

## An advanced optimization technique for considering reliability issues and unit commitment problems simultaneously

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**Abstract:** This paper presents an advanced optimization technique to solve unit commitment problems and reliability issues simultaneously for thermal generating units. To solve unit commitment, generalized Benders decomposition along with a genetic algorithm are proposed to include minimum up/down time constraints, and for reliability issues consideration, a fuzzy stochastic-based technique is presented. To implement the problem into an optimization program, MATLAB software and CPLEX and KNITRO solvers are applied. To verify the proposed technique and algorithm, two case studies, the IEEE 14- and 118-bus systems, are implemented for optimal generation scheduling and reliability issues. Finally, a comparison with other solution techniques is given.

**Key words:** Benders decomposition, fuzzy programming, genetic algorithm, optimization technique, reliability issues, unit commitment

### 1. Introduction

Reliability-constrained unit commitment is applied to minimize costs economically and schedule unit reserves like spinning reserves to provide system reliability. On the other hand, loss of load probability must be taken into consideration to obtain customers satisfactorily for the power distribution. Many optimization methods and modeling techniques have been proposed to solve security-constrained unit commitment (SCUC) [1–5]. In [6], a unit commitment solution was considered based on uncertainty, and a combination of Benders decomposition and the outer approximation technique was proposed. In [7], a unit commitment solution was developed integrating wind power and demand response uncertainties with the aid of Benders decomposition. In [8], multiobjective unit commitment with fuzzy membership design variables was tuned. In [9], unit commitment and reliability were proposed under uncertain forecasting based on fuzzy credibility theory. In [10], a unified stochastic and robust unit commitment problem along with reliability was developed based on the Benders decomposition algorithm. In [11], a Benders decomposition approach was proposed for a combined heat and power system. In [12], a fuzzy radial interval linear programming model was developed for robust planning of energy management systems with environmental consideration. In [13], security-constrained self-scheduling of generating companies in day-ahead electricity markets was considered.

Among these techniques and methods, Benders decomposition [14–16] is applied more often because of the nature of power system problems that are mixed-integer, like on/off states of generating units. Benders

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decomposition is a decomposition technique separating the main problem and subproblem(s) such that solving the whole problem involves less computational burden.

In this paper, as the master problem, the minimum up and down time constraints are nonlinear [17] and may lower the program speed; therefore, a modified genetic algorithm (GA) is used to solve these constraints. Based on [1,18–20] GAs are adaptive search methods that obtain their characteristics from the genetic processes of biological organisms based on evolutionary facts.

In power system operations, there are two other methods for distributing energy and system reserves: sequential dispatch and simultaneous dispatch [21]. As [21] proposed, the better solution of the problem from the optimization viewpoint is found when all the constraints and limitations are considered simultaneously rather than sequentially. Furthermore, [22] proposed a mixed deterministic-probabilistic structure for system reserves with a market-clearing algorithm and unit commitment. However, [22] just ran the algorithm for one time period. Other studies, like [21] and [23], considered system reserves like interruptible loads.

For reliability issues, loss of load probability (LOLP) along with system spinning reserves are included, and a fuzzy programming (FP) method is implemented to consider the stochastic nature of reliability issues because of the nature of the problem that is stochastic-based.

The studies in [24] and [25] proposed new a decomposition approach for the thermal unit commitment problem. In [25], extended comparisons are available.

With a review of the literature, the gap needs to be filled with a robust and advanced optimization technique. This study proposes a technique solving the SCUC problem and reliability issues simultaneously with the aid of existing and advanced optimization techniques having less computation burden, yielding robust, reliable results that are comparable to other existing techniques.

The main contribution of this paper is to apply some existing optimization techniques, Benders decomposition, a GA, and FP all together to solve a problem that is not only based on unit commitment but is also based on reliability issues. It is noted that, in this definition, reliability issues are considered as spinning reserves and the ability of the power system under study to supply loads (LOLP).

The reason for choosing these methods, generalized Benders decomposition (GBD), the GA, and FP, was that they have the ability to tackle these kinds of problems based on the literature, so the authors made a decision to optimize these methods based on new challenges for each part of the problem separately and all together.

