

## Stability analysis of an asymmetrical six-phase synchronous motor

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**Abstract:** This paper deals the stability analysis of a six-phase synchronous motor using eigenvalue criteria from its linearized model, so that the small displacement stability during steady-state condition can be accessed. Eigenvalue provides a simple but effective means for the stability analysis of a motor. An association between eigenvalues and motor parameters has been established, followed by the formulation of transfer functions and plotting the root locus of system, employing the dq0 approach in the MATLAB environment.

**Key words:** Six-phase synchronous motor, stability analysis, linearized model

### 1. Introduction

The area of multiphase (more than three phases) motor drives has been found to experience accelerated growth over the last two decades. This is because of their use in different applications like electric ship propulsion, EV/HEV vehicle propulsion, aircraft, thermal power plants to drive induce draft fans, nuclear power plants, and higher power applications. A multiphase motor possesses several potential advantages over conventional three-phase motors, such as reduction of space and time harmonics, lower dc link current harmonics, reduced amplitude and increased frequency of torque pulsations, reduction of current per phase without increase in voltage per phase, increase in power to weight ratio, and higher reliability [1–3].

As far as six-phase synchronous motors are concerned, general modelling is available in the literature [4–8]. Analysis of a six-phase synchronous alternator with its two three-phase stator windings displaced by an arbitrary angle was carried out by Fuchs and Rosenberg [4], where an orthogonal transformation of the phase variables into a new set of  $(d - q)$  variables was employed. In the analysis, it was concluded that the advantage of partial elimination of time dependence of the coefficients from system differential equations can be achieved by employing orthogonal transformation. Furthermore, a phasor can be utilized for the steady-state analysis of generator having two sets of stator windings. Consideration of the mutual leakage coupling between two sets of three-phase stator windings in the mathematical model of a six-phase synchronous machine was presented by Schiferl and Ong [5]. Moreover, the transformer and motor mode of power transfer during steady-state was also examined. In their companion paper [6], formulation of the relationships of mutual leakage inductances with winding displacement angle and pitch was carried out, applicable for different winding configurations of six-phase and proposed a scheme of a single machine uninterruptible power supply using a six-phase synchronous machine. Along with the presentation of modelling and simulation of a high power drives using a double star synchronous motor, the effect of winding displacement angle on torque ripple and harmonic effect was highlighted by Terrien

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and Benkhoris [7]. Authors in ref. [8] briefly presented the analysis of a six-phase synchronous machine with emphasis on its redundancy property, operation during fault conditions, machine behavior under nonsinusoidal voltage input, and sensitivity of the design parameters.

Operation of any electrical machine during steady-state depends on many design factors that directly affect the stability of the machine. Stability of the machine is an important factor considered by design engineers. Among the various aspects of stability, small signal stability is an important one. It determines the stable operation of the machine during a small disturbance. This needs a comprehensive stability analysis of the machine to ensure a stable operation of the system. As far as stability of an AC motor is concerned, very limited studies are available. Rogers [9] examined induction machine operation that gives a frequency dependent oscillatory response, and analyzed its stability using the root locus technique. Stability of an induction motor fed by a variable frequency inverter was examined in ref. [10], wherein the effect of harmonics was neglected, using Nyquist stability criteria [11] and the root locus technique [12]. The technique of transfer function for stability of controlled current induction motor was carried out by Cornell and Lipo [13], whereas a linearized model of the current source inverter fed induction motor drive for stability analysis was developed by Macdonald and Sen [14], followed by the formulation of transfer for different control strategies. Stability analysis of a double-cage induction motor was carried out by Tan and Richard [15]; the eigenvalues were calculated using a boundary layer model, whereas the Lyapunov's first method by employing the placement of eigenvalues was carried out in detail for a three-phase induction motor in ref. [16]. As far as stability of a synchronous machine is concerned, small signal stability analysis was carried out for a reluctance-synchronous machine in ref. [17], and the variable frequency operation of synchronous machine in ref. [18] using the Nyquist criteria technique, whereas the root locus study of a synchronous machine was carried out in ref. [19].

The only available papers on small signal stability of multiphase motor are [20, 21]. These deal with the stability issue for six-phase and five-phase induction motors, respectively, with no available literature as far as multiphase synchronous motor is concerned to the best of the authors' knowledge. Therefore, the main purpose of the present paper is to report the small signal stability analysis of a six-phase synchronous motor, by applying the eigenvalues criterion for small excursion behavior through the developed linearized version of motor equations, followed by the establishment of eigenvalues association with motor parameters. Further, a transfer function will be formulated between input and output variables, followed by the stability plots of root locus. Analysis has been carried out using  $dq0$  approach in the MATLAB environment.

## 2. Linearization of motor equations for stability analysis

While developing the linearized model of the motor, some important simplifying assumptions are made:

- Both sets of stator windings ( $abc$  and  $xyz$ ) are symmetrical so as to have a perfect sinusoidal distribution along the air-gap,
- Existence of space harmonics is neglected, ensuring the flux and mmfs are sinusoidal in space,
- Saturation and hysteresis effects are neglected,
- Skin effect is neglected, windings resistance are not dependent on frequency.

A schematic representation of stator and rotor axes is shown in Figure 1. The six stator phases of the motor consist of two sets of star-connected three-phase windings, namely,  $abc$  and  $xyz$ , such that their magnetic

axes are displaced by an angle  $\xi = 30$  electrical degrees (for asymmetrical winding configuration). This is because, with an asymmetrical winding configuration, advantages of the elimination of many lower order time harmonics such as 5th, 7th, 17th, and 19th are ensured from the contribution to the air gap flux and torque pulsation. The rotor circuit is equipped with a field winding  $F_R$  and a damper winding  $K_D$  along the  $d$ -axis and a damper winding  $K_Q$  along the  $q$ -axis. By adopting the motor convention, the equations of voltage and electromagnetic torque when written in machine variables will result in sets of nonlinear differential equations. Nonlinearity is introduced due to the presence of the inductance term, which is the function of rotor position and is time dependent. In order to achieve the simplified equations with constant inductance terms, reference frame theory will be applied, and all the sets of equations will be written in rotor reference frame (Park's equation). The equations of voltages and flux linkage per second of a six-phase synchronous motor in Park's variables are stated in the Appendix [22,23], which are the function of current and flux linkage per second. It is to be noted that either current or flux linkage per second can be selected as state variable, because they are related to each other (as seen in Eqs. (A8)–(A16)). This choice is generally application dependent. In this paper, current has been selected as state variable. Treating current as independent variable, the flux linkage per second is replaced by currents and the voltage–current relation of motor can be written in matrix form. An explanation of the nomenclature used is given in Table 1.

$$[v] = [z][i], \quad (1)$$

where

$$[v] = [v_{Q1}, v_{D1}, v_{Q2}, v_{Q2}, v_{KQ}, v_{FR}, v_{KD}] \quad (2)$$

$$[i] = [i_{Q1}, i_{D1}, i_{Q2}, i_{Q2}, i_{KQ}, i_{FR}, i_{KD}] \quad (3)$$

$$z = \begin{bmatrix} r_1 + \frac{p}{\omega_B}(x_{L1} + x_{LM} + x_{MQ}) & x_{L1} + x_{LM} + x_{MD} & \frac{p}{\omega_B}(x_{LM} + x_{MQ}) + x_{LDQ} \\ -(x_{L1} + x_{LM} + x_{MQ}) & r_1 + \frac{p}{\omega_B}(x_{L1} + x_{LM} + x_{MD}) & \frac{p}{\omega_b}x_{LDQ} - (x_{LM} + x_{MQ}) \\ \frac{p}{\omega_B}(x_{LM} + x_{MQ}) - x_{LDQ} & \frac{p}{\omega_b}x_{LDQ} + (x_{LM} + x_{MD}) & r_2 + \frac{p}{\omega_B}(x_{L2} + x_{LM} + x_{MQ}) \\ -\frac{p}{\omega_B}x_{LDQ} - (x_{LM} + x_{MQ}) & \frac{p}{\omega_B}(x_{LM} + x_{MD}) - x_{LDQ} & -(x_{L1} + x_{LM} + x_{MQ}) \\ \frac{p}{\omega_B}x_{MQ} & 0 & \frac{p}{\omega_B}x_{MQ} \\ 0 & \frac{p}{\omega_B} \frac{x_{MD}^2}{r_{FR}} & 0 \\ 0 & \frac{p}{\omega_B}x_{MD} & 0 \end{bmatrix}$$

