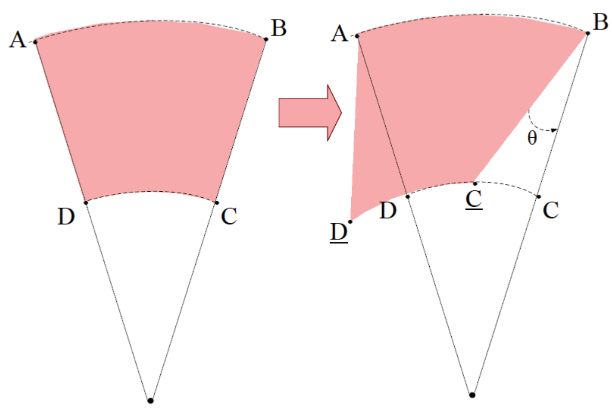



**Figure 5.** Schema of a fan-shaped magnet and conventional skew magnet.

Analyses for various values of mechanical skew angles was carried out and it was found that 20 mechanical degrees of skew angle is the optimum value for the magnets. As shown earlier, the reference AFPM motor has an 8 Nm peak cogging torque and the rotor with conventional skewed magnets with optimum skew angle has 1.54 Nm peak cogging torque. Cogging torque waveforms of reference and conventional skewed magnets are shown in Figure 6. The peak cogging torque was reduced by 80.75% with respect to the reference AFPM motor by simply finding the optimum skew angle.

### 4.2. Cogging torque reduction by magnet/pole shifting (grouping)

Another technique for reducing the cogging torque component in AFPM motors is magnet/pole shifting or grouping [24]. In this method, no modification is done on magnet shape [25–27], but the positions of the magnets are carefully selected based on an optimization study. In other words, magnets are shifted or positioned with harmony. The goal of this method is to illustrate that the cogging levels of the AFPM disc motor can be reduced even more than with conventional skew magnet structure.

The reference AFPM motor has 8 poles in each rotor and the magnets can be shifted-in-2. Shifted-in-2 and shifted-in-4 are both suitable for this AFPM motor. Therefore, the aim of the shifting technique is grouping the magnets into 2 or 3 magnets based on shifting angle, $\theta$. In this study, shifted-in-4, which is more effective for reducing the cogging torque compared to shifted-in-2, was applied to a conventional skewed magnet with 2 degrees of shifting angles to improve the torque quality and reduce the cogging component of the motor. A schema of shifted-in-4 in conventional skewed magnets is shown in Figure 7.