The rest of the paper is organized as follows: Section 2 proposes the formulation and methodology. Section 3 gives two case studies, the IEEE 14- and 118-bus systems, to verify the proposed algorithm, and finally Section 4 concludes the study.

## 2. Formulation and methodology

### 2.1. Formulation of SCUC and reliability issues

To formulate the SCUC and reliability issues mathematically, the constraints and formulations are as follows: power balance, minimum up and down time constraints, ramp rate limits, unit reserves, LOLP, startup cost, and shutdown cost.

In this paper, the whole problem is a mixed-integer nonlinear program (MINLP) problem, and it is solved with the GBD method along with considering the minimum up and down time constraints by applying the GA. In this technique, unit commitment is the master problem assigning on/off states of generating units; in the

next step, the subproblem solves the economic dispatch problem, and finally reliability issues are solved by applying FP.

All formulations and constraints are as follows [23,26–29]:

Power balance:

$$\sum_{i=1}^{N_g} [P_{i,t}u_{i,t}] = Pload(t) \quad t = 1, \dots, Nt \quad (1)$$

Eq. (1) indicates that each running generating unit must supply the active power demanded by the loads at each specified hour (t). In the above equation, i and t are indices standing for generating units and time period, respectively. P is the active power of generating units. Pload is the consumed active power at load buses, and u is the on/off state of generating units: 1 for running units, and 0 for decommitted units.

Limits of generating units:

$$P_{\min}(i) \leq P_{i,t} \leq P_{\max}(i) \quad i = 1, \dots, Ngt = 1, \dots, Nt \quad (2)$$

Eq. (2) indicates that the supplied active power must be between min and max values because of physical properties of the turbine and generating units. In the above equation,  $P_{\min}$  and  $P_{\max}$  are limitations on generated active power.

Minimum up/down time constraints:

$$\begin{aligned} [X_{i,t}^{on} - T^{on}(i)] \times [u_{i,t-1} - u_{i,t}] &\geq 0 \\ [X_{i,t-1}^{off} - T^{off}(i)] \times [u_{i,t-1} - u_{i,t}] &\geq 0 \end{aligned} \quad (3)$$

Eq. (3) defines minimum up/down time constraints. Minimum up time is defined such that once the unit is running it should not be turned off immediately. Minimum down time is defined such that once the unit is decommitted there is a minimum time before it can be recommitted. In the above equations,  $T^{on}$  and  $T^{off}$  are minimum up time and minimum down time of unit I, respectively, and  $X^{on}$  and  $X^{off}$  are the ON time and OFF time of unit i at time t before the beginning of the specified time. This means that X depends on the elapsed time that the generating units were running.

Ramp-up rate limits:

$$P_{i,t} - P_{i,t-1} \leq Rup(i) \quad (4)$$

In Eq. (4), the traditional model for ramping is considered; the ramp rates are fixed at all loading levels and the ramping delay is not considered. In the above equation, Rup stands for ramp-up rate limit.

Inequality of generating units' active power:

$$P_{i,t} \geq 0 \quad (5)$$

Eq. (5) is a mathematical constraint on generated active power.

Objective function of minimization problem for SCUC:

$$\sum_{i=1}^{N_g} \sum_{t=1}^{Nt} [F_i(P_{i,t})u_{i,t} + s_{i,t} + sd_{i,t}] \quad (6)$$

Where:

$$F_i(P_{i,t}) = CP_{i,t}^2 + BP_{i,t} + A$$

Eq. (6) defines the objective function of the operational part of programming. It includes three sums: the fuel cost depending on a nonlinear curve  $F(P)$ , startup cost, and shutdown cost.

In the above equation,  $s$  represents the startup cost,  $sd$  stands for shutdown cost, and  $u$  is a binary value assigning on/off states of generating units.  $A$ ,  $B$ , and  $C$  are constants applied for cost functions of fuels for generating units.

All details and formulations of reliability issues are as follows:

Spinning reserves limits [21,22]:

$$\begin{aligned} 0 \leq SR_i^T &\leq u_i P_i^{\max} - P_i \\ SR_i^T &\leq Rup_i^T \end{aligned} \quad (7)$$

Eq. (7) indicates the spinning reserve of generating units, and that is the ability of generating units to supply for reliability issues when they cannot supply loads normally. In this equation,  $SR$  stands for spinning reserves in MW.