$$\left. \begin{array}{llll} -\frac{p}{\omega_B}x_{LDQ} + (x_{LM} + x_{MD}) & \frac{p}{\omega_b}x_{MQ} & x_{MD} & x_{MD} \\ \frac{p}{\omega_b}(x_{LM} + x_{MD}) + x_{LDQ} & -x_{MQ} & \frac{p}{\omega_b}x_{MD} & \frac{p}{\omega_b}x_{MD} \\ x_{L2} + x_{MD} + x_{LM} & \frac{p}{\omega_b}x_{MQ} & x_{MD} & x_{MD} \\ r_2 + \frac{p}{\omega_B}(x_{L2} + x_{LM} + x_{MD}) & -x_{MQ} & \frac{p}{\omega_b}x_{MD} & \frac{p}{\omega_b}x_{MD} \\ 0 & r_{KQ} + \frac{p}{\omega_B}(x_{LKQ} + x_{MQ}) & 0 & 0 \\ \frac{p}{\omega_B} \frac{x_{MD}^2}{r_{FR}} & 0 & \frac{x_{MD}}{r_{FR}} \left\{ r_{FR} + \frac{p}{\omega_B}(x_{LFR} + x_{MD}) \right\} & \frac{p}{\omega_B} \frac{x_{MD}^2}{r_{FR}} \\ \frac{p}{\omega_B}x_{MD} & 0 & \frac{p}{\omega_B}x_{MD} & r_{KD} + \frac{p}{\omega_B}(x_{LKD} + x_{MD}) \end{array} \right] \quad (4)$$

Developed motor electromagnetic torque is given by

$$T_E = \frac{3P}{2} \frac{1}{\omega_B} \left[ \begin{array}{l} (i_{Q1} + i_{Q2})x_{MD}(i_{D1} + i_{Q2} + i_{KD} + i_{FR}) \\ -(i_{D1} + i_{D2})x_{MQ}(i_{Q1} + i_{Q2} + i_{KQ}) \end{array} \right] \quad (5)$$

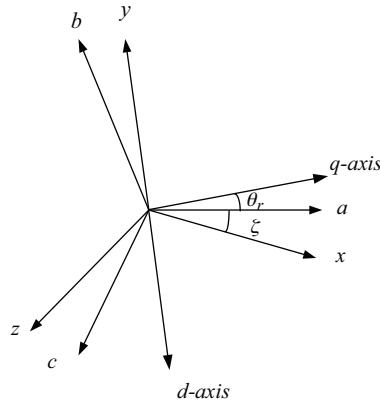


Figure 1. Stator and rotor axes of six-phase motor.

Table 1. Explanation of nomenclature used.

Notation	Explanation
$v_{D1}, v_{Q1}$	$d - q$ voltage of winding set $abc$
$v_{D2}, v_{Q2}$	$d - q$ voltage of winding set $xyz$
$i_{D1}, i_{Q1}$	$d - q$ current of winding set $abc$
$i_{D2}, i_{Q2}$	$d - q$ current of winding set $xyz$
$v_{FR}$	Field excitation voltage
$i_{FR}$	Field excitation current
$v_{KQ}, v_{KD}$	Voltage along damper windings $K_Q$ and $K_D$ , respectively
$i_{KQ}, i_{KD}$	Current along damper windings $K_Q$ and $K_D$ , respectively
$r_1, r_2$	Stator resistance per phase of winding sets $abc$ and $xyz$ , respectively
$x_{L1}, x_{L2}$	Leakage reactance per phase of winding sets $abc$ and $xyz$ , respectively
$x_{LKq}, x_{LKd}$	Leakage reactance of damper windings $K_Q$ and $K_D$ , respectively
$x_{LFR}$	Leakage reactance of field winding
$x_{MD}, x_{MQ}$	Magnetizing inductance along $d - q$ axes respectively
$x_{LM}$	Common mutual leakage reactance between winding sets $abc$ and $xyz$
$x_{LDQ}$	Cross mutual coupling reactance between $d - q$ axes of stator windings
$\psi_{D1}, \psi_{Q1}$	Flux linkage per second along $d - q$ axes of stator winding $abc$
$\psi_{D2}, \psi_{Q2}$	Flux linkage per second along $d - q$ axes of stator winding $xyz$
$\psi_{MD}, \psi_{MQ}$	Magnetizing flux linkage per second along $d - q$ axes, respectively
$T_E, T_L$	Motor electromagnetic torque and load torque, respectively
$\omega_R, \omega_B$	Rotor speed and base speed, respectively
$J$	Moment of inertia

The relationship between torque and rotor speed is given by Eq. (6), whereas the expression of rotor angle (load angle) is given by Eq. (7).

$$T_E = \frac{P}{2J} \frac{\omega_R}{\omega_B} + T_L \tag{6}$$

$$\delta = \frac{\omega_B}{p} \left( \frac{\omega_R - \omega_E}{\omega_B} \right) \tag{7}$$

The relationship between the variables  $f_Q^E$  and  $f_D^E$  in synchronously reference frame and variables  $f_Q^R$  and  $f_D^R$  in rotor reference frame will be helpful in the linearization process, which is given by Eq. (8).

$$\begin{bmatrix} f_{kQ}^R \\ f_{kD}^R \end{bmatrix} = \begin{bmatrix} \cos \delta_k & -\sin \delta_k \\ \sin \delta_k & \cos \delta_k \end{bmatrix} \begin{bmatrix} f_{kQ}^E \\ f_{kD}^E \end{bmatrix}, \tag{8}$$

where  $k = 1$  (for winding set  $abc$ ) and  $2$  (for winding set  $xyz$ ).

$$\delta_1 = \delta_0(\text{load angle})$$

$$\delta_2 = \delta_0 - \gamma - \xi$$

$\gamma$  is the phase difference between phases  $a$  and  $x$  voltages and  $\xi$  is the phase shift between both winding sets  $abc$  and  $xyz$ . Both the angles have the magnitude of 30 electrical degrees.

The procedure of linearization about small departure will be carried out by making use of the well-known Taylor series expansion, wherein each variable  $x$  is replaced by its reference value plus a deviation ( $x_0 + \Delta x$ ). For example, consider an equation  $z = xy$ ; this is written as

$$\begin{aligned} z_0 + \Delta z &= (x_0 + \Delta x)(y_0 + \Delta y) \\ &= x_0 y_0 + y_0 \Delta x + x_0 \Delta y + \Delta x \Delta y \end{aligned}$$

Terms in the reference level can be eliminated from both sides of equation ( $z_0 = x_0 y_0$ ); also the last term ( $\Delta x \Delta y$ ) is neglected, considering the deviations are very small, resulting in an equation between the deviations with coefficient that can be considered constant over a limited region. Simplification incorporated in Taylor series expansion can be quantitatively included by having the following restrictions imposed on motor equations:

(a) Under small perturbation, there is a little variation in flux levels, such that the inductances can be regarded as constant.

(b) Under small perturbation, the terms involving the product of two (or more) deviations can be neglected, being small with respect to other terms.

Linearization of Eqs. (1), (5)–(8) yields an equation that can be written in matrix form as

$$\begin{bmatrix} \Delta v_{1DQ} \\ \Delta v_{2DQ} \\ \Delta v_{RR} \end{bmatrix} = \begin{bmatrix} W_1 & X_1 & Y_1 \\ X_2 & W_2 & Y_2 \\ Q_1 & Q_2 & S \end{bmatrix} \begin{bmatrix} \Delta i_{1DQ} \\ \Delta i_{2DQ} \\ \Delta i_{RR} \end{bmatrix} \tag{9}$$

where

$$\begin{aligned} (x)^T &= [(\Delta i_{1DQ})^T \ (\Delta i_{2DQ})^T \ (\Delta i_{RR})^T] \\ &= \left[ \Delta i_{Q1}, \Delta i_{D1}, \Delta i_{Q2}, \Delta i_{D2}, \Delta i_{KQ}, \Delta i_{FR}, \Delta i_{KD}, \frac{\Delta \omega_R}{\omega_B}, \Delta \delta \right] \end{aligned} \tag{10}$$

$$\begin{aligned} (u)^T &= [(\Delta v_{1DQ})^T \ (\Delta v_{2DQ})^T \ (\Delta v_{RR})^T] \\ &= [\Delta v_{Q1}, \Delta v_{D1}, \Delta v_{Q2}, \Delta v_{D2}, \Delta v_{KQ}, \Delta v_{FR}, \Delta v_{KD}, \Delta T, 0] \end{aligned} \tag{11}$$

Other elements of the matrix are explained in a later part of the section.

