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Fractional control and generalized synchronization for a nonlinear electromechanical chaotic system and its circuit simulation with Multisim

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Abstract: Two fractional differential controllers, described and validated by using the fractional order stability theorem and the Gershgorin circle theorem for a self-sustained electromechanical system consisting of a van der Pol–Duffing coupled oscillator, were studied in this paper. Based on the idea of a nonlinear observer, a new method for generalized synchronization (GS) of this system is proposed. Finally, the circuit simulation results demonstrate the correctness and the effectiveness of the proposed control and GS strategy.

Key words: Fractional order controller, generalized synchronization, electromechanical system, circuit simulation

1. Introduction

With the development of nonlinear system theory, the dynamics of coupled oscillators [1–3], which appeared in various fields of natural science and engineering technology, have been researched by many scientists. Especially for electromechanical systems (ESs) that are described by van der Pol oscillators and Duffing oscillators [4–6], due to the fact that these coupled oscillators systems can exhibit various types of complex chaotic behaviors, the dynamics of these ESs have been widely discussed. In [7–9], some authors considered the chaotic dynamics for some self-sustained ESs (SSESs) consisting of Duffing oscillators and van der Pol oscillators. To avoid unexpected behaviors arising from chaotic ESs, many chaos control methods based on different strategies have been presented in experiments and applications, such as robust control [10], adaptive backstepping control [6], passive control [11], and delay feedback control [12,13]. Meanwhile, different kinds of synchronization methods such as complete synchronization (CS), phase synchronization (PS), and generalized synchronization (GS) [14–16] have been described. However, reviewing these control and synchronization methods, most of them either have cancelled out the nonlinear parts of coupled systems or have concentrated on studying CS. Furthermore, we know that CS is difficult to achieve, and there always exist parameter mismatches and distortions in the physical world. Therefore, the control and GS for ESs will become an important issue.

On the other hand, the applications of fractional calculus have attracted much attention in recent years [17–20]. Many fractional order chaotic systems have been discussed and some fractional controllers have been designed. It is verified that fractional controllers have a strong ability to eliminate chaotic oscillations compared to traditional controllers [21,22]. Following these ideas, and motivated by previous works [21,22], this paper presents a fractional controller for a SSES based on the fractional stability theories, and a GS method based

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on the idea of a nonlinear observer. The paper is organized as follows. Fractional calculus for the fractional differential equation (FDE) and stability theorem in an incommensurate FDE system is presented in Section 2. In Section 3, the fractional differential controller is designed and proved by using the Gershgorin circle theorem. Based on the nonlinear observer and the pole assignment technique, a GS scheme of the chaotic system is also proposed in this section. In Section 4, circuit simulations are provided to illustrate the performance of the proposed control strategy together with GS. Finally, some concluding remarks are presented in the final section.

2. Fractional calculus

In the following, we introduce some definitions for the general fractional differentiation and integration. The first is the Grünwald–Letnikov (GL) definition.

$$ {}_aD_t^q f(t) = \lim_{h\to 0} \frac{1}{h^q} \sum_{j=0}^{[(t-q)/h]} (-1)^j \begin{pmatrix} q \\ j \end{pmatrix} f(t-jh) \tag{1} $$

The second is the Riemann–Liouville (RL) definition.

$$ {}_aD_t^q f(t) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau \tag{2} $$

In this paper, we use the third definition of the differintegral introduced by Caputo.

$$ {}_aD_t^q f(t) = \frac{1}{\Gamma(q-n)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{q-n+1}} d\tau,\ n-1 < q < n \tag{3} $$

Consider the Cauchy problem:

$$ D_t^q x(t) = f(t, x(t)), \quad 0 < t \le T, \quad x^{(i)}(0) = x_0^{(i)}, \quad i = 0, 1, \cdots, m-1, \tag{4} $$

where $m-1 < q \le m \in \mathbb{N}$. The numerical calculation of a FDE is as follows.

Transform Eq. (4) into an equivalent Volterra integral equation:

$$ x(t) = \sum_{i=0}^{m-1} \frac{t^i}{i!} x_0^{(i)} + \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} f(\tau, x(\tau)) d\tau. \tag{5} $$

Set $h = \frac{T}{N-1}$, $N \in \mathbb{N}$, $t_n = nh$, $n = 0, \cdots N-1$. Then Eq. (5) can be discretized as follows:

$$ x_h(t_{n+1}) = \sum_{i=0}^{m-1} \frac{t_{n+1}^i}{i!} x_0^{(i)} + \frac{h^q}{\Gamma(q+2)} [f(t_{n+1}, x_h^p(t_{n+1})) + \sum_{j=0}^{n} a_{j,n+1} f(t_j, x_h(t_j))], $$

where

$$ a_{j,n+1} = \begin{cases} n^{q+1} - (n-q)(n+1)^q, \text{if } j = 0 \\ (n-j+2)^{q+1} + (n-j)^{q+1} - 2(n-j+1)^{q+1}, \text{if} 1 \le j < n \end{cases}, $$

$$x_h^p(t_{n+1}) = \sum_{i=0}^{m-1} \frac{t_{n+1}^i}{i!} x_0^{(i)} + \frac{1}{\Gamma(q)} \sum_{j=0}^{n} b_{j,n+1} f(t_j, x_h(t_j)),$$

$$b_{j.n+1} = \frac{h^q}{q}[(n+1-j)^q - (n-j)^q], \quad 0 \le j \le n.$$

Lemma 1 [23] *Linear incommensurate FDE system:*

$$\begin{cases} \frac{d^\alpha X}{dt^\alpha} = AX, X \in R^n, A \in R^{n\times n} \\ \frac{d^\alpha}{dt^\alpha} = \left[\frac{d^{\alpha_1}}{dt^{\alpha_1}}, \frac{d^{\alpha_2}}{dt^{\alpha_2}}, \cdots, \frac{d^{\alpha_n}}{dt^{\alpha_n}}\right]^{\mathrm{T}}, 0 < \alpha_i < 1 \end{cases}.$$

Let $\alpha_i = \frac{v_i}{u_i}$, $(v_i, u_i) = 1$, $v_i, u_i \in Z^+$ *for* $i = 1, 2, \cdots, n$, *and assume* M *to be the lowest common multiple of all the denominators* u_i. *Define*

$$\Delta(\lambda) = diag(\lambda^{M\alpha_1}, \lambda^{M\alpha_2}, \cdots, \lambda^{M\alpha_n}) - A.$$

Then the zero solution of the system is globally asymptotically stable in the Lyapunov sense if all roots λ *of equation* $\det(\Delta(\lambda)) = 0$ *satisfy* $|\arg(\lambda)| > \frac{\pi}{2M}$ *or* $|\arg(\lambda)| > \frac{\Lambda\pi}{2}$, *where* $\Lambda = \max\{\alpha_1, \alpha_2, \cdots, \alpha_n\}$.