To consider the LOLP, it is a constraint and it must be satisfied in the reliability issues part of the problem.

LOLP can be defined classically as [22]:

$$LOLP = P\left[\sum_{i=1}^n u_i(P_i + SR_i) < Pload\right] \quad (8)$$

In other words, LOLP is the probability ( $P$ ) that the available generation, including spinning reserve, cannot meet the system load for all generating units.

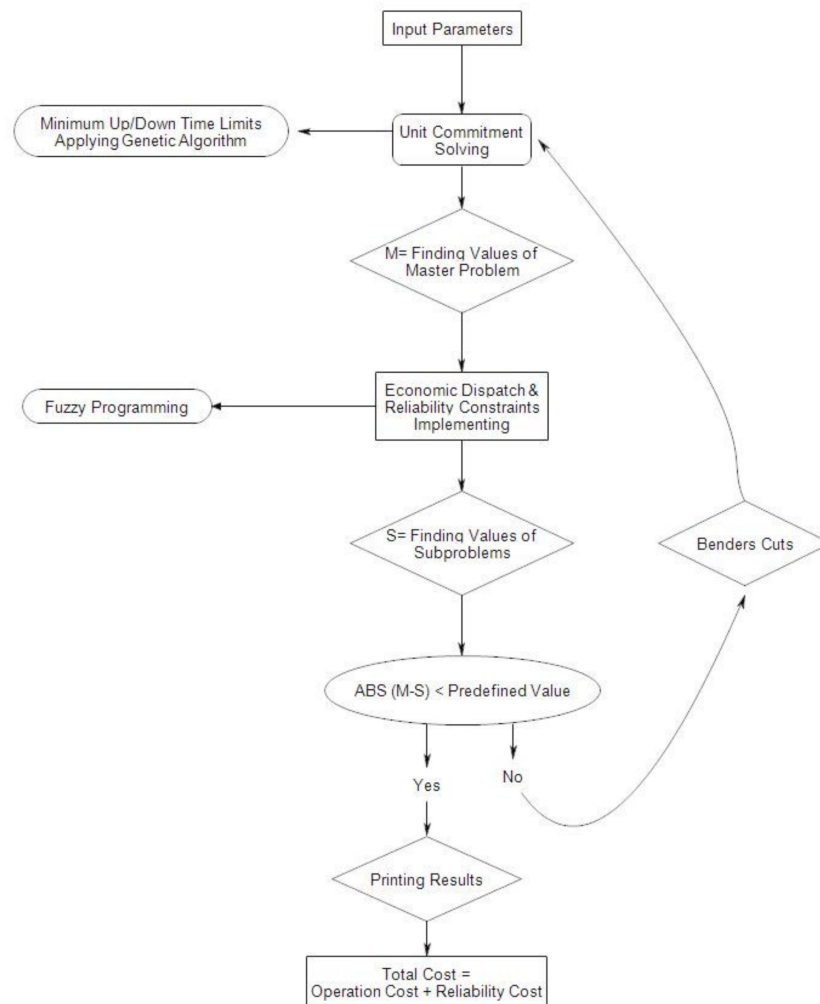
Finally, the objective function of the reliability issues section is added to Eq. (6).

$$\sum (F(P_{i,t}) \times u_{i,t} + s_{i,t} + sd_{i,t}) + \sum (P_{SR} \times SR \times u_{i,t}) \quad (9)$$

$P_{SR}$  is the cost for each MW produced in money units. It is noted that LOLP is an obligatory constraint, and it must be satisfied for the problem to be solved.

## 2.2. Algorithm

The algorithm that is implemented in this paper is shown in Figure 1. The algorithm is based on a mixed optimization technique that solves the running program in each iteration. As shown, in the first step, GBD solves the unit commitment while GAs help solve the nonlinear part of problem, i.e. the minimum up and down time constraints. After finding the minimum cost of the operational section, FP based on the min-max method is used to solve the reliability issues part of the problem. In each iteration, FP and the GA work under the support of GBP until an absolute minimum point is found yielding the minimum cost of the power system and satisfying reliability issue constraints.