A synchronous motor is usually connected to an infinite bus, where the existence of input supply of constant magnitude and frequency is assumed. Therefore, it will be advantageous to relate the synchronously rotating reference frame variables, where independent input supply exists, to the variables in the rotor reference frame. This has to be taken into account with the help of Eq. (8), which is nonlinear. This equation will be linearized before incorporating it into a set of linear set motor differential equations. Therefore, suitable approximation is taken,  $\cos \Delta\delta_k = 1$  and  $\sin \Delta\delta_k = \Delta\delta_k$ , such that linearization of Eq. (8) yields

$$\Delta f_{kDQ}^R = T_k \Delta f_{kDQ}^E + F^R \Delta\delta \tag{12}$$

Linearization of inverse transformation yields

$$\Delta f_{kDQ}^E = (T_k)^{-1} \Delta f_{kDQ}^R + F^E \Delta\delta, \tag{13}$$

where

$F^R$  and  $F^E$  are the steady-state  $d-q$  operating indices in the rotor and synchronously rotating reference frame, respectively.

$$T_k = \begin{bmatrix} \cos \delta_k & -\sin \delta_k \\ \sin \delta_k & \cos \delta_k \end{bmatrix} \tag{14}$$

$$(T_k)^{-1} = \begin{bmatrix} \cos \delta_k & \sin \delta_k \\ -\sin \delta_k & \cos \delta_k \end{bmatrix} \tag{15}$$

Substitution of Eqs. (12) and (13) into Eq. (9) yields

$$\begin{bmatrix} T_1 \Delta v_{1DQ}^E \\ T_2 \Delta v_{2DQ}^E \\ \Delta v_{RR} \end{bmatrix} = \begin{bmatrix} W_1 & X_1 & Y_1 \\ X_2 & W_2 & Y_2 \\ Q_1 & Q_2 & S \end{bmatrix} \begin{bmatrix} T_1 \Delta i_{1DQ}^E \\ T_2 \Delta i_{2DQ}^E \\ \Delta i_{RR} \end{bmatrix}, \tag{16}$$

which can be further arranged and written as

$$\begin{bmatrix} \Delta v_{1DQ}^E \\ \Delta v_{2DQ}^E \\ \Delta v_{RR} \end{bmatrix} = \begin{bmatrix} (T_1)^{-1} W_1 T_1 & (T_1)^{-1} X_1 T_2 & (T_1)^{-1} Y_1 \\ (T_2)^{-1} X_2 T_1 & (T_2)^{-1} W_2 T_2 & (T_2)^{-1} Y_2 \\ Q_1 T_1 & Q_2 T_2 & S \end{bmatrix} \begin{bmatrix} \Delta i_{1DQ}^E \\ \Delta i_{2DQ}^E \\ \Delta i_{RR} \end{bmatrix} \tag{17}$$

The above equation can be expressed in following form:

$$Epx = Fx + u, \tag{18}$$

where

$$\begin{aligned} (x)^T &= [(\Delta i_{1DQ}^E)^T \ (\Delta i_{2DQ}^E)^T \ (\Delta i_{RR})^T] \\ &= [\Delta i_{Q1}^E, \ \Delta i_{D1}^E, \ \Delta i_{Q2}^E, \ \Delta i_{D2}^E, \ \Delta i_{KQ}, \ \Delta i_{FR}, \ \Delta i_{KD}, \ \frac{\Delta\omega_R}{\omega_B}, \ \Delta\delta] \end{aligned} \tag{19}$$

$$\begin{aligned} (u)^T &= [(\Delta v_{1DQ}^E)^T \ (\Delta v_{2DQ}^E)^T \ (\Delta v_{RR})^T] \\ &= [\Delta v_{Q1}^E, \ \Delta v_{D1}^E, \ \Delta v_{Q2}^E, \ \Delta v_{D2}^E, \ \Delta v_{KQ}, \ \Delta v_{FR}, \ \Delta v_{KD}, \ \Delta T, \ 0] \end{aligned} \tag{20}$$

$$E = \begin{bmatrix} (T_1)^{-1}W_{1P}T_1 & (T_1)^{-1}X_{1P}T_2 & (T_1)^{-1}Y_{1P} \\ (T_2)^{-1}X_{2P}T_1 & (T_2)^{-1}W_{2P}T_2 & (T_2)^{-1}Y_{2P} \\ Q_{1P}T_1 & Q_{2P}T_2 & S_P \end{bmatrix} \quad (21)$$

$$F = - \begin{bmatrix} (T_1)^{-1}W_{1K}T_1 & (T_1)^{-1}X_{1K}T_2 & (T_1)^{-1}Y_{1K} \\ (T_2)^{-1}X_{2K}T_1 & (T_2)^{-1}W_{2K}T_2 & (T_2)^{-1}Y_{2K} \\ Q_{1K}T_1 & Q_{2K}T_2 & S_K \end{bmatrix} \quad (22)$$

In Eq. (18), coefficient matrix  $E$  is associated with the derivative part with the elements having subscript  $p$ . Similarly, the coefficient matrix  $F$  whose elements have subscript  $K$  are associated with the remaining terms of the linearized motor equation. Elements of the matrices  $E$  and  $F$  are defined as

$$W_{1P} = \left(\frac{1}{\omega_B}\right) \begin{bmatrix} (x_{L1} + x_{LM} + x_{MQ}) & 0 \\ 0 & (x_{L1} + x_{LM} + x_{MD}) \end{bmatrix} \quad (23)$$

$$W_{2P} = \left(\frac{1}{\omega_B}\right) \begin{bmatrix} (x_{L2} + x_{LM} + x_{MQ}) & 0 \\ 0 & (x_{L2} + x_{LM} + x_{MD}) \end{bmatrix} \quad (24)$$

$$X_{1P} = \left(\frac{1}{\omega_B}\right) \begin{bmatrix} (x_{MQ} + x_{LM}) & -x_{LDQ} \\ x_{LDQ} & (x_{MD} + x_{LM}) \end{bmatrix} \quad (25)$$

$$X_{2P} = \left(\frac{1}{\omega_B}\right) \begin{bmatrix} (x_{MQ} + x_{LM}) & x_{LDQ} \\ -x_{LDQ} & (x_{MD} + x_{LM}) \end{bmatrix} \quad (26)$$

$$Q_{1P} = Q_{2P} = \left(\frac{1}{\omega_B}\right) \begin{bmatrix} x_{MQ} & 0 \\ 0 & \frac{x_{MD}^2}{r_{FR}} \\ 0 & x_{MD} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (27)$$

$$Y_{1P} = \left(\frac{1}{\omega_B}\right) \begin{bmatrix} x_{MQ} & 0 & 0 & 0 & (x_{L1} + x_{LM} + x_{MQ})i_{D10} - (x_{MQ} + x_{LM})i_{D20} \\ 0 & x_{MD} & x_{MD} & 0 & (x_{L1} + x_{LM} + x_{MD})i_{Q10} + (x_{MD} + x_{LM})i_{Q20} \end{bmatrix} \quad (28)$$

$$Y_{2P} = \left(\frac{1}{\omega_B}\right) \begin{bmatrix} x_{MQ} & 0 & 0 & 0 & (x_{L2} + x_{LM} + x_{MQ})i_{D20} - (x_{MQ} + x_{LM})i_{D10} \\ 0 & x_{MD} & x_{MD} & 0 & (x_{L2} + x_{LM} + x_{MD})i_{Q20} + (x_{MD} + x_{LM})i_{Q10} \end{bmatrix} \quad (29)$$