3. Control and synchronization

3.1. Control chaos via fractional order derivative

The SSES consisting of a van der Pol–Duffing coupled oscillator [6,13] can be written as shown below.

$$\begin{cases} \ddot{x} - \varepsilon_1(1 - x^2)\dot{x} + \omega_1^2 x + p\ddot{y} = 0 \\ \ddot{y} + \varepsilon_2\dot{y} + \omega_2^2 y + cy^3 - qx = 0 \end{cases} \tag{6}$$

The dynamics of this system can be seen in [6]. In order to analyze this expediently, we rewrite the system of Eq. (6) as

$$\dot{Z} = f(Z), \tag{7}$$

where $f(Z) = \begin{pmatrix} z_2 \\ -(\omega_1^2 + pq)z_1 + \varepsilon_1(1 - z_1^2)z_2 + p\omega_2^2 z_3 + pcz_3^3 + p\varepsilon_2 z_4 \\ z_4 \\ qz_1 - \varepsilon_2 z_4 - \omega_2^2 z_3 - cz_3^3 \end{pmatrix}$. Let Z_0 be any point but not equilibrium in the chaos attractor of the system of Eq. (7), and consider the following control system in order to stabilize the point Z_0 via fractional order derivative:

$$\dot{Z} = f(Z) + V(t), \tag{8}$$

where $V(t) = -f(Z_0) - \frac{d^\alpha Z}{dt^\alpha} + \dot{Z}$.

Theorem 1 *Suppose the Jacobian matrix of the system of Eq. (7) on point* Z_0 *does not have positive real eigenvalues; then there exists* $0 < \alpha_0 < 1$, *and when* $0 < \alpha < \alpha_0 < 1$, *the control system of Eq. (8) will be asymptotically stable on point* Z_0.

Proof According to the system of Eq. (8), we can see that

$$\frac{d^\alpha Z}{dt^\alpha} = f(Z) - f(Z_0). \tag{9}$$

The Jacobian matrix of this system on point Z_0 equals the Jacobian matrix of the system of Eq. (7) on the same point. Since the eigenvalues of the Jacobian matrix of this system on point Z_0 do not have positive real eigenvalues, then there exists $0 < \alpha_0 < 1$ such that $|\arg(\lambda)| \geq \frac{\alpha_0 \pi}{2}$ for all eigenvalues. Therefore, there exists α that satisfies $0 < \alpha < \alpha_0 < 1$, and the zero solution of the system of Eq. (9) will be globally asymptotically stable in the Lyapunov sense, i.e. the control system of Eq. (8) will be stable on point Z_0. □

3.2. Control chaos via fractional differential controller

Lemma 2 (Gershgorin circle theorem) [24] *Let* $A = (a_{ij})_{n\times n} \in \mathbb{C}^{n\times n}$*, and then the eigenvalue* λ *lies in one of the circles* $|t - a_{ii}| \leq \sum\limits_{j\neq i} |a_{ij}|$.

Let $\hat{Z} = Z - Z_0$ and $Z_0 = (z_{10}, z_{20}, z_{30}, z_{40})^{\mathrm{T}}$; obviously, $z_{20} = z_{40} = 0$, and we rewrite the system of Eq. (7) as shown below.

$$\begin{cases} \dot{\hat{z}}_1 = \hat{z}_2 \\ \dot{\hat{z}}_2 = -(\omega_1^2 + pq)\hat{z}_1 + \varepsilon_1(1 - (\hat{z}_1 + z_{10})^2)\hat{z}_2 - \varepsilon_1(\hat{z}_1^2 + 2\hat{z}_1 z_{10})z_{20} + p\omega_2^2\hat{z}_3 \\ +pc(\hat{z}_3^3 + 3\hat{z}_3^2 z_{30} + 3\hat{z}_3 z_{30}^2) + p\varepsilon_2\hat{z}_4 \\ \dot{\hat{z}}_3 = \hat{z}_4 \\ \dot{\hat{z}}_4 = q\hat{z}_1 - \varepsilon_2\hat{z}_4 - \omega_2^2\hat{z}_3 - c(\hat{z}_3^3 + 3\hat{z}_3^2 z_{30} + 3\hat{z}_3 z_{30}^2) \end{cases} \tag{10}$$

Consider the following control system.

$$\begin{cases} \dot{\hat{z}}_1 = \hat{z}_2 + u_1 - k_1\hat{z}_1 \\ \dot{\hat{z}}_2 = -(\omega_1^2 + pq)\hat{z}_1 + \varepsilon_1(1 - (\hat{z}_1 + z_{10})^2)\hat{z}_2 - \varepsilon_1(\hat{z}_1^2 + 2\hat{z}_1 z_{10})z_{20} + p\omega_2^2\hat{z}_3 \\ +pc(\hat{z}_3^3 + 3\hat{z}_3^2 z_{30} + 3\hat{z}_3 z_{30}^2) + p\varepsilon_2\hat{z}_4 + u_2 - k_2\hat{z}_2 \\ \dot{\hat{z}}_3 = \hat{z}_4 + u_3 - k_3\hat{z}_3 \\ \dot{\hat{z}}_4 = q\hat{z}_1 - \varepsilon_2\hat{z}_4 - \omega_2^2\hat{z}_3 - c(\hat{z}_3^3 + 3\hat{z}_3^2 z_{30} + 3\hat{z}_3 z_{30}^2) + u_4 - k_4\hat{z}_4 \\ \frac{d^\alpha u_1}{dt^\alpha} = -u_1 - k_5\hat{z}_1 \\ \frac{d^\beta u_2}{dt^\beta} = -u_2 - k_6\hat{z}_2 \\ \frac{d^\gamma u_3}{dt^\gamma} = -u_3 - k_7\hat{z}_3 \\ \frac{d^r u_4}{dt^r} = -u_4 - k_8\hat{z}_4 \end{cases} \tag{11}$$

Here, $u_i(0) = 0$, $k_i > 0$, and $0 < \alpha, \beta, \gamma, r < 1$. Obviously, the system of Eq. (11) can be transformed to Eq.

