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# Wideband analysis of planar scalable antennas and PEC bodies using CBFM

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**Abstract:** In this paper, scattering and radiation from scalable planar bodies are investigated in a wide frequency band. Characteristic basis functions (CBFs) are utilized for this purpose with a different approach. The CBFs for the body are calculated at the highest frequency of interest and reused at other frequencies. As the frequency decreases, the unnecessary CBFs are removed from the basis set. The reduced impedance matrices generated at lower frequencies are also used for the calculation of the ones at higher frequencies. As a result, a significant reduction in CPU time is obtained with the proposed method. Numerical results for the problems of scattering from a square plate and radiation from a bowtie antenna are presented to demonstrate the accuracy and efficiency of the proposed method.

**Key words:** Characteristic basis function method, method of moments, integral equations

## 1. Introduction

The electromagnetic properties of an arbitrarily shaped perfect electric conductor (PEC) could be obtained by the solution of the electric field integral equation (EFIE). The method of moments (MoM) is one of the most popular numerical methods used for the solution of the EFIE [1]. In the MoM, a matrix equation is obtained by dividing the PEC body into subdomains and defining unknown current densities on them. The number of subdomains and hence the size of the matrix increase as the electrical dimensions of the PEC body increase. This leads to prohibitive CPU times for the solution of the matrix equation for electrically large bodies. Many types of fast solution methods have been proposed [2] in order to circumvent this problem. Some of these methods are the fast multipole method [3], the impedance matrix localization method [4], the matrix decomposition algorithm [5], and wavelet transform methods [6].

Another method that is used for the solution of electromagnetic scattering or radiation problems is the characteristic basis function method (CBFM). This method is based on the characteristic basis functions defined on macrodomains (blocks) [7,8]. These CBFs are generated taking the self-interactions and the mutual coupling effects of the blocks into account. Since the number of unknowns decreases, the matrix equation obtained in this method could be solved directly, instead of using iterative methods.

Many electromagnetic applications require the solution of the radiation from an antenna or scattering problem over a wide frequency band rather than at a single frequency. However, the solution of the problem using either the conventional CBFM or other fast solution methods mentioned above requires the calculations to be executed at each frequency. In the CBFM, the CBFs and the reduced matrices should be recalculated at each frequency over the band of interest. This creates a heavy burden on CPU time, especially for wideband analysis

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of electrically large objects. Existing methods for wideband EM analysis are based on interpolating the MoM matrix [9] or utilizing frequency and frequency derivative data [10] with reduced frequency samples. However, when the electrical size of the body is large, the CPU time for these methods may become prohibitively high.

In [11] it was shown that the CBFS derived for the highest frequency in the range of interest could be used at lower frequencies, too. The use of these ultrawide band characteristic basis functions (UCBFs) enables one to solve the scattering for any frequency sample in the band without going through the time-consuming process of generating the CBFs anew. However, as indicated in [11], since these UCBFs have been generated at the highest frequency, their number is higher than necessary for lower frequencies. This leads to a reduced matrix whose dimensions are larger than needed. As the frequency decreases, the number of unnecessary CBFs increases. As a result, unless the number of CBFs is decreased at lower frequencies in some way, the time consumed for the calculation and the solution of the reduced matrix will be much more than needed.

In this work, we present CBFs that are generated with a different approach for the solution of radiation and scattering problems from planar scalable antennas and PEC bodies. These CBFs are calculated at the highest frequency of interest and could be used over a wide frequency band as in the method described in [11]. However, unlike the UCBFs mentioned above, the unnecessary CBFs are removed from the basis set as the frequency decreases. Since the reduced matrix dimension is decreased, the time required for the calculation of the reduced matrix and the solution of the new matrix equation decreases in this method. Unlike the method in [11], the reduced matrices generated for lower frequencies are used in the calculation of the ones at higher frequencies in the presented method. This reduces the CPU time significantly since most of the calculations are not repeated at each frequency as in the conventional CBFM and the version of the CBFM mentioned above.

The theory of the conventional CBFM and CBFM for scalable planar bodies is described in Sections 2.1 and 2.2, respectively. In Section 3, numerical results of the scattering problem from a square PEC plate and the radiation problem from a bowtie antenna are presented to demonstrate the accuracy and the computational efficiency of the proposed method.

## 2. Theory

### 2.1. Characteristic basis function method

The application of MoM formulation for the electromagnetic scattering problem results in a matrix equation as follows:

$$Z \cdot J = V, \tag{2.1}$$

where $V$ is the $N \times 1$ excitation vector, $J$ is the $N \times 1$ current density vector, and $Z$ is the $N \times N$ impedance matrix, $N$ being the number of unknowns. As the electrical dimensions of the object increase, the number of unknowns increases. This leads to a significant increase in required CPU time and memory consumption since they are proportional to $O(N^3)$ and $O(N^2)$, respectively. The increase in the size of the moment matrix necessitates the use of iterative solvers instead of direct solvers. However, if the solution is needed for more than one excitation vector, iterative solvers would be inefficient since they need to be started again for each excitation vector. In addition, they might suffer from convergence difficulties [7].