$$S_p = \left(\frac{1}{\omega_B}\right) \begin{bmatrix} (x_{LKQ} + x_{MQ}) & 0 & 0 & 0 & -(x_{MQ}i_{D10} + x_{MQ}i_{D20}) \\ 0 & \frac{x_{MD}(x_{LFR} + x_{MD})}{r_{FR}} & \frac{x_{MD}^2}{r_{FR}} & 0 & \frac{x_{MD}^2}{r_{FR}}(i_{Q10} + i_{Q20}) \\ 0 & x_{MD} & (x_{LKD} + x_{MD}) & 0 & x_{MD}(i_{Q10} + i_{Q20}) \\ 0 & 0 & 0 & -\frac{2J\omega_B^2}{P} & 0 \\ 0 & 0 & 0 & 0 & -\omega_B \end{bmatrix} \quad (30)$$

$$W_{1K} = \begin{bmatrix} r_1 & (x_{L1} + x_{LM} + x_{MD}) \\ -(x_{L1} + x_{LM} + x_{MQ}) & r_1 \end{bmatrix} \quad (31)$$

$$W_{2k} = \begin{bmatrix} r_2 & (x_{L2} + x_{LM} + x_{MD}) \\ -(x_{L2} + x_{LM} + x_{MQ}) & r_2 \end{bmatrix} \quad (32)$$

$$X_{1k} = \begin{bmatrix} x_{LDQ} & (x_{LM} + x_{MD}) \\ -(x_{LM} + x_{MQ}) & x_{LDQ} \end{bmatrix} \quad (33)$$

$$X_{2k} = \begin{bmatrix} x_{LDQ} & (x_{LM} + x_{MD}) \\ -(x_{LM} + x_{MQ}) & -x_{LDQ} \end{bmatrix} \quad (34)$$

$$Q_{1K} = Q_{2K} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ (x_{MD}(i_{D10} + i_{D20} + i_{FR0}) - x_{MQ}(i_{D10} + i_{D20}))f_{QS} & (x_{MD}(i_{Q10} + i_{Q20}) - x_{MQ}(i_{Q10} + i_{Q20}))f_{QS} \end{bmatrix} \quad (35)$$

$$Y_{1K} = \begin{bmatrix} 0 & x_{MD} & x_{MD} & (x_{L1}i_{D10} + x_{MD}(i_{D10} + i_{D20} + i_{FR0})) \\ -x_{MQ} & 0 & 0 & -(x_{L1}i_{Q10} + x_{MQ}(i_{Q10} + i_{Q20})) \end{bmatrix}$$

$$\begin{bmatrix} (-r_1i_{D10} + (x_{L1} + x_{LM} + x_{MD})i_{Q10} - x_{LDQ}i_{D20} + (x_{MD} + x_{LM})i_{Q20} + v_{D10}) \\ (r_1i_{Q10} + (x_{L1} + x_{LM} + x_{MQ})i_{D10} + (x_{MQ} + x_{LM})i_{D20} - v_{Q10}) \end{bmatrix} \quad (36)$$

$$Y_{2K} = \begin{bmatrix} 0 & x_{MD} & x_{MD} & (x_{L2}i_{D20} + x_{MD}(i_{D10} + i_{D20} + i_{FR0})) \\ -x_{MQ} & 0 & 0 & -(x_{L1}i_{Q10} + x_{MQ}(i_{Q10} + i_{Q20})) \end{bmatrix}$$

$$\begin{bmatrix} (-r_2i_{D20} + (x_{L2} + x_{LM} + x_{MD})i_{Q20} + x_{LDQ}i_{D10} + (x_{MD} + x_{LM})i_{Q10} + v_{D20}) \\ (r_2i_{Q20} + (x_{L2} + x_{LM} + x_{MQ})i_{D20} + (x_{MQ} + x_{LM})i_{D10} - v_{Q20}) \end{bmatrix} \quad (37)$$

$$S_K = \begin{bmatrix} r_{KQ} & 0 & 0 & 0 & 0 \\ 0 & r_{FR} \left(\frac{x_{MD}}{r_{FR}}\right) & 0 & 0 & 0 \\ 0 & 0 & r_{Kd} & 0 & 0 \\ -x_{MQ}(i_{D10} + i_{D20})f_{QS} & x_{MD}(i_{Q10} + i_{Q20})f_{QS} & x_{MD}(i_{Q10} + i_{Q20})f_{QS} & 0 & f_{QS}s_{45} \\ 0 & 0 & 0 & \omega_b & 0 \end{bmatrix}, \quad (38)$$



where

$$s_{45} = -i_{D10} (x_{MD} (i_{D10} + i_{D20} + i_{FR0}) - x_{MQ} (i_{D10} + i_{D20})) + i_{Q10} (x_{MD} (i_{D10} + i_{D20}) - x_{MQ} (i_{Q10} + i_{Q20})) \\ -i_{D2} (x_{MD} (i_{D10} + i_{D20} + i_{FR0}) - x_{MQ} (i_{D10} + i_{D20})) + i_{Q20} (x_{MD} (i_{Q10} + i_{Q20}) - x_{MQ} (i_{Q10} + i_{Q20}))$$

$$f_{QS} = 3P/4\omega_B$$

Variables with additional subscript ‘0’ ( $i_{D10}$ ,  $i_{D20}$ ,  $i_{FR0}$ ,  $i_{Q10}$ ,  $i_{Q20}$ ) show its value during steady-state operating condition. Eq. (18) may be written in the fundamental form:

$$px = Ax + Bu, \tag{39}$$

where

$$A = (E)^{-1} F \tag{40}$$

$$B = (E)^{-1} \tag{41}$$

It has to be noted that in the developed linearized motor model, the effect of mutual leakage reactance has been considered. Results discussed in the following section are only for the asymmetrical motor (practical case where  $\xi = 30$  electrical degrees).

### 3. Stability analysis with eigenvalues

An effective but simple means of stability analysis under small disturbance is provided by the eigenvalues of a system characteristic equation, given by

$$\det (A - \lambda I) = 0, \tag{42}$$

where  $I$  is the identity matrix and  $\lambda$  are the roots of the characteristic equation.

Eigenvalues may be either real or complex; when complex, they occur as conjugate pairs signifying a mode of oscillation of the state variables. The system is said to be stable if all the real and/or real component of eigenvalues are negative. This is because a negative real part represents a damped oscillation to a finite value of system response and the roots with positive real parts lead to an infinite output response, indicating an unstable system. Thus the sign of real parts of the roots of a system characteristic equation (location of roots in s-plan) can be used for the determination of motor absolute stability.

The state equation (39) of a six-phase synchronous motor is described by nine state variables. Therefore, nine eigenvalues will be obtained, out of which there will be three complex conjugate pairs and the remaining will be real. The dependency of eigenvalues on motor parameters are difficult to relate analytically [20,24]; therefore, this dependency has been established by calculating the eigenvalues by varying the motor parameters with each parameter varied at a time within a certain interval, keeping the other parameters constant at its normal value. Calculations of motor eigenvalues have been carried out for the motor operating at the load torque of 50%, maintaining the phase voltage of 160 V at power factor of 0.88 (lagging). The motor parameters are given in Table 2.

**Table 2.** Parameter of 3.7 kW, 36 slots, 6-poles six-phase synchronous motor.

$x_{MQ} = 3.9112\Omega$	$x_{L1} = x_{L2} = 0.1758\Omega$	$r_1 = 0.181\Omega$
$x_{MD} = 6.1732\Omega$	$x_{LDQ} = 0$	$r_2 = 0.210\Omega$
$x_{LKQ} = 0.66097\Omega$	$x_{LM} = 0.001652\Omega$	$r_{KQ} = 2.535\Omega$
$x_{LKD} = 1.550\Omega$	$r_{FR} = 0.056\Omega$	$r_{KD} = 140.0\Omega$
$x_{LFR} = 0.2402\Omega$		

### 3.1. Change in stator parameters

Variations in eigenvalues with the change in stator parameters are tabulated in Table 3a for the change in stator resistance  $r_s$  and in Table 3b for the change in stator leakage reactance  $x_{LS}$  (assuming that stator resistance and leakage reactance are the same for both the winding sets  $abc$  and  $xyz$ , i.e.  $r_s = r_1 = r_2$  and  $x_{LS} = x_{L1} = x_{L2}$ ). The complex conjugate pairs, which are affected by the change in stator parameters, are termed ‘Stator eigenvalue’ I and II, as shown in the Tables, where other eigenvalues almost remain unchanged. As the value of stator resistance of both winding sets  $abc$  and  $xyz$  is increased, the real part of both stator eigenvalue I and II is becoming more negative, resulting in the system being more stable and lowering the time constant. The pattern of the variation in the real part of stator eigenvalue I and II was found to be reversed and become less negative, as far as the variation in leakage reactance  $x_{LS}$  is concerned, hence taking the system towards instability. It has to be noted that for both the variation in stator parameters  $r_s$ ,  $x_{LS}$ , there is no change in the imaginary part of stator eigenvalue I and small variation in stator eigenvalue II (close to base speed  $\omega_B$ ), indicating that the damped frequency of oscillation will be at base frequency, approximately.