(7), and the coefficient matrix is

$$A(Z)=\begin{pmatrix} -k_1 & 1 & 0 & 0 & 1 & 0 & 0 & 0\\ A_{21} & A_{22} & A_{23} & p\varepsilon_2 & 0 & 1 & 0 & 0\\ 0 & 0 & -k_3 & 1 & 0 & 0 & 1 & 0\\ q & 0 & A_{43} & -\varepsilon_2-k_4 & 0 & 0 & 0 & 1\\ -k_5 & 0 & 0 & 0 & -1 & 0 & 0 & 0\\ 0 & -k_6 & 0 & 0 & 0 & -1 & 0 & 0\\ 0 & 0 & -k_7 & 0 & 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -k_8 & 0 & 0 & 0 & -1 \end{pmatrix},$$

where $A_{21}=-\omega_1^2-pq-2\varepsilon_1(\hat{z}_1+z_{10})(\hat{z}_2+z_{20})$, $A_{22}=-[\varepsilon_1((\hat{z}_1+z_{10})^2-1)+k_2]$, $A_{23}=p(\omega_2^2+3c(\hat{z}_3+z_{30})^2)$, and $A_{43}=-\omega_2^2-3c(\hat{z}_3+z_{30})^2$. Because the variable of the chaotic system is bounded, we can let $M_2=|A_{21}|+|A_{23}|$, $M_4=|A_{43}|$, and we have the following theorem.

Theorem 2 *The zero solution of the incommensurate fractional system of Eq. (11) is asymptotically stable if* $k_1>2$, $k_2>M_2+p\varepsilon_2+\varepsilon_1+1>0$, $k_3>2$, $k_4>1+M_4-\varepsilon_2>0$, *and* $0<k_{5,6,7,8}<1$.

Proof By Lemma 2, the eigenvalue of $A(Z)$ lies in the circles

$|\lambda_1+k_1|\le 2$, $\left|\lambda_2+[\varepsilon_1((\hat{z}_1+z_{10})^2-1)+k_2]\right|\le M_2+p\varepsilon_2+1, |\lambda_3+k_3|\le 2$, $|\lambda_4+\varepsilon_2+k_4|\le 1+M_4$, $|\lambda_i+1|\le k_i (i=5,6,7,8)$.

According to the condition, when $k_1>2$, $k_2>M_2+p\varepsilon_2+\varepsilon_1+1>0$, $k_3>2$, $k_4>1+M_4-\varepsilon_2>0$, and $0<k_{5,6,7,8}<1$, we can see that all the circles lie in the left of the imaginary axis, and the real value of all the eigenvalues of $A(Z)$ is less than zero, i.e. all λ satisfy $|\arg(\lambda)|>\frac{\pi}{2}$. By Lemma 1, we know that the zero solution of the control system of Eq. (11) is asymptotically stable. □

3.3. GS with observer

Rewrite the system as

$$\frac{dZ}{dt}=AZ+BH(Z), \tag{12}$$

where $A=\begin{pmatrix} 0 & 1 & 0 & 0\\ -(\omega_1^2+pq) & \varepsilon_1 & p\omega_2^2 & p\varepsilon_2\\ 0 & 0 & 0 & 1\\ q & 0 & -\omega_2^2 & -\varepsilon_2 \end{pmatrix}$, $B=\begin{pmatrix} 0 & 0\\ -\varepsilon_1 & pc\\ 0 & 0\\ 0 & -c \end{pmatrix}$, $H(Z)=\begin{pmatrix} z_1^2 z_2\\ z_3^3 \end{pmatrix}$.

Consider the observer

$$\frac{dY}{dt}=P^{-1}APY+P^{-1}B(KZ+H(Z)-KPY), \tag{13}$$

where $Rank(P)=4$.

Theorem 3 *If* (A,B) *is controllable, and* $\lambda_i(A-BK)<0$, $i=1,2,3,4$, *then* $\lim_{t\to\infty}\|PY-X\|=0$, *i.e. the systems of Eqs. (12) and (13) will approach GS with the observer of Eq. (13).*

Proof Let $e = PY - X$, and then

$$\frac{de}{dt} = P\frac{dY}{dt} - \frac{dX}{dt} = (A - BK)e. \tag{14}$$

By [25], the system of Eq. (14) is globally asymptotically stable under the condition $\lambda_i(A - BK) < 0$, $i = 1, 2, 3, 4$, i.e. $\lim_{t\to\infty} \|e\| = 0$. □

Remark 1 *When $P = I$, where I is an identical matrix, the systems of Eqs. (12) and (13) are in CS. When $P = -I$, the two systems are antisynchronized. When $P = kI$ and $k \neq \pm 1$ is constant, the two systems are in GS.*

4. Simulation experiments

4.1. Control chaos via fractional order derivative

Taking $\varepsilon_2 = 0.00987$, $\varepsilon_1 = 2.466$, $\omega_1 = \omega_2 = 1$, $c = 0$, $p = 3.518$, and $q = 0.808$, we can see that the system of Eq. (7) has a chaotic attractor from [13]. Letting $Z_0 = (0, 1, 2, 1)^{\mathrm{T}}$, we can calculate that all the eigenvalues of the Jacobian matrix of the system of Eq. (9) on the point Z_0 are $\lambda_{1,2} = 0.954339 \pm 1.604902i$, $\lambda_{3,4} = 0.273726 \pm 0.460323i$, and then there exists $0 < \alpha_0 < 1$ such that $|\arg(\lambda)| \geq \frac{\alpha_0 \pi}{2}$ for all eigenvalues, so we can select $\alpha_0 \doteq 0.658473$ and also take $\alpha = 0.5$ as satisfying $0 < \alpha < \alpha_0 < 1$. By Theorem 1, we know that the system of Eq. (9) will be stable on point Z_0.

Generally, since the calculations of fractional calculus are very difficult in the time domain, people will transform the time domain into the complex frequency domain. Upon considering the initial conditions to be zero, the Laplace transform of the Caputo fractional derivative is $L\{{}_aD_t^\alpha f(t)\} = s^\alpha L\{f(t)\}$, so the fractional integral operator of order "α" can be represented by the transfer function $H(s) = 1/s^\alpha$, and we can use the approximations for $1/s^\alpha$ with α from 0.1 to 0.9 in steps of 0.1 with errors of approximately 2 dB [26]. The fractional $\frac{1}{s^{0.5}}$ circuit unit is designed in Figure 1, and the corresponding circuit equation is $\frac{1}{s^{0.5}} \doteq \frac{R_1}{sR_1C_1+1} + \frac{R_2}{sR_2C_2+1} + \frac{R_3}{sR_3C_3+1} + \frac{R_4}{sR_4C_4+1} + \frac{R_5}{sR_5C_5+1} + \frac{R_6}{sR_6C_6+1}$. Figure 2 shows the circuit of the control system of Eq. (9), and the corresponding circuit equation is as given below.