The CBFM is a technique introduced by Prakash and Mittra to circumvent these problems by reducing the size of the impedance matrix in Eq. (2.1) [7]. The CBFM begins by dividing the object into $M$ distinct blocks. For the sake of illustration, a square plate divided into 16 blocks is shown in Figure 1. The blocks are then extended as shown in Figure 1, and the CBFs for each extended block are calculated. The CBFs, which consist of primary CBFs arising from the self-interactions of the blocks and secondary CBFs arising from the mutual coupling effects with other blocks, are then truncated to the original block size.

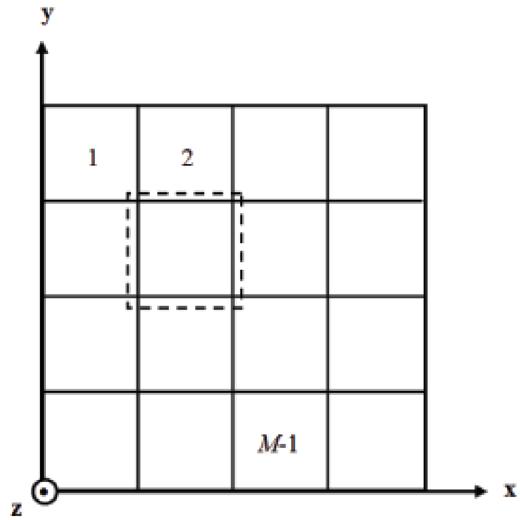


**Figure 1.** Square plate divided into 16 blocks.

The primary and secondary CBFs are generated by solving the following equations.

$$Z^{ii} \cdot J_i^i = V^i \text{ for } i = 1, 2, \ldots, M \tag{2.2}$$

$$Z^{ii} \cdot J_k^i = -Z^{ik} J_k^k \text{ for } k = 1, 2, \ldots, i-1, i+1, \ldots, M \tag{2.3}$$

Here, $Z^{ik}$ is the submatrix formed from the MoM matrix $Z$ by selecting the testing location at block $i$ and source location at block $k$, and $V^i$ is the subvector formed from $V$ by selecting the testing location at block $i$. $J_i^i$ is the primary CBF for block $i$, and $J_k^i$ is the $k$th secondary CBF for block $i$. The number of primary CBFs for a geometry divided into $M$ blocks is equal to $M$ while the number of secondary CBFs is equal to $M \times (M-1)$. Once Eq. (2.2) is solved by LU factorization of the block matrix $Z^{ii}$ for each of the $M$ blocks, Eq. (2.3) could be solved easily using the already factored matrix $Z^{ii}$. The CBFs calculated for the extended blocks using Eqs. (2.2) and (2.3) are then truncated to the original size and orthonormalized. The current density vector now could be expressed as follows.

$$J = \sum_{k=1}^{M} \alpha_k^1 \begin{bmatrix} [J_k^1] \\ [0] \\ \vdots \\ [0] \end{bmatrix} + \sum_{k=1}^{M} \alpha_k^2 \begin{bmatrix} [0] \\ [J_k^2] \\ \vdots \\ [0] \end{bmatrix} + \ldots + \sum_{k=1}^{M} \alpha_k^M \begin{bmatrix} [0] \\ [0] \\ \vdots \\ [J_k^M] \end{bmatrix} \tag{2.4}$$

After substituting Eq. (2.4) in Eq. (2.1), a reduced matrix equation could be formed whose solution gives the unknown coefficients $\alpha_k^i$ and hence the solution vector $J$. Since the MoM matrix dimension is reduced to $M^2 \times M^2$, the matrix equation could be solved using direct methods even if the number of unknowns $N$ is large.

This approach becomes more effective when the solution is desired for multiple excitations because the block matrices $Z_{ik}^{mn}$ need to be factorized only once and could be used for several excitations [7].

### 3. Characteristic basis function method for scalable bodies

Let us now consider an open PEC surface for which electromagnetic properties such as RCS are needed to be calculated over a wide frequency band instead of a single frequency. In order to obtain electromagnetic properties over a frequency band, Eq. (2.1) has to be generated and solved repeatedly for each frequency over the band of interest. In the CBFM, the solution of the problem requires the calculation of new primary and secondary CBFs for each frequency, which in turn are used to form the reduced order matrix equations at these frequencies.

Instead of using conventional CBFs, the open surface geometry could be divided into blocks that have shapes such that using the subsets of these blocks one could obtain the electrical equivalent of the object at lower frequencies. As an example, a square plate divided into 3 blocks, whose edge lengths are defined in terms of wavelength $(\lambda)$, is shown in Figure 2. In this example, the blocks have outer dimensions scaled as we move from the outer boundary towards the center of the object. The edge lengths of the blocks are chosen to be proportional to the frequencies for which the solution is required.