**Table 3a.** Variation in eigenvalue with the change in stator resistance.

Value of stator resistance, $r_s$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
0.1538	$-91.6 \pm j 104.7$	$-14.3 \pm j 100.3$	$-11.5 \pm j 58.2$	-9135.9,	-698.5,	-16.3
0.1629	$-97.0 \pm j 104.7$	$-15.2 \pm j 100.0$	$-11.4 \pm j 58.2$	-9136.0,	-699.1,	-16.3
0.1719	$-102.4 \pm j 104.7$	$-16.1 \pm j 99.7$	$-11.3 \pm j 58.2$	-9136.2,	-699.7,	-16.3
0.1810*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
0.1901	$-113.2 \pm j 104.7$	$-17.8 \pm j 99.1$	$-11.1 \pm j 58.3$	-9136.5,	-701.0,	-16.4
0.1991	$-118.6 \pm j 104.7$	$-18.7 \pm j 98.8$	$-11.0 \pm j 58.3$	-9136.7,	-701.6,	-16.4
0.2081	$-124.0 \pm j 104.7$	$-19.6 \pm j 98.5$	$-10.9 \pm j 58.3$	-9136.8,	-702.2,	-16.5

**Table 3b.** Variation in eigenvalue with the change in stator reactance.

Value of stator leakage reactance, $x_{LS}$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
0.1406	$-134.8 \pm j 104.7$	$-17.9 \pm j 100.3$	$-11.3 \pm j 58.6$	-9192.3,	-717.0,	-17.3
0.1582	$-119.8 \pm j 104.7$	$-17.7 \pm j 99.2$	$-11.3 \pm j 58.4$	-9163.4,	-708.5,	-16.8
0.1758*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
0.1934	$-98.0 \pm j 104.7$	$-16.5 \pm j 99.6$	$-11.2 \pm j 58.1$	-9110.9,	-692.3,	-16.0
0.2110	$-89.8 \pm j 104.7$	$-16.1 \pm j 99.8$	$-11.2 \pm j 57.9$	-9086.8,	-684.6,	-15.6

### 3.2. Change in field parameters

Variations in the motor eigenvalues for the change in parameters of the field circuit, i.e. field resistance  $r_{FR}$ , and field leakage reactance  $x_{LFR}$ , are tabulated in Tables 4a and 4b, respectively. The real eigenvalues are

associated with the offset current decay in rotor circuits and therefore associated with inverse of the effective time constant. The time constant of the field winding circuit has the largest value and therefore gives rise to the smallest real eigenvalue. It has been confirmed by noting the variation in the smallest real eigenvalue with the variation in field circuit parameters. As the field circuit resistance  $r_{FR}$  is increased (or decreased) up to 20% of its normal value, variation in the real eigenvalue III is noted. This value becomes more negative (or less negative), moving the system towards more stable region, i.e. field offset current will decay more rapidly. This pattern of variation in real eigenvalue III was found to be reversed by having the variation in field leakage reactance  $x_{LFR}$  by the same percentage of amount, i.e. field offset current will decay less rapidly and the system moves towards instability.

**Table 4a.** Variation in eigenvalue with the change in field circuit resistance.

Value of field resistance, $r_{FR}$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
0.0448	$-107.8 \pm j 104.7$	$-17.1 \pm j 100.0$	$-11.0 \pm j 58.2$	-9136.3,	-700.3,	-13.0
0.0504	$-107.8 \pm j 104.7$	$-17.0 \pm j 99.7$	$-11.1 \pm j 58.2$	-9136.3,	-700.3,	-14.7
0.0560*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
0.0616	$-107.8 \pm j 104.7$	$-16.8 \pm j 99.1$	$-11.4 \pm j 58.3$	-9136.4,	-700.3,	-18.1
0.0672	$-107.8 \pm j 104.7$	$-16.7 \pm j 98.8$	$-11.5 \pm j 58.3$	-9136.4,	-700.3,	-19.8

**Table 4b.** Variation in eigenvalue with the change in field circuit reactance.

Value of field leakage reactance, $x_{LFR}$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
0.1922	$-107.8 \pm j 104.7$	$-19.5 \pm j 98.0$	$-11.3 \pm j 59.0$	-9158.5,	-700.3,	-19.1
0.2162	$-107.8 \pm j 104.7$	$-18.1 \pm j 99.7$	$-11.3 \pm j 58.6$	-9146.5,	-700.3,	-17.6
0.24021*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
0.2642	$-107.8 \pm j 104.7$	$-15.9 \pm j 99.9$	$-11.2 \pm j 58.0$	-9127.6,	-700.3,	-15.3
0.2883	$-107.8 \pm j 104.7$	$-15.0 \pm j 100.3$	$-11.2 \pm j 57.7$	-9119.9,	-700.3,	-14.4

### 3.3. Change in damper winding parameters

The time constant of the damper winding  $K_D$  has a smaller value ( $1.76 \times 10^{-4} s^{-1}$ ) than the other rotor circuit; therefore, it will give rise to the largest real eigenvalue. This has been confirmed by having the variation in real eigenvalue I by changing the damper winding resistance  $r_{KD}$  and leakage reactance  $x_{LKD}$ , as tabulated in Table 5a and Table 5b, respectively. The value of real eigenvalue I becomes more negative for the change in resistance  $r_{KD}$  from lower to higher (20% of normal) value. Therefore, the system will become more stable. However, the system moves towards instability for the variation in leakage reactance  $x_{LKD}$  from lower to higher value. It can be noted that other eigenvalues almost remain unaffected while changing the above parameters. The time constant of damper winding  $K_Q$  was found to be  $2.87 \times 10^{-3} s^{-1}$ . This value is higher than the time constant of damper winding  $K_D$  but lower than the field circuit  $F_R$ . Therefore, it causes the variation in real eigenvalue II with the change in resistance  $r_{KQ}$  and leakage reactance  $x_{LKQ}$ , as tabulated in Tables 6a and 6b, respectively. Variation in the parameter of this damper winding also shows the same pattern of variation in eigenvalue II as discussed in the above cases.

**Table 5a.** Variation in eigenvalue with the change in damper winding  $KD$  resistance.

Value of damper winding resistance, $r_{KD}$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
112.56	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-7309.7,	-700.3,	-16.4
126.63	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.6$	-8223.0,	-700.3,	-16.4
140.70*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
154.77	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-10050.0,	-700.3,	-16.4
168.84	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-10963.0,	-700.3,	-16.4

**Table 5b.** Variation in eigenvalue with the change in damper winding  $KD$  reactance.

Value of damper leakage reactance, $x_{LKD}$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
1.2397	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-11309.0,	-700.3,	-16.4
1.3946	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-10107.0,	-700.3,	-16.4
1.5496*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
1.7045	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-8335.7,	-700.3,	-16.4
1.8595	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-7664.0,	-700.3,	-16.4

**Table 6a.** Variation in eigenvalue with the change in damper winding  $KQ$  resistance.

Value of damper leakage reactance, $x_{LKQ}$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
4.0568	$-107.8 \pm j 104.7$	$-16.8 \pm j 98.8$	$-14.4 \pm j 58.5$	-9136.3,	-552.0,	-16.3
4.5639	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.1$	$-12.6 \pm j 58.4$	-9136.3,	-626.6,	-16.3
5.071*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
5.5781	$-107.8 \pm j 104.7$	$-17.0 \pm j 99.6$	$-10.1 \pm j 58.1$	-9136.3,	-773.6,	-16.4
6.0852	$-107.8 \pm j 104.7$	$-17.0 \pm j 99.8$	$-9.2 \pm j 58.1$	-9136.3,	-846.5,	-16.4