**Figure 1.** The algorithm that was implemented.

The reason why these mixed optimization techniques are applied is because of the structure that the program deals with and the structural properties of the power system under study, i.e. being mixed-integer (GBD) and nonlinear (GAs), and having a probabilistic structure (FP).

As shown, TC, standing for the total cost of the power system, is the sum of operational costs that relates to unit commitment and reliability issues cost.

As shown in Figure 1, R stands for reliability functions and the running program stops if and only if the absolute difference value of the master problem and the subproblem is less than a prespecified tolerance. The equations that link master problems and subproblem constraints are Benders cut equations when the NO box in Figure 1 is obtained.

The main advantage of the proposed algorithm is its ability to take care of the unit commitment problem that is a traditional problem and the reliability issue constraints that are less traditional ones and apply modern and advanced optimization techniques that have some properties: applying several optimization methods in spite of just one optimization program that may have some deficits; less computational burden; and applying stochastic properties of FP and evolutionary properties of GAs under the support of GBD that is a robust optimization program.

### 2.3. Optimization program

#### 2.3.1. GBD

The GBD problem to be implemented in the program is as follows [14]:

$$\begin{aligned}
 & \min_{x_1, \dots, x_n; y_1, \dots, y_m} f(x_1, \dots, x_n; y_1, \dots, y_m) \\
 & \text{subject to} \\
 & h_k(x_1, \dots, x_n; y_1, \dots, y_m) = 0; k = 1, \dots, q \\
 & g_l(x_1, \dots, x_n; y_1, \dots, y_m) \leq 0; l = 1, \dots, r \\
 & y_j^{down} \leq y_j \leq y_j^{up}, y_j \in R; j = 1, \dots, m \\
 & x_i^{down} \leq x_i \leq x_i^{up}, x_i \in N; i = 1, \dots, n
 \end{aligned} \tag{10}$$

As in Eq. (10), we have a mathematical optimization program aiming at minimizing the  $f$  function. The  $f$  function stands for the cost function of the subproblem, including  $x_i$  representing on/off states of plants and  $y_j$  representing variables that need to be constrained like P (generated active power). It should be noted that all the equations stated in this paper are implemented by Eq. (10), including equalities and inequalities.

In Eq. (10),  $x_i$  are integer parameters and  $y_j$  are noninteger parameters.  $h_k$  defines equalities and  $g_l$  defines inequalities.  $f$  is the objective function of the optimization problem. Note that upper and lower bounds are imposed on optimization variables to reflect physical limits.

In the method applied in this paper, the program written by the authors in MATLAB applies a branch-and-cut method [14] to obtain a feasible solution based on cutting the extra space, searching for the desired minimum or maximum point. The property of this method is its iteration: if there is no feasible solution at the first iteration, with the aid of the Benders cut, it loops for the second iteration, and so on until searching and finding the minimized or maximized objective function. It is noted that the GA and FP are subsets of GBD and run under the main program.

#### 2.3.2. FP

A simple way of converting a stochastic model to a deterministic model using fuzzy set theory is to take its expected value:

$$\bar{F} = E(\text{Reliability Constraints}) \tag{11}$$

Where  $E$  is the expected value

Putting reliability constraints together:

$$\begin{aligned}
 \bar{F} = & \sum_{j=1}^{Ns} s_j (u_j P_j^{\max} - P_j - SR_j^T) + \sum_{j=1}^{Ns} s_j (Rup_j^T - SR_j^T) - \sum_{k=1}^{Ns} s_k (P[\sum_{i=1}^n u_i (P_i + SR_i) < Pload]) \\
 & - \sum_{k=1}^{Ns} s_k (\sum_{j=1}^n P_{Rj} LOSS_j (Pload_t - C_{Rj}), t \in Nt)
 \end{aligned} \tag{12}$$

In the above equation,  $s_i$  are slack variables;  $i, j,$  and  $k$  are indices; and  $\bar{F}$  defines the objective function of the reliability constraints. As written, equations of the reliability section are applied. Eq. (12) is based on Eqs. (7), (8), and (9).