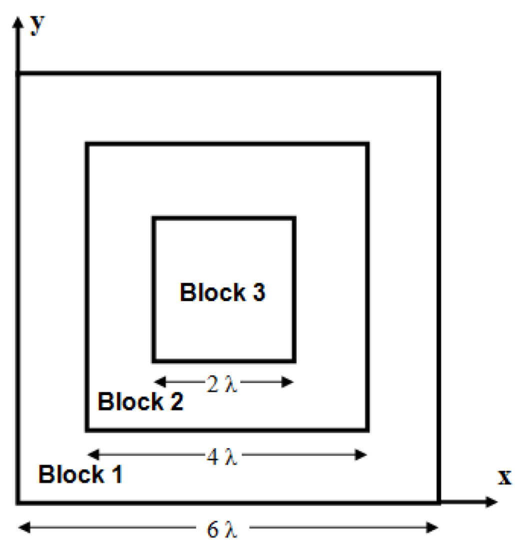


**Figure 2.** Square plate divided into 3 blocks.

After defining the blocks as shown in Figure 2, primary and secondary CBFs for each of these blocks are calculated. In order to obtain wideband CBFs, we first define the coefficient matrices $Z_{ik}$ which are the submatrices of the MoM matrix. The coefficient matrices $Z_{ik}$ are generated using the MoM matrix elements that have the testing location at block i and the source location at block k.

After defining the coefficient matrices $Z_{ik}$, CBFs can be calculated in a way similar to that mentioned in the previous section. The extent of the frequency band over which these CBFs could be used depends on the properties of the defined blocks. Let us consider the square plate in Figure 2 again, and suppose that the solution at three frequencies, which are proportional to 1, 2, and 3, are needed. Denote these frequencies by $f_1$, $f_2$, and $f_3$ from the lowest to the highest. Defining three blocks that have outer edge lengths proportional to 3, 2, and 1 is the first step. The edge lengths of the blocks would then be equal to 6 $\lambda$, 4 $\lambda$, and 2 $\lambda$ for the square plate of Figure 2 at the highest frequency of interest, i.e. $f_3$. Next, the CBFs for these blocks at $f_3$ are calculated. The CBFs obtained in this manner not only will span the solution at the highest frequency $f_3$ but will also span the solution at the other two lower frequencies $f_1$ and $f_2$.

Although each CBF is needed to represent the solution at the highest frequency, as the frequency decreases the blocks in the outer regions would be redundant. For example, while CBFs of block 1, block 2, and block 3 are needed to represent the current distribution on the square plate at $f_3$, CBFs of block 2 and block 3 would be sufficient to represent the solution at $f_2$. For the smallest frequency $f_1$, the CBFs for block 1 would be equal to the solution itself.

Once the CBFs are calculated at the highest frequency of interest, the reduced matrix equations for each frequency need to be generated. This time, the reduced matrices calculated for lower frequencies are used for the calculation of the ones at higher frequencies. In this way, the calculations related to the inner blocks are not repeated for the calculation of the larger bodies that include these blocks. The only calculations carried out are the ones related to the outer blocks we add to get the equivalent electrical dimensions at higher frequencies. The calculation of the reduced matrix for the lowest frequency is done first, and then the ones for higher frequencies follow using the reduced matrix we obtained in the previous step. Finally, the reduced matrix equations can be solved using direct solvers as in the conventional CBFM.

As mentioned before, the method proposed here allows the use of the subset of the CBFs generated at the highest frequency for lower frequencies. Furthermore, it also allows the use of reduced matrices generated at lower frequencies for the higher ones. This will save CPU time significantly since the most time-consuming calculations of the CBFM are not repeated at each frequency. The final step in this method is the solution of the reduced matrix equations. As the electrical size of the object increases, the savings increase, and the method becomes more effective.

In the next section, we give some numerical results to show the effectiveness of the proposed method.

## 4. Numerical examples and discussion

To demonstrate the efficiency and accuracy of the method presented above, the analysis results of scattering from a square plate and radiation from a bowtie antenna will be presented. The simulation results obtained with the method proposed here are given in the following sections together with the MoM results. The comparisons of the CPU times are also given in the following sections.

## 5. Square plate

First, scattering from a PEC plate of 20 cm $\times$ 20 cm illuminated by a normally incident plane wave polarized along the $x$ axis as shown in Figure 3 is analyzed. The bistatic radar cross-section at 36 frequencies with frequency increments of 0.3 GHz over a frequency range of 1:8 (1.5 GHz to 12 GHz) is calculated and the results are compared with MoM solutions. Pulse basis functions and razor blade testing functions are used in the calculation of the MoM matrix equation.