**Table 6b.** Variation in eigenvalue with the change in damper winding  $KQ$  reactance.

Value of damper leakage reactance, $x_{LKQ}$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
0.5288	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.1$	-9136.3,	-856.0,	-16.4
0.5949	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-770.6,	-16.4
0.6610*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
0.7271	$-107.8 \pm j 104.7$	$-17.0 \pm j 99.4$	$-11.3 \pm j 58.3$	-9136.3,	-641.5,	-16.4
0.7932	$-107.8 \pm j 104.7$	$-17.0 \pm j 99.4$	$-11.3 \pm j 58.2$	-9136.3,	-591.5,	-16.4

### 3.4. Change in moment of inertia

The remaining eigenvalue has been termed the ‘rotor eigenvalue’ of the synchronous motor. It indicates the oscillatory behavior of the motor, particularly referred to as hunting or swing mode, i.e. the primary mode of oscillation of the rotor with respect to electrical angular velocity, in an electromechanical system. Oscillatory

behavior of the rotor is affected by the variation in moment of inertia, as tabulated in Table 7. It is to be noted that the frequency of rotor oscillation is decreased by varying the value of moment of inertia  $J$  from lower to higher (20% of normal) value. Moreover, it also tends to move the system towards instability as the real component of eigenvalue is becoming less negative.

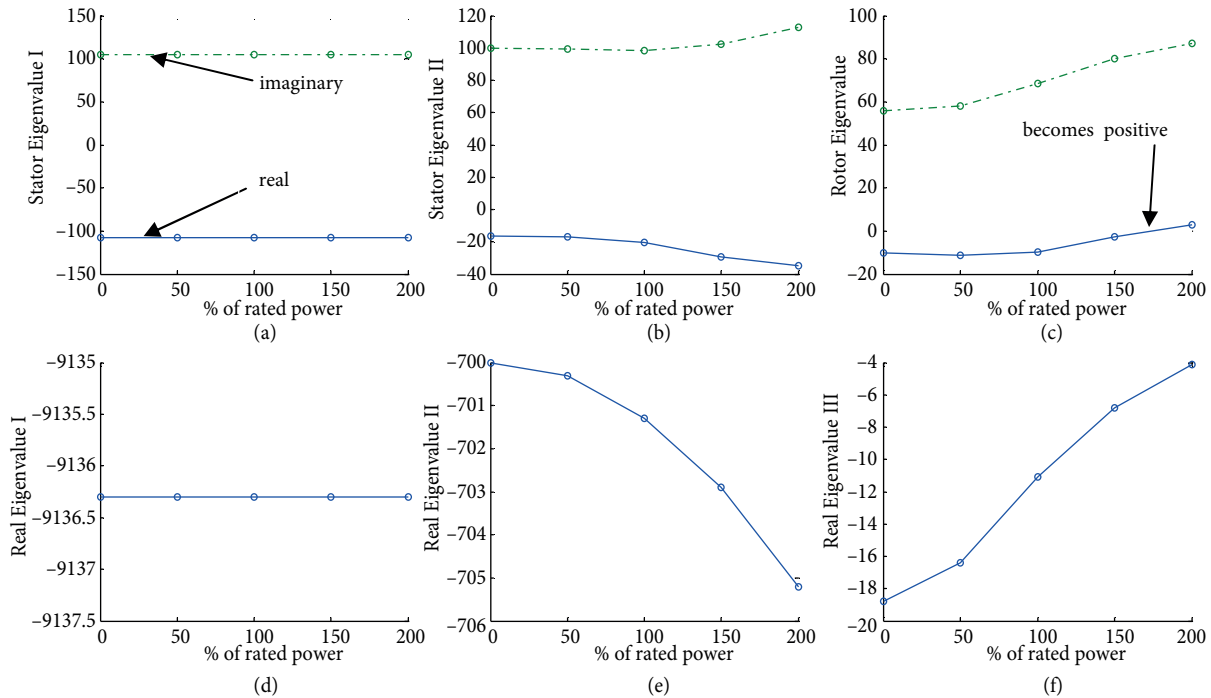
**Table 7.** Variation in eigenvalue with the change in moment of inertia.

Value of moment of inertia, $J$	Stator eigenvalue I	Stator eigenvalue II	Rotor eigenvalue	Real eigenvalue		
				I	II	III
0.4224	$-107.8 \pm j 104.7$	$-17.4 \pm j 98.8$	$-13.7 \pm j 65.6$	-9136.3,	-694.5,	-16.3
0.4752	$-107.8 \pm j 104.7$	$-17.1 \pm j 99.2$	$-12.4 \pm j 61.6$	-9136.3,	-697.8,	-16.4
0.5280*	$-107.8 \pm j 104.7$	$-16.9 \pm j 99.4$	$-11.2 \pm j 58.2$	-9136.3,	-700.3,	-16.4
0.7271	$-107.8 \pm j 104.7$	$-16.8 \pm j 99.6$	$-10.3 \pm j 55.4$	-9136.3,	-702.4,	-16.4
0.7932	$-107.8 \pm j 104.7$	$-16.7 \pm j 99.7$	$-9.6 \pm j 53.0$	-9136.3,	-704.2,	-16.4

Note: ‘\*’ indicates the normal value.

**3.5. Effect of load variation**

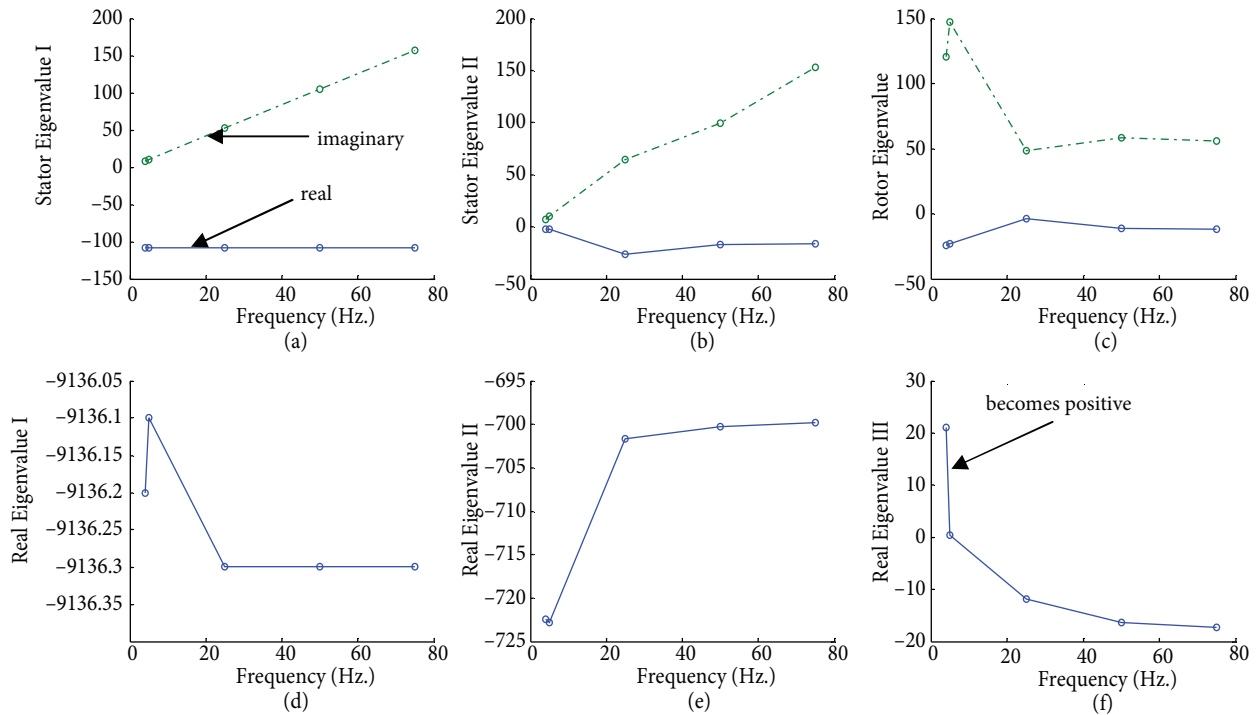
The effect of load variation in eigenvalues is graphically presented in Figure 2. The solid line indicates the real component and the dashed line indicates the imaginary component of eigenvalue. The real components of both stator eigenvalue I and II are found to be numerically constant with the imaginary component almost equal to base speed  $\omega_B$ , i.e. stator eigenvalues are almost unaffected by the variation in load. The motor was found to become unstable at the load of 1.7 times the rated load/torque. This has been depicted by the rotor eigenvalue, where the real component is becoming positive, associated with the increase in oscillation in the rotor circuit, shown by the imaginary component in Figure 2c. The real eigenvalues remain negative with increase and decrease in its magnitude for real eigenvalue II and III, respectively.