$$\begin{cases} \frac{d^{0.5}U_1}{d^{0.5}t} = \frac{R_{13}}{R_{12}R_{10}}U_2 - 5\frac{R_{13}}{R_{11}R_{10}} \\ \frac{d^{0.5}U_2}{d^{0.5}t} = -\frac{R_{29}R_{24}}{R_{27}R_{22}R_{20}}U_1^2U_2 - \frac{R_{29}R_{24}}{R_{27}R_{23}R_{20}}U_1 + \frac{R_{29}}{R_{25}R_{20}}U_2 \\ +\frac{R_{29}}{R_{26}R_{20}}U_3 + \frac{R_{29}}{R_{28}R_{20}}U_4 - 5\frac{R_{29}R_{24}}{R_{27}R_{21}R_{20}} \\ \frac{d^{0.5}U_3}{d^{0.5}t} = \frac{R_{33}}{R_{32}R_{30}}U_4 - 5\frac{R_{33}}{R_{31}R_{30}} \\ \frac{d^{0.5}U_4}{d^{0.5}t} = -\frac{R_{43}R_{37}}{R_{41}R_{45}R_{40}}U_3 - \frac{R_{43}R_{37}}{R_{42}R_{45}R_{40}}U_4 + 5\frac{R_{37}}{R_{44}R_{40}} + \frac{R_{37}}{R_{46}R_{40}}U_1 \end{cases} \tag{15}$$

By Theorem 1, we can see that the state of the system of Eq. (9) is stable, i.e. the chaotic ES (7) is stable by the controller $V(t) = -F(Z_0) - \frac{d^\alpha Z}{dt^\alpha} + \dot{Z}$. Figure 3 shows the stable waveform by circuit simulation.

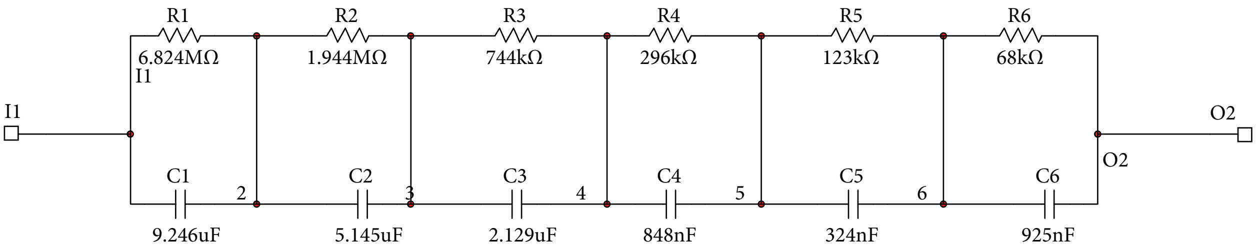


Figure 1. The $\frac{1}{s^{0.5}}$ circuit unit.

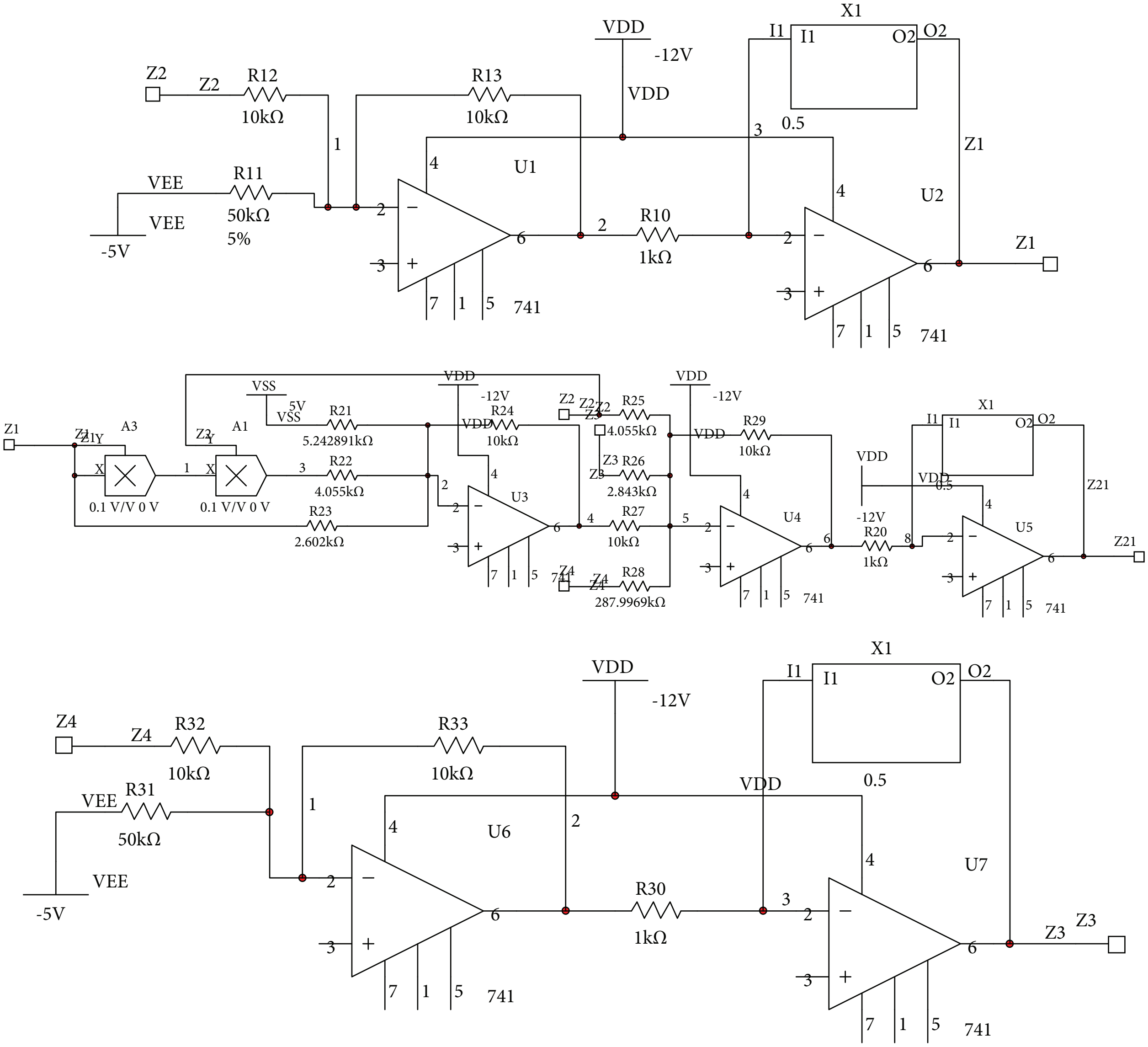


Figure 2. Circuit schematic for the control system of Eq. (9): a) circuit schematic for state z_1 of Eq. (9), b) circuit schematic for state z_2 of Eq. (9), c) circuit schematic for state z_3 of Eq. (9), d) circuit schematic for the state z_4 of Eq. (9).