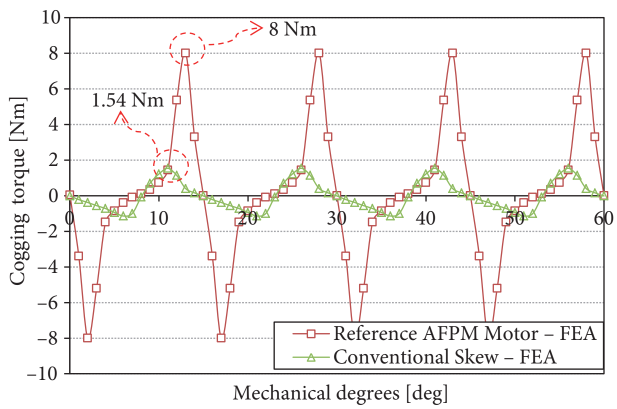



**Figure 6.** Cogging torque variation of reference and conventional skew magnets.

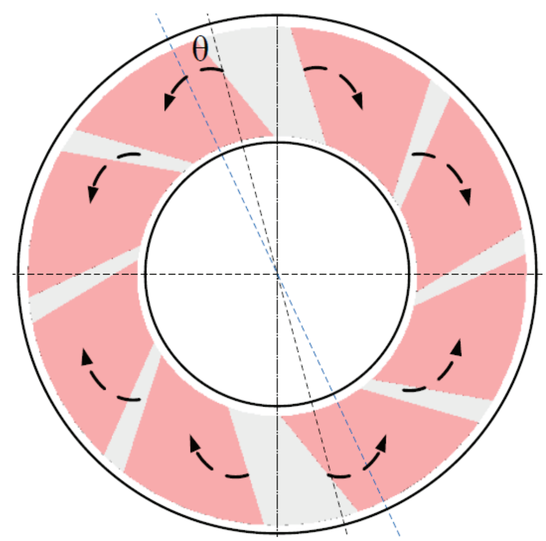



**Figure 7.** Schema of shifted-in-4 technique for a conventional skew AFPM motor.

As illustrated earlier, a conventional skewed magnet has a 1.54 Nm peak cogging level, which means an 80.75% reduction with respect to the reference AFPM motor. In order to further reduce the cogging, the magnet shifting methods explained above were applied to the skewed rotor. The cogging torque waveform of the shifted-in-4 option obtained from 3D FEA is shown in Figure 8 with the cogging variations of the conventional skewed rotor and reference motor. The shifted-in-4 AFPM motor has a 0.22 Nm peak torque and an 85.21% reduction with respect to conventional skewed was accomplished. This also means that, due to the shifting technique used, peak cogging torque was reduced by 97.25% with respect to the reference AFPM motor, which is a significant improvement in reducing cogging torque and improving torque quality of AFPM motors. It has to be mentioned that a 0.22 Nm peak cogging level is very hard to notice in such AFPM motors and is acceptable in most applications.

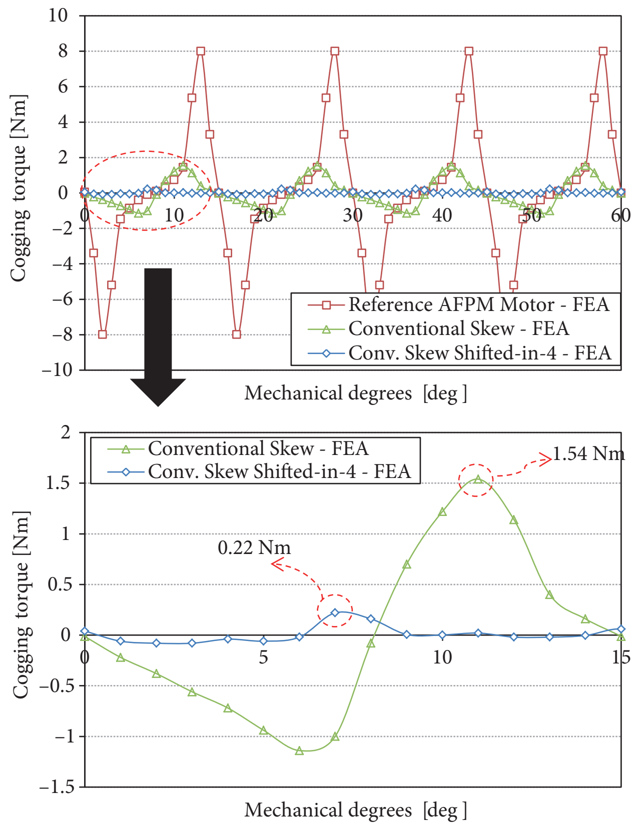



**Figure 8.** Cogging torque waveforms of shifted-in-4, conventional skewed, and reference AFPM motor.

Variations of average torque, cogging torque, and torque ripple for the reference AFPM motor, conventional skewed motor, and shifted-in-4 with conventional skewed magnets are displayed in Figure 9. The average torque of the reference AFPM motor is 37.73 Nm, with 25.43% ripple and peak cogging torque equal to roughly 21% of the average torque. After optimizing the magnet skew angle, average torque of AFPM motor was reduced to 35.83 Nm with 4.74% ripple, but the cogging torque was reduced to 3.3% of average torque. To further improve the output torque quality, a magnet shifting method was applied to conventional skewed magnets. Average torque of the AFPM motor became 31.37 Nm, with 3.63% torque ripple and cogging torque reduced even further to 0.63% of the average torque. The analyses show that shifting or grouping magnets is quite effective and can be used to reduce the cogging component of the AFPM motors with some sacrifice of the average torque. This can be tolerable for applications where low-speed control is critical.

### 4.3. Cogging torque reduction by rotor shifting

Another technique for reducing the cogging torque for AFPM motors is rotor shifting. This method can be used only for double and greater number of rotor AFPM motors. The rotor shifting method is different from skewing, since no modification is done on magnet shape whatsoever [24]. Nevertheless, one rotor is shifted based on the calculated shifting angle, $\gamma$, according to the other rotor. How the rotor shifting is utilized for the reference double rotor AFPM motor is displayed in Figure 10. The aim of this technique is to change the flux paths along the no-load line and reduce the cogging components. Rotor shifting technique is relatively easy to implement by changing the mechanical contacts on the shaft and it can be applied to other types of disc motors as well.