The authors applied the Fuzzy Logic toolbox of MATLAB applying the FIS editor based on Eqs. (11) and (12). First of all, Eq. (14) has been linearized, and state variables were picked as desired reliability parameters

that are spinning reserve (SR) and LOLP. Limitations of these parameters have been given in Eqs. (7), (8), and (9), and the GA and FP are part of the outer optimization program and it is in a loop. On the other hand, the capacity outage probability table (COPT) was formed using the data given.

The method applied for this part of problem was “Mamdani”, and the defuzzification method was “centroid”. The fuzzy set was considered as [NB NS ZR PS PB], standing for negative big, negative small, zero, positive small, and positive big, respectively. The membership function was considered as a triangle, and the structure used was the min–max method.

### 2.3.3. GA

$$F_{GA} = \sum_{g=1}^{Ng} h_g ([X_{i,t}^{on} - T^{on}(i)] \times [u_{i,t-1} - u_{i,t}])^2 + \sum_{g=1}^{Ng} h_g ([X_{i,t-1}^{off} - T^{off}(i)] \times [u_{i,t-1} - u_{i,t}])^2 + \sum_{g=1}^{Ng} h_g^2 \quad (13)$$

In Eq. (13), we apply a GA for minimum up/down time constraints, with a slack variable  $h_g$ . This equation aims at satisfying the parameters like  $T^{on}$  and variables  $u$  based on the main GBD optimization program.

In Eq. (13),  $h_g$  are slack variables,  $g$  is the index for integer binary parameters, and  $F_{GA}$  is the objective function of this part of the problem. The GA is designed for the solution of the maximization problem, so the fitness function is defined as the inverse of Eq. (13):

$$F_{fitness} = \frac{1}{F_{GA}} \quad (14)$$

It is noted that the GA does not solve the objective function solely, and it is a subset of an outer optimization program.

As Eqs. (13) and (14) propose, the GA converts minimum up/down time constraints to an objective function and searching fitness function with inverting of the objective function. To solve this part of the problem, an m-file was written based on the Genetic Algorithm and Direct Search toolbox of MATLAB.

Finally, the GA and FP are converted into two separate m-files; each m-file is called in a module by the m-file written by the GBD.

## 3. Results and discussion

In this section, two case studies, the IEEE 14- and 118-bus test systems, are implemented to verify the proposed algorithm for a multiperiod optimization problem. The master problem is a mixed-integer programming problem along with the GA applying the CPLEX solver, and the subproblem is an MINLP problem and fuzzy stochastic-based problem applying the KNITRO solver. The proposed method was implemented on a DELL VOSTRO 1320 with an Intel Core 2 Duo CPU 2.53 GHz and 4 GB RAM using a MATLAB programming file (m-files) and MATLAB toolboxes for FP and the GA.

### 3.1. IEEE 14 bus system

Figure 2 shows the IEEE 14-bus system [30]. As shown in Figure 2, this system has five generating units at buses 1, 2, 3, 6, and 8. There are three tap-changing transformers named T1, T2, and T3. All data for loads and generating units are given in the Appendix.

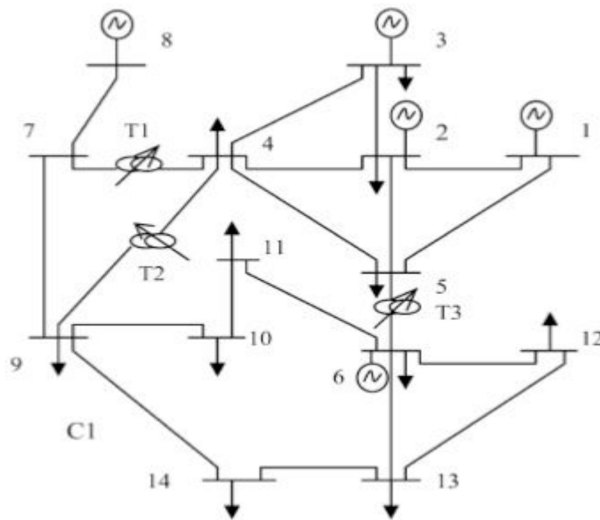


Figure 2. IEEE 14-bus system [28].