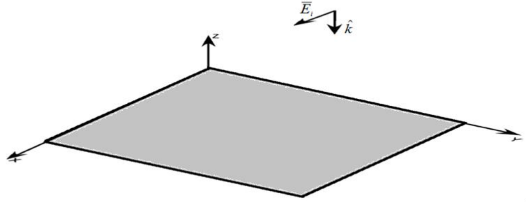

**Figure 3.** PEC plate illuminated by a normally incident plane wave.

The number of unknowns defined on the plate is 12,640. The plate is divided into 36 frames with the edge lengths of 20 cm to 2.5 cm from the outer boundary towards the center. Two blocks are defined on each frame that include current sources along $x$ and $y$ directions. Thus, the total number of blocks is equal to 72. The bistatic RCS values calculated with the proposed method at 1.5 GHz, 6 GHz, and 12 GHz are presented in Figure 4 along with the MoM results. The corresponding current densities obtained with this method and MoM at 1.5 GHz, 6 GHz, and 12 GHz are shown in Figures 5–7. It is seen that the two results are very close, especially for the RCS. The $L_2$ norm of the current density and RCS errors referred to the MoM solutions are shown in Figures 8 and 9, respectively. The RCS error maximum is 1.7% at 2.4 GHz, and it steadily decreases to 0.0002%. The corresponding current density error maximum is 6.6% at 2.4 GHz and it decreases to 0.04%. The CPU time is 3812 s for MoM and 344 s for the proposed method to calculate the RCS at 36 frequencies between 1.5 GHz and 12 GHz. The total CPU time needed for the calculations at the highest frequency is the sum of the CPU times required to calculate the CBFs, to carry out the matrix vector products after substituting Eq. (2.4) in Eq. (2.1), to generate the reduced matrix, and to solve the new matrix equation in the CBFM. As mentioned in the previous section, using the conventional CBFM, each of these calculations has to be made at each frequency. In the current method, on the other hand, the CBFs are calculated only at the highest