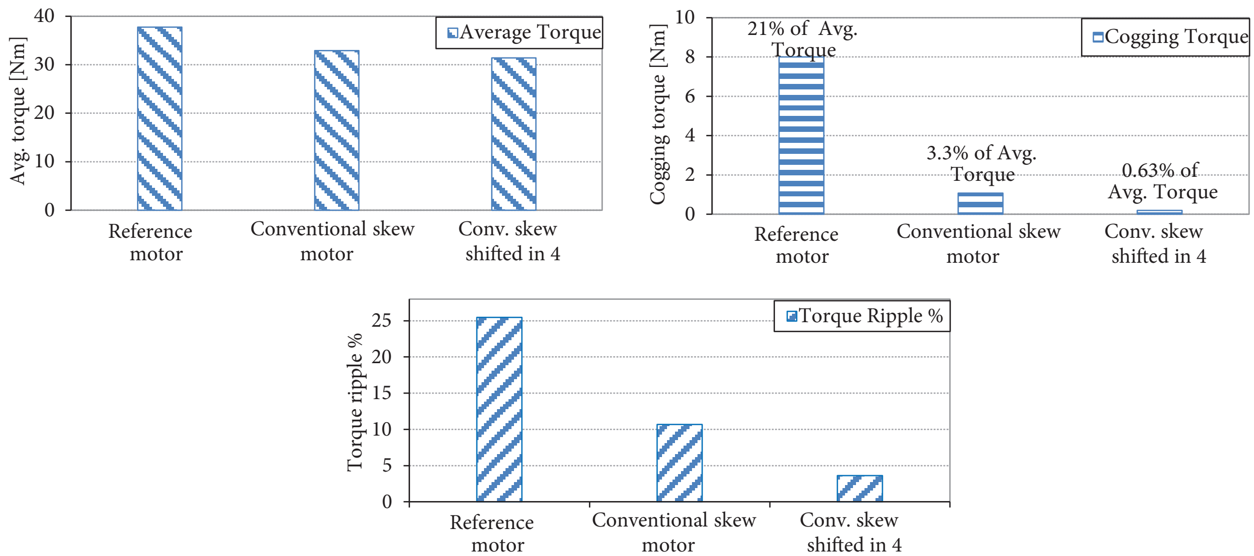
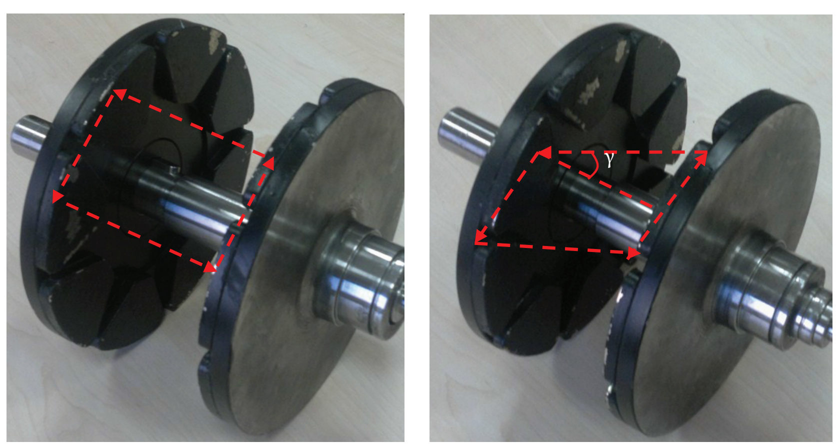



**Figure 9.** Variation of average torque, cogging torque, and torque ripple for reference AFPM motor, conventional skewed motor, and conventional skewed shifted-in-4.