### 3.1.1. Unit commitment results

Running the optimization program yields on/off states of generating units,  $u$ , and  $P$ , active generated power in MW. Tables 1 and 2 show data obtained from the algorithm.

Table 1. On/off states of generating units,  $u$ .

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
Unit 1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Unit 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
Unit 3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
Unit 4	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0
Unit 5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0

As shown in Tables 1 and 2, unit 1, the cheapest generating unit, generates all 24 h. Unit 2, the next cheapest unit, generates 23 h with respect to minimum up and down time constraints. It is noted that all the constraints have been satisfied. The GA satisfies the nonlinear constraints, minimum up and down time constraints. Minimum power and maximum power have been satisfied, and the minimum cost is obtained.

The number of iterations for this part of the case study is 3, and time elapsed is 1.5240 s such that 0.9872 s is spent in the GA loop.

Minimum operational cost with respect to Eq. (6) including startup and shutdown cost is 11,149 in money units.

### 3.1.2. Reliability issue results

In this section,  $LOLP_{max}$  is assumed to be 0.01. It is noted that this constraint is a constraint on the whole program. For COPT it is assumed that loss is 5% of each load based on MW.  $P_{SR}$  is 1% of each generating unit's active power cost.

For reliability issues, two variables, SR and LOLP, are obtained. Tables 3 and 4 show data obtained from the program.



**Table 2.** P, generated active power in MW.

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
1	96.4	26.6	15	0	10
2	117.8	30.2	15	0	10
3	156.7	37	16.3	0	10
4	176.6	40.2	17.2	0	10
5	189	42.2	17.8	0	10
6	179.9	40.7	17.4	0	10
7	162.5	37.9	16.6	0	10
8	141.9	34.5	15.6	0	10
9	120.3	30.7	15	0	10
10	84.5	24.5	15	0	10
11	55	20	15	0	10
12	81.1	23.9	15	0	10
13	104.1	27.9	15	0	10
14	103.5	29.5	15	10	10
15	126.2	33.5	15.3	10	10
16	150.8	37.6	16.6	10	10
17	166.6	40.2	17.2	10	10
18	164.1	39.8	17.1	10	10
19	155	38.3	16.7	10	10
20	138.4	30.6	16	10	10
21	120.3	30.7	15	10	0
22	114.1	27.9	15	0	0
23	110.9	27.1	0	0	0
24	103	0	0	0	0

**Table 3.** System spinning reserves for generating units in MW.

	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
1	10	2.66	0	0	0
2	10	3.02	0	0	0
3	10	3.7	1.63	0	0
4	10	4.02	1.72	0	0
5	10	4.22	1.78	0	0
6	10	4.07	1.74	0	0
7	10	3.79	1.66	0	0
8	10	3.45	1.56	0	0
9	10	3.07	0	0	0
10	10	2.45	0	0	0
11	10	2.0	0	0	0
12	10	2.39	0	0	0
13	10	2.79	0	0	0
14	10	2.95	0	0	0
15	10	3.35	1.53	0	0
16	10	3.76	1.66	0	0
17	10	4.02	1.72	0	0
18	10	3.98	1.71	0	0
19	10	3.83	1.67	0	0
20	10	3.06	1.6	0	0
21	10	3.07	0	0	0
22	10	2.79	0	0	0
23	10	2.71	0	0	0
24	10	0	0	0	0

**Table 4.** Loss of load probability (LOLP).

1	0.00177	9	0.00177	17	0.0051
2	0.00177	10	0.00176	18	0.0071
3	0.00177	11	0.00176	19	0.0073
4	0.00177	12	0.0097	20	0.0094
5	0.00177	13	0.0082	21	0.0094
6	0.00177	14	0.0078	22	0.00107
7	0.00177	15	0.0066	23	0.00176
8	0.00177	16	0.0051	24	0.00176

As shown in Tables 3 and 4, LOLP is a constraint that was satisfied. System reserves also help the generating units be able to satisfy system reliability.

The number of iterations for this part of the case study is 7, and time elapsed is 3.3250 s. These results were obvious because of the time-consuming properties of FP.

Finally, the total cost (that is, the sum of the reliability cost and operational cost) from Eq. (11) is 11,183.08 in money units.