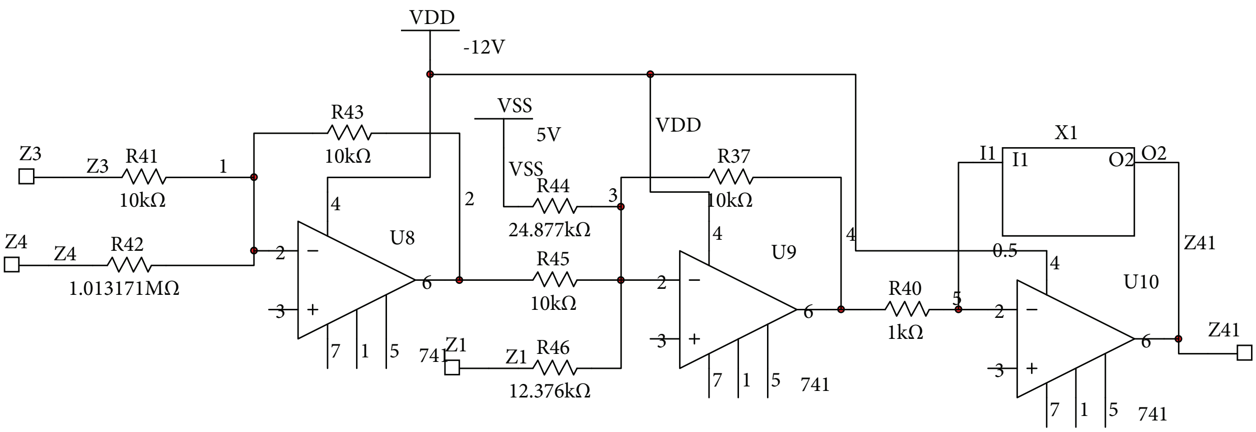
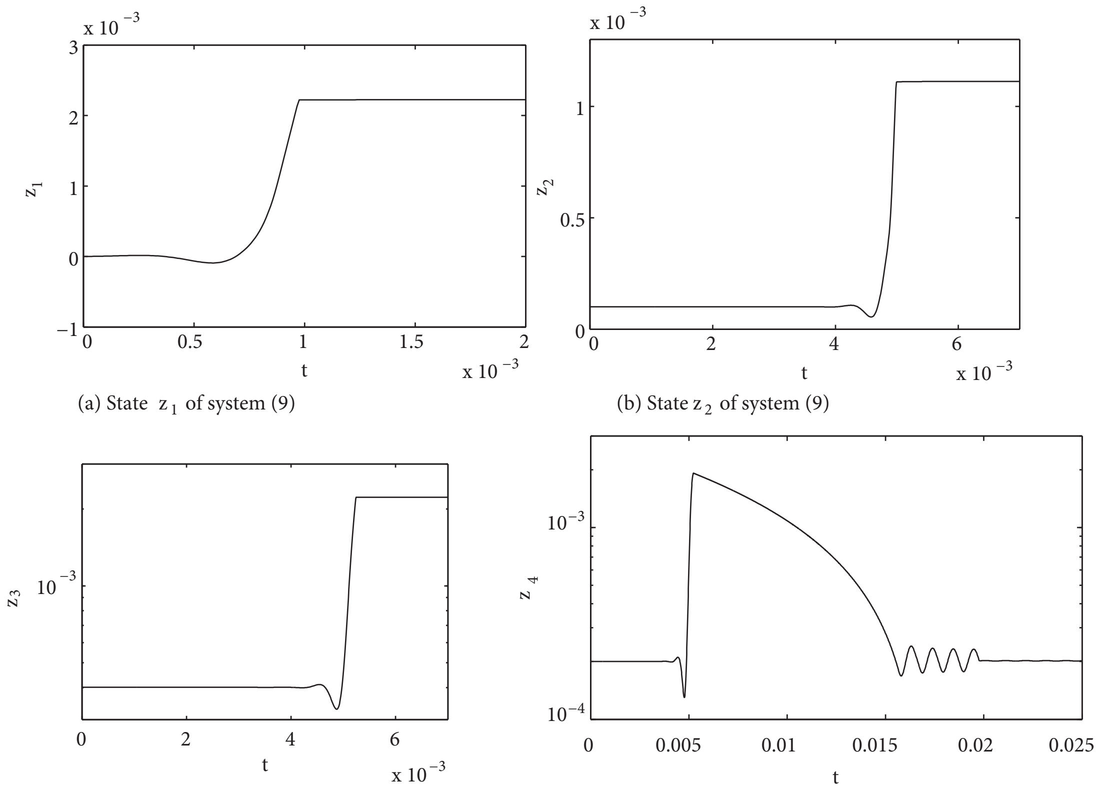


Figure 2. Continued.



(a) State z_1 of system (9)

(b) State z_2 of system (9)

(c) State z_3 of system (9)

(d) State z_4 of system (9)

Figure 3. Stable states of the system of Eq. (9): a) state z_1 of Eq. (9), b) state z_2 of Eq. (9), c) state z_3 of Eq. (9), d) state z_4 of Eq. (9).

4.2. Control chaos via fractional differential controller

Since the fractional differential controller is suitable for arbitrary alpha order systems, in order to facilitate the circuit simulation, we take $\alpha = \beta = \gamma = r = 0.9$. Figure 4 shows the fractional $\frac{1}{s^{0.9}}$ circuit unit [27], and its circuit equation is $\frac{1}{s^{0.9}} \doteq \frac{R_1}{sR_1C_1+1} + \frac{R_2}{sR_2C_2+1} + \frac{R_3}{sR_3C_3+1}$. The circuit schematic for every state of the control system of Eq. (11) is shown in Figure 5, and the circuit equation is described by the system given in Eq. (16).

By Theorem 2, we can see that the states of the system of Eq. (11) are stable, and the stable waveform by circuit simulation is given in Figure 6.

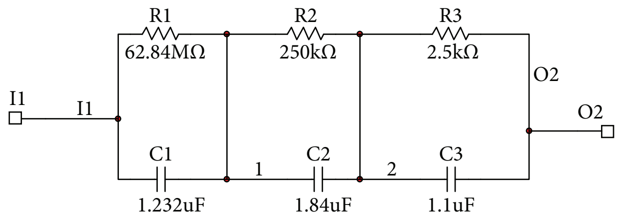


Figure 4. The $\frac{1}{s^{0.9}}$ circuit unit.