(a) (b)

**Figure 10.** Schema of rotor shifting technique for flux path of unshifted (a) and shifted (b) rotor AFPM motor.

As mentioned before, the reference AFPM motor had 24 slots and cogging torque periodicity was 15 mechanical or 60 electrical degrees. Therefore, the shifting angle, $\gamma$, must have 15 degrees of periodicity, too. In this study, rotor shifting was applied to both reference and conventional skewed AFPM motors and cogging torque variations obtained from FEA for both structures were investigated. As displayed in Figure11, cogging torque periodicity had 15 mechanical degrees and the lowest cogging torque was obtained at nearly half of the period. Peak cogging torque of 7 degrees rotor shifting for the reference motor had 3.63 Nm, which means a 54.62% reduction with respect to the original reference motor. When the same study was repeated for the skewed magnet rotor, peak cogging torque with shifted rotor became 0.5 Nm and cogging was reduced by 67.53% with respect to the unshifted conventional skewed motor. A 93.75% reduction of cogging with respect to the reference AFPM motor was achieved by 7 degrees rotor shifting with conventional skewed magnets. This shows that rotor shifting can effectively be used in disc motors without too many penalties on the motor performance.

## 5. Test setup and experimental verification

The experimental system to test the cogging torque component of the rotors and the back-EMF waveforms of AFPM motor is displayed in Figure 12. The system consisted of a step motor, torque meter, and test motor. The system was specifically designed for this test and neither alignment problems nor other mechanical issues existed in the system. No gear box was used in the test system. Cogging torque test results were obtained at 1 rpm and data were collected from the torque meter and d-Space system. In addition, it has to be pointed out that the driving motor must have no cogging or torque ripple component whatsoever and must have precise speed control capability.

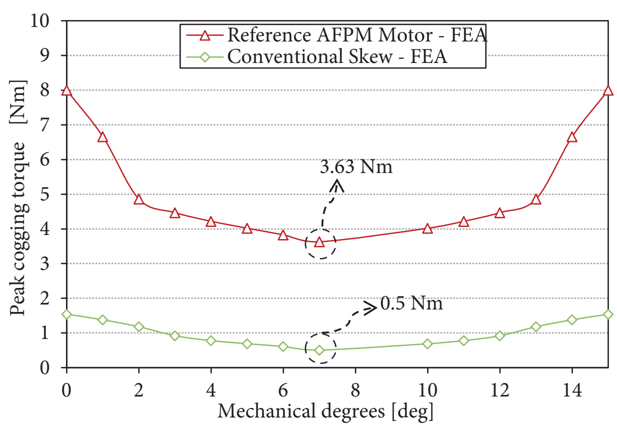



**Figure 11.** Peak cogging torque variation of shifting rotor for reference and conventional skewed motor.

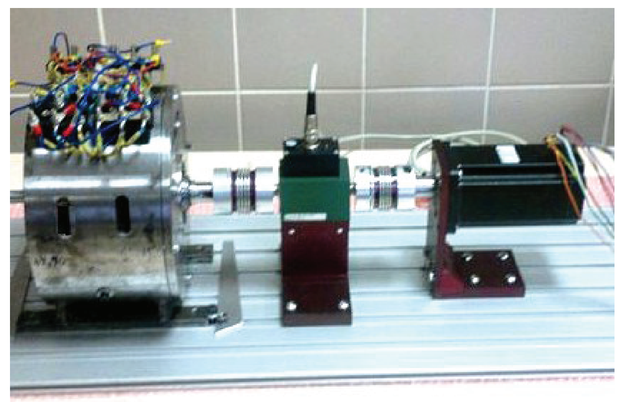


**Figure 12.** Test setup system for the AFPM disc motor.

Two different rotors, based on FEA results, were also built and tested. Built prototype rotor structures were the unskewed reference AFPM motor, a conventional skewed rotor, and a conventional skewed shifted-in-4 rotor; all are illustrated in Figure 13.

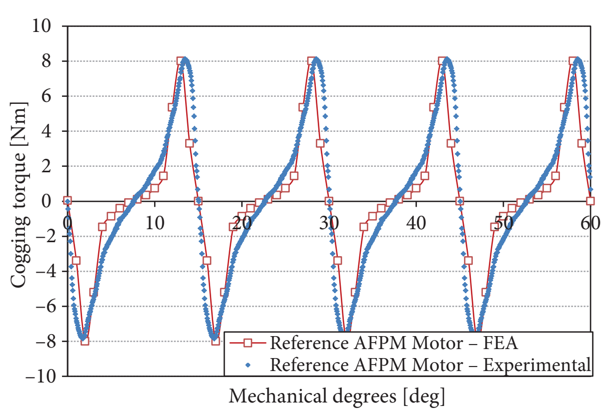



**Figure 13.** Built prototype rotor structures: reference unskewed AFPM rotor (a), conventional skewed rotor (b), and conventional skewed and shifted-in-4 rotor (c).