**Figure 2.** Effect of load variation on motor’s eigenvalues.

**3.6. Effect of frequency/speed variation**

Frequency and voltage can be individually varied to study the stability of the motor. Practically, this practice is not adopted. In fact, in variable speed systems, the amplitude of applied voltages is varied in proportion with the frequency in order to avoid saturation. Therefore, in this section, stability of the motor has been observed for the change in frequency and voltage of the same ratio. The effect on motor eigenvalues with the change in frequency/speed is shown graphically in Figure 3. With change in the eigenvalue, both real and imaginary components were found to have almost the same variation patterns for both stator eigenvalue I and II. The real component of rotor eigenvalue was also found to be almost the same, irrespective with the change in frequency. However, for lower values of frequency (less than 20 Hz), the imaginary component was found to increase, showing a larger frequency of oscillation in the rotor circuit, whereas the magnitude of real eigenvalue II was found to increase at lower frequency operation. The motor at very low frequency operation is becoming unstable. This is because the real eigenvalue III under very low frequency operation is becoming positive, as shown in Figure 3f. This result is the same as discussed by the authors in [9,11–13] for the operation of a three-phase motor at very low frequency.



**Figure 3.** Effect of frequency variation on motor’s eigenvalues.

**4. Formulation of transfer function**

The linearized equations of a system are more often used for the analysis and design of controllers, like a speed controller for a variable speed drive system, which needs the formulation of transfer function. This is because the transfer function when written from linearized system equations yields a relation between the output variable to be controlled to the input variable. Moreover, it also facilitates the study of small displacement behavior of the system about a steady-state operating point rather than to use the detailed nonlinear equations [24,25]. Consequently, the different control theory approach can be applied like plotting of root locus, Bode plot, and

Nyquist plot. Hence, this greatly simplifies the control analysis of a system, applicable for motor operation.

In this section, the formulation of a transfer function will be outlined, wherein the linearized equations of a six-phase synchronous motor will be utilized. As an illustration, transfers function of the change in reactive power w.r.t. the change in field excitation, i.e.  $\frac{\Delta Q(s)}{\Delta E_{FR}(s)}$  is derived in a simplified manner, followed by the plotting of root locus for stability analysis.

The state equations of a linear dynamic system are

$$\dot{x} = Ax + Bu \quad (43)$$

$$y = Cx + Du, \quad (44)$$

where

$x$  is state vector defined by Eq. (19)

$u$  is input vector defined by Eq. (20)

$y$  is a output variable (or a set of output variables)

$A$  is a matrix defined by Eq. (39)

$C$  and  $D$  matrices are defined below.

Solving Eq. (43) for  $x$ , considering the initial condition to be zero and substituting the result in Eq. (44) yields

$$Y(s) = [C(sI - A)^{-1}B + D]U(s) \quad (45)$$

Therefore, the transfer function is

$$G(s) = \frac{Y(s)}{U(s)} = [C(sI - A)^{-1}B + D] \quad (46)$$

For the purpose to derive the transfer function of  $\frac{\Delta Q(s)}{\Delta E_{FR}(s)}$ , the linearized expression for reactive power is given by

$$\begin{aligned} \Delta Q(s) &= v_{Q1}^E \Delta i_{D1}^E + i_{D1}^E \Delta v_{Q1}^E - v_{D1}^E \Delta i_{Q1}^E - i_{Q1}^E \Delta v_{D1}^E + v_{Q2}^E \Delta i_{D2}^E + i_{D2}^E \Delta v_{Q2}^E - v_{D2}^E \Delta i_{Q2}^E - i_{Q2}^E \Delta v_{D2}^E \\ &= Cx + Du, \end{aligned} \quad (47)$$

where

$$C = [-v_{D1}^E, v_{Q1}^E, -v_{D2}^E, v_{Q2}^E, 0, 0, 0, 0, 0] \quad (48)$$

$$D = [i_{D1}^E, -i_{Q1}^E, i_{D2}^E, -i_{Q2}^E, 0, 0, 0, 0, 0] \quad (49)$$

Therefore, the transfer function will be in following form:

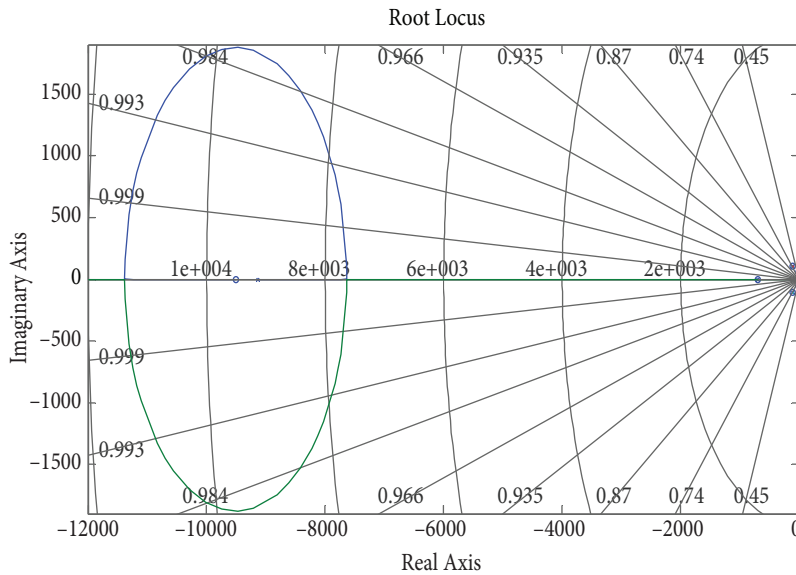
$$\frac{\Delta Q(s)}{\Delta E_{FR}(s)} = K \frac{(s - z_1)(s - z_2) \dots (s - z_m)}{(s - p_1)(s - p_2) \dots (s - p_n)}, \quad (50)$$

where the values of gain  $K$ , zeros  $(z_1, z_2, \dots, z_m)$ , and poles  $(p_1, p_2, \dots, p_n)$  were calculated for the motor operation at 50% of rated load. These values are shown in Table 8. Root locus of the transfer function, given

by Eq. (50) can be drawn by the well-known procedure [25] and is shown in Figure 4. It also shows that the system is stable for all values of gain  $K$ , as all the poles are lying in the negative half of  $s$ -plan. Necessary steps used to perform the small signal stability analysis of a six-phase synchronous motor are shown in the form of a flowchart in Figure 5. Steps have to be followed for different combinations of input and output variables, while formulating the transfer function to study the stability of a six-phase synchronous motor under small disturbance.