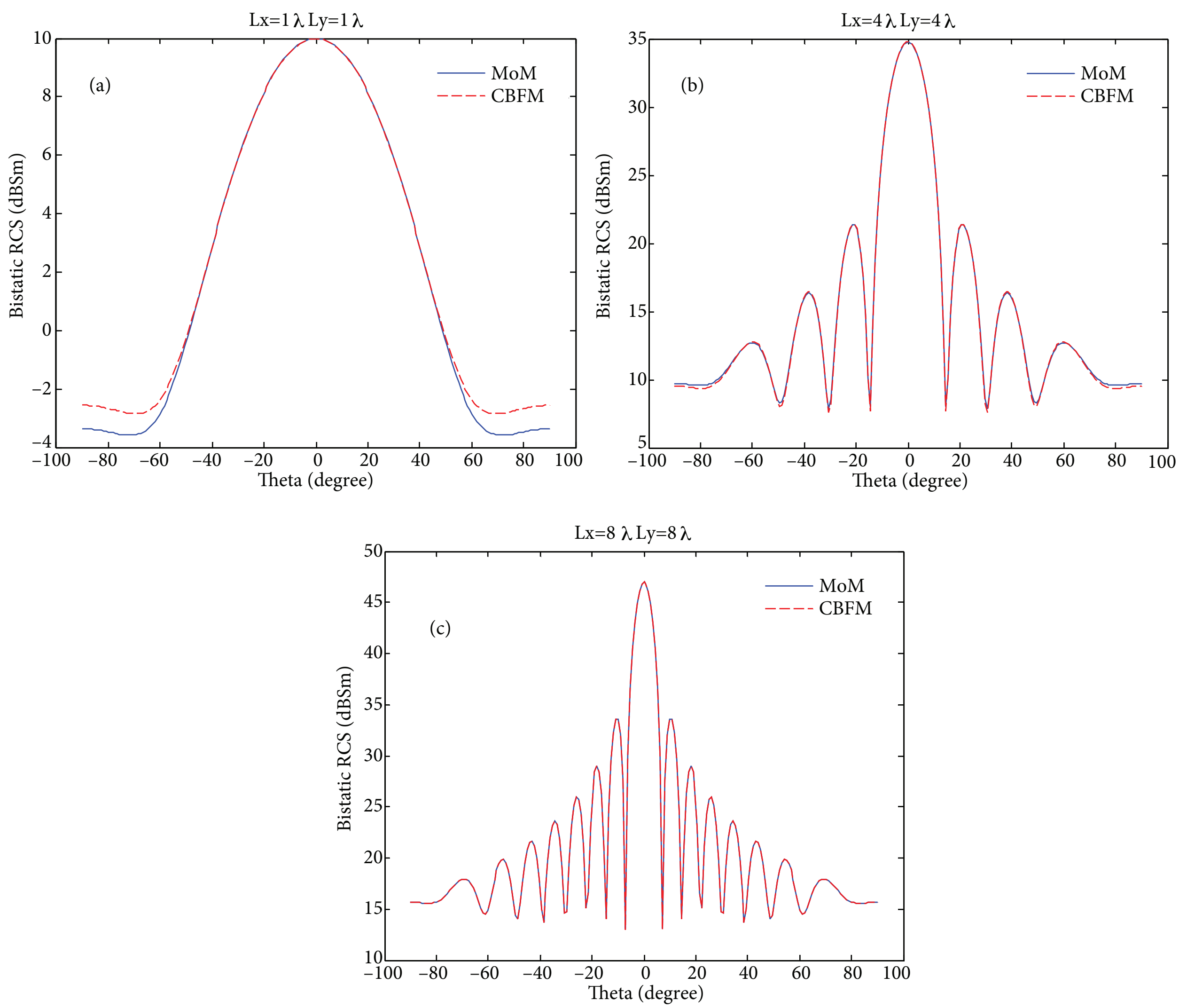


**Figure 4.** Bistatic RCS of normally illuminated PEC plate at (a) 1.5 GHz, (b) 6 GHz, (c) 12 GHz.

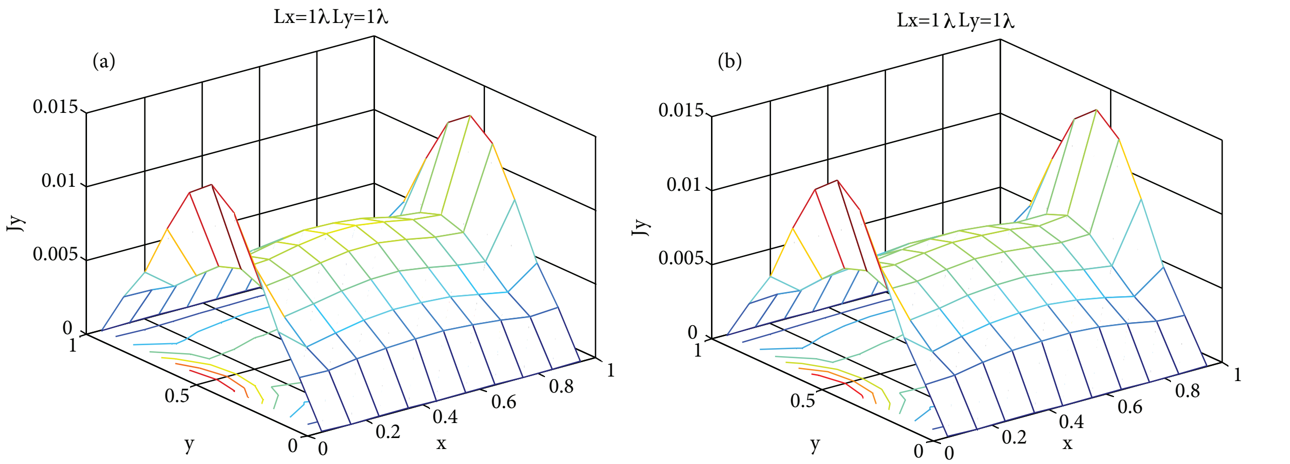


**Figure 5.** Current densities along y obtained with (a) CBFM and (b) MoM for normally illuminated PEC body at 1.5 GHz.

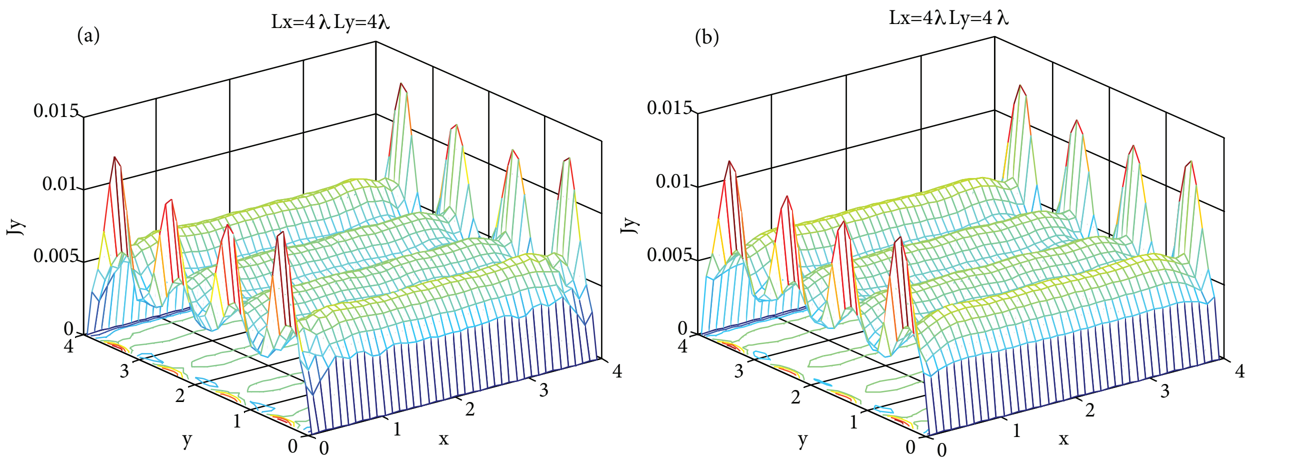


**Figure 6.** Current densities along y obtained with (a) CBFM and (b) MoM for normally illuminated PEC body at 6 GHz.

frequency of interest and are reused at lower frequencies. Reduced matrices obtained at lower frequencies are used for the generation of new ones at higher frequencies also, contributing to the savings of the method. As a result, the CPU time for the frequencies other than the highest frequency involves only the time needed for the calculations of the reduced matrix components, which are related to the outer blocks that are added as the frequency increases, and the time consumed for the solution of the reduced matrix equation.

## 6. Bowtie antenna

The second example is the bowtie antenna shown in Figure 10. The height of the antenna is 6 cm and the flare angle is 90° . Triangular patch segments and Rao–Wilton–Glisson [12] basis functions are used in the calculation of the MoM matrix equation.