Figure 14 displays the simulated and measured cogging torque waveform of the reference AFPM motor as well as skewed and shifted-in-4 rotors. Experimental and FEA results were in good agreement in terms of their peak values and the shape of the waveform. It can also be seen that the cost-effective magnet skewing and shifting approach is enough to practically eliminate the cogging component of the AFPM motors.

Back-EMF tests were also performed for the three built rotors at 100 rpm. Back-EMF waveforms and their harmonics are shown in Figure 15. Since magnet volumes of all 3 rotors were the same, the fundamental harmonics had similar peak values. As expected, high order harmonics were reduced in the back-EMF waveforms as the skewing and shifting were introduced. The harmonic contents of the shifted-in-4 were nearly zero and much better than the reference and conventional skewed AFPM motor.

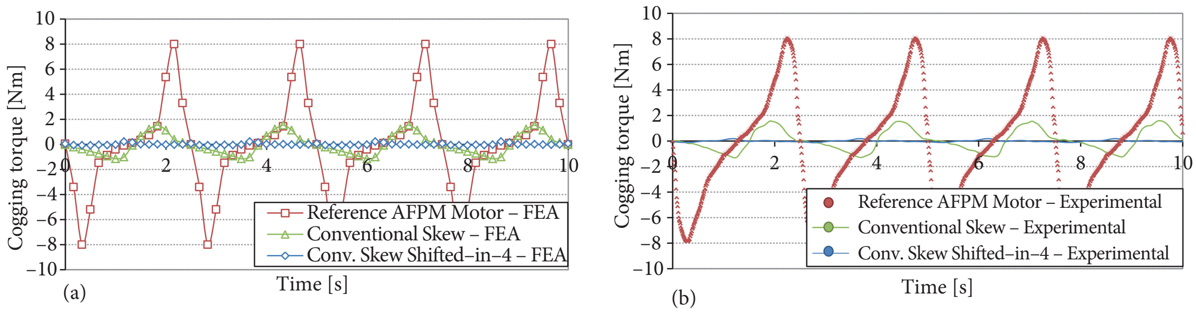



**Figure 14.** Comparison of FEA and experimental results of cogging torque waveforms for reference, conventional skewed, and shifted-in-4 AFPM motors.