**Table 8.** Zeros and poles of the transfer function  $\frac{\Delta Q(s)}{\Delta E_{FR}(s)}$ .

Zeros	Poles
$z_1 = -91.6 + j 104.7$	$p_1 = -91.6 + j 104.7$
$z_2 = -91.6 - j 104.7$	$p_2 = -91.6 - j 104.7$
$z_3 = -1.3 + j 103.5$	$p_3 = -14.3 + j 100.3$
$z_4 = -1.3 - j 103.5$	$p_4 = -14.3 - j 100.3$
$z_5 = -11.3 + j 54.7$	$p_5 = -11.5 + j 58.3$
$z_6 = -11.3 - j 54.7$	$p_6 = -11.5 - j 58.3$
$z_7 = -9508.4$	$p_7 = -9135.9$
$z_8 = -698.1$	$p_8 = -698.5$
Gain $K = 619.6$	$p_9 = -16.3$



**Figure 4.** Root locus for the transfer function  $\Delta Q(s)/\Delta E_{FR}(s)$ .

### 5. Conclusion

A linearized mathematical model of a six-phase synchronous motor has been developed, where the mutual leakage between both the stator winding sets  $abc$  and  $xyz$ , was considered, by employing the  $dq\theta$  approach. This results in a set of linear differential equations describing the dynamic behavior of small displacement/excursion about a steady-state operating point, so that the basic linear control system theory can be applied to evaluate the eigenvalues. An eigenvalue criterion was used to study the stability of a six-phase synchronous motor. An association between the eigenvalues and motor parameters has been established by calculating the eigenvalue at



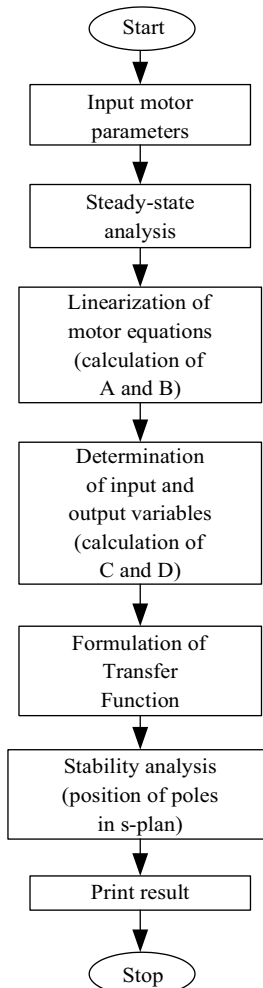


Figure 5. Steps to perform stability analysis of six-phase synchronous motor.

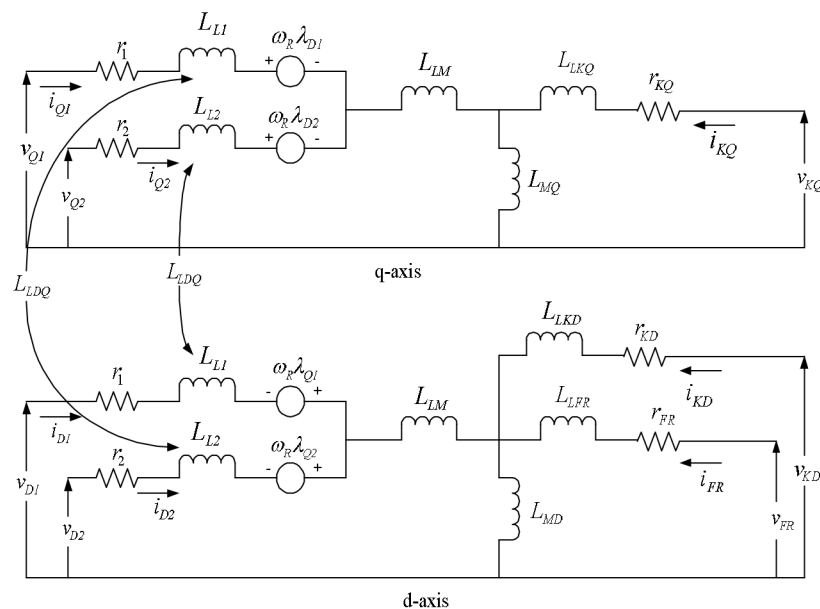


Figure 6. An equivalent circuit of a six-phase synchronous machine (motor).

an operating point by changing the motor parameter. It was found that the two eigenvalues (stator eigenvalue I and II) are almost unaffected by the variation in rotor parameter and load. The other complex conjugate pair is affected by the variation in moment of inertia  $J$  and load. It actually indicates “settling out” rotor oscillation of the synchronous motor during hunting or swing mode. The remaining three real eigenvalues signify the offset currents decay in rotor circuit hence associated with their effective time constant.

The developed linearized model can be easily used to formulate the transfer function between input and output variables, where different stability plots (like root locus, Nyquist plot, Bode plot) can be easily drawn, thus greatly simplifying the control analysis of a system applicable for a six-phase synchronous motor.

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## A. Appendix

### A.1. Voltage equation:

$$v_{Q1} = r_1 i_{Q1} + \frac{\omega_R}{\omega_B} \psi_{D1} + \frac{p}{\omega_B} \psi_{Q1} \quad (\text{A1})$$

$$v_{D1} = r_1 i_{D1} - \frac{\omega_R}{\omega_B} \psi_{Q1} + \frac{p}{\omega_B} \psi_{D1} \quad (\text{A2})$$

$$v_{Q2} = r_2 i_{Q2} + \frac{\omega_R}{\omega_B} \psi_{D2} + \frac{p}{\omega_B} \psi_{Q2} \quad (\text{A3})$$

$$v_{D2} = r_2 i_{D2} - \frac{\omega_R}{\omega_B} \psi_{Q2} + \frac{p}{\omega_B} \psi_{D2} \quad (\text{A4})$$

$$v_{KQ} = r_{KQ} i_{KQ} + \frac{p}{\omega_B} \psi_{KQ} \quad (\text{A5})$$

$$v_{KD} = r_{KD} i_{KD} + \frac{p}{\omega_B} \psi_{KD} \quad (\text{A6})$$

$$v_{FR} = \frac{x_{MD}}{r_{FR}} \left( r_{FR} i_{FR} + \frac{p}{\omega_B} \psi_{FR} \right), \quad (\text{A7})$$

where  $p$  denotes the differentiation function with respect to time.

#### A.1.1. Flux linkage per second:

$$\psi_{Q1} = x_{L1} i_{Q1} + x_{LM} (i_{Q1} + i_{Q2}) - x_{LDQ} i_{D2} + \psi_{MQ} \quad (\text{A8})$$

$$\psi_{D1} = x_{L1} i_{D1} + x_{LM} (i_{D1} + i_{D2}) + x_{LDQ} i_{Q2} + \psi_{MD} \quad (\text{A9})$$

$$\psi_{Q2} = x_{L2} i_{Q2} + x_{LM} (i_{Q1} + i_{Q2}) + x_{LDQ} i_{D1} + \psi_{MQ} \quad (\text{A10})$$

$$\psi_{D2} = x_{L2} i_{D2} + x_{LM} (i_{D1} + i_{D2}) - x_{LDQ} i_{Q1} + \psi_{MD} \quad (\text{A11})$$

$$\psi_{KQ} = x_{LKQ} i_{KQ} + \psi_{MQ} \quad (\text{A12})$$

$$\psi_{KD} = x_{LKD} i_{KD} + \psi_{MD} \quad (\text{A13})$$

$$\psi_{FR} = x_{LFR} i_{FR} + \psi_{MD}, \quad (\text{A14})$$

where

$$\psi_{MQ} = x_{MQ} (i_{Q1} + i_{Q2} + i_{KQ}) \quad (\text{A15})$$

$$\psi_{MD} = x_{MD} (i_{D1} + i_{D2} + i_{KD} + i_{FR}) \quad (\text{A16})$$

Parameters of the rotor circuit are referred to one of the stator windings ( $abc$  windings). These voltage and flux linkage equations suggest the equivalent circuit as shown in Figure 6, where  $L_{LM}$  and  $L_{LDQ}$  are known

as common mutual leakage inductance and cross mutual coupling inductance between  $d$  and  $q$ -axis of stator respectively given by:

$$x_{LM} = x_{Lax} \cos(\xi) + x_{Lay} \cos(\xi + 2\pi/3) + x_{Laz} \cos(\xi - 2\pi/3) \quad (\text{A17})$$

$$x_{LDQ} = x_{Lax} \sin(\xi) + x_{Lay} \sin(\xi + 2\pi/3) + x_{Laz} \sin(\xi - 2\pi/3) \quad (\text{A18})$$

The common mutual leakage reactance  $x_{lm}$  signifies the mutual coupling due to leakage flux between two sets of three-phase stator windings ( $abc$  and  $xyz$ ) occupying the same slot. It has an important effect on the harmonic coupling of both sets of stator windings, but negligible effect on transient except some changes in voltage harmonic distortion [22,23]. The value of common mutual leakage reactance  $x_{lm}$  depends on the winding pitch and displacement angle between two stator winding sets ( $abc$  and  $xyz$ ). An explanation in detail along with a technique for finding the slot reactance is given in [26]. Standard test procedures for the determination of machine parameters are given in [27,28].