The number of triangles defined on the antenna is 7440 and the number of unknowns is 10,979. The antenna is divided into 40 blocks as shown in Figure 10. The height of the blocks is 0.5 mm. The CBFs are calculated at 30 GHz. As the frequency decreases, the CBFs calculated for the outer blocks are extracted from the CBF set to obtain the electrical equivalence of the antenna at lower frequencies as in the plate example.

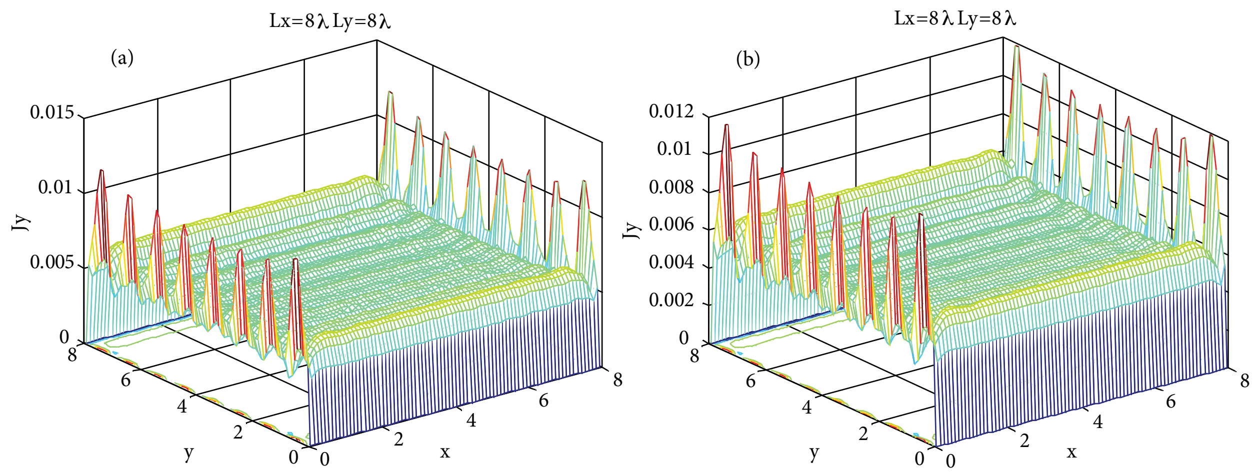


**Figure 7.** Current densities along y obtained with (a) CBFM and (b) MoM for normally illuminated PEC body at 12 GHz.

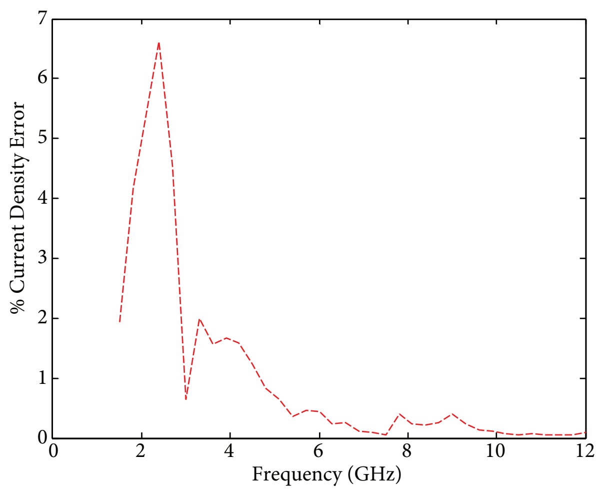


**Figure 8.** $L_2$ norm of the current density error.

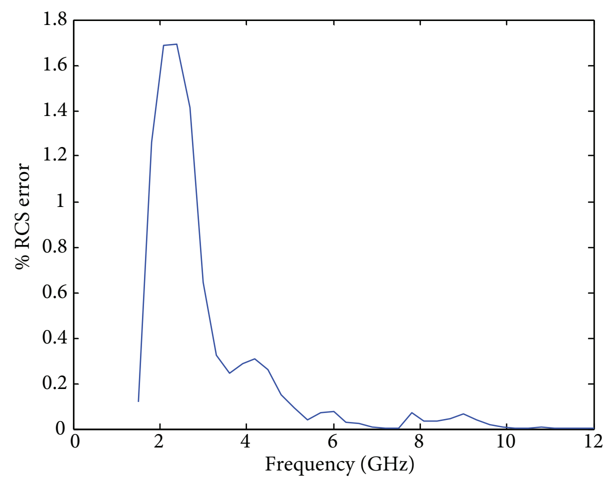


**Figure 9.** $L_2$ norm of the RCS error.

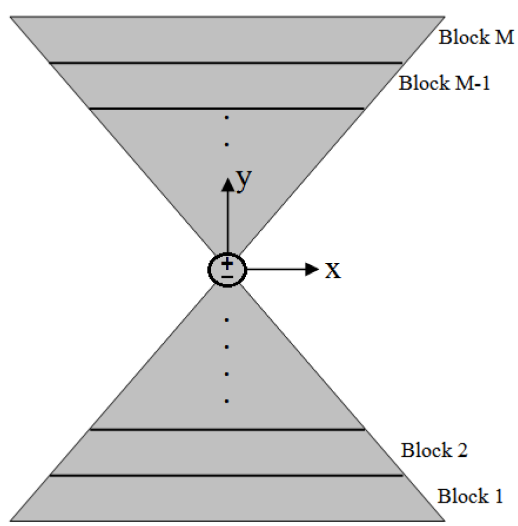


**Figure 10.** Bowtie antenna divided into M blocks.

The CBF calculation for the radiation problem is slightly different than the calculation of that for the scattering problem [13]. For the radiation problem, first the primary CBFs for the excited patches are found. Here the primary CBF for the horizontal edge at the center of the antenna is calculated first. The secondary CBFs for the other blocks, which occur as a result of the impact of the primary CBFs, are then calculated. Finally tertiary CBFs are calculated, which occur as a result of the impact of the secondary CBFs on the other blocks.