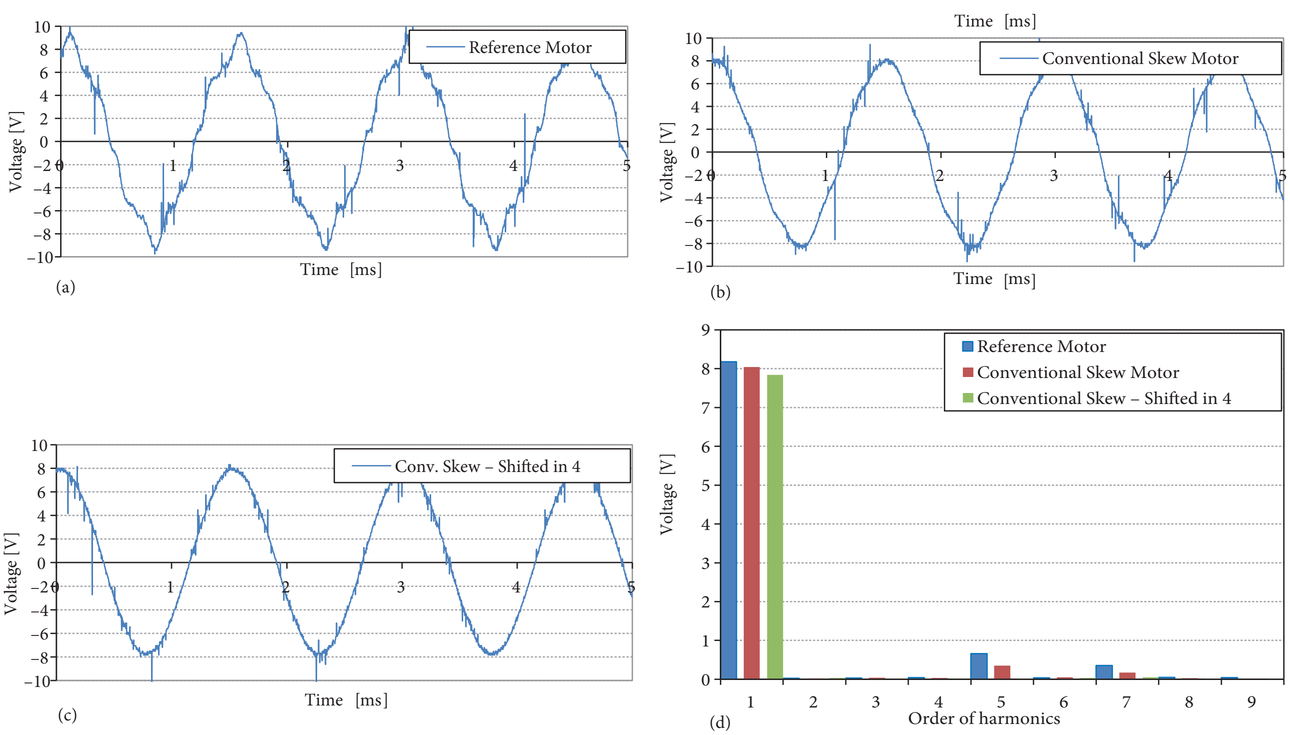



**Figure 15.** Back EMF waveform and harmonic contents of reference (a), conventional (b), and shifted-in-4 (c) AFPM motor under 100 rpm and harmonic contents of built rotors.

Rotor shifting technique was also carried out for the reference and conventional skewed AFPM motor in this study. Figure 16 shows the comparison of FEA and experimental results of rotor shifting method. Several independent tests were carried out for the conventional skewed rotor with different mechanical shifting angles. Excellent agreement between the FEA and experimental data was attained. A slight difference between FEA and experimental data after 7.5 mechanical sifting angles was observed due to the shifting angle alignment issues of the system.

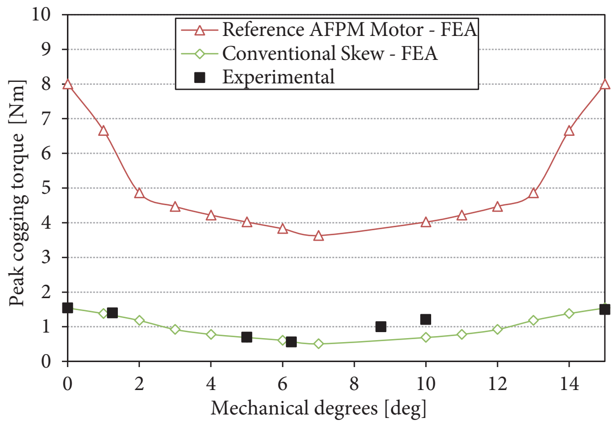



**Figure 16.** Variation of cogging torque for nonshifted and shifted rotor AFPM motors.

Back-EMF test was also performed at 100 rpm for all the shifted rotors of conventional skewed AFPM rotors. Back-EMF waveforms and harmonics contents of shifted rotors are given in Figure 17. As expected, the fundamental harmonics of back-EMF were the same in all shifted rotors and high-order harmonics decreased as more shifting was introduced, which resulted in more sinusoidal back-EMF variation of the motor.

## 6. Conclusions

In this paper, various cogging torque minimization techniques, namely magnet skewing, magnet grouping, and shifting for AFPM synchronous disc motors, have been proposed and presented. These techniques were examined by 3D FEA in detail and the results were compared with a reference AFPM motor with unskewed magnets. A prototype motor with 3 different rotors (reference, conventional skewed, and conventional skewed shifted-in-4) was built to confirm the analyses. Good agreement between the experimental results and FEA was observed and significant reduction on cogging torque was achieved. As displayed in the paper, magnet skewing and shifting techniques were both very effective and low-cost approaches in minimizing the cogging torque components of AFPM motors and they helped improve the torque quality of disc motors, especially at low-speed applications. It was shown that rotor shifting can decrease the cogging torque peak values without complicating the manufacturing process. In addition, it was demonstrated that the back-EMF of the reference AFPM motor, which had a nonsinusoidal shape, was improved significantly by some modifications to the magnet and rotor, so that a perfectly sinusoidal back-EMF waveform with less than 1% THD was obtained and better motor control capability at low speeds could be developed.