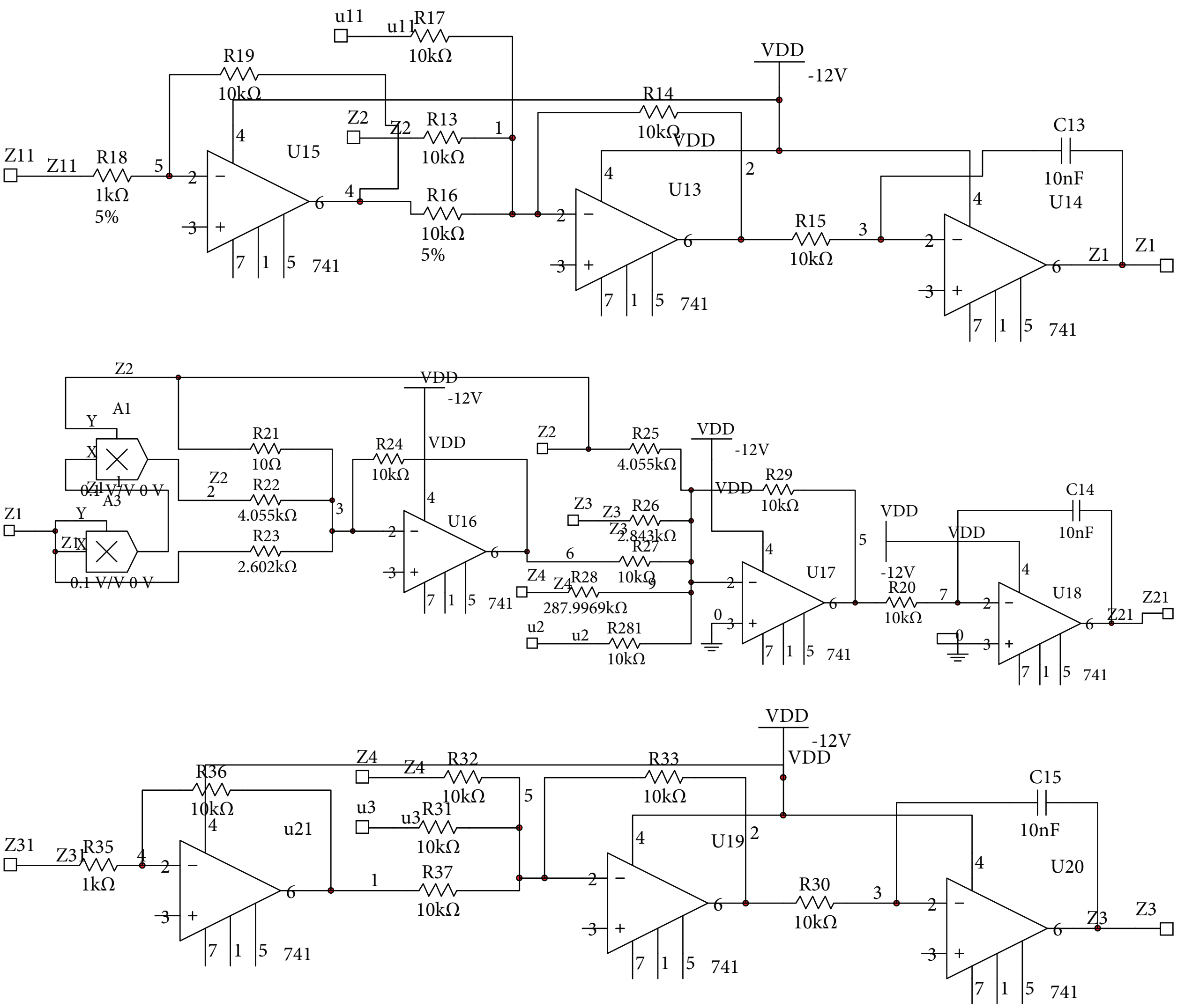


Figure 5. Circuit schematic for the control system of Eq. (11): a) circuit schematic for state $\hat{z}_1$ of Eq. (11), b) circuit schematic for state $\hat{z}_2$ of Eq. (11), c) circuit schematic for state $\hat{z}_3$ of Eq. (11), d) circuit schematic for state $\hat{z}_4$ of Eq. (11), e) circuit schematic for state u_1 of Eq. (11), f) circuit schematic for state u_2 of Eq. (11), g) circuit schematic for state u_3 of Eq. (11), h) circuit schematic for state u_4 of Eq. (11).