### 3.2. IEEE 118-bus system

The IEEE 118-bus test system has 54 thermal generators, 186 branches, and 91 demand sides. The parameters of the generators, transmission network, and load profiles are given at

[http://www.ee.washington.edu/research/pstca/pf118/pg\\_tca118fig.htm](http://www.ee.washington.edu/research/pstca/pf118/pg_tca118fig.htm).

Table 5. P, generated active power in MW.

Units	Hours (1-24)																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	203	180	150	150	150	150	203	270	245	270	270	264	225	195	270	270	270	270	285	300	300	270	270	245
5	200	180	140	100	100	160	200	260	240	280	280	260	240	200	280	280	277	280	280	300	300	280	280	240
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	24	40	40	24	24	24	40	40	24	40	55	62.5	70	40	40	24
8-9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	200	180	140	100	100	157	200	260	240	280	280	260	240	200	280	280	260	280	280	300	300	280	280	240
11	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360	360
12-13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	24	39	39	24	24	24	39	39	24	39	55	62.5	70	39	39	24
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	24	39	39	24	24	24	39	39	24	39	55	62.5	70	39	32	24
17-18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	24	39	39	24	24	24	39	39	24	39	55	62.5	70	39	24	24
20	239	239	239	134	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239
21	239	239	239	131	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239	239
22-23	0	0	0	0	0	0	0	0	24	39	39	24	24	24	39	39	24	39	55	62.5	70	39	24	24
24	200	200	200	100	155	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
24	200	200	200	100	151	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
26	0	0	0	0	0	0	0	0	24	32	39	24	24	24	32	39	24	39	55	62.5	70	39	24	24
27-28	420	388	366	178	292	366	420	420	420	420	420	420	420	420	420	420	420	420	420	420	420	420	420	420
29	212	189	124	80	80	146	212	246	246	278	278	246	234	205	278	278	278	278	278	310	310	278	278	246
31-33	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	24	39	39	24	24	24	39	39	24	39	55	62.5	70	39	24	24
36	0	0	0	0	0	0	0	0	24	39	39	24	24	24	39	39	24	39	55	60	70	39	24	24
36	195	180	150	150	150	150	195	264	244	270	270	245	224	195	270	270	270	270	285	310	310	278	278	246
37	0	0	0	0	0	0	0	0	24	39	39	24	24	24	39	39	24	39	55	67.5	70	39	24	24
38	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
39	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310	310
39	200	185	124	50	50	80	155	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200	200
41-42	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
43	200	180	139	100	100	139	200	260	239	280	280	260	231	200	280	280	260	280	280	310	310	280	280	239
44	200	180	129	100	100	139	200	260	239	280	280	260	220	200	280	280	260	280	280	310	310	280	280	239
45	200	180	120	100	100	139	200	260	239	280	280	260	220	200	280	280	260	280	280	310	310	280	280	239
46	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
47	0	0	0	0	0	0	0	0	24	24	32	24	24	24	39	39	24	32	55	55	62.5	39	24	24
48	0	0	0	0	0	0	0	0	24	24	24	24	24	24	39	39	24	32	54.5	55	62.5	39	24	24
49-50	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
51-52	0	0	0	0	0	0	0	0	24	24	24	24	24	24	39	39	24	24	47.5	55	62.5	39	24	24
53	0	0	0	0	0	0	0	0	24	24	24	24	24	24	32	32	24	24	47.5	55	62.5	32	24	24
54	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

### 3.2.1. Unit commitment results

Conducting another optimization program for the IEEE 118-bus test system gives P, the generated active power in MW. Table 5 shows data obtained from the algorithm.

As shown in Table 5, it should be noted that all the constraints have been satisfied. The GA satisfies the nonlinear constraints, the minimum up and down time constraints. Minimum power and maximum power have been satisfied, and the minimum cost is obtained.

Operational cost with respect to Eq. (6) including startup and shutdown cost is 1,643,818 in money units.

### 3.2.2. Reliability issue results

For reliability issues,  $LOLP_{max}$  is assumed to be 0.1. It is noted that this constraint is a constraint on the whole program. For COPT it is assumed that loss is 5% of each load based on MW.  $P_{SR}$  is 10% of each generating unit's active power cost.