To conclude, it is shown that it is possible to reduce the cogging component of such integer slot permanent magnet axial-gap synchronous disc motors and practically eliminate unwanted cogging components, although the motor is intentionally designed for very high cogging levels. In addition, the approaches used to reduce the cogging components in this paper can help improve the back-EMF harmonic content so that better control

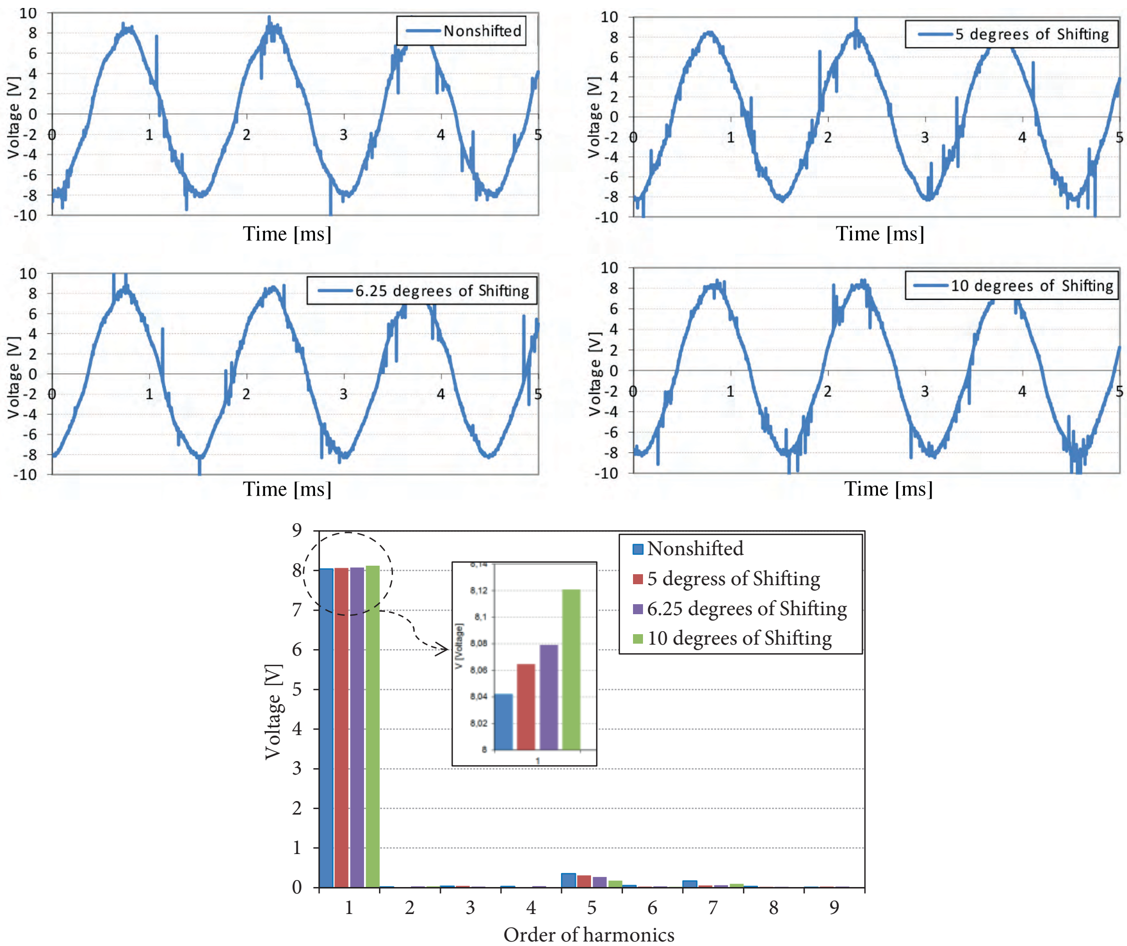



**Figure 17.** Variation of back-EMF waveform for nonshifted and shifted rotor AFPM motors.

capability can be obtained. The high-frequency harmonics for such integer slot AFPM motors can be reduced to very low levels so that almost zero cogging torque with a perfectly sinusoidal back-EMF AFPM motor can be designed without sacrificing the average torque.

**Acknowledgments**

The authors are indebted to ENPAY for providing strip cores of the motor prototype and Cedrat Co. for providing the Flux software. This research was supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) under grant number 108E051 and the Kocaeli University Scientific Research Unit.

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