The radiation patterns at 41 frequencies with frequency increments of 0.5 GHz over a frequency range of 1:3 (10 GHz to 30 GHz) are calculated with the proposed method. The radiation patterns obtained with the

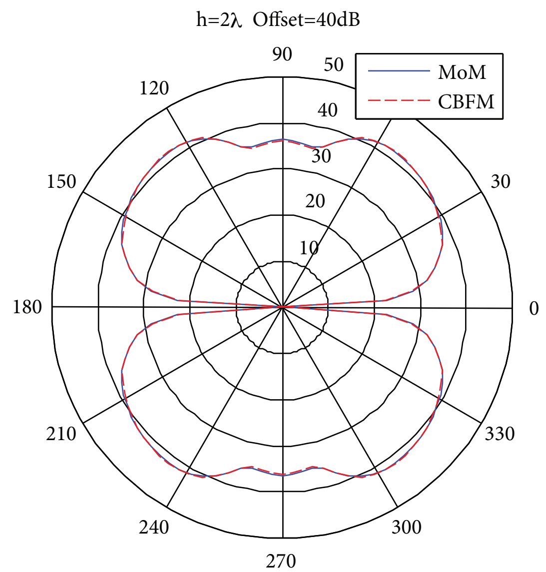


**Figure 11.** Radiation pattern of the bowtie antenna at 10 GHz.

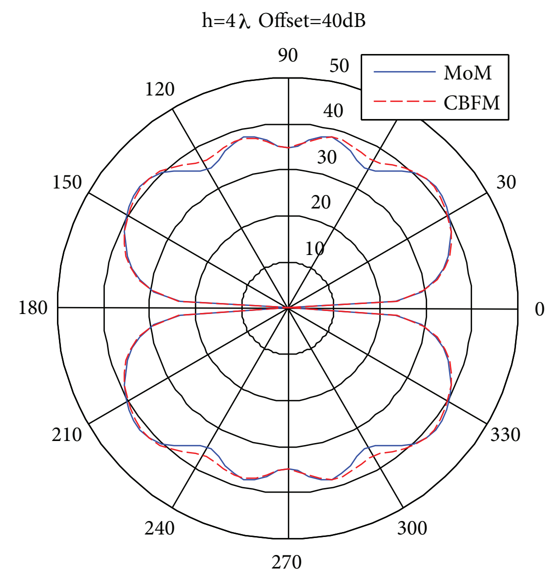


**Figure 12.** Radiation pattern of the bowtie antenna at 20 GHz.

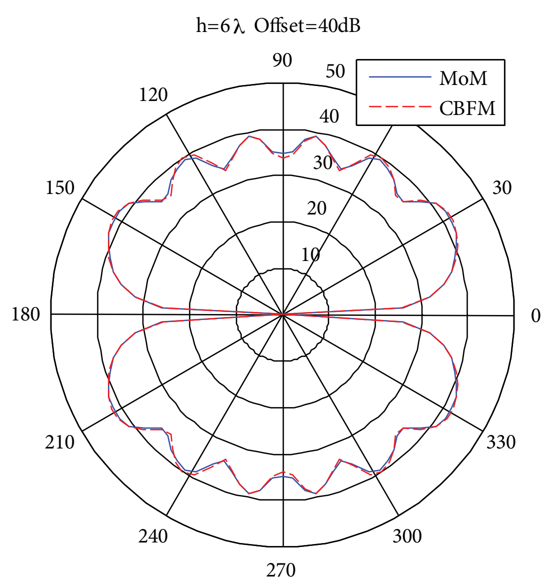


**Figure 13.** Radiation pattern of the bowtie antenna at 30 GHz.

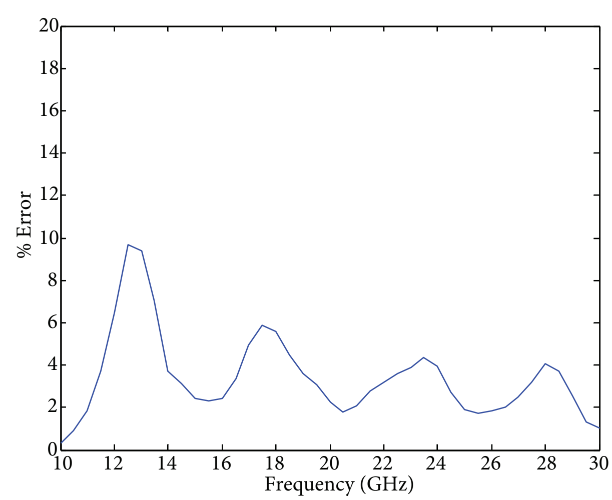


**Figure 14.** $L_2$ norm of the radiation pattern error.

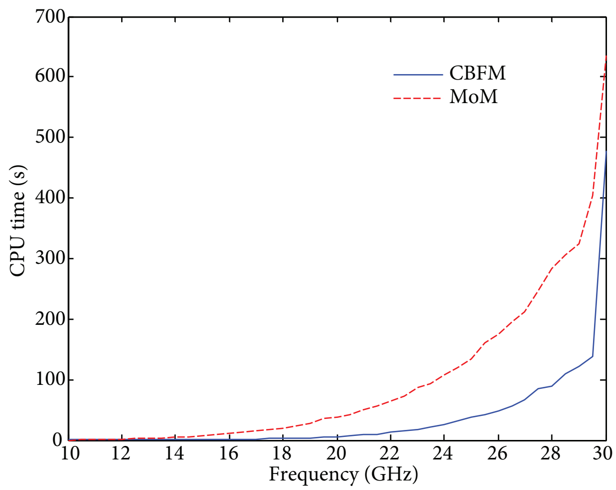


Figure 15. CPU time comparison of MoM and CBFM for bowtie antenna.

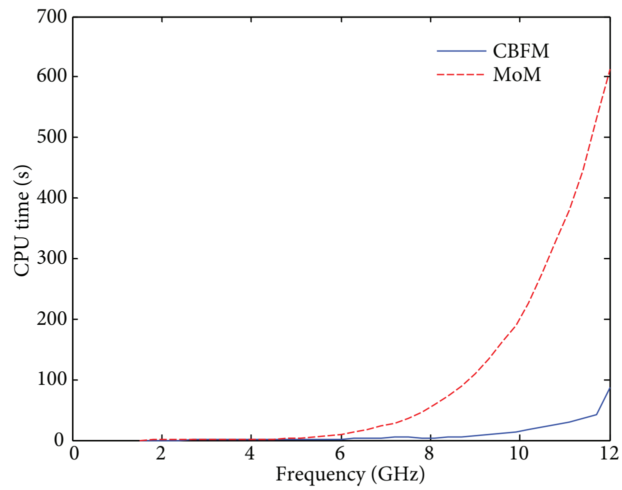


Figure 16. CPU time comparison of MoM and CBFM for square plate.

proposed method for the y-z plane at 10 GHz, 20 GHz, and 30 GHz are shown in Figures 11–13 along with the MoM results. It is seen that the two results are very close to each other. The $L_2$ norm of the radiation pattern errors referred to the MoM solutions is shown in Figure 14. The error maximum for the radiation pattern is 9.69% at 12.5 GHz, and it steadily decreases to 1.001% at 30 GHz. The total CPU time is 4009 s for MoM and 1460 s for the proposed method to calculate the radiation pattern at 41 frequencies between 10 GHz and 30 GHz.

The CPU time comparison for a bowtie antenna between the proposed method and MoM is given in Figure 15. The peak at 30 GHz for the CPU time of our method is due to the calculation of CBFs, which is carried out only once. If the problem for lower frequencies were solved with the CBFM, the CPU time would be better than that for MoM but it would be worse compared to our method. The computational cost of one time calculation of CBFs is a function of block size and the number of elements in blocks. The CPU time comparison for the square plate between our method and MoM, shown in Figure 16, demonstrates this fact, where the peak is smaller. It is worth repeating here that the conventional CBFM would require such a calculation at each frequency.