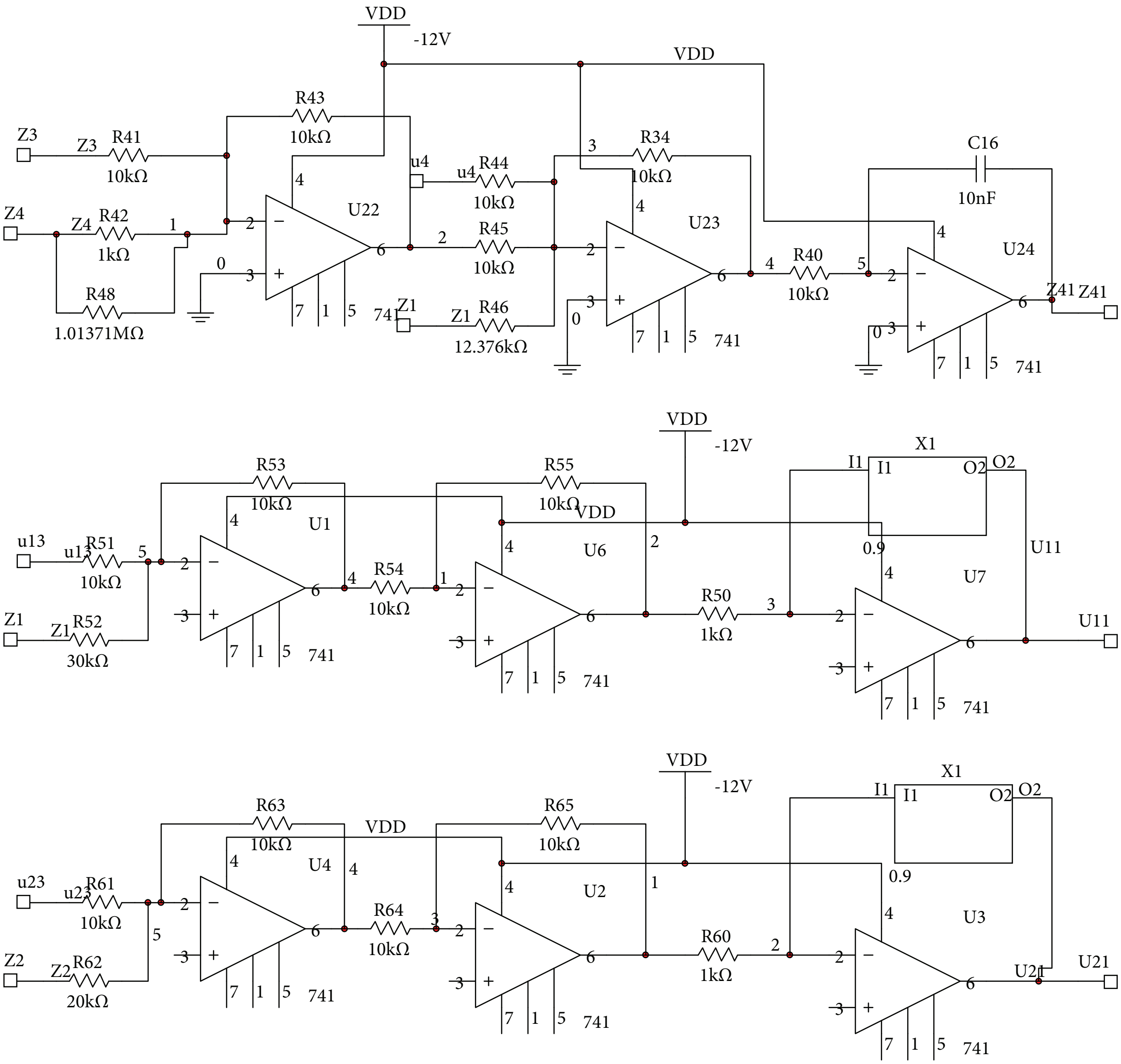
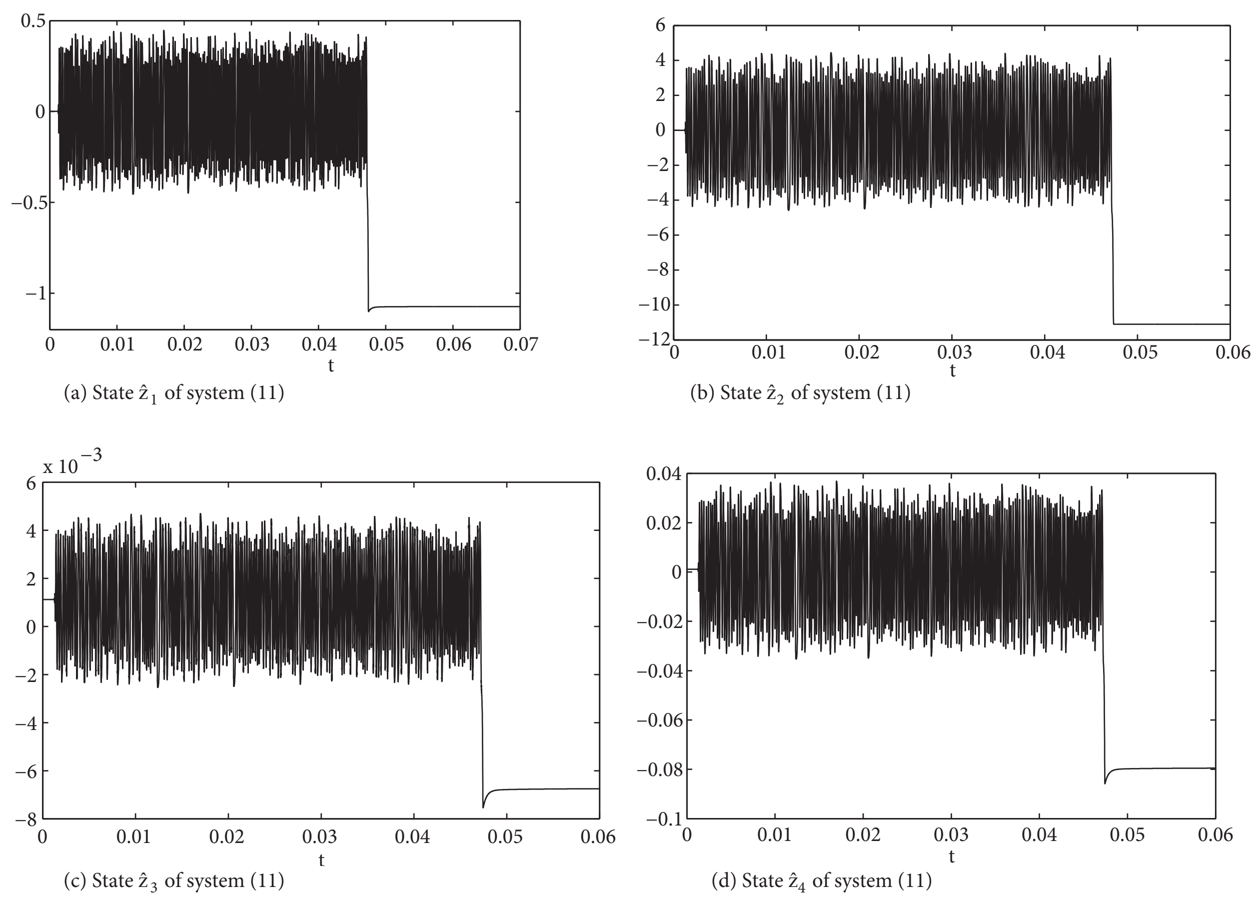


Figure 5. Continued.

$$\left\{\begin{array}{l}
\frac{dU_1}{dt} = \frac{R_{14}}{R_{13}R_{15}C_{13}}U_2 + \frac{R_{14}}{R_{17}R_{15}C_{13}}u_1 - \frac{R_{19}R_{14}}{R_{18}R_{16}R_{15}C_{13}}U_1 \\
\frac{dU_2}{dt} = -\frac{R_{29}R_{24}}{R_{27}R_{22}R_{20}C_{14}}U_1^2U_2 - \frac{R_{29}R_{24}}{R_{27}R_{23}R_{20}C_{14}}U_1 + \frac{R_{29}}{R_{26}R_{20}C_{14}}U_3 \\
+\frac{R_{29}}{R_{28}R_{20}C_{14}}U_4 + \frac{R_{29}}{R_{281}R_{20}C_{14}}u_2 + (\frac{R_{29}}{R_{25}} - \frac{R_{29}R_{24}}{R_{27}R_{21}})\frac{1}{R_{20}C_{14}}U_2 \\
\frac{dU_3}{dt} = \frac{R_{34}}{R_{33}R_{35}C_{15}}U_4 + \frac{R_{34}}{R_{37}R_{35}C_{15}}u_3 - \frac{R_{39}R_{34}}{R_{38}R_{36}R_{35}C_{15}}U_3 \\
\frac{dU_4}{dt} = -\frac{R_{43}R_{34}}{R_{41}R_{45}R_{40}C_{16}}U_3 - (\frac{R_{43}R_{34}}{R_{42}R_{45}R_{40}C_{16}} + \frac{R_{43}R_{34}}{R_{48}R_{45}R_{40}C_{16}})U_4 \\
+\frac{R_{34}}{R_{44}R_{40}C_{16}}u_4 + \frac{R_{34}}{R_{46}R_{40}C_{16}}U_1 \\
\frac{d^{0.9}u_1}{d^{0.9}t} = -\frac{R_{53}R_{55}}{R_{51}R_{54}R_{50}}u_1 - \frac{R_{53}R_{55}}{R_{52}R_{54}R_{50}}U_1 \\
\frac{d^{0.9}u_2}{d^{0.9}t} = -\frac{R_{63}R_{65}}{R_{61}R_{64}R_{60}}u_2 - \frac{R_{63}R_{65}}{R_{62}R_{64}R_{60}}U_2 \\
\frac{d^{0.9}u_3}{d^{0.9}t} = -\frac{R_{73}R_{75}}{R_{71}R_{74}R_{70}}u_3 - \frac{R_{73}R_{75}}{R_{72}R_{74}R_{70}}U_3 \\
\frac{d^{0.9}u_4}{d^{0.9}t} = -\frac{R_{83}R_{85}}{R_{81}R_{84}R_{80}}u_3 - \frac{R_{83}R_{85}}{R_{82}R_{84}R_{80}}U_4
\end{array}\right. \tag{16}$$