For reliability issues, two variables, SR and LOLP, are obtained. Tables 6 and 7 show the data obtained from the program.

As Table 6 shows, for SR studies, the IEEE 118-bus test system has been converted into 3 zones [31]: A, B, and C. Zone A includes the left side of the figure, Zone B includes the bottom side, and zone C includes the top side. As shown in Table 7, LOLP is a constraint that was satisfied.

**Table 6.** Spinning reserves in MW.

	Maximum available spinning reserves (MW)
Area A	500
Area B	1024.4
Area C	700

**Table 7.** Loss of load probability (LOLP).

1	0.0911	9	0.0911	17	0.0808
2	0.0911	10	0.0809	18	0.0808
3	0.0912	11	0.0803	19	0.0808
4	0.0912	12	0.0804	20	0.0808
5	0.0912	13	0.0803	21	0.0904
6	0.0912	14	0.0808	22	0.0911
7	0.00177	15	0.0066	23	0.00176
8	0.00177	16	0.0051	24	0.00176

Finally, the total cost (namely, sum of reliability cost and operational cost) from Eq. (11) is 1,644,039.44 in money units.

Table 8 shows the number of iterations and time elapsed to conduct the IEEE 118-bus system.

**Table 8.** IEEE 118-bus test system, iterations and elapsed time.

Solution techniques		No. of iterations	Time elapsed	Program time cond. (s)
Unit commitment	Benders decomposition	17	6.43	9.93
	Genetic algorithm		1.2	
Reliability	Fuzzy programming	8	2.3	

### 3.3. Comparison with other solution techniques

In this section, the results obtained with the proposed algorithm are compared with other algorithms and optimization programs to verify the results. Table 9 shows the results.

It should be noted that Table 9 just includes the unit commitment problem, and the cost of considering reliability issues must be added to this operating cost.

**Table 9.** IEEE 118-bus test system comparisons.

	Minimum operating cost (\$)
Genetic algorithm (GA) [31]	1,644,434.90
Particle swarm optimization (PSO) [31]	1,644,321.20
Binary real coded firefly algorithm (BRCFF) [31]	1,644,141.00
Semidefinite programming-based method (SDP) [32]	1,645,445.00
Artificial bee colony algorithm (ABC-LR) [33]	1,644,269.70
The proposed method (GA-MINLP-FP)	1,643,118.00

## 4. Conclusion

In this paper, optimal generation scheduling in two power systems, the IEEE 14- and 118-bus systems, was implemented for both SCUC and reliability issues for 24 time period horizons. This implementation applied an advanced and mixed optimization technique including GBD, the GA, and FP. The advantage of these methods is searching for and finding a feasible solution matching the proposed algorithm and decreasing computational burden. In other words, these methods have good convergence based on the size of the given problem. The SCUC problem and the reliability issue constraints based on the proposed technique were considered simultaneously. The results obtained from the case studies presented good convergence with the proposed algorithm, and in comparison with other solution techniques, the proposed method shows superiority. The paper proposes to satisfy system reliability issues and economy simultaneously, though some extra costs must be paid. It also proposes that this advanced optimization technique is a suitable technique to address this kind of power system problems as well as lowering computational burden.

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**Appendix.** IEEE 14-bus system.**Table A1.** Load data (MW) for 24 h.

1	2	3	4	5	6	7	8
148	173	220	244	259	248	227	202
9	10	11	12	13	14	15	16
176	134	100	131	157	168	195	225
17	18	19	20	21	22	23	24
244	241	231	210	176	157	138	103

**Table A2.** Generating units' data.

	Pmax	Pmin	A	B	C	Min. up	Min. down	Startup cost	Shutdown cost	In. state
Unit 1	250	10	0.00315	2.0	0	1	1	70	176	1
Unit 2	139	20	0.01750	1.75	0	2	1	74	187	-3
Unit 3	100	15	0.06250	1.0	0	1	1	50	113	-2
Unit 4	120	10	0.00834	3.25	0	2	2	110	267	-3
Unit 5	45	10	0.0250	3.0	0	1	1	72	180	-2