## 7. Conclusion

In this paper, the CBFM is presented for wideband analysis of scalable PEC structures. It is shown that this method could be used for the solution of the radiation and scattering problems of scalable planar bodies in a very wide frequency band. Although the scalability requirement prohibits the application of the method to arbitrary planar structures, many important designs such as tapered slot antennas and ultrawideband printed or planar monopole antennas in rectangular, circular, elliptical, or trapezoidal shapes can be carried out with the proposed method.

In the proposed method, the PEC body is divided into a number of blocks whose shape and dimensions are defined in such a manner that the combination of the subset of these blocks forms the electrical equivalent of the body at lower frequencies. Once the CBFs are calculated at the highest frequency, the reduced matrices are generated step by step for each frequency using these CBFs. The algorithm runs from the lowest to the highest frequency of interest, where at each step the reduced matrix of the next step is calculated using that of the previous step. In this way, a significant reduction in CPU time is achieved, since the most time-consuming parts of the conventional CBFM are not repeated at each frequency.

In the first example, the surface current distribution and RCS for a PEC square plate over a frequency range of 1:8 are computed using the proposed method. In the second example, radiation characteristics are analyzed for a bowtie antenna over a frequency range of 1:3. The results are compared with MoM solutions for both of the problems. The numerical results were found to be in good agreement with those obtained by MoM.

Although geometries in air are investigated, the reported method can be easily used for microstrip structures by simply utilizing the corresponding Green's function.

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