(a) State $\hat{z}_1$ of system (11)

(b) State $\hat{z}_2$ of system (11)

(c) State $\hat{z}_3$ of system (11)

(d) State $\hat{z}_4$ of system (11)

Figure 6. Stable states of the system of Eq. (11): a) state $\hat{z}_1$ of Eq. (11), b) state $\hat{z}_2$ of Eq. (11), c) state $\hat{z}_3$ of Eq. (11), d) state $\hat{z}_4$ of system (11).

4.3. GS with observer

Supposing the closed-loop poles at $[-2,-1,-4,-3]$, we can obtain the feedback gain matrix $K = \begin{pmatrix} -12.1751 & -5.0511 & 3.4819 & -19.6827 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ by the pole assignment algorithm, and we let $P = 0.3I$. From Figures 7–9, we can see that by the observer of Eq. (13) and the original system of Eq. (12) we have GS.

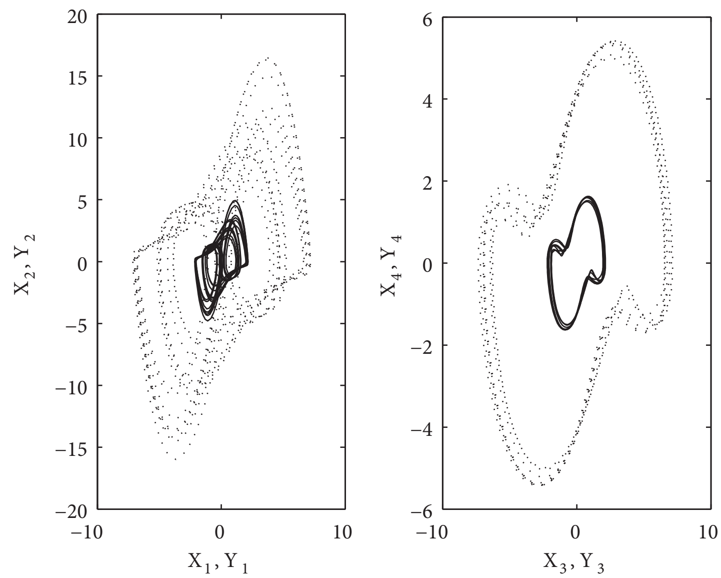


Figure 7. GS of the system of Eq. (12) and the observer of Eq. (13) (X: real line, Y: dotted line).

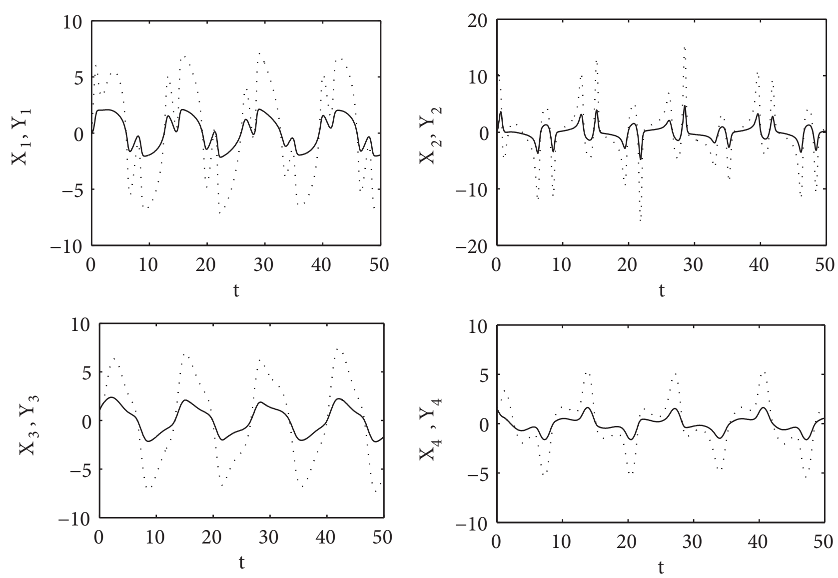


Figure 8. GS of states for the system of Eq. (12) and the observer of Eq. (13) (X: real line, Y: dotted line).

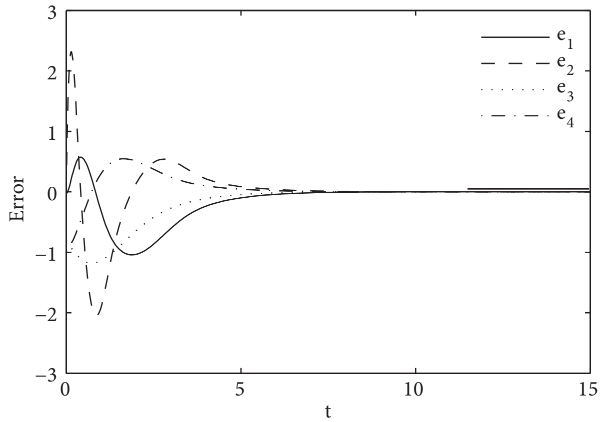


Figure 9. State diagram of the error system of Eq. (14).

5. Conclusion

In this paper, based on fractional calculus and the Gershgorin circle theorem, we construct two control laws for chaos suppression in a SSES consisting of a van der Pol–Duffing coupled oscillator. Any desired point in the chaos attractor of the ES can be stabilized via the proposed controller. Meanwhile, we offer an observer for GS of the ES. Circuit simulations confirm the efficiency of the proposed controller. The main contribution of this paper is selecting the order of the fractional order control system (see Eq. (9)) to control the integer order electromechanical chaotic system. It is a novel technique compared with the other control methods such as robust control [12], adaptive backstepping control [6], passive control [13], and delay feedback control [14–16]. The traditional control technique does not change the order of the control system, but this new control method (the first controller) is not the same. Other topics such as the influences of the noise in the parameters and the structure of the controlled system can be investigated for future